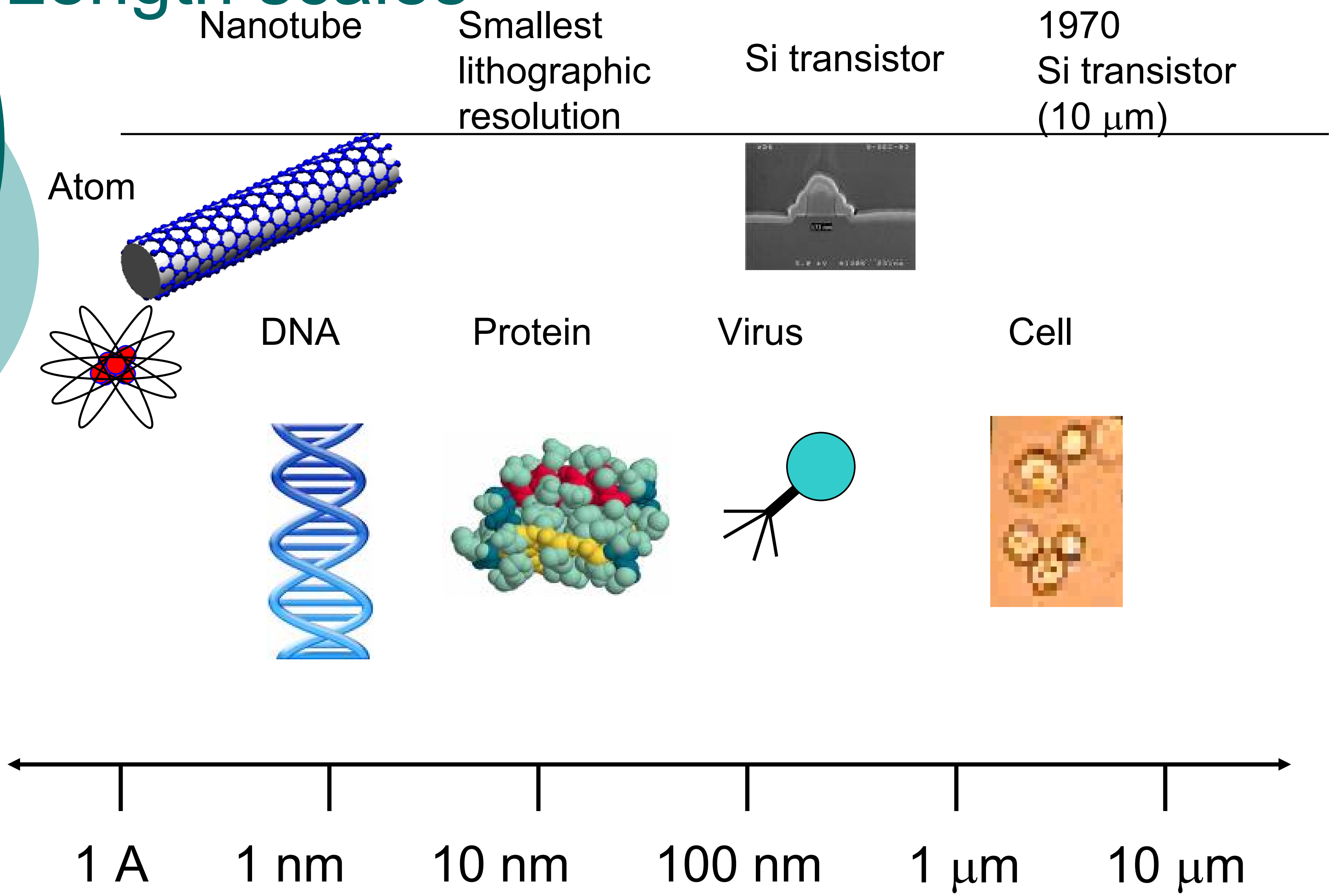
Nanotechnology

- Nanofabrication techniques
- Characterization techniques
- Single electron transistors
- Quantization of electrical resistance
- Nanotubes, nanowires

Units

- Meter (m)
- Millimeter (mm) = 10^{-3} m
- Micrometer (μm) = 10^{-6} m
- Nanometer (nm) = 10^{-9} m
- Picometer (pm) = 10^{-12} m
- Femtometer (fm) = 10^{-15} m

Length scales



What is nanotechnology?

- "Top down" approach
  - Micron scale lithography
    - optical, ultra-violet
    - Focused Ion Beam
  - 10-100 nm
    - Electron-beam lithography
- "Bottom up" approach
  - Chemical self-assembly
    - Man-made synthesis (e.g. carbon nanotubes)
    - Biological synthesis (DNA, proteins)
  - Manipulation of individual atoms
    - Atomic Force Microscopy
    - Scanning Tunneling microscopy

A brief history of nanotechnology

- Democritus in ancient Greece: concept of atom
- Rutherford, 1900: discovery of atomic nucleus
- Feynman, 1960: speech at Caltech
- Drexler, 1986, 1992: *Engines of Creation*, *Nanosystems*
- Clinton, speech, Caltech, 2000
- *National Nanotechnology Initiative* since 2000

Feynman challenges

- "There's Plenty of Room at the Bottom"
- Feynman, Caltech 1960 set two challenges
  - Construct a 1/64 cubic inch motor
  - claimed in 1960
  - On display at Caltech today
- Encyclopedia Britanica on head of a pin
  - Actualy on page in 10 microns^2
  - Claimed in 1985
  - Used electron-beam lithography

Foresight challenges

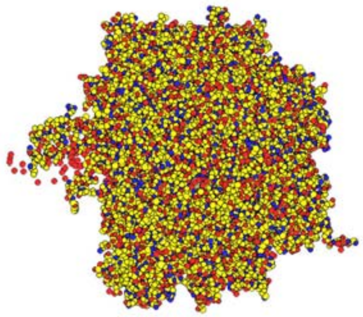
- Drexler wrote two books:
  - 1986: *Engines of Creation: The Coming Era of Nanotechnology*
  - 1992: *Nanosystems: Molecular Machinary, Manufacturing, and Computation*
- Foresight/Feynman $250,000 prize
  - 100 nm arm nano-robot
  - 50 nm^3 8-bit adder

Biosystems

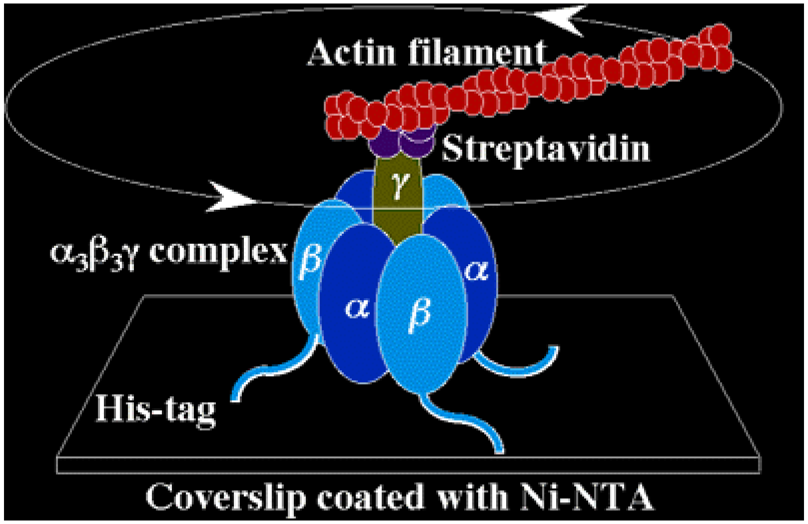
- DNA
  - 2-3 nm per base pair
  - Human genome contains ~ 10^9 base pairs
- Proteins
  - typically 1-10 nm in size
  - ~ 100,000 different proteins in human genetic code
  - all are synthesized enzymatically (bottom up)
- Biological Nano-motors
  - ATP synthase
  - Kinesin, Actin important for muscle movement
- *Nanotechnology is important for life itself*

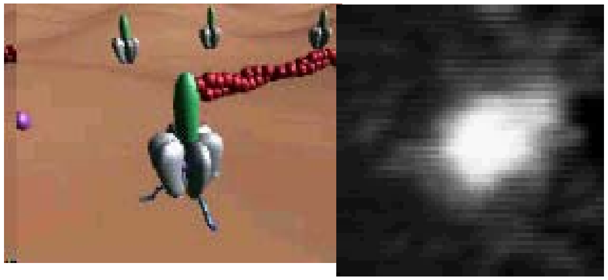
ATP Synthase

- 10 nm nanomachine at the mitochondria membrane
- Uses proton gradient to convert ADP to ATP
- Extremely important for metabolism



10 nm





Movie source: www.res.titech.ac.jp

References: Boyer, Annu. Rev. Biochem. 1997
Yoshida, Nature Rev. Mol. Cell Bio. 1997
Soong, Montemagno, Nature, 2000

Nano-manufacturing

- Lithography can do 10 nm
- Tricks to 2 nm
- Biosystems can add 2 carbon atoms at a time
  - typical in lipid biosynthesis
  - enzymes are nano machines
- We do not know how to design enzymes, only copy them
- As such, nanotechnology does not yet exist according to Drexler's definition

Readings this lecture covers

- Ferry, pp. 1-5
- Feynman,
  "*There's plenty of room at the bottom*"
- Moore's law original paper
- Moore's law slides
- Drexler ch. 2
- Hanson p. 1-14

Course themes

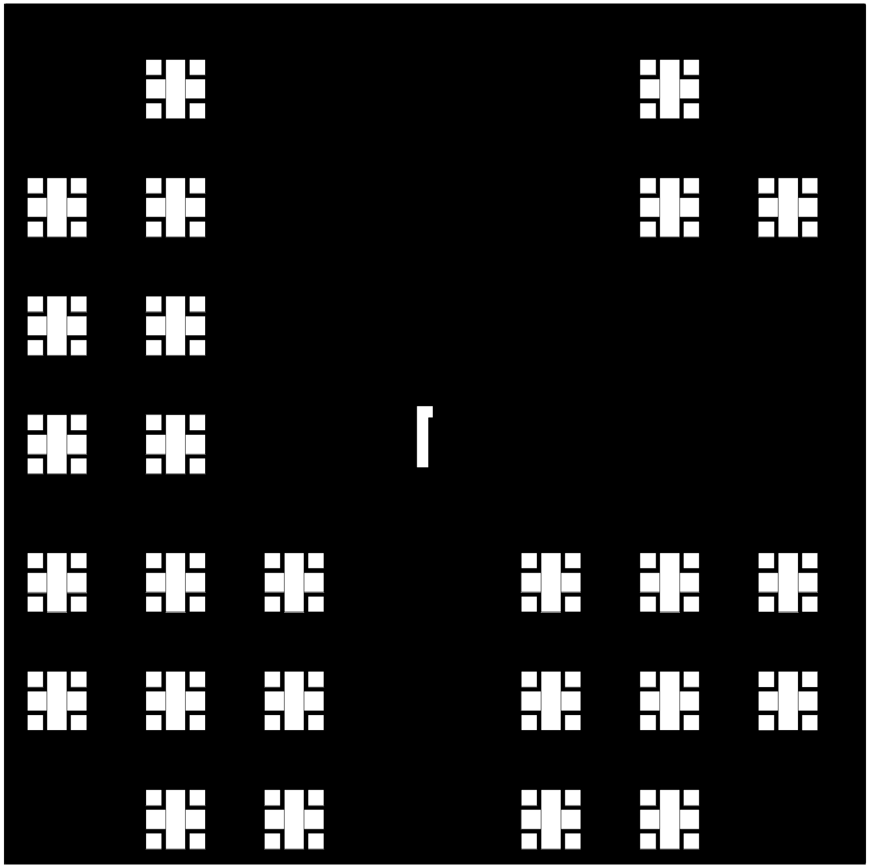
- *Nano-electronics:* Wave/particle duality
- Particle:
  - Charging energy e^2/C
    (single electron transistor)
- Wave:
  - Gradually reduced dimensions:
    - 3 (bulk)
    - 2 (2DEG)
    - 1 (nanowire)
    - 0 (quantum dot).
  - Quantization of electrical resistance: e^2/h

Fabrication

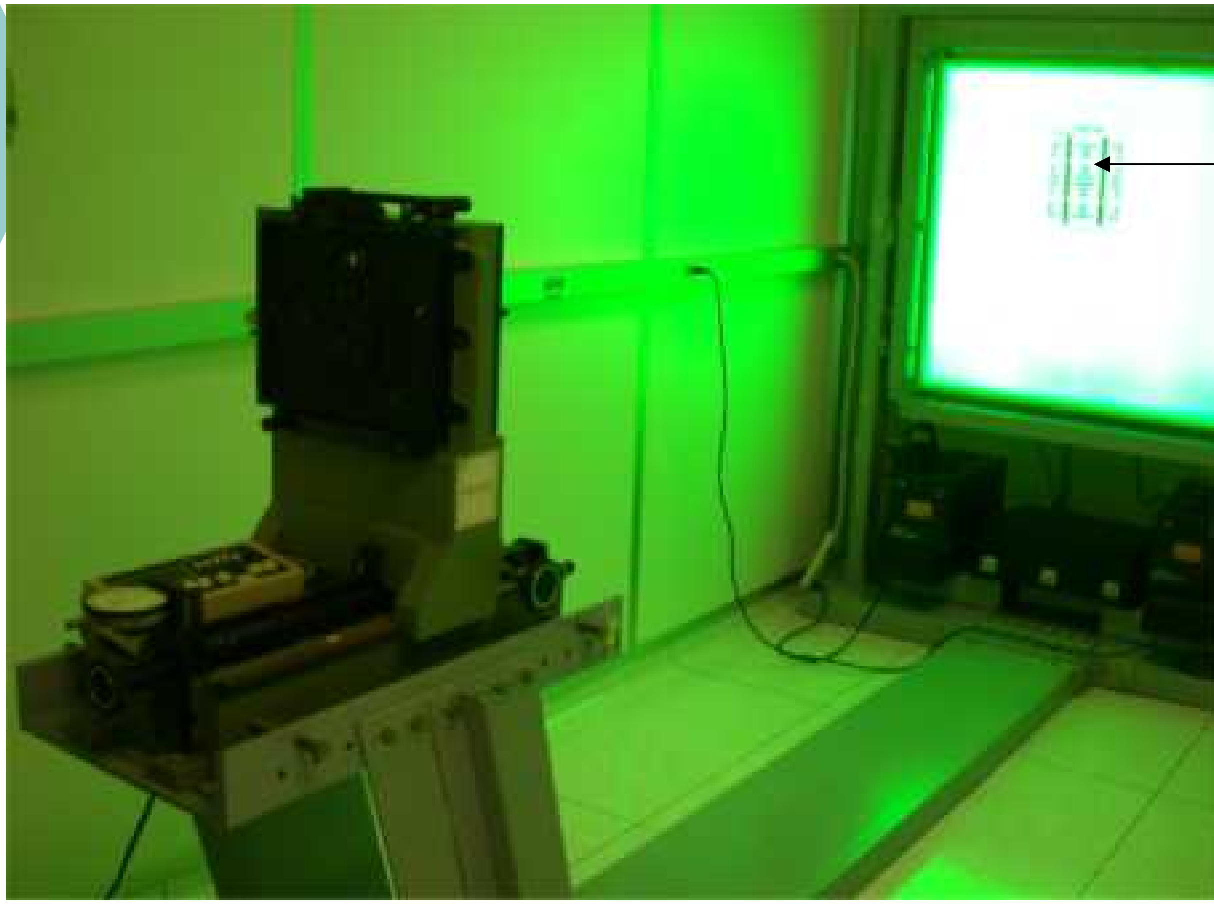
- "Top down" approach to nanotechnology
- This is overview, for more details take MAE courses by Marc Madou, Andre Shkel
- Thanks to Sungmu Kang for INRF images

Photomasks

Design geometry on computer.



Mask fabrication



transparency

After Exposure
Developer
Stop bath
Fixer

Dark room (1/20 reduction)

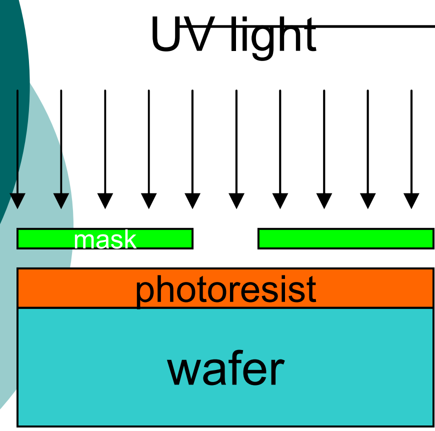
Spin on photoresist

Photo resist spinner

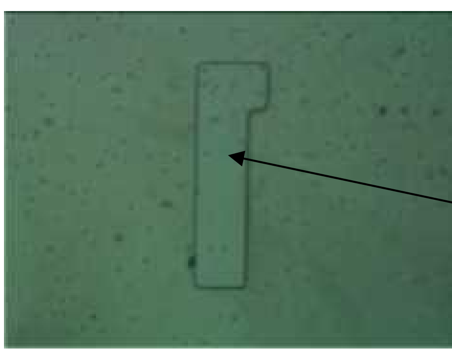
Soft bake

Oven for soft baking of photo resist (at 90C for 30 min)

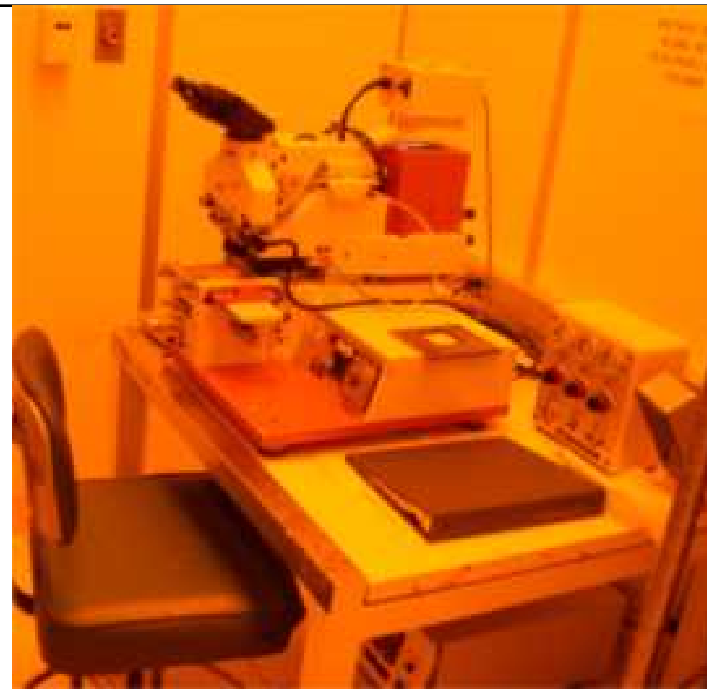
Expose to UV light



Development
For Shipley 1827
Water : MF351 = 5.5 : 1



Mask Aligner



Exposed regions dissolved in developer leaving bare wafer

This is the step which limits the spatial resolution.

