EECS277C Spring 11 **Midterm #1**

4/20/2011 3:30 to 4:50 pm

Professor Peter Burke

Name: Solutions

ID no.: ______

1	2	3a	Total
/40	/30	/30	/100

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Helpful constants for you:

$c = 3\ 10^{8}$ m/s

$e = 1.6\ 10^{-19}$ coulombs

$h = 6.63\ 10^{-34}$ J-s

$m = 9.1\ 10^{-31}$ kg

$k_B = 1.38\ 10^{-23}$ J/K

1. Calculate the density of states in a 1 dimensional world.

(30 pts)

$$E = \frac{\hbar^2}{2m} K^2 = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \quad n = 0, 1, 2, \ldots$$

$$\Rightarrow K = \frac{\sqrt{2mE}}{\hbar}$$

$$D(k) = 2 \text{ states per } \pi/L = \frac{2}{\pi/L} \quad \text{(50 pts)}$$

$$D(E)\,dE = D(k)\,dk$$

$$\Rightarrow D(E) = D(k)\frac{dk}{dE}$$

$$= \frac{2}{\pi/L}\frac{d}{dE}\frac{\sqrt{2mE}}{\hbar} \quad \text{(20 pts)}$$

$$= \frac{2}{\pi/L}\frac{\sqrt{2m}}{\hbar}\frac{1}{2\sqrt{E}} = \frac{1}{\hbar\pi}\sqrt{\frac{2m}{E}}\,L$$

$$\rho(E) = \frac{1}{\hbar\pi}\sqrt{\frac{2m}{E}}$$

2. Calculate the size of the box (actually wire) needed to get into the 1d limit for free electrons at room temperature. I.E.: L_z is infinity. What must L_x, L_y be? (Use Fermi energy of 10 eV).

$$E = \frac{\hbar^2}{2m}\pi^2\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]$$

Need $E_{2,1,\text{anything}} > 10\text{eV}$ by several kT

$\Rightarrow$ $\boxed{\frac{\hbar^2}{2m}\pi^2\frac{5}{L_x^2} > 10\text{eV}}$ 10 pts

Assume $L_x = L_y$

$$E_{2,1,\text{anything}} = \frac{\hbar^2}{2m}\pi^2\frac{1}{L_x^2}5 > 10\text{eV}$$

$$\Rightarrow L_x < \sqrt{\frac{\hbar^2}{2m}\pi^2 5\frac{1}{10\text{eV}}}$$ (10 pts)

$$= \sqrt{\frac{6^2(10^{-34})^2}{2^2\pi^2}\frac{\pi^2}{2\; 9\; 10^{-31}}\frac{5}{10\; 1.6\times10^{-19}}}$$

-34, -34, -1, -1 → -70; 31, $+1$, $+19$ → $+51$

$$= \sqrt{\frac{3.6}{4}\frac{5}{2}\frac{1}{1.6}\;10^{-34-34-1+31-1+1+19}}$$

$6^2 = 3.6\times10^1$

≈ 2

$$= \sqrt{2}\sqrt{10^{-19}}\text{ m} = \sqrt{2\cdot10\times10^{-20}}\text{ m}$$

$\approx$ ~~1.4~~ $= \sqrt{20}\times10^{-10}$ m

$\simeq 4\; 10^{-10}$ m

$\simeq \boxed{0.5\text{ nm}}$ (10 pts)

Exact answer w/calculator is $\boxed{0.39\text{ nm}}$

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3) List the requirements for observing Coulomb blockade behavior for a single tunnel junction system.

Requirement 1: $kT < e^2/C$

Requirement 2: $R_T > R_Q$

Requirement 3: $Z(\omega) > R_T$ up to $\omega \sim \frac{1}{RC}$