Note: This HW is a "toy model" of graphene.
Does not get at 1) Valley degeneracy 2) 2D–1D nanoribbon effects

1) In graphene, we have a linear relationship between energy and momentum:

$$E = \hbar v_F k = \hbar v_F \sqrt{(k_x)^2 + (k_y)^2} = \hbar v_F \sqrt{\left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_x}\right)^2} \quad \Rightarrow k = \frac{1}{v_F} E$$

↑ typo

Derive the density of states vs. energy in graphene.

$$\Rightarrow \frac{dk}{dE} = \frac{1}{v_F}$$

2) Now imagine you have a graphene nanoribbon. $L_y$ is small. Calculate the density of states vs. energy of a 1d graphene nanoribbon.

$$E = \hbar v_F \sqrt{k_x^2 + k_{y0}^2} \qquad k_{y0} = \frac{\pi}{L_y}$$

1) $D(E)\,dE = D(k)\,dk$

$$D(k)\,dk = \left[\frac{1 \text{ state}}{(\pi/L)^2} \times 2 \text{ (spin)}\right] \times \text{area of disk of radius } k \text{ in } k\text{-space } k_x > 0\ k_y > 0$$

$$= \left[\frac{1}{(\pi/L)^2} \times 2\right] 2\pi k\,dk\,\frac{1}{4}$$

$$= L^2 \frac{1}{\pi} k\,dk = L^2 \frac{1}{\pi} \frac{E}{\hbar v_F} dk$$

$$\Rightarrow D(k) = L^2 \frac{1}{\pi} k = L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F}$$

$$D(E) = D(k)\frac{dk}{dE} = \boxed{L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F}\frac{1}{\hbar v_F} = D(E)}$$

$$\boxed{\rho(E) = \frac{E}{\hbar^2 \pi v_F^2}}$$

2) $E = \hbar v_F k$

$D(E)\,dE = D(k)\,dk$

$$D(k)\,dk = \left[\frac{1}{\pi/L} \times 2 \text{ (spin)}\right] \times \text{distance in } k\text{-space between } k, k+dk$$

$$= \frac{2}{\pi/L} \times dk \quad \Rightarrow D(k) = L\frac{2}{\pi}$$

$$D(E) = D(k)\frac{dk}{dE} = L\frac{2}{\pi}\frac{1}{v_F} \quad \Rightarrow \boxed{\rho(E) = \frac{2}{\pi}\frac{1}{v_F}}$$