Thermal evaporation

Thermo evaporator

Alumina coated W boat

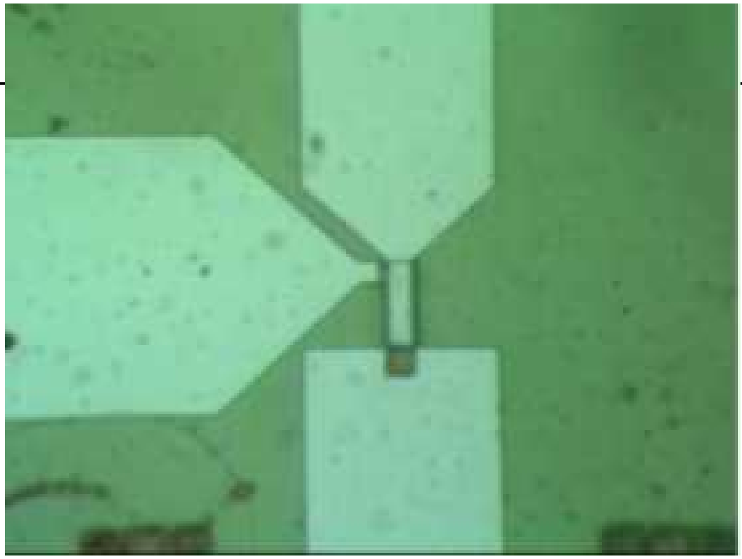
Useful for e.g.
Al, Ni, Au, Cr, Ti, NiCr, Pb, Sn

E-beam evaporation

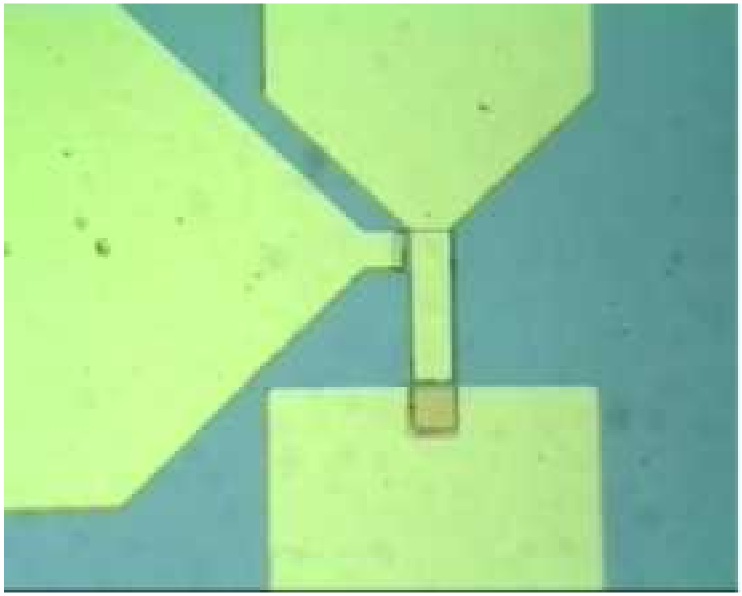
Electron beam evaporator

Au

Liftoff



Opening of photo resist
for Ti/Au gate



After deposition of Ti/Au,
then soaking in acetone

Resolution of optical lithography

$$R = \frac{3}{2}\sqrt{\frac{\lambda z}{2}}$$

Contact printing

z is resist thickness
(typically 0.1-1 μm)

$$R = 0.61\frac{\lambda}{NA}$$

Projection printing

NA is numerical aperture
(typically 0.5)

Light sources

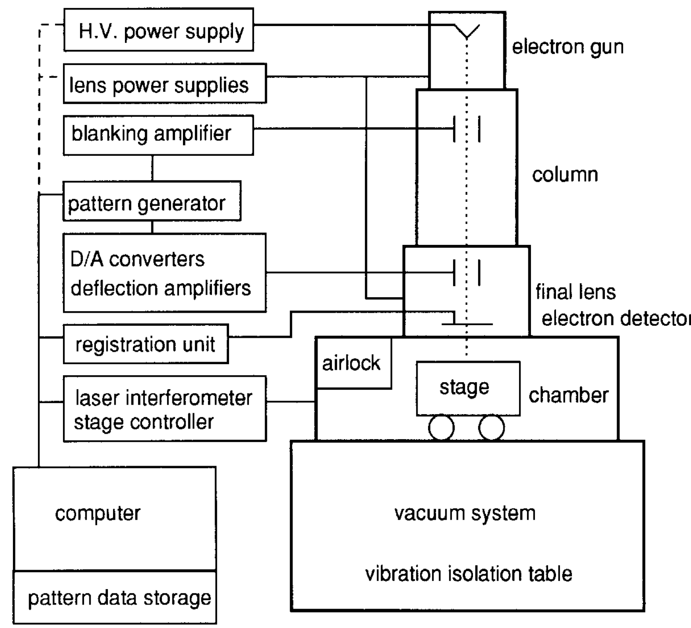
Source	λ	Resolution
Hg lamp (g-line)	436 nm	400 nm
Hg lamp (i-line)	365 nm	350 nm
KrF	248 nm	150 nm
ArF	193 nm	80 nm
F_2	157 nm	research

increasing cost

Extreme UV, x-ray lithography research topics.
Difficulties lie in sources, and materials for optics and masks.

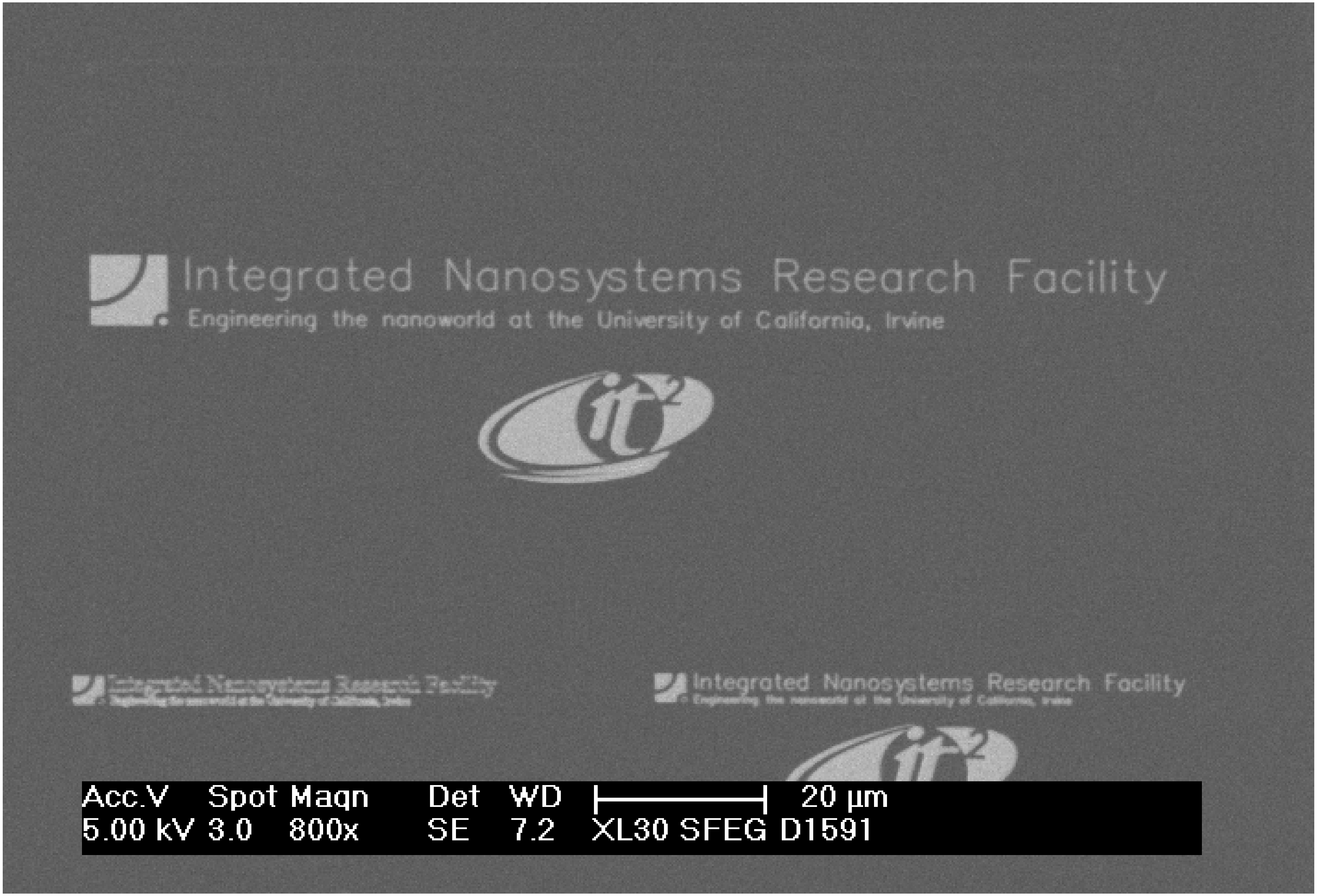
Electron Beam Lithography

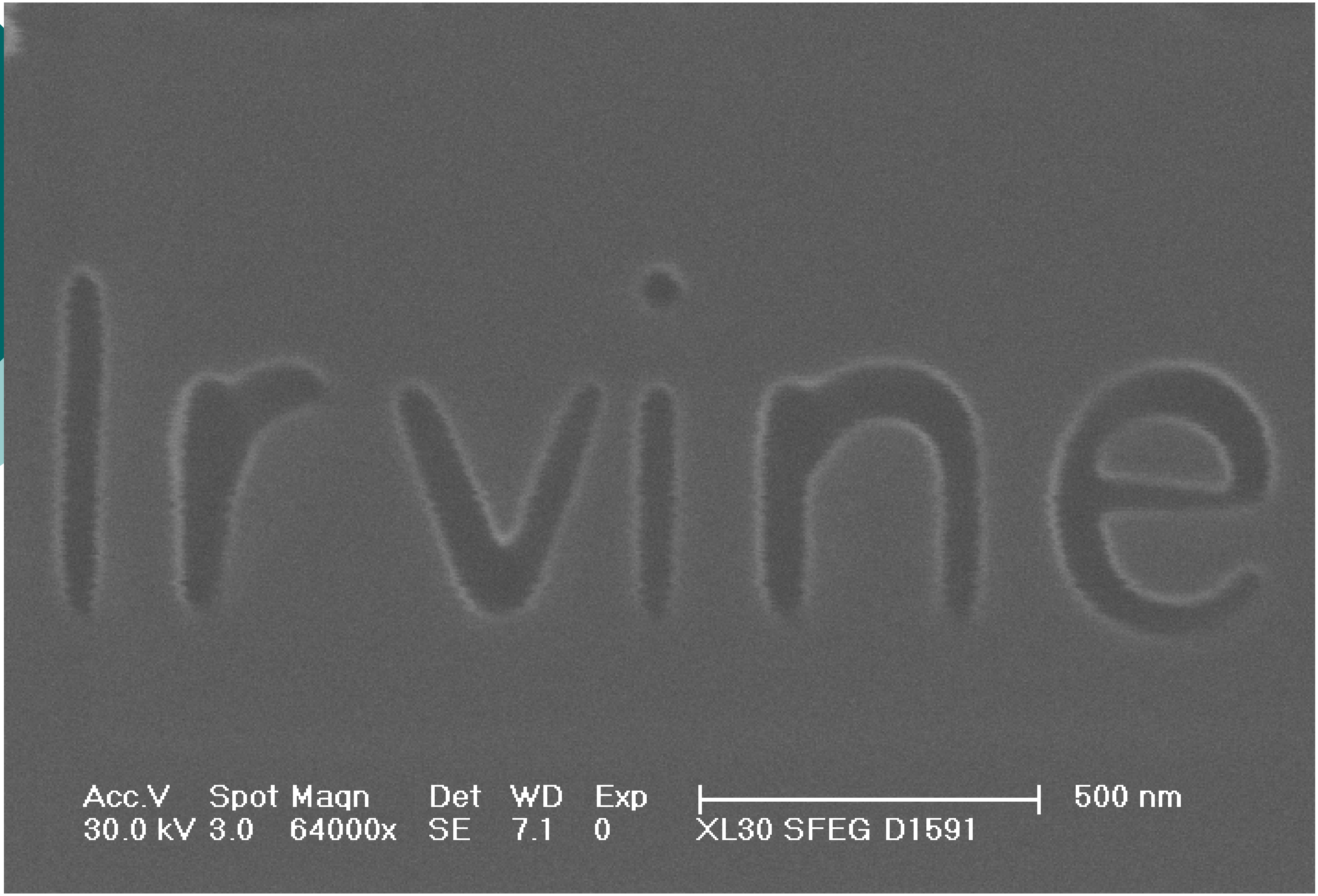
- Advantages
  - Resolution
    - electron wavelength small
    - beamsize 1 nm
    - resolution from scattering typically 10 nm
  - Flexibility
    - All patterns under computer control
- Disadvantages
  - Cost
    - Need high vacuum
    - Need precision electron focusing magnets
  - Throughput
    - Only one pixel exposed at a time
    - Not commercially viable except for a few applications

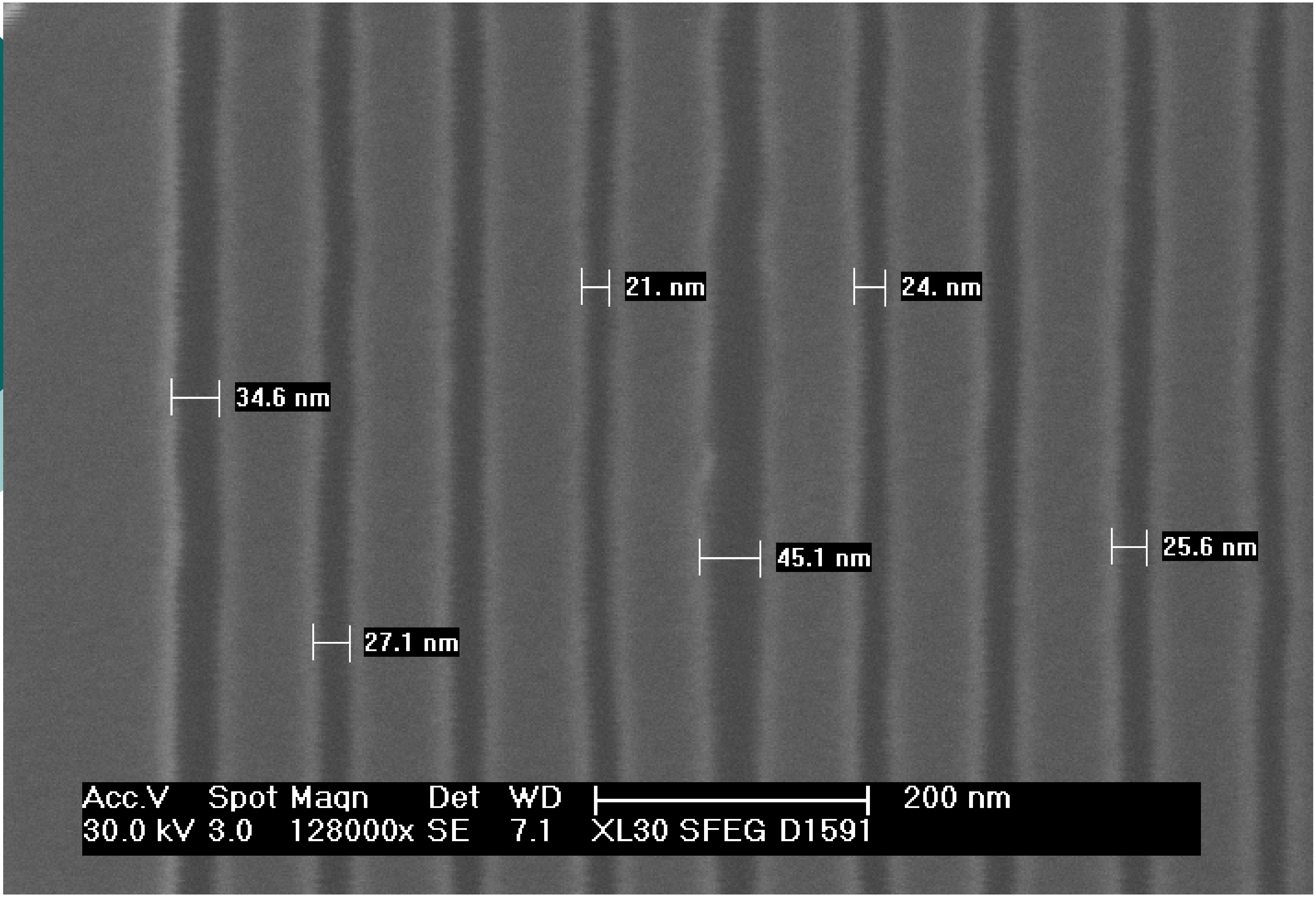


Reference: SPIE Handbook of Microlithography, Micromachining, and Microfabrication available at http://www.cnf.cornell.edu/spiebook/toc.htm

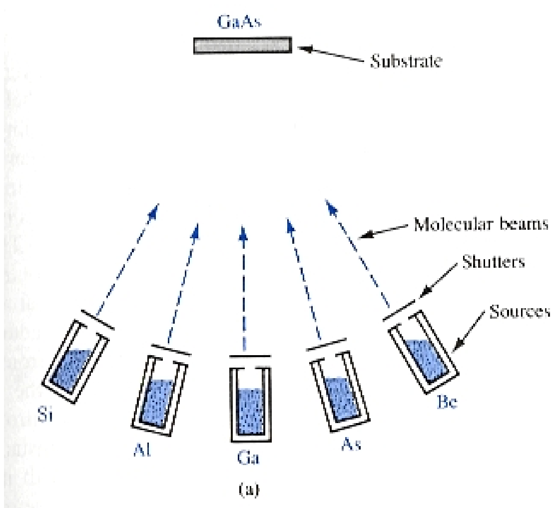
In spite of its disadvantages,
e-beam lithography is the main tool for nanotechnology research.

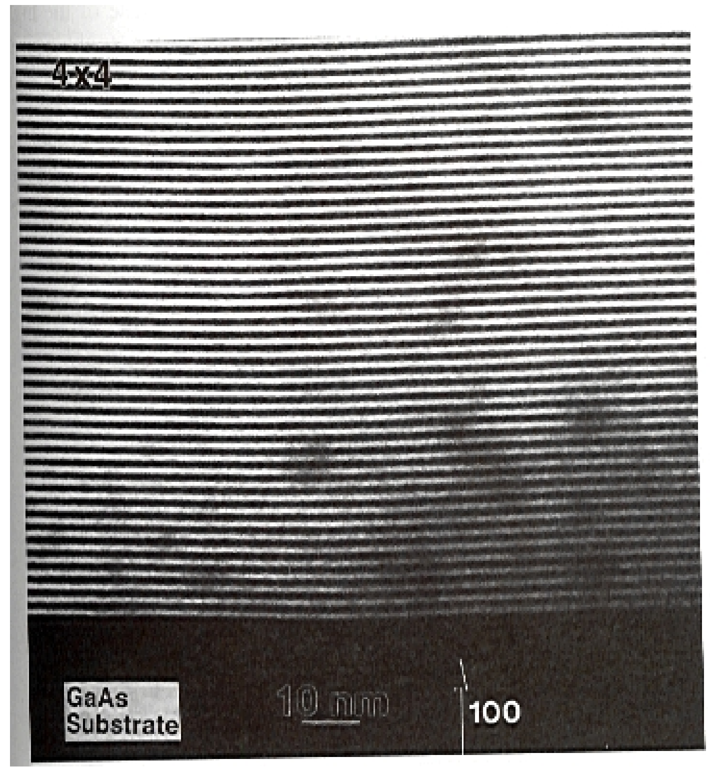

Integrated Nanosystems Research Facility
Engineering the nanoworld at the University of California, Irvine
Integrated Nanosystems Research Facility
Acc.V Spot Magn Det WD 20 µm
5.00 kV 3.0 800x SE 7.2 XL30 SFEG D1591


Irvine
Acc.V Spot Magn Det WD Exp
30.0 kV 3.0 64000x SE 7.1 0
500 nm
XL30 SFEG D1591


21. nm
24. nm
34.6 nm
45.1 nm
25.6 nm
27.1 nm
Acc.V Spot Magn Det WD 200 nm
30.0 kV 3.0 128000x SE 7.1 XL30 SFEG D1591

Molecular Beam Epitaxy (MBE)



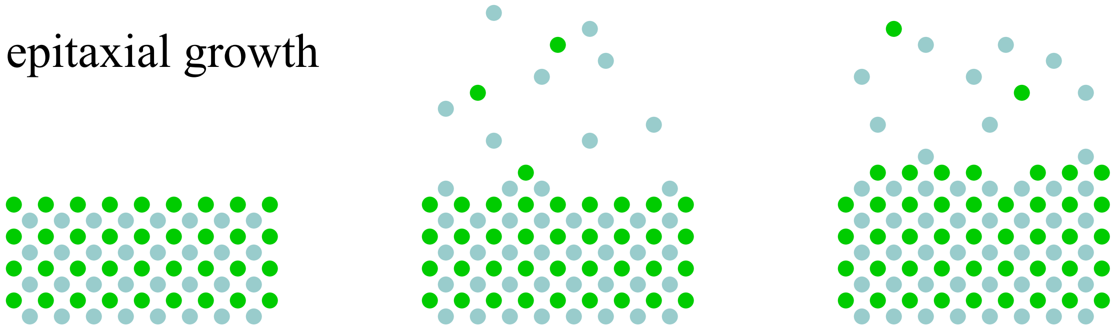


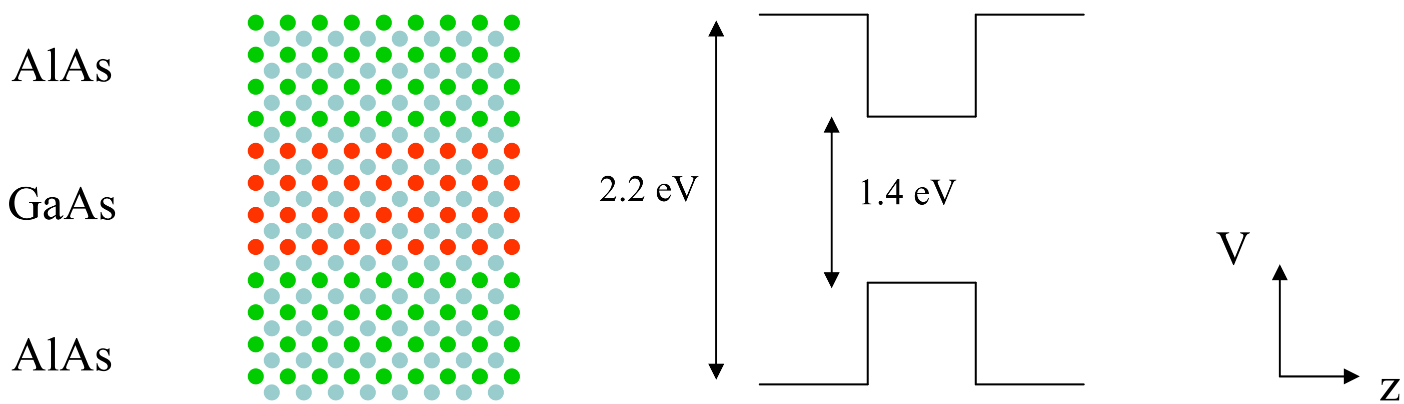
4 atom per layer!

(From Streetman, Solid State Electronic Devices)

MBE

epitaxial growth



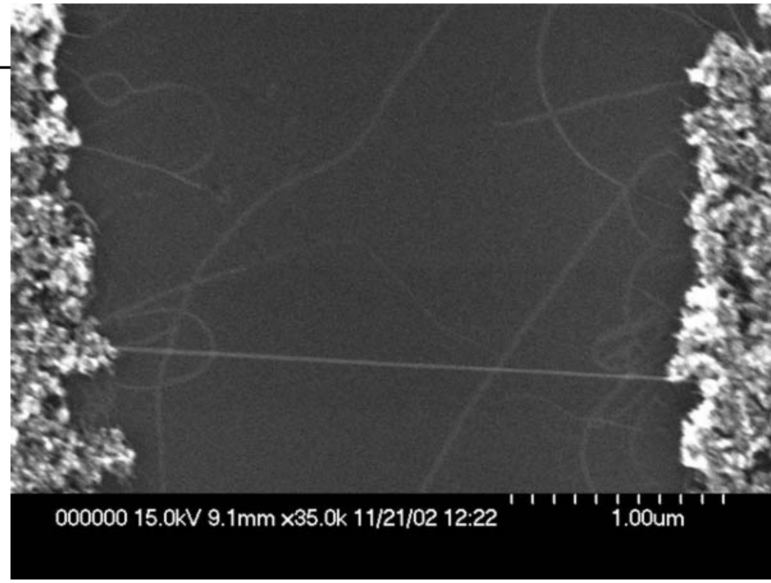


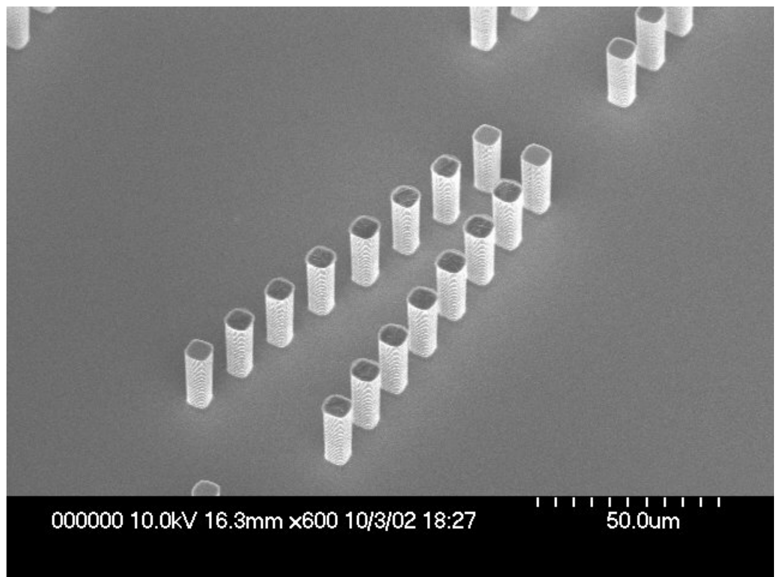
Also InP, InGaAs, InAlAs, InGaAsP …

Characterization

- Optical microscopy cannot see better than wavelength of light, ~ 1 μm
- Scanning electron microscope (SEM)
- Transmission electron microscope (TEM)
- Scanning probe microscopy (SPM)
- Atomic force microscope (AFM)

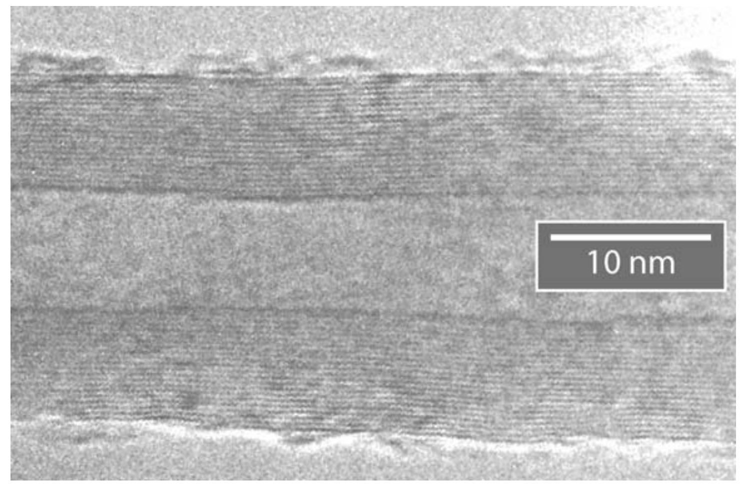
SEM





- Advantages:
  - resolution to 1 nm
  - fast
  - 3d structures visible
  - back-scattered x-ray spectrum gives compositional information
- Disadvantage
  - must be in vacuum environment (not good for bio)
  - expensive
  - samples must be conductive

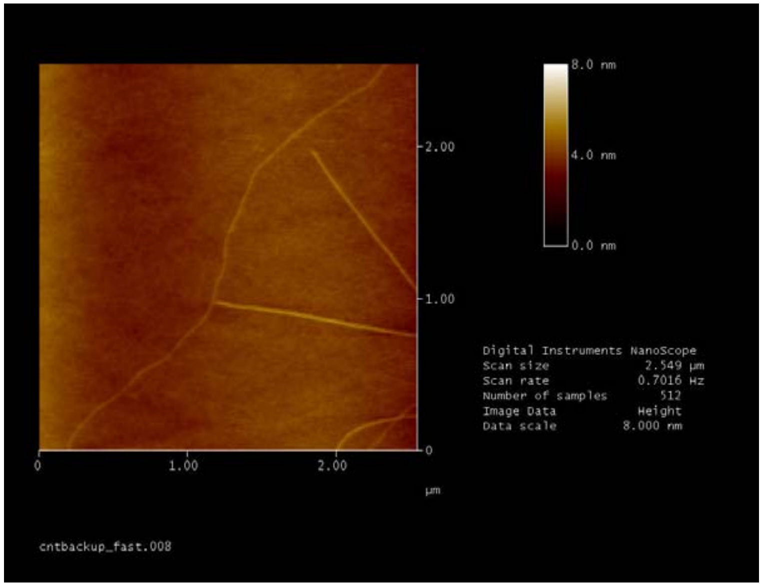
Multiwalled carbon nanotube



Shengdong Li, submitted

- Advantages
  - resolution < 1 nm
  - fast
  - diffraction pattern gives crystolographic info
- Disadvantages
  - expensive
  - high vacuum
  - sample must be thinned

SPM/AFM



- Mode of operation
  - non-contact
  - tunneling
- Advantages
  - works in air or liquid
  - angstrom resolution possible
  - can image individual atoms
  - probes various quantities
    - conductance
    - magnetism
- Disadvantages
  - extremely slow
  - many minutes for one image

Length scales

- Atoms
  - ~ angstrom 10^{-10} m
- Light
  - wavelength ~ μm
- Electrons
  - De Broglie wavelength = h/p (quantum mechanics)

  = sqrt (150/V) in angstroms (V is energy in volts)

  ~0.1-10 nm
  - If circuit element is about the size of an electron wavelength, wave nature will be *crucial*
  - Conductance quantized at these small scales in units of e^2/h
- Mean free path (MFP)
  - 10^{-10} m in metals at room temperature
  - 10^{-4} m in ultra high quality semiconductors at low temperatures

Energies

- Electronic transition energies
  - ~ 1-10 eV
- Fermi energy
  - 1-10 eV in metals
  - 1-10 meV in semiconductors
- kT
  - 30 meV at room temperature

Quantum mechanics of free electrons

- Important for quantized resistance calculation
- Important for single electron transistors
- Density of states
  - 3 dimensions
  - 2 dimensions
  - 1 dimensions
  - 0 dimensions
- Dimensionality (effective)
  - Set by size of nano-device compared to electron wavelength

Readings for this lecture

- Ferry, *Quantum Mechanics for Electrical Engineering*, ch. 1 (in handout packet)
- Hanson p. 16-44,62-69,85-101,chapter 8
- Good references:
  - Brandsen and Joachian, *Introduction to Quantum Mechanics*, Longman Scientific, 1989
  - Kittel, *Introduction to Solid State Physics*, Wiley, 1996
  - Ashcroft/Mermin, *Solid State Physics*, Saunders College, 1976

Quantum mechanics of free particles

$$\left|\Psi(\vec{r},t)\right|^2$$ is probability of finding an electron at point r at time t.

Ψ is complex, and both real and imaginary parts are physical.

Quantum mechanics of free particles:

$$\left|\Psi(\vec{r},t)\right|^2$$ is probability of finding an electron at point r at time t.

Ψ is complex, and both real and imaginary parts are physical.

$$\omega = E/\hbar$$

For a free particle:

$$\Psi(\vec{r},t) \sim e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Momentum:

$$\vec{p} = \hbar\vec{k}$$

Energy:

$$E = \frac{p^2}{2m} = \frac{(\hbar k)^2}{2m}$$

Schrodinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

(1 dimension)

(Time dependent)

Let

$$\Psi(x,t) = A\cdot e^{i(kx-\omega t)}$$

A is a (complex) constant.

Then

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}A\cdot e^{i(kx-\omega t)} = i\hbar(-i\omega)A\cdot e^{i(kx-\omega t)}$$

$$= E\cdot A\cdot e^{i(kx-\omega t)} = E\cdot\Psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(A\cdot e^{i(kx-\omega t)}\right) = \left(-\frac{\hbar^2}{2m}\right)(ik)^2\left(A\cdot e^{i(kx-\omega t)}\right)$$

$$= \frac{\hbar^2 k^2}{2m}\left(A\cdot e^{i(kx-\omega t)}\right) = \frac{p^2}{2m}\Psi(x,t)$$

Schrodinger equation:

(3 dimensions)

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=-\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r},t)=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Psi(\vec{r},t)$$

Let $\Psi(\vec{r},t)=A\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}=A\cdot e^{i\left((k_x\cdot x+k_y\cdot y+k_z\cdot z)-\omega t\right)}$

Then $i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=i\hbar(-i\omega)\Psi(\vec{r},t)=E\cdot\Psi(\vec{r},t)$ as before.

But:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Psi(\vec{r},t)=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(A\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)$$

$$=\left(-\frac{\hbar^2}{2m}\right)\left((ik_x)^2+(ik_y)^2+(ik_z)^2\right)\left(A\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)=\left(\frac{\hbar^2(k_x{}^2+k_y{}^2+k_z{}^2)}{2m}\right)\Psi(\vec{r},t)$$

$$=\frac{\hbar^2k^2}{2m}\left(A\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)=\frac{p^2}{2m}\Psi(\vec{r},t)$$

Quantum mechanics of free particles:

$$\Psi(\vec{r},t) \sim e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Generally,

$$\Psi(\vec{r},t) = \sum_n A_n e^{i(k_n x-\omega_n t)} \rightarrow \int dk A(k) e^{i(kx-\omega t)}$$

is also a possibility.

Time-independent Schrodinger equation

$$\Psi(\vec{r},t) = A\cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= A\cdot e^{i\left((k_x\cdot x+k_y\cdot y+k_z\cdot z)-\omega t\right)} = \underbrace{A\cdot e^{i(k_x\cdot x+k_y\cdot y+k_z\cdot z)}}\cdot e^{-i\omega t}$$

Call this $\psi(\vec{r})$

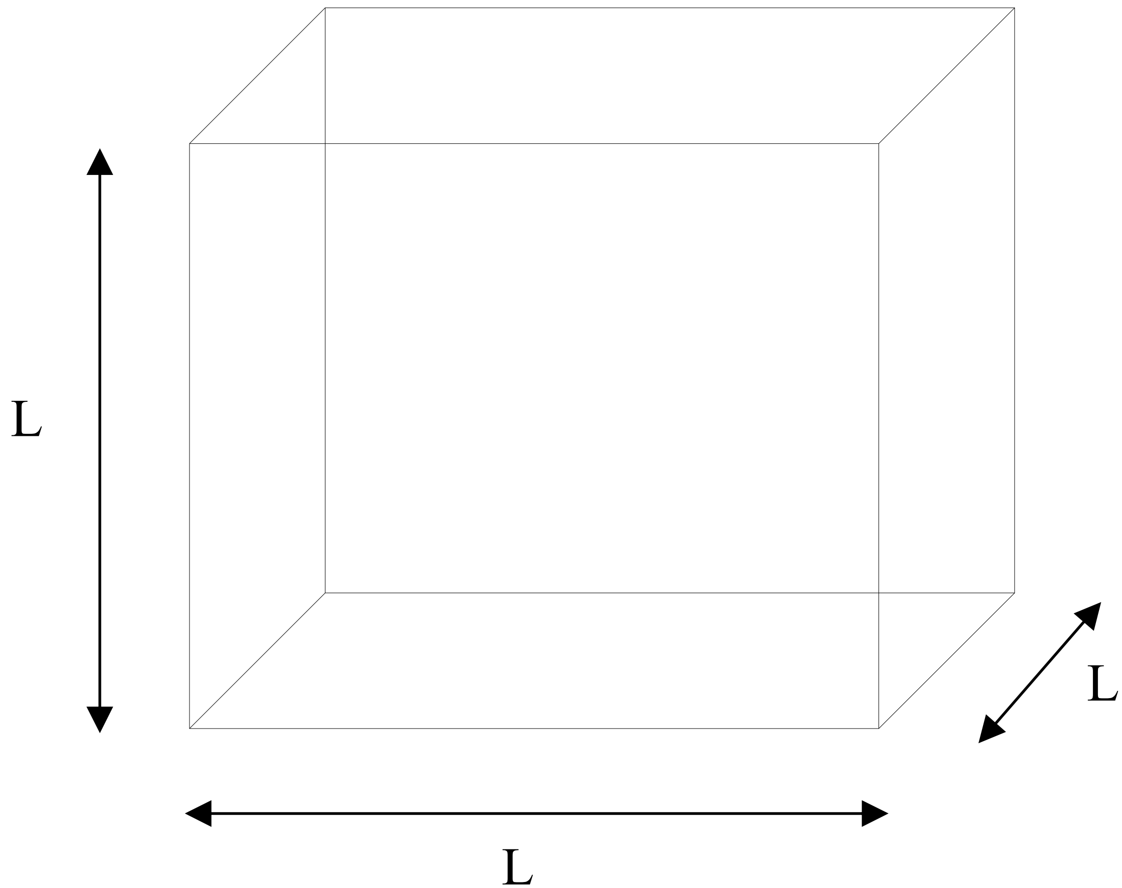
$$\Rightarrow \Psi(\vec{r},t) = \psi(\vec{r})\cdot e^{-i\omega t}$$

From: $$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r},t)$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r})\cdot e^{-i\omega t} = i\hbar(-i\omega)\psi(\vec{r})\cdot e^{-i\omega t} = E\cdot\psi(\vec{r})\cdot e^{-i\omega t} = -\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{r})\cdot e^{-i\omega t}$$

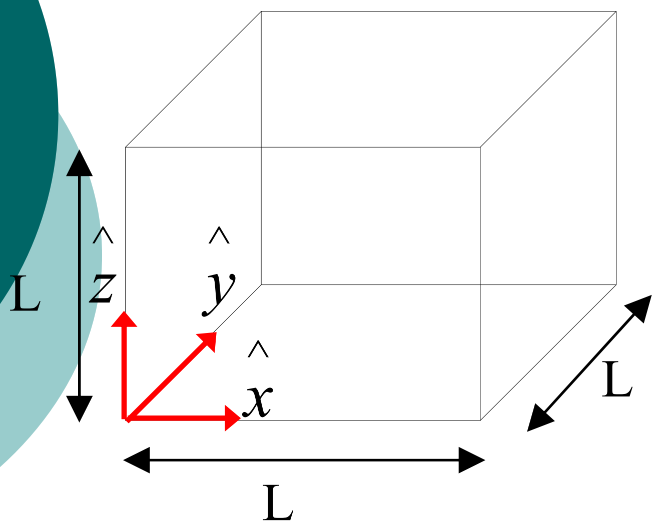
$$\boxed{\Rightarrow -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{r}) = E\cdot\psi(\vec{r})}$$

Confined particles: A box



Goal: find $\psi(\vec{r})$

Similar to electric field inside the box.



Goal: find $\psi(\vec{r})$

Everywhere outside the box

$$|\psi(\vec{r})|^2 = 0$$

In particular,

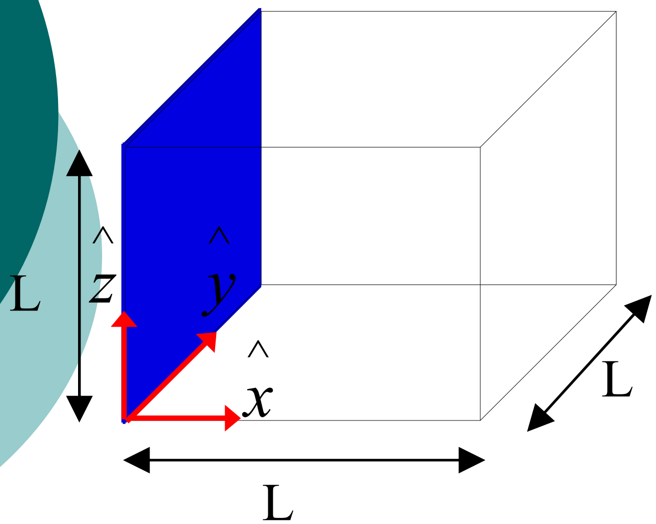
$$|\psi(\vec{r})|^2 = 0$$

on the boundaries.

As before, we will consider all six surfaces:

Boundary conditions:

The plane x=0:



Try:

$$\psi(\vec{r}) = A \cdot e^{i(k_x \cdot x + k_y \cdot y + k_z \cdot z)}$$

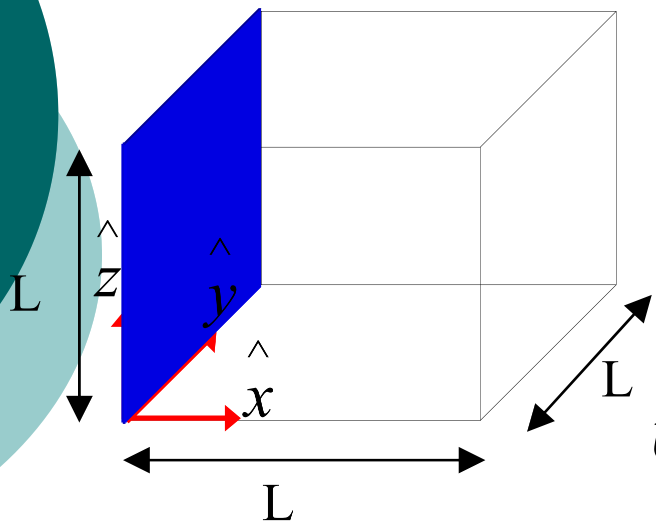
$$\psi(x=0, y, z) = A \cdot e^{i(k_x \cdot x + k_y \cdot y + k_z \cdot z)} = A \cdot e^{i(k_y \cdot y + k_z \cdot z)}$$

(arrow pointing from the $k_x \cdot x$ term to: 0)

Does not solve boundary condition!!!

Boundary conditions:

The plane x=0:



Let's try something:

$$\psi(\vec{r}) = A \cdot e^{i(k_x \cdot x + k_y \cdot y + k_z \cdot z)}$$

$$- A \cdot e^{i(-k_x \cdot x + k_y \cdot y + k_z \cdot z)}$$

$$\psi(\vec{r}) = A \cdot \left(e^{ik_x \cdot x} - e^{-ik_x \cdot x} \right) \cdot e^{i(k_y \cdot y + k_z \cdot z)}$$

$$e^{a \cdot b} = e^a \cdot e^b$$

$$\psi(x=0, y, z) = A \cdot \left(e^{ik_x \cdot x} - e^{-ik_x \cdot x} \right) \cdot e^{i(k_y \cdot y + k_z \cdot z)}$$

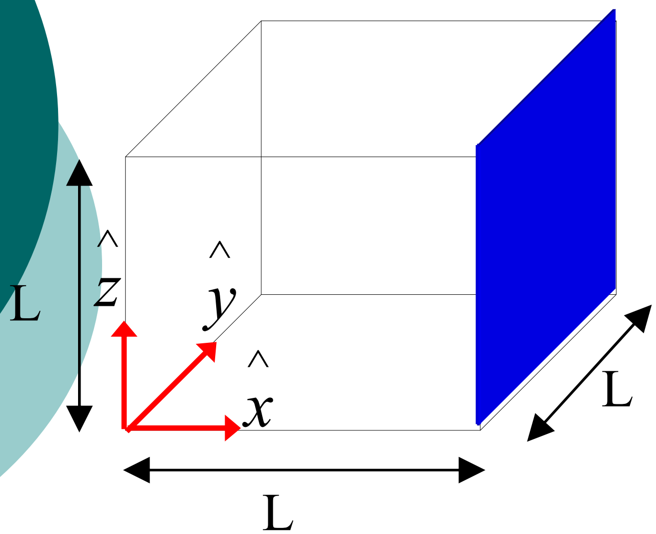
0 0

$$= A \cdot \left(e^0 - e^0 \right) \cdot e^{i(k_y \cdot y + k_z \cdot z)} = 0$$

Does solve boundary condition!!!

Boundary conditions:

The plane x=L:



$$\psi(\vec{r}) = A\cdot\left(e^{ik_x\cdot x} - e^{-ik_x\cdot x}\right)\cdot e^{i(k_y\cdot y+k_z\cdot z)}$$

$$= 2iA\cdot\sin(k_x x)\cdot e^{i(k_y\cdot y+k_z\cdot z)}$$

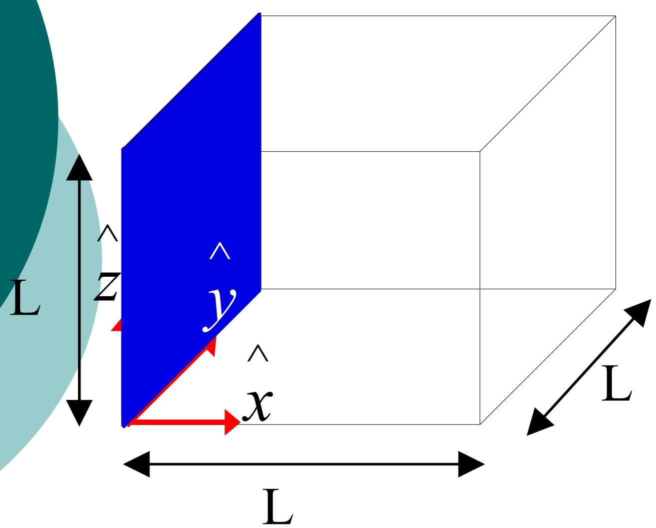
$$\sin(\theta) = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

$$\psi(x=L,y,z) = 2iA\cdot\sin(k_x L)\cdot e^{i(k_y\cdot y+k_z\cdot z)} = 0\,?$$

If and only if:

$$k_n = n\pi / L \quad n = 1,2,3\ldots$$

Boundary conditions:



We can do the same for y, z:

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

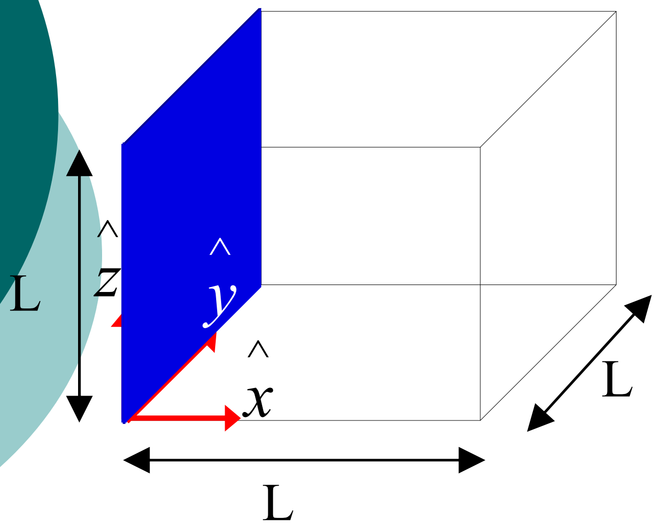
$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or "quantum states"

Many electrons:

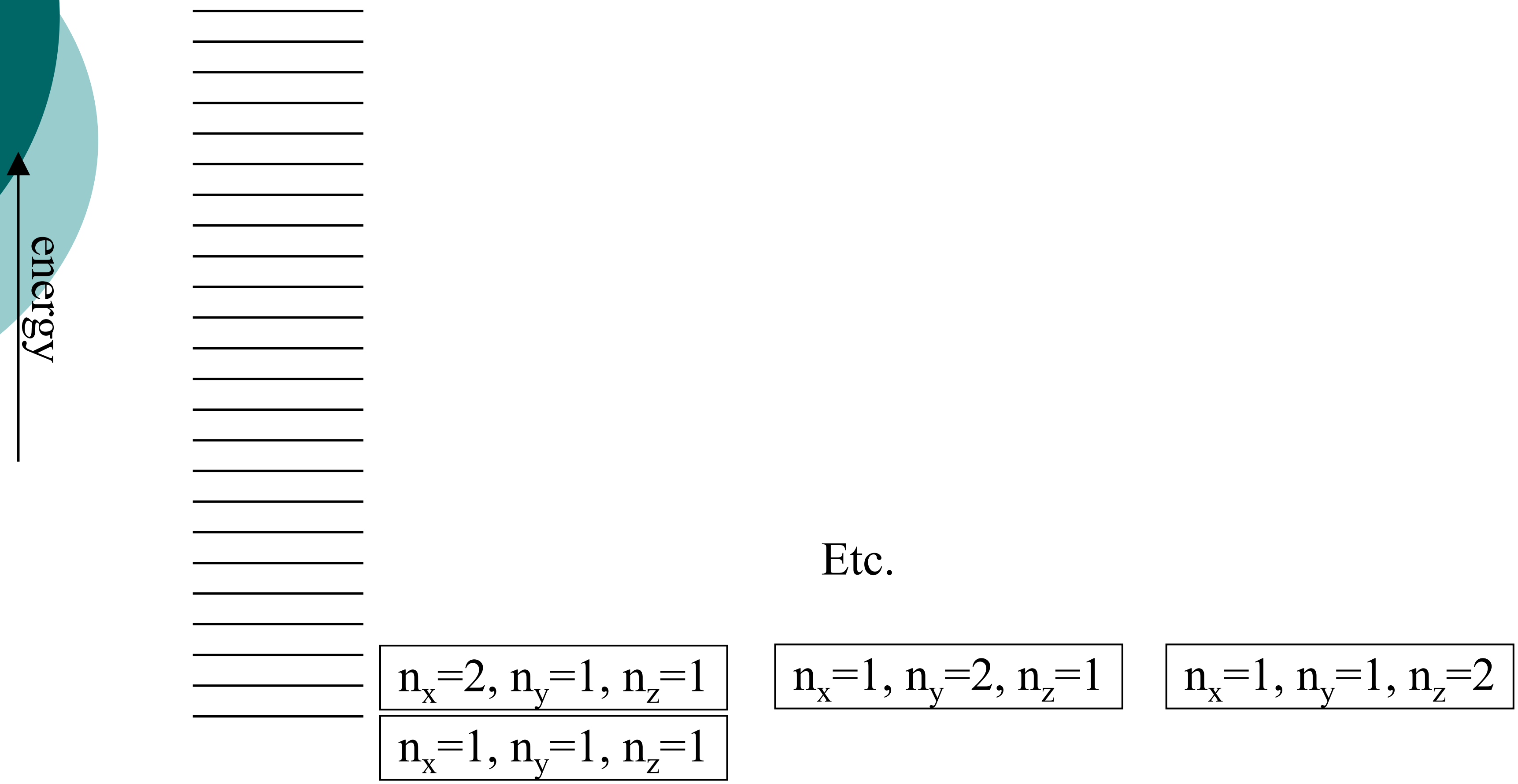


$$E = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x^{\ 2} + n_y^{\ 2} + n_z^{\ 2})$$

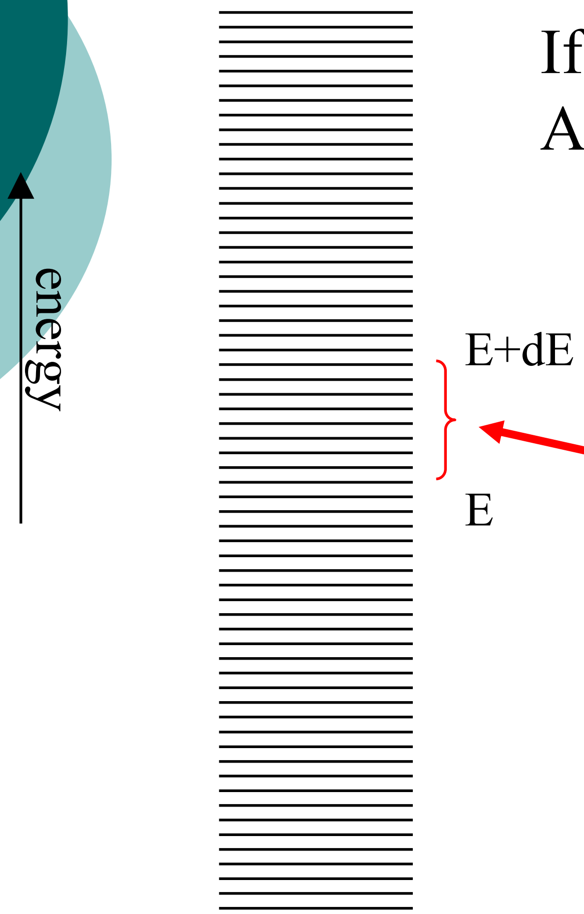
These are the allowed energy levels, or "quantum states"

Pauli exclusion principle: Each unique combination of n_x, n_y, n_z can only have two electrons (spin up, spin down).

Energy spectrum of free particles



Density of states



If L is large, states are very close together.
Approximate as a continuum.

How many states?

$$N_E dE = ?$$

Number of states with energy between E and E + dE

$$\rho(E)dE = ?$$

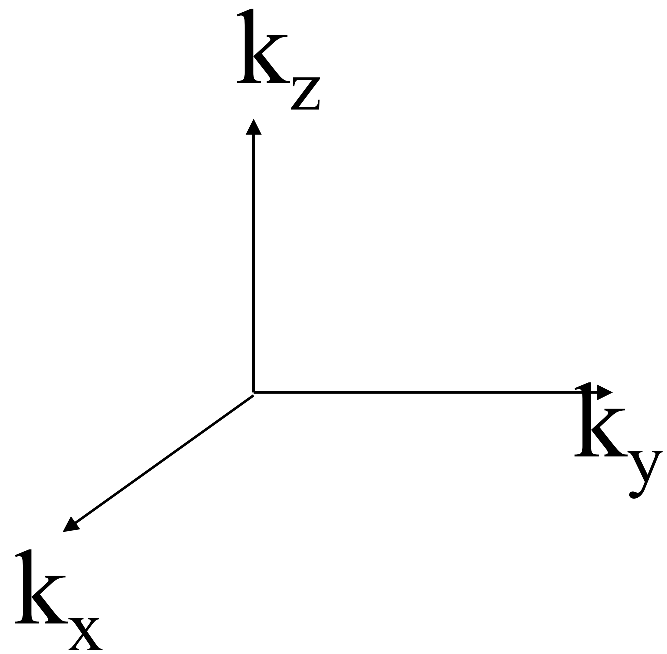
Number of states with energy between E and E + dE *per volume.*

Density of states

Easier first to think of in k-space:

Density of states in k-space is uniform:

One state per $(\pi/L)^3$:



Density of states

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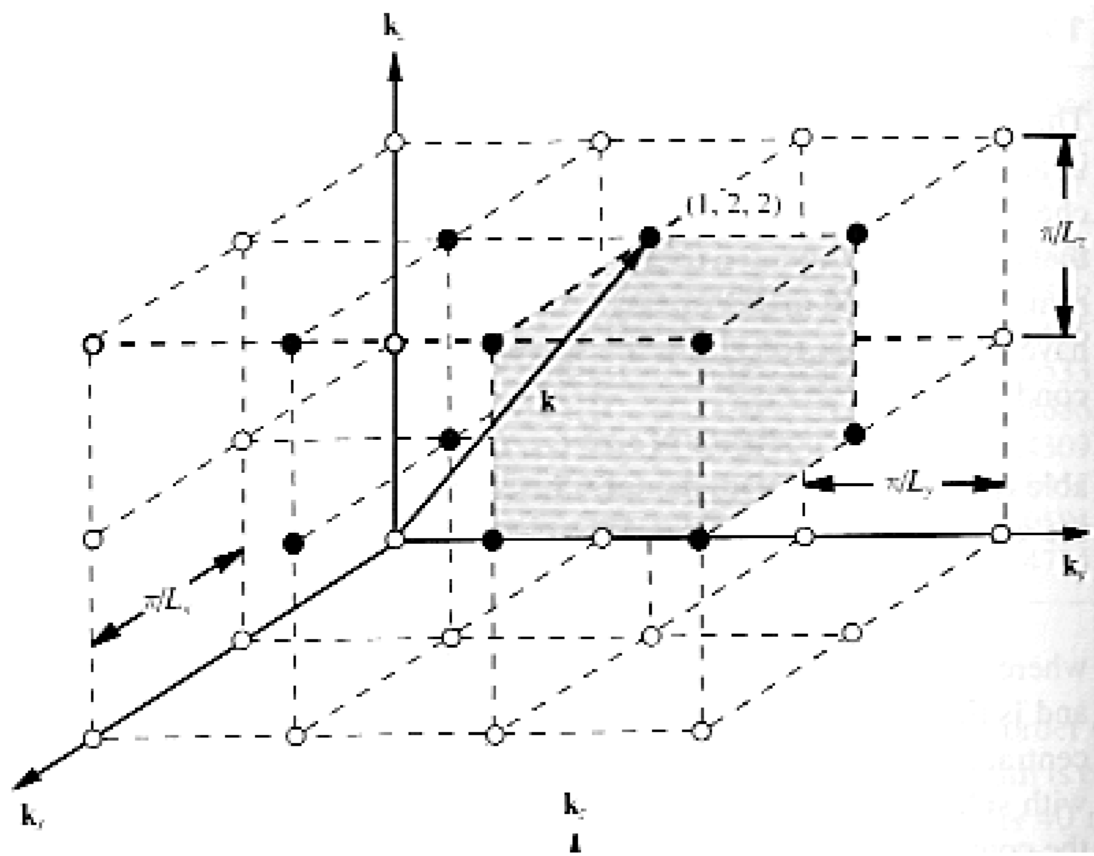
One state per $(\pi/L)^3$:

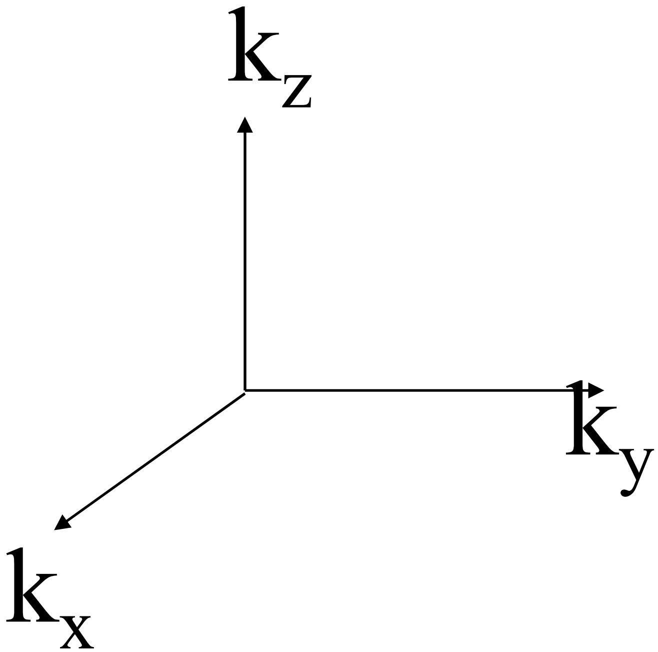


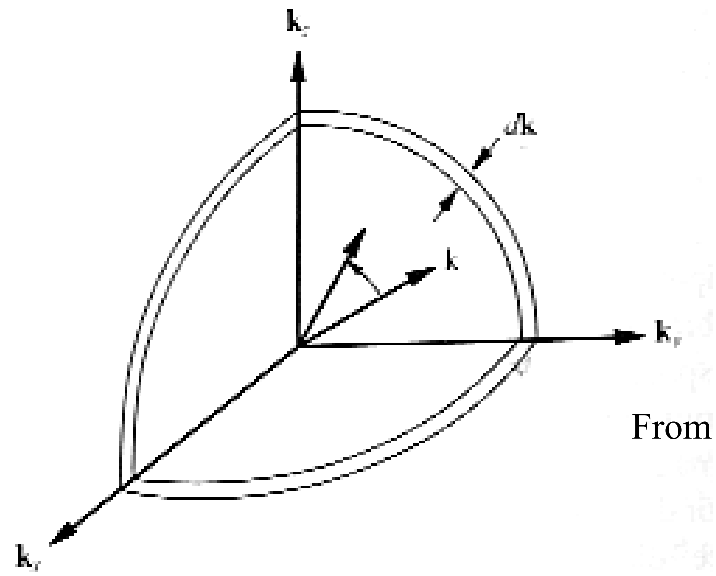
From Verdeyen

Density of states

Number of states between k, k+dk:

$$N_k dk = ?$$





From Verdeyen

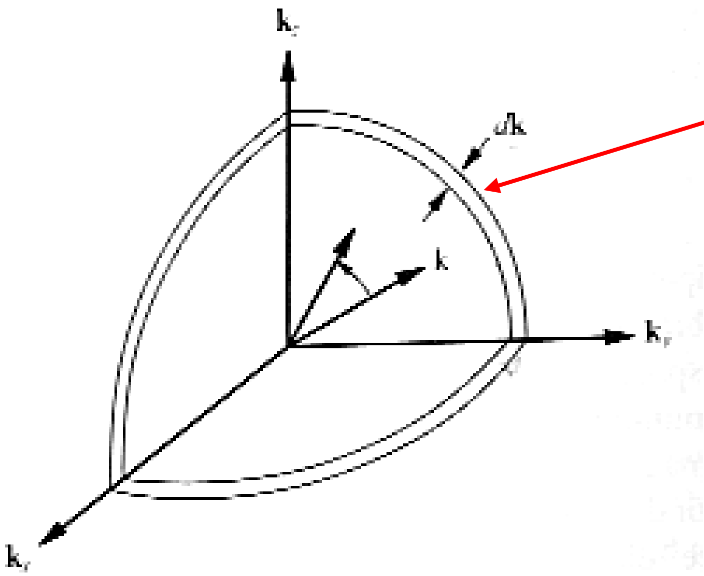
$$k \equiv \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$N_k dk = ?$



Volume of spherical shell
$=4\pi k^2 dk/8$
8 is for upper right quadrant

Number of states in volume=
Volume x States/volume

States/volume = $1 / (\pi/L)^3$:

$$N_k dk = \left(4\pi k^2 dk / 8\right) \cdot \left(\frac{1}{(\pi / L)^3}\right) \cdot 2 = L^3 \frac{k^2 dk}{\pi^2}$$

$$\rho_k dk \equiv \frac{N_k dk}{\text{volume}} = \frac{k^2 dk}{\pi^2}$$

HW you will do calculation for 2 dimensional world.

$\rho(E)dE = ?$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{k^2 dk}{\pi^2}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

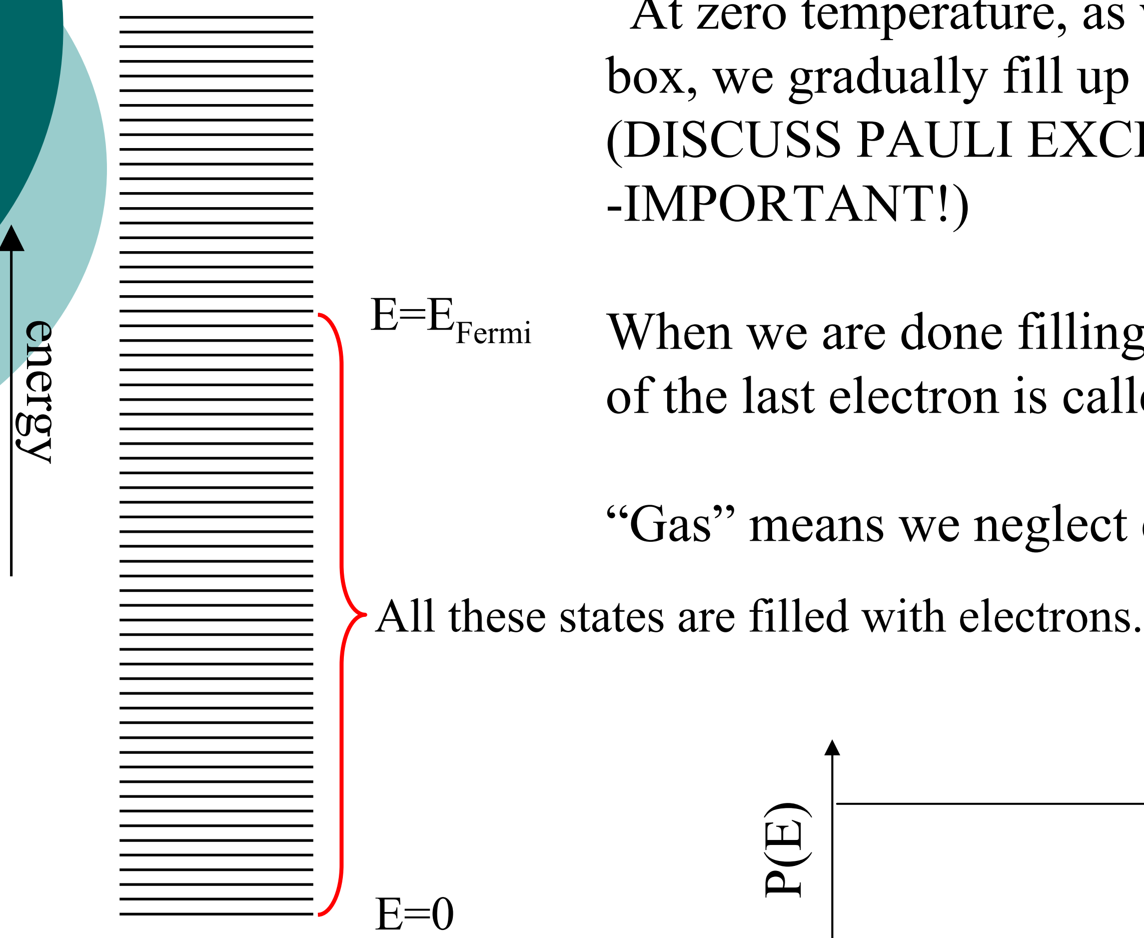
$$\rho(E)dE = \frac{2^{3/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \cdot E^{1/2} dE$$

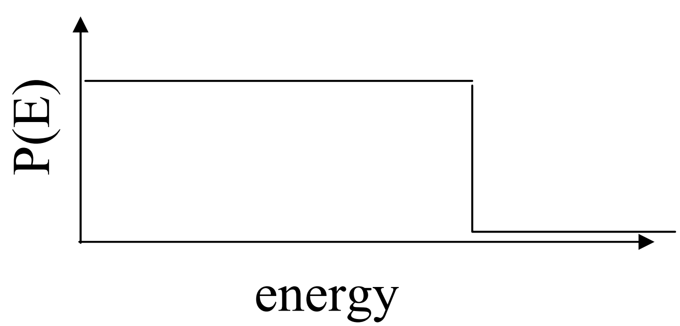
Fermi gas

At zero temperature, as we add electrons to the box, we gradually fill up all the states. (DISCUSS PAULI EXCLUSION PRINCIPLE -IMPORTANT!)

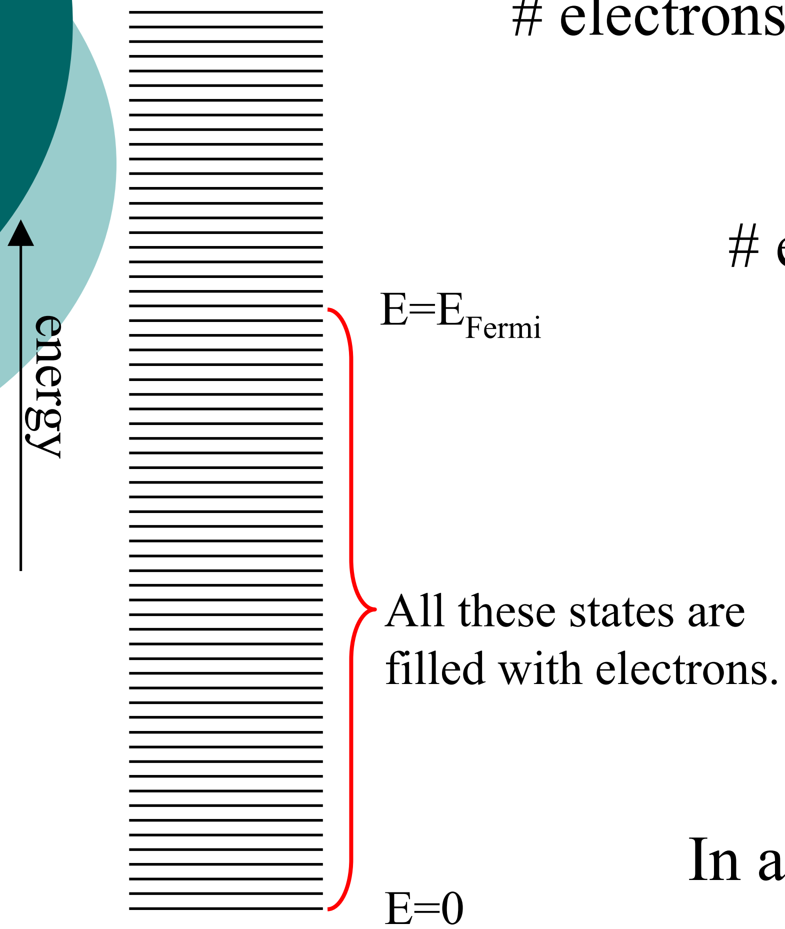
When we are done filling the box, the energy of the last electron is called the "Fermi energy."

"Gas" means we neglect electron-electron interactions.





Fermi energy



$$\#\text{ electrons} = \int_0^{E_f} N_E \, dE = \int_0^{E_f} L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \cdot E^{1/2} dE$$

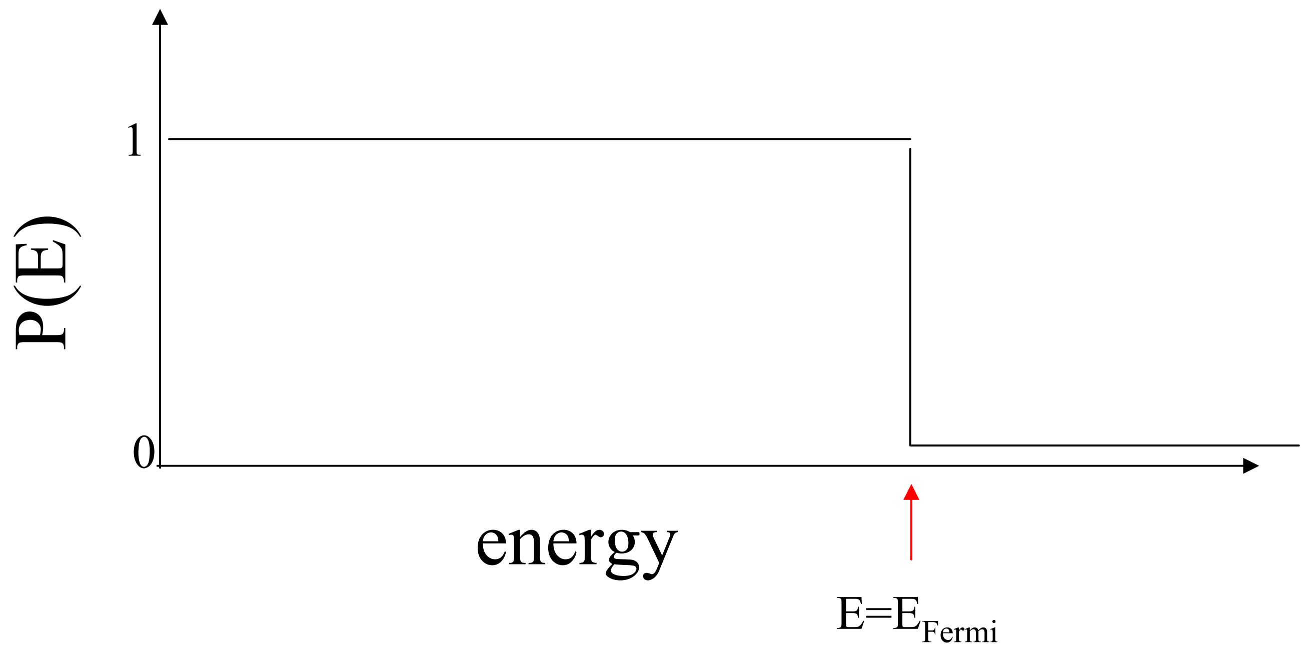
$$\#\text{ electrons} = L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \frac{2}{3} E_f^{3/2}$$

$$\Rightarrow E_f = \frac{\hbar^2 3^{2/3} \pi^{4/3}}{2m} \left(\frac{\#\text{ electrons}}{L^3} \right)^{2/3}$$

In a typical metal, 1 electron /(0.1 nm)^3.

$E_f \sim 10$ eV

Occupation probability



P(E) = probability of occupying a state with energy E

What about finite temperature?

Boltzmann

Recall Boltzmann factor P(ε):

"The probability for a physical system to be in a state with energy ε is proportional to ."

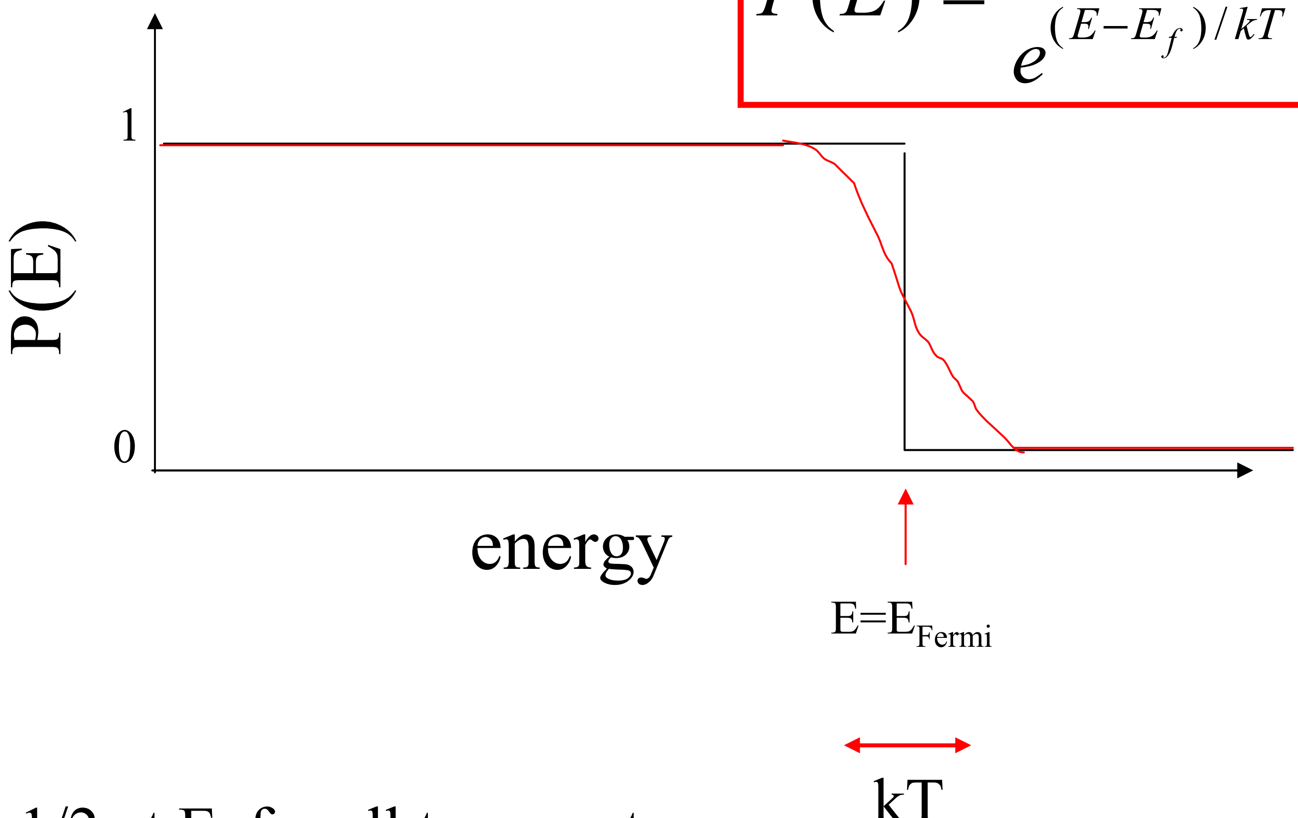
This is actually not quite true. It is classical. A quantum calculation shows for electrons:

$$P(E) = \frac{1}{e^{(E-E_f)/kT} + 1}$$

Called Fermi-Dirac distribution function.
Boltzman is high-energy limit (discuss!)

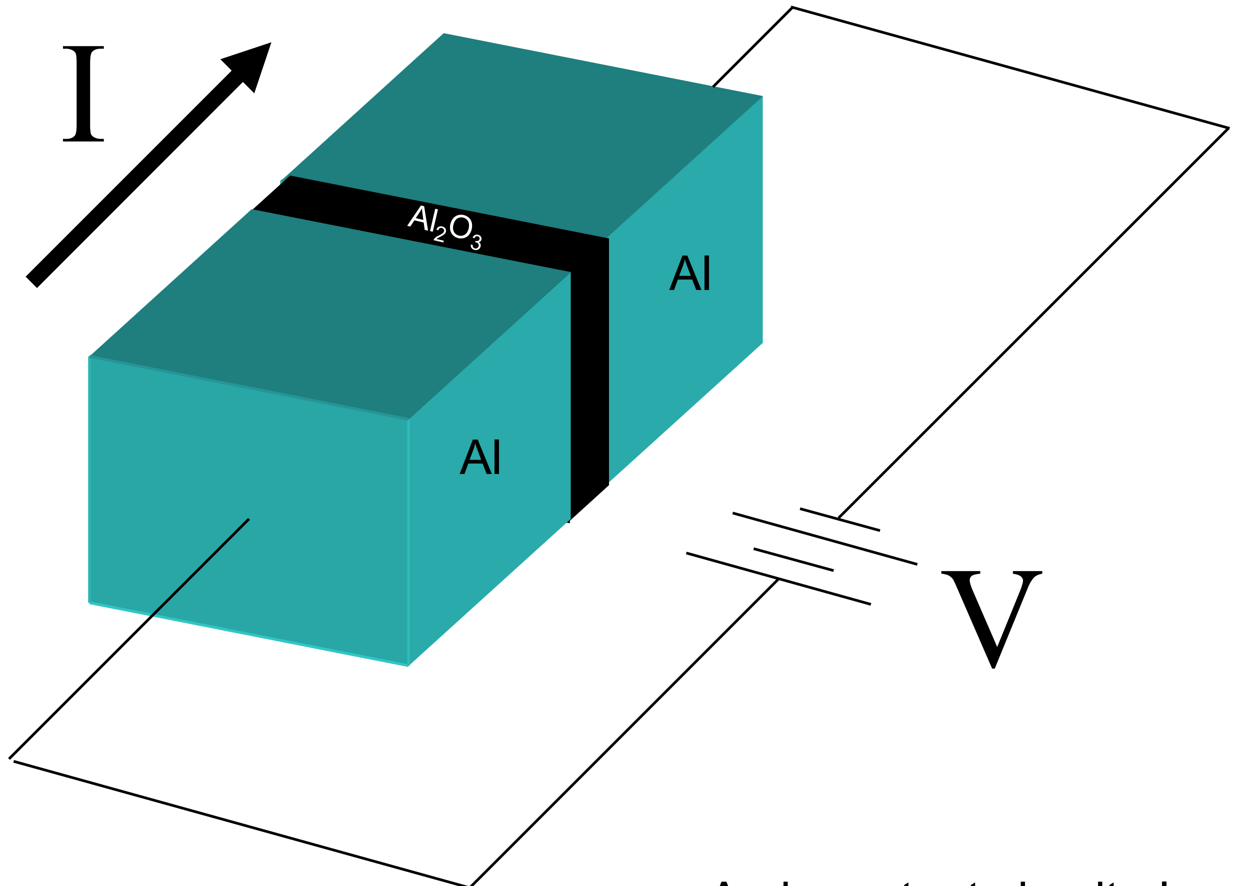
Fermi-Dirac

$$P(E) = \frac{1}{e^{(E-E_f)/kT}+1}$$



P=1/2 at E_f for all temperatures.

Tunnel junctions

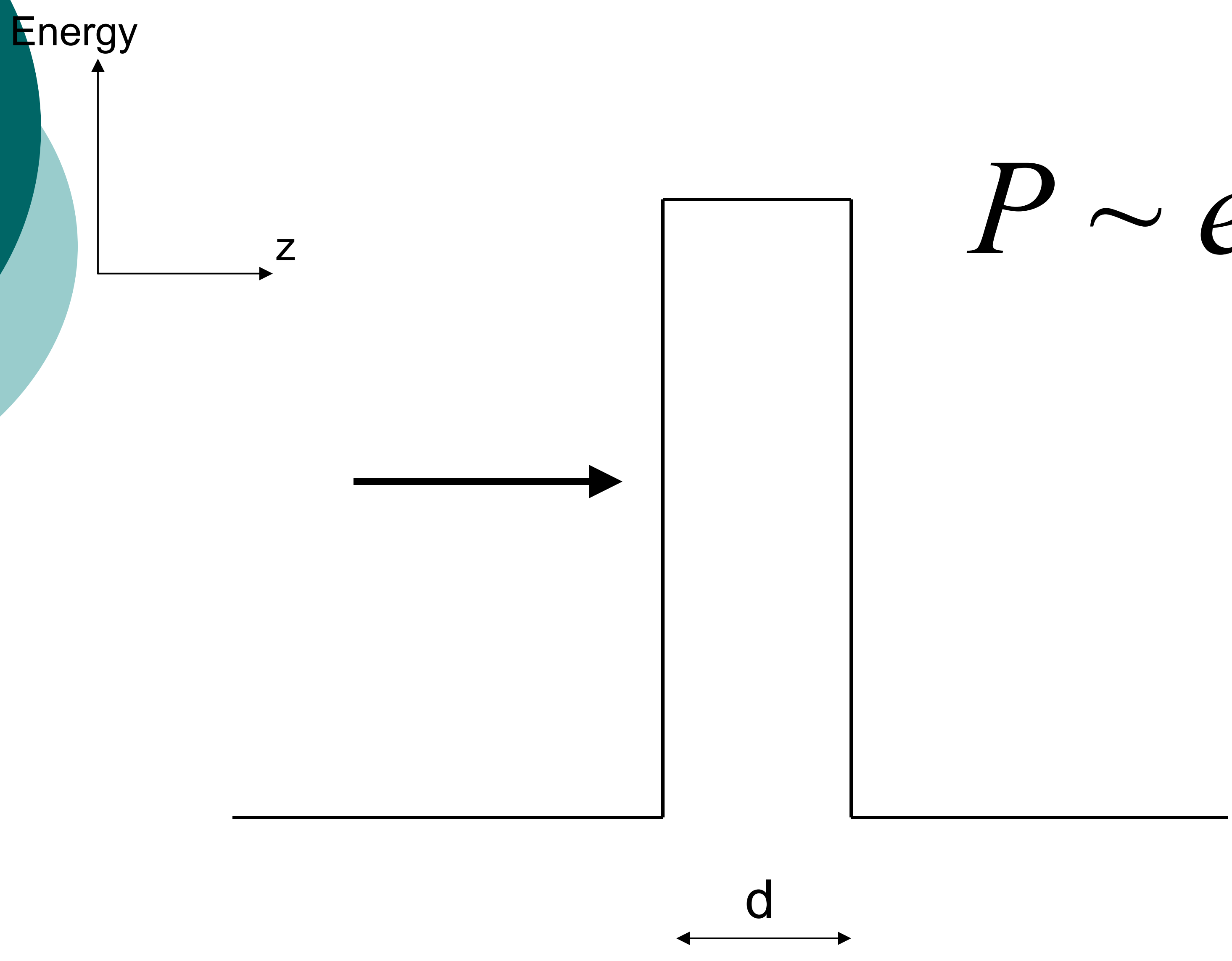


An important circuit element in *single electron transistors*.

Readings this lecture covers

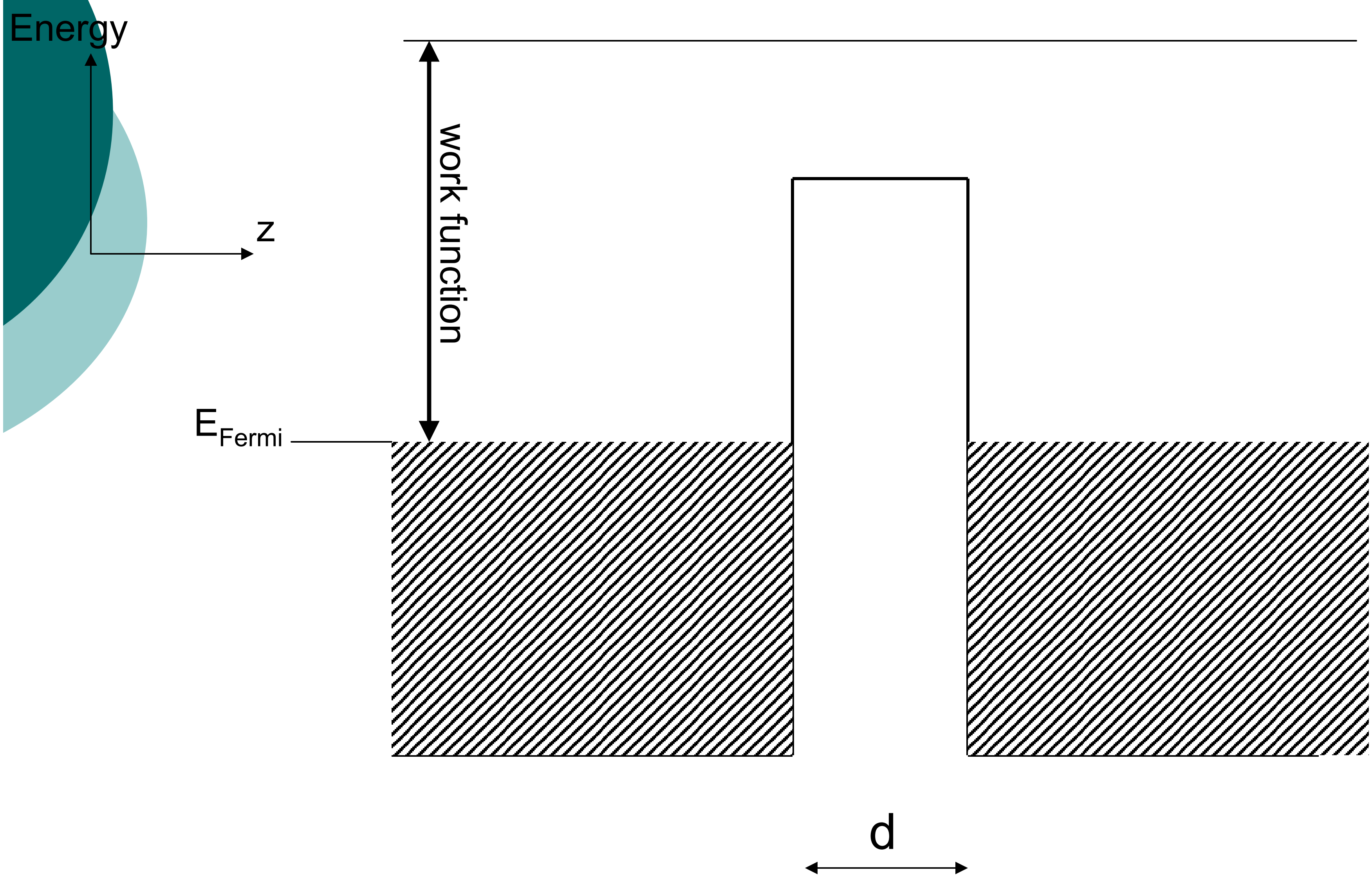
- Ferry pp. 91-101, 114-117
- Reference: Hanson Ch. 6

Quantum tunnel probability

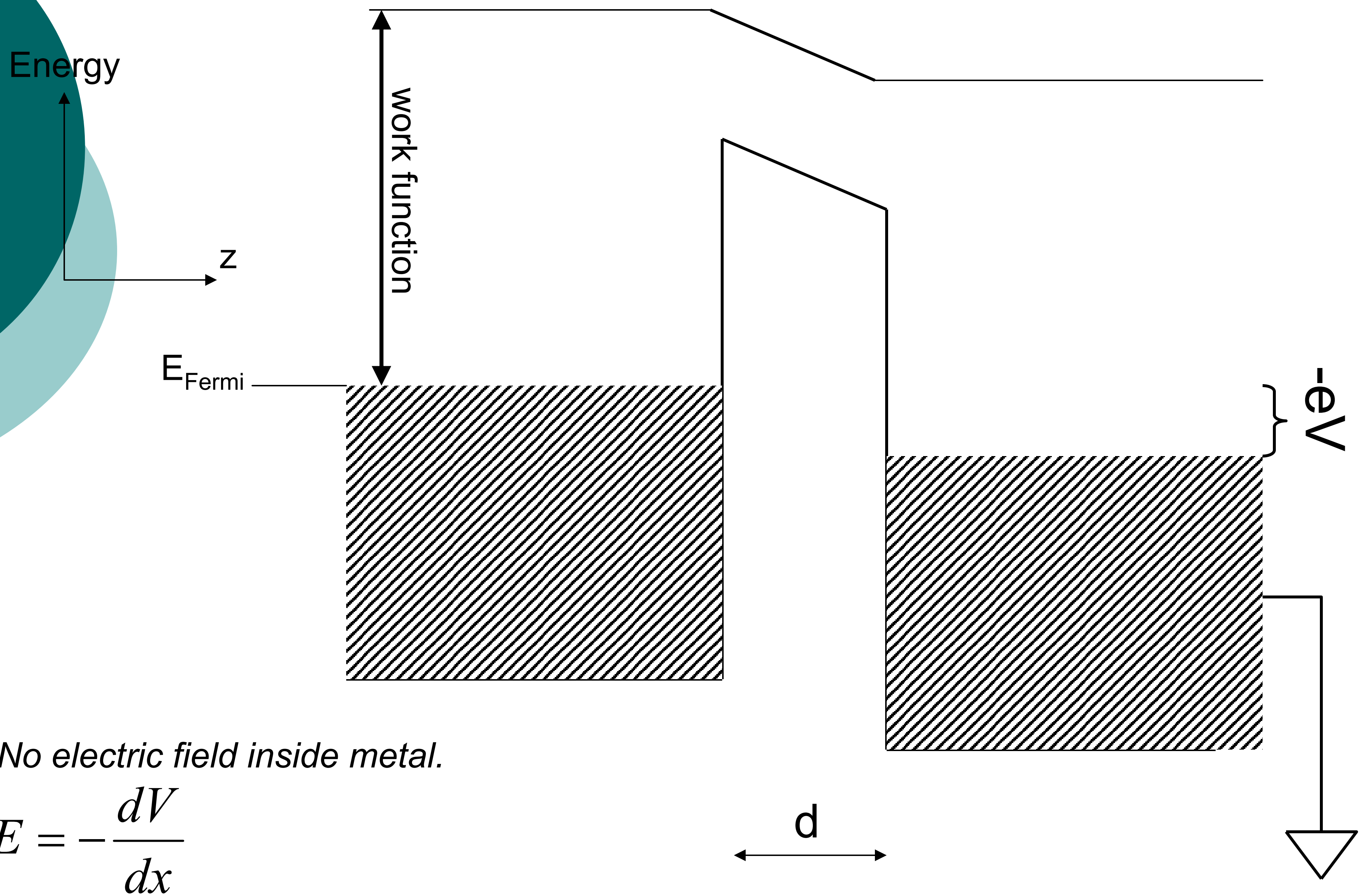


$$P \sim e^{-d}$$

Band diagram for tunnel junction



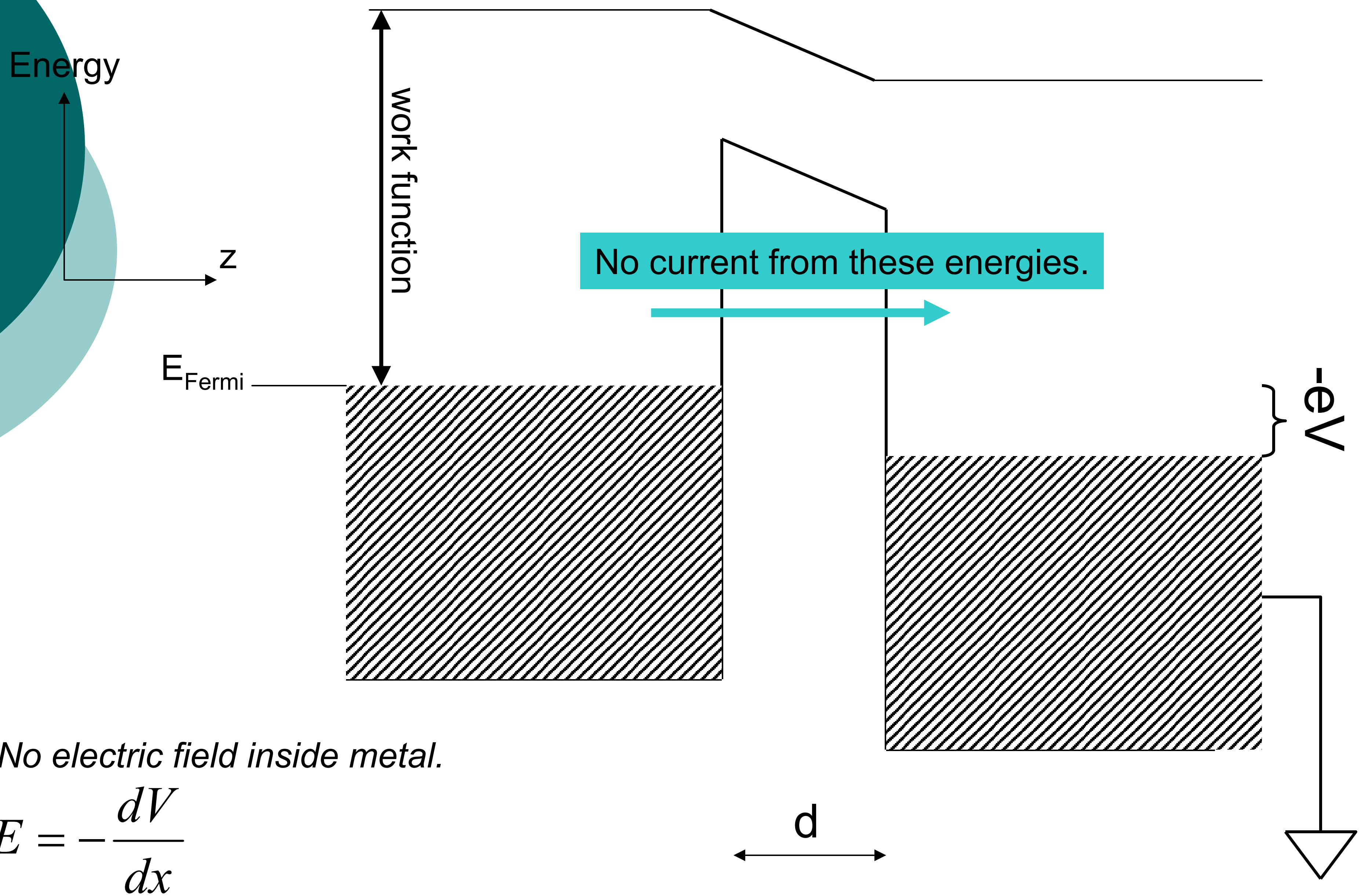
Band diagram under bias



No electric field inside metal.

$$E = -\frac{dV}{dx}$$

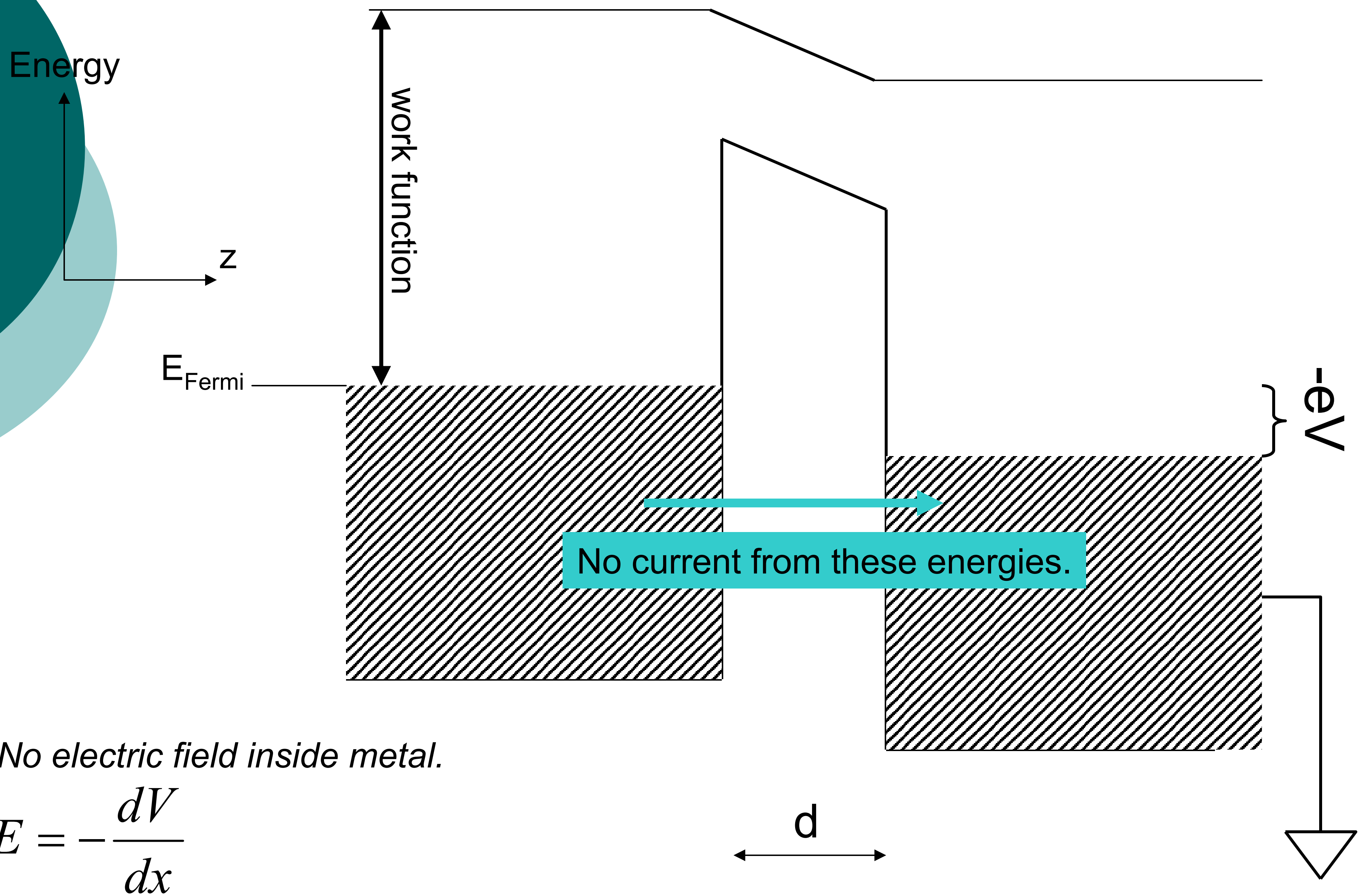
Band diagram under bias



No electric field inside metal.

$$E = -\frac{dV}{dx}$$

Band diagram under bias



No electric field inside metal.

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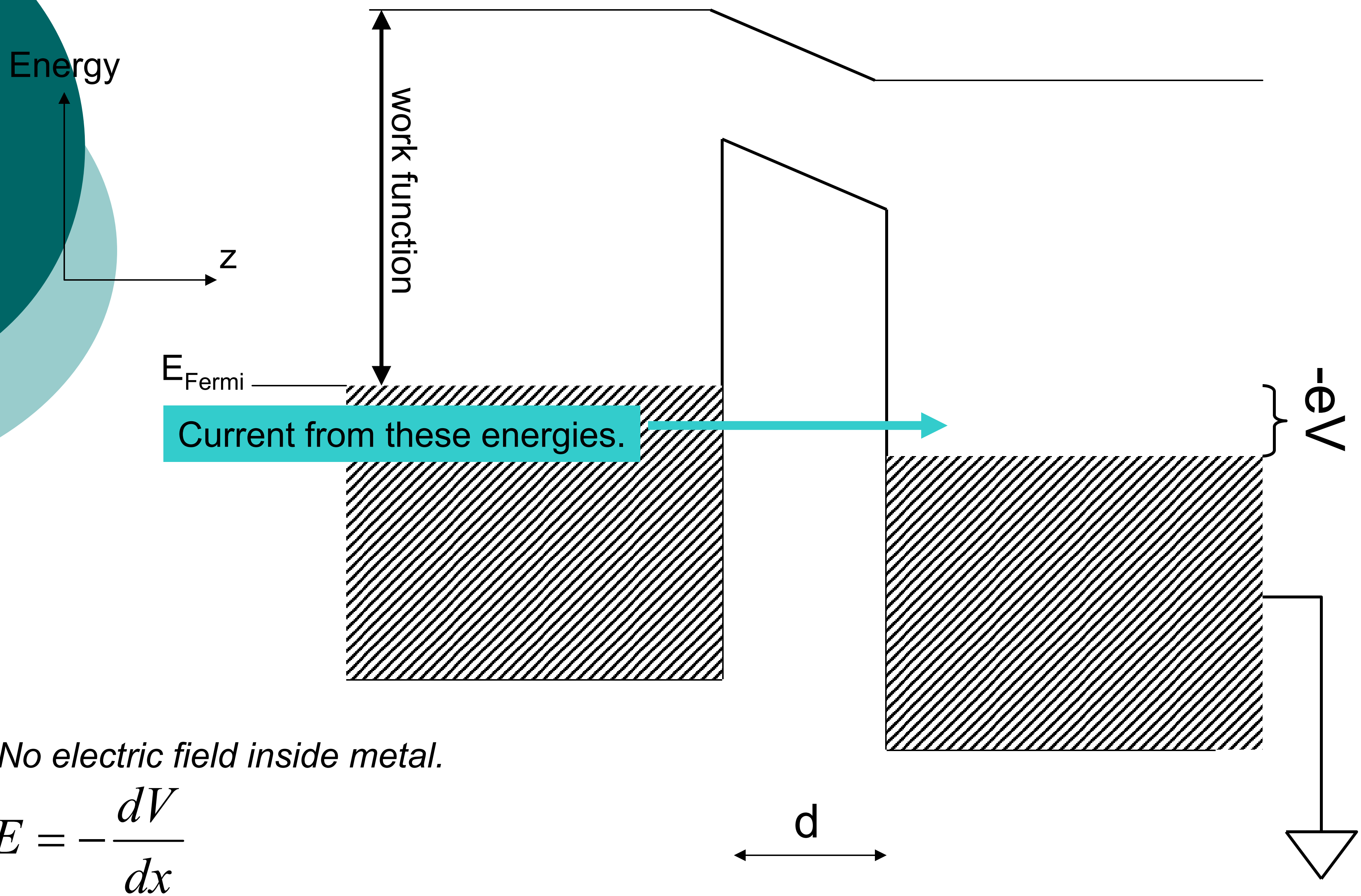
Band diagram under bias



No electric field inside metal.

$$E = -\frac{dV}{dx}$$

I-V curve

$$I = e\left(\left.\frac{\#electrons}{second}\right|_{R-L} - \left.\frac{\#electrons}{second}\right|_{L-R} \right)$$

$$\left.\frac{\#electrons}{second}\right|_{L-R} = \sum_{leftelectronstates} \sum_{rightelectronstates} \left(\Pr ob_{leftelectronstateoccupied}\right)\left(\Pr ob_{rightelectronstateempty}\right) T$$

Treat particles in left as "particle in a box"
Recall our way of labeling states, and each state has energy:

$$E = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x^{\ 2} + n_y^{\ 2} + n_z^{\ 2})$$

$$\left.\frac{\#electrons}{second}\right|_{L-R} \rightarrow \sum_{n_x, n_y, n_z} \sum_{m_x, m_y, m_z} \left(\Pr ob_{leftelectronstateoccupied}\right)\left(\Pr ob_{rightelectronstateempty}\right) T$$

$$\rightarrow \sum_{n_x, n_y, n_z} \sum_{m_x, m_y, m_z} \left(\mathrm{P}_{n_x, n_y, n_z}\right)\left(1 - \mathrm{P}_{m_x, m_y, m_z}\right) T$$

I-V curve

$$\left.\frac{\# electrons}{second}\right|_{L-R} = \sum_{n_x,n_y,n_z} \sum_{m_x,m_y,m_z} \left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{m_x,m_y,m_z}\right)T$$

Energy and momentum are conserved in physics so:

$$T = 0 \text{ unless}$$

$$n_x = m_x$$

$$n_y = m_y$$

$$E_{left} - eV = E_{right}$$

$$E_{left} - eV = E_{right}$$

$$\Rightarrow \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x^{\ 2}+n_y^{\ 2}+n_z^{\ 2}\right) - eV = \frac{\hbar^2(\pi/L)^2}{2m}\left(m_x^{\ 2}+m_y^{\ 2}+m_z^{\ 2}\right)$$

$$\Rightarrow \frac{\hbar^2(\pi/L)^2}{2m} n_z^{\ 2} - eV = \frac{\hbar^2(\pi/L)^2}{2m} m_z^{\ 2}$$

I-V curve

$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} = \sum_{n_x,n_y,n_z}\sum_{m_x,m_y,m_z}\left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{m_x,m_y,m_z}\right)T$$

$$\rightarrow \sum_{n_x,n_y,n_z}\left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{n_x,n_y,m_z}\right)T$$

$$\mathrm{P}_{n_x,n_y,n_z} = \frac{1}{1+e^{\left(\frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2+n_y{}^2+n_z{}^2\right)-E_f\right)/kT}} = f(E_L)$$

$$\mathrm{P}_{n_x,n_y,m_z} = \frac{1}{1+e^{\left(\frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2+n_y{}^2+m_z{}^2\right)-E_f\right)/kT}} = \frac{1}{1+e^{\left(\frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2+n_y{}^2+n_z{}^2\right)+eV-E_f\right)/kT}}$$

$$= \frac{1}{1+e^{\left(E_L+eV-E_f\right)/kT}} = f(E_L+eV)$$

$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} \rightarrow \sum_{n_x,n_y,n_z}\left(f(E_L)\right)\left(1-f(E_L+eV)\right)T$$

I-V curve

$$\left.\frac{\#electrons}{second}\right|_{L-R} \rightarrow \sum_{n_x,n_y,n_z} \big(f(E_L)\big)\big(1-f(E_L+eV)\big)T$$

A similar calculation shows:

$$\left.\frac{\#electrons}{second}\right|_{R-L} \rightarrow \sum_{n_x,n_y,n_z} \big(f(E_L+eV)\big)\big(1-f(E_L)\big)T$$

Since:

$$I = e\left(\left.\frac{\#electrons}{second}\right|_{R-L} - \left.\frac{\#electrons}{second}\right|_{L-R}\right)$$

We have:

$$I = e\sum_{n_x,n_y,n_z} \Big[\big(f(E_L) - f(E_L+eV)\big)\Big]T$$

A nice, simple result.

I-V curve

$$I = e \sum_{n_x, n_y, n_z} \left[\left(f(E_L) - f(E_L + eV) \right) \right] T$$

$$I = e \sum_{n_x, n_y} \sum_{n_z} \left[\left(f(E_L) - f(E_L + eV) \right) \right] T$$

In the macro world, states are very finely spaced and we have (discuss):

(Later in the class we will see that this fails in nanosized circuits.)

$$\sum_{n_x} \rightarrow \int dn_x \qquad \sum_{n_y} \rightarrow \int dn_y \qquad \sum_{n_z} \rightarrow \int dn_z$$

$$I \rightarrow e \int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV) \right) \right] T$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

$$I \to e\int dn_x \int dn_y \int \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_L - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

$$I \approx e\int dn_x \int dn_y \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_F - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} T \int dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right]$$

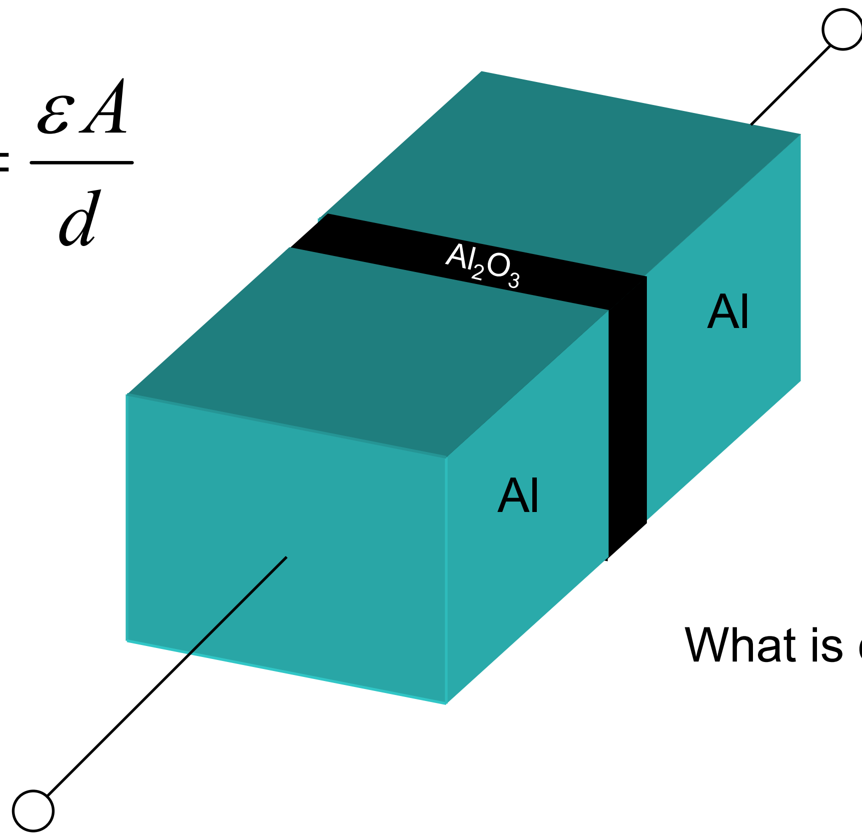
$\int dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right] \approx eV$ (show on board)

$$I \approx (eV) eT \frac{m}{\hbar^2(\pi/L)^2} \int_0^\infty dn_x \int_0^\infty dn_y \frac{1}{\sqrt{E_F - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}}$$

$$\boxed{I \approx (eV)(\text{constant})}$$

Lecture 5: Coulomb blockade

$$C = \frac{\varepsilon A}{d}$$

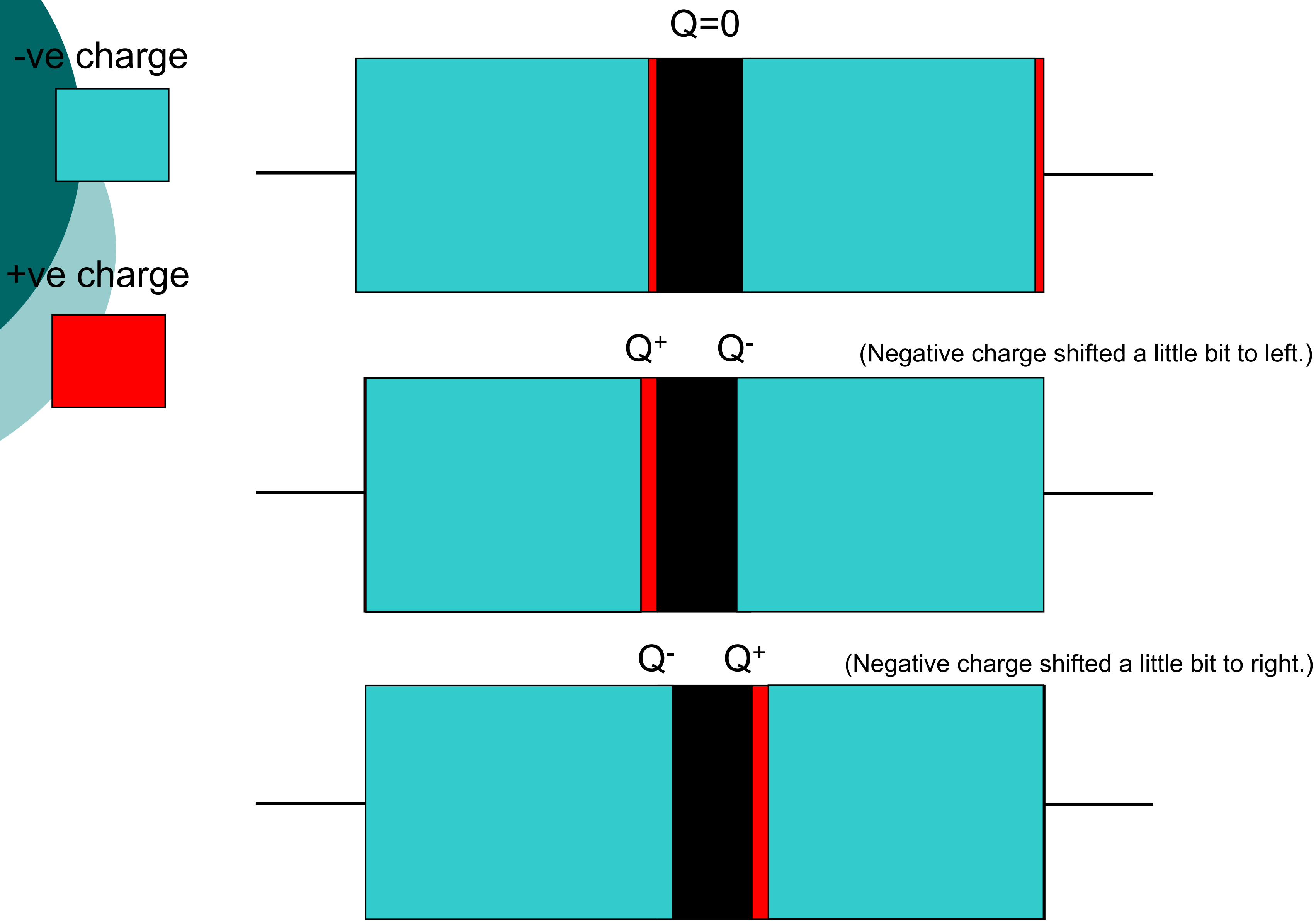


What is charge on this capacitor?

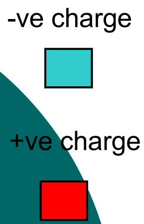
Readings this lecture covers

- Ferry pp. 226-244
- Hanson, pp. 212-244
- Cleland PRL, PRB (reading packet)
- Devoret chapter in *Single Charge Tunneling* (reading packet)
- Grabert chapter (reading packet)
- These chapters are covered all the way to (and including) lecture 8

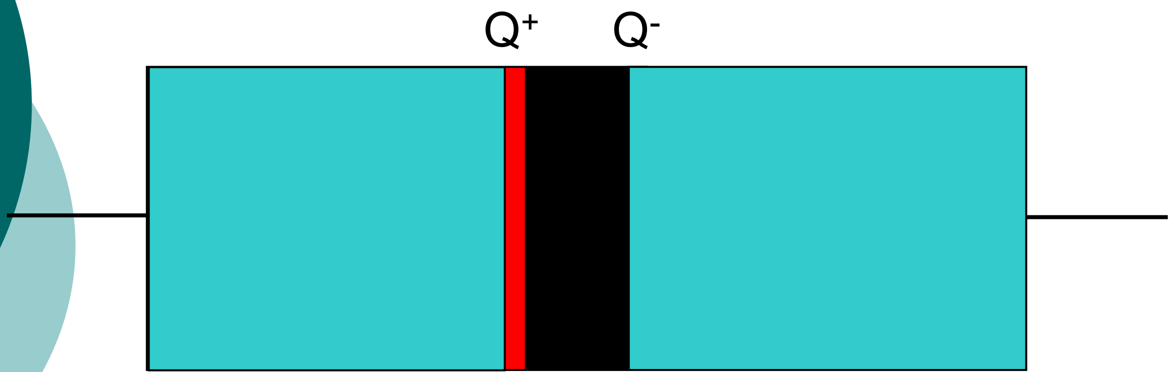
Charge on capacitor continuous



Is tunneling allowed?



(Negative charge shifted a little bit to left.)



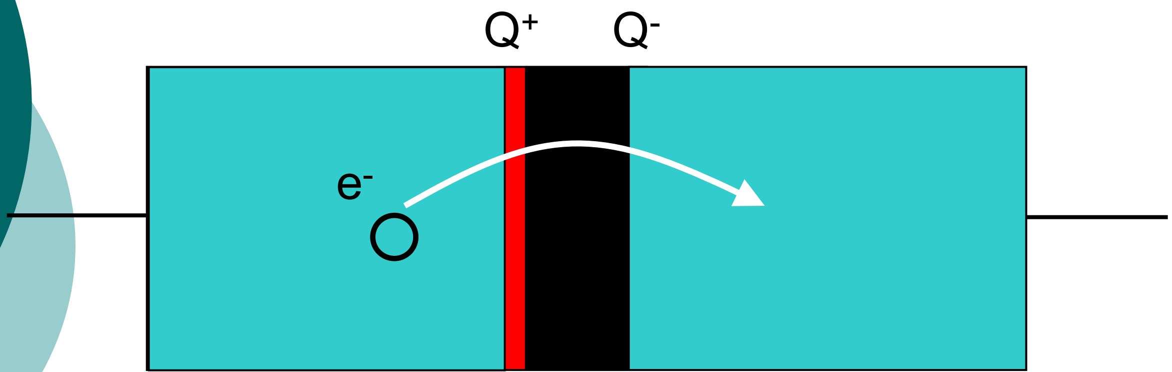
$$E = \frac{Q^2}{2C}$$

Is tunneling allowed?

-ve charge

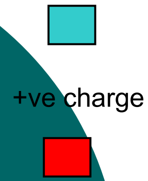
+ve charge

(Negative charge shifted a little bit to left.)

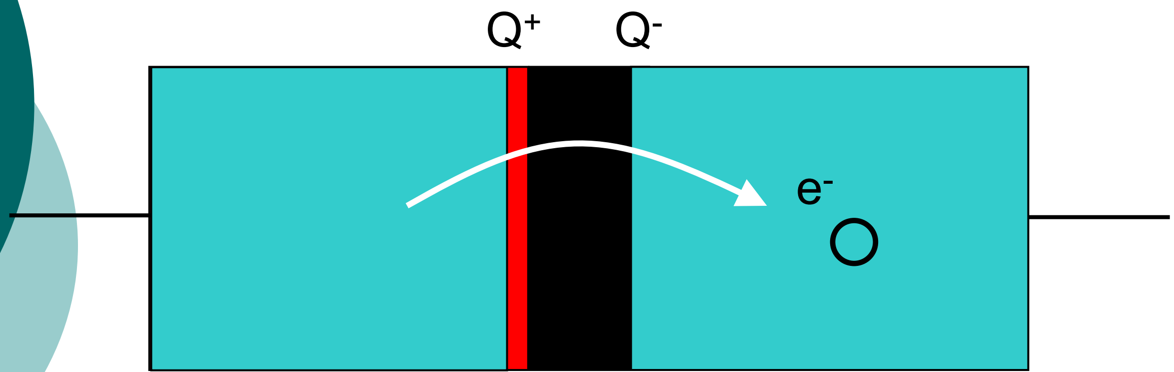


$$E = \frac{Q^2}{2C}$$

Is tunneling allowed?

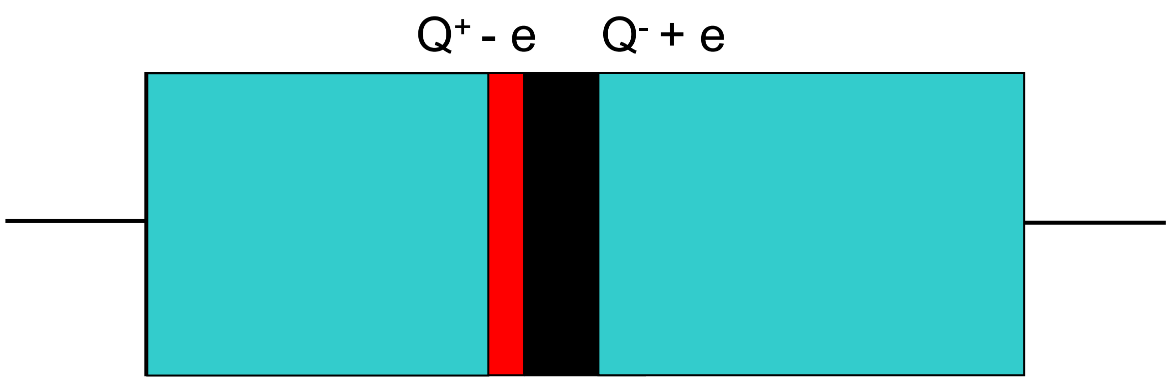


(Negative charge shifted a little bit to left.)



$$E = \frac{Q^2}{2C}$$

After electron tunnels:



$$E = \frac{(Q-e)^2}{2C}$$

$$\Delta E = \frac{e(Q - e/2)}{C}$$

-ve charge

+ve charge

Coulomb gap

$$\Delta E = \frac{e(Q - e/2)}{C} > 0$$

$$\Rightarrow Q - e/2 > 0$$

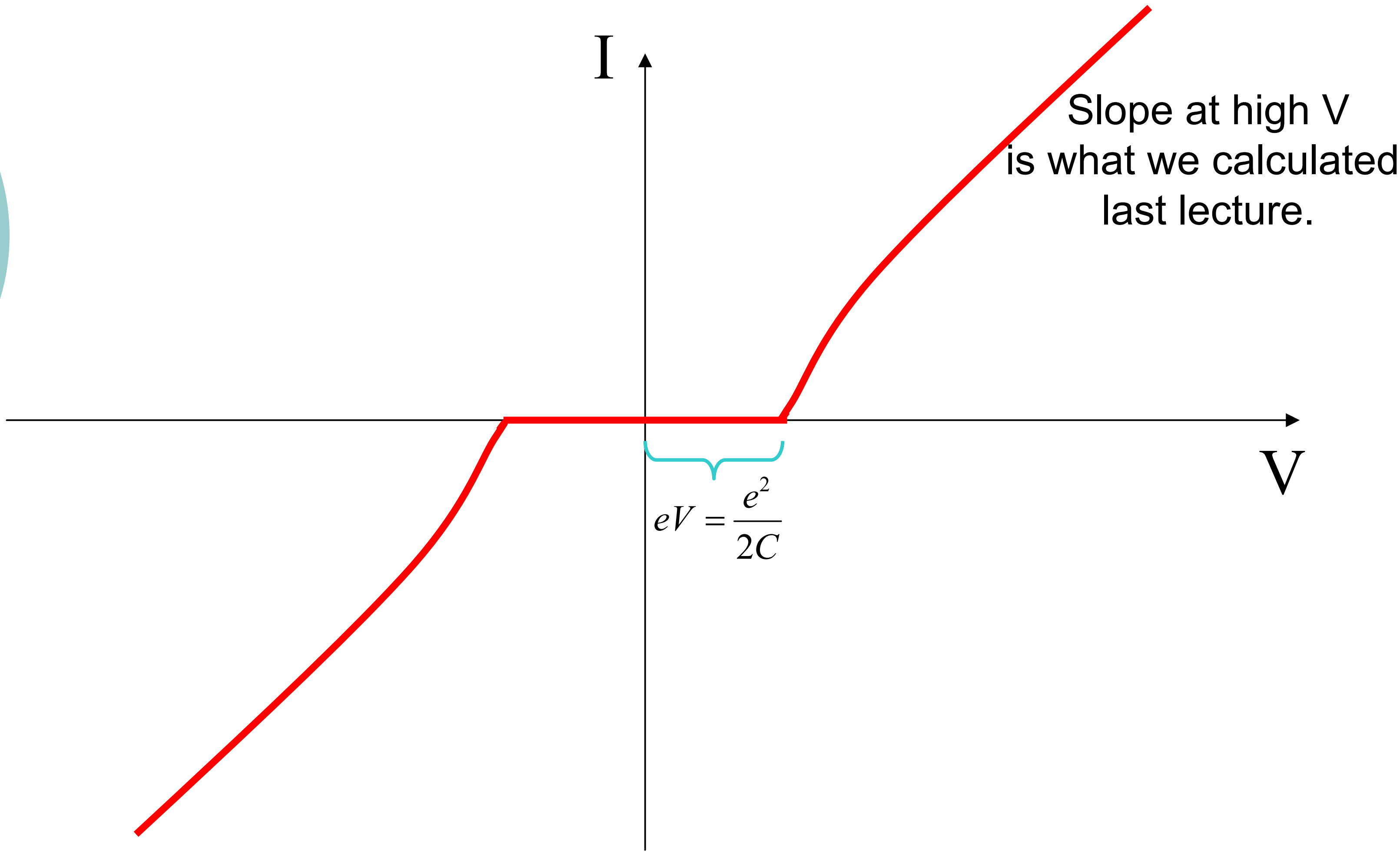
$$\Rightarrow Q > e/2$$

$$\Rightarrow C|V| > e/2 \Rightarrow |V| > \frac{e}{2C}$$

$$\boxed{-\frac{e}{2C} < V < \frac{e}{2C}}$$

Tunneling only under these conditions, *otherwise no tunneling!*

I-V curve


I
V
$eV = \frac{e^2}{2C}$
Slope at high V
is what we calculated
last lecture.

Temperature

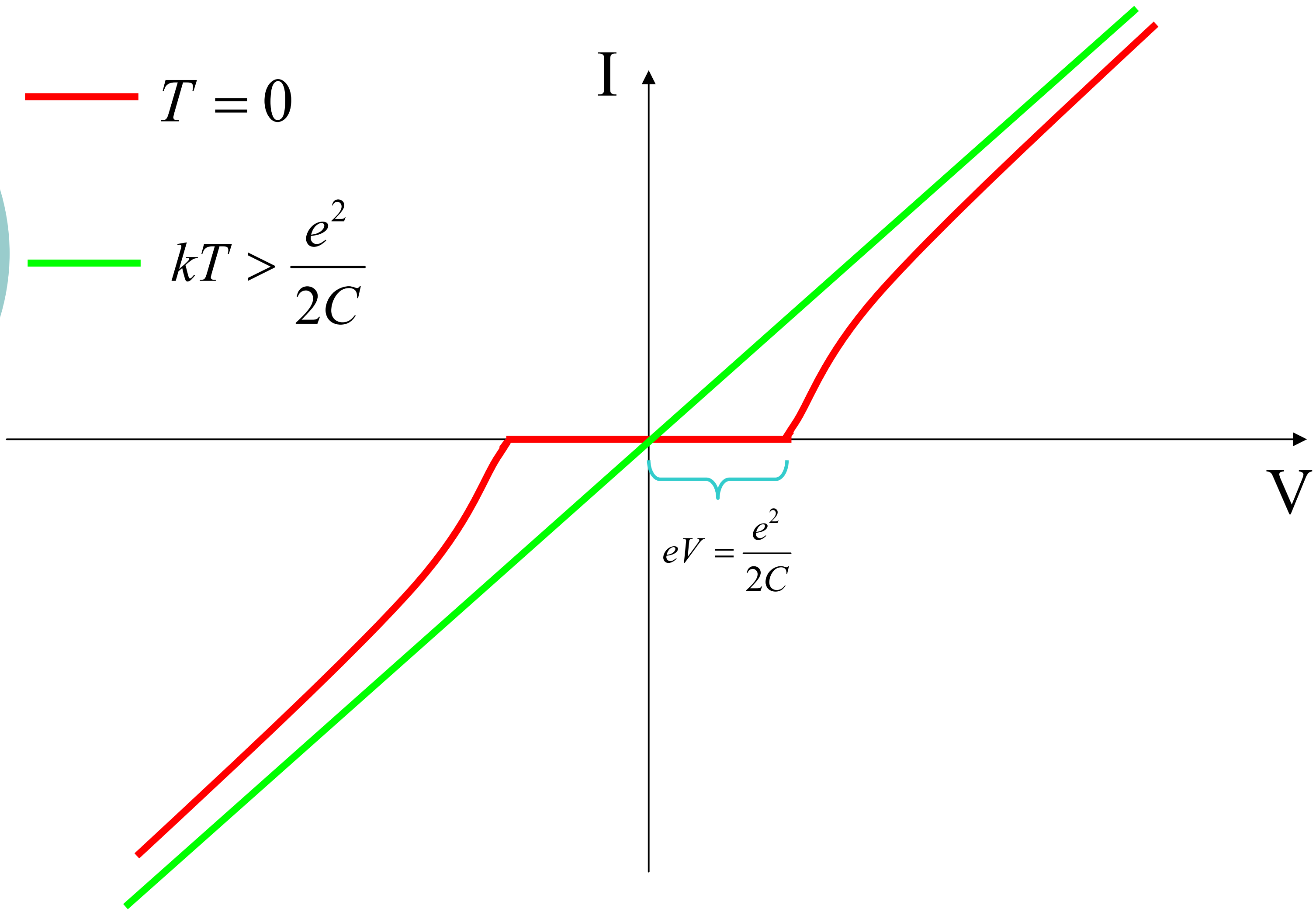
$$\Delta E = \frac{e(Q - e/2)}{C}$$

can be less than 0 if thermal energy available

Criteria to observe coulomb gap behavior:

$$\frac{e^2}{C} > kT$$

I-V curve vs. temperature



Numbers

Class demo:
1 nm barrier, 1 mm x 1 mm junction:

$$C = \frac{\varepsilon A}{d} = \frac{10 \cdot 8.85 \cdot 10^{-12} F/m (10^{-3} m)^2}{10^{-9} m} \approx 10^{-7} F$$

$$\frac{e^2}{C} > kT \Rightarrow T < \frac{e^2}{Ck} = \frac{(1.6 \cdot 10^{-19} coulomb)^2}{10^{-7} F \cdot 1.38 \cdot 10^{-23} J/K} \approx 10^{-8} K$$

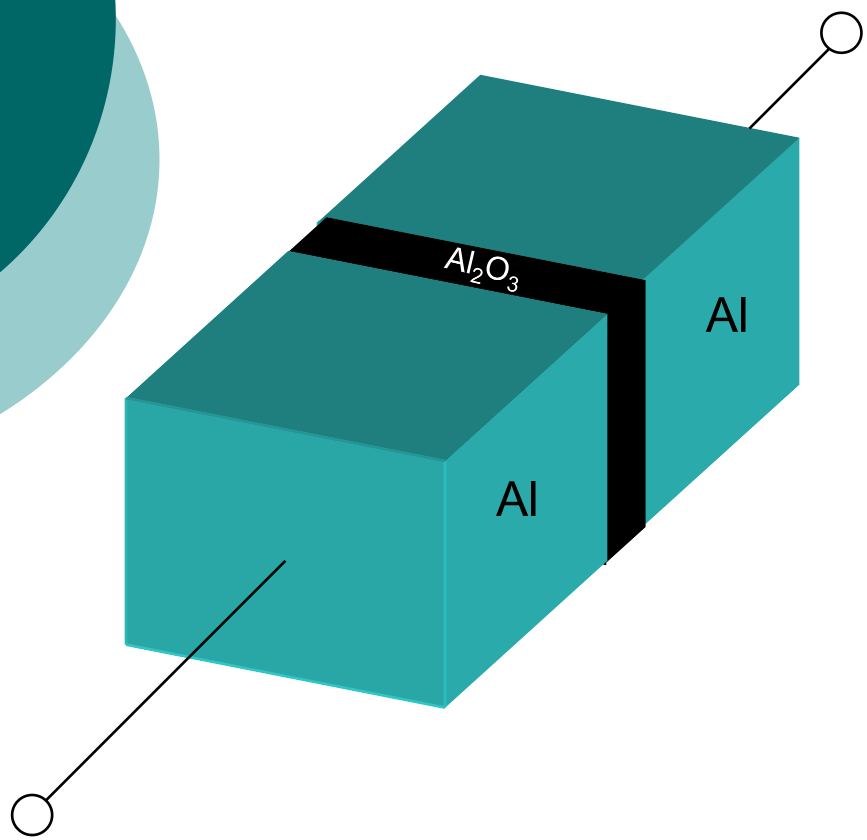
Practically impossible.

Best lithographic junction:
1 nm barrier, 100 nm x 100 nm junction:

$$C \approx 10^{-15} F \Rightarrow T < 1K$$

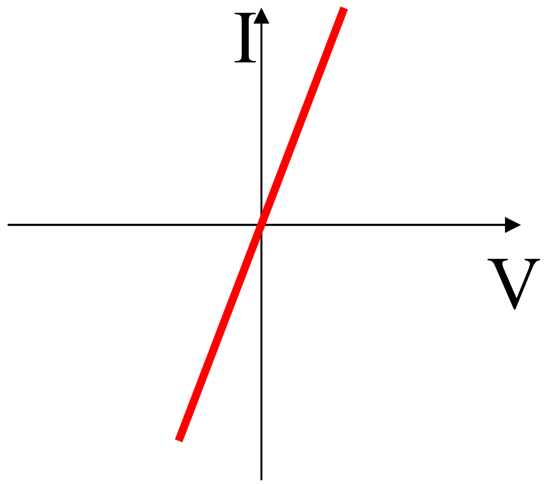
Possible to achieve in the lab.

Quantum mechanics



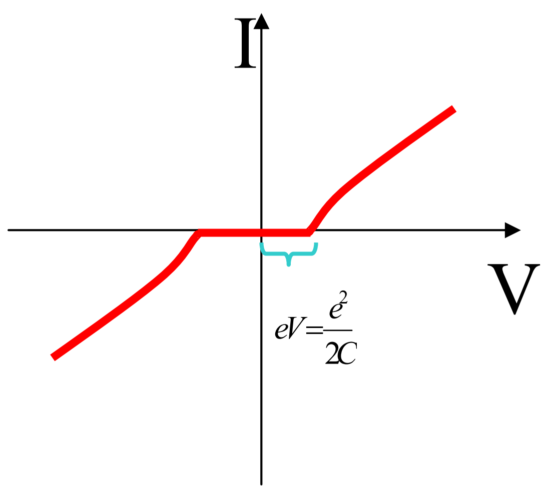
- For strong tunneling, electron can have a large probability to be on both sides at the same time.
- This means the system energy cannot be defined by localizing the electron on only one side.
- This makes coulomb blockade irrelevant.

I-V curve vs. tunnel strength



Strong tunneling

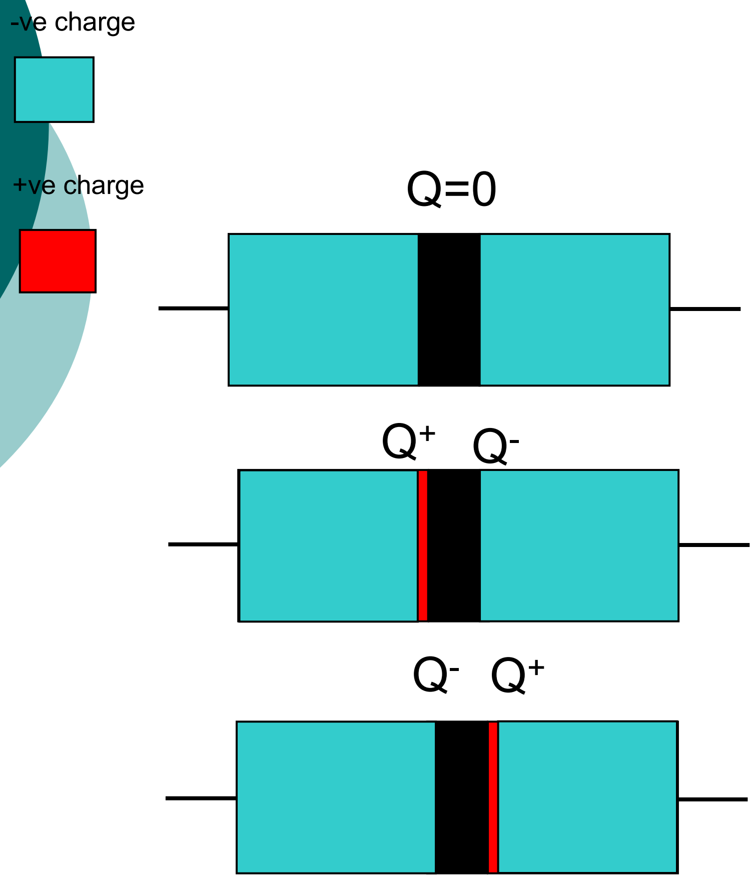
$$R_T < R_K \equiv \frac{h}{e^2} = 25\ \mathrm{k}\Omega$$



Weak tunneling

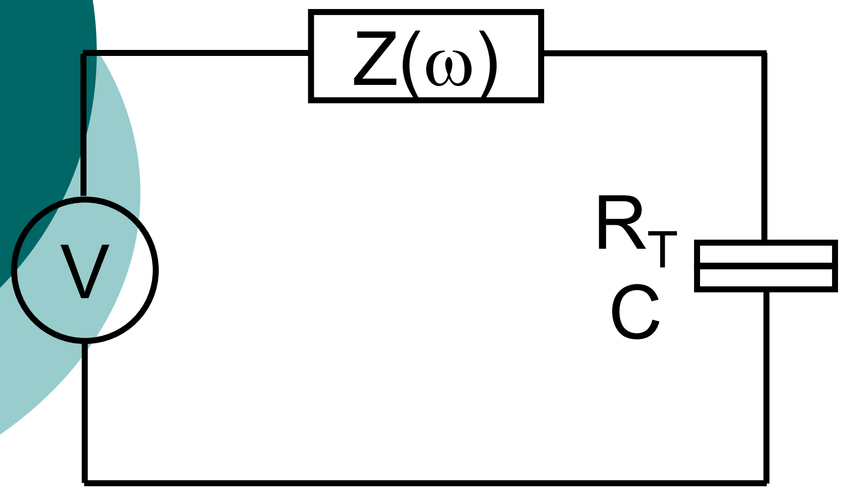
$$R_T > \frac{h}{e^2} = 25\ \mathrm{k}\Omega$$

Charge on capacitor is a *quantum variable*



- We don't always know what Q is.
- Treating Q as a quantum variable, there is a certain probability for the system to have a certain value of Q.
- Should describe a "wave function" for Q: $\Psi(Q)$ just like wave function for position $\Psi(x)$
- Now, we need quantum theory of electric circuits.

Quantum theory of electric circuits



- At DC, can have current bias or voltage bias depending on Z(dc) vs. R_T.
- At AC, almost always have $Z(\omega) < R_T$ because of lead capacitance (typically pF).

Full quantum treatment beyond the scope of this class.

In order to see Coulomb blockage,
need current bias all the way up to $1/(R_K C)$ which is typically 10 GHz, i.e.:

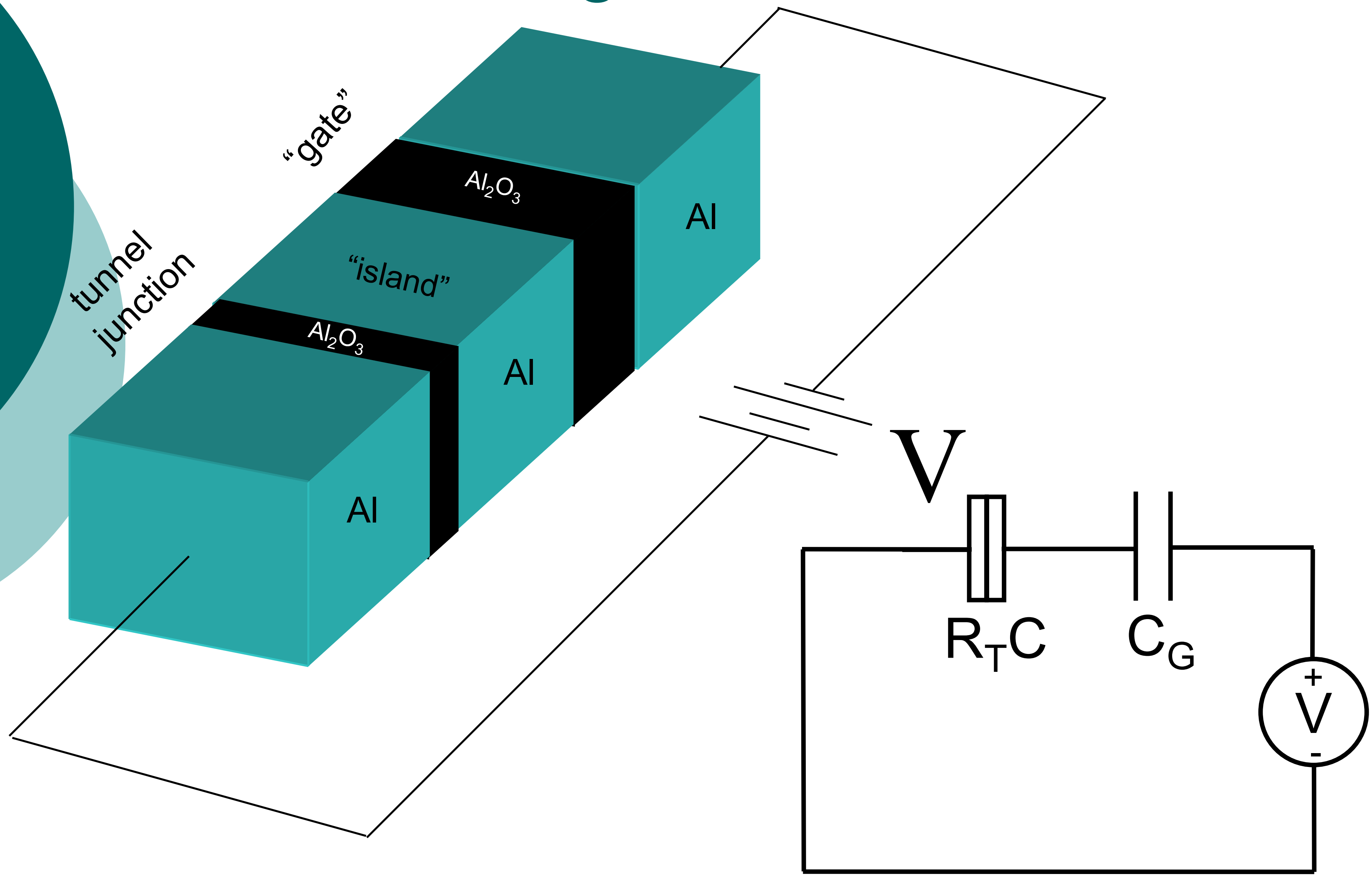
$$Z(\omega) > R_T \text{ for all } \omega \leq \frac{1}{R_K C} \sim 10 \text{ GHz}$$

Requirements for Coulomb blockade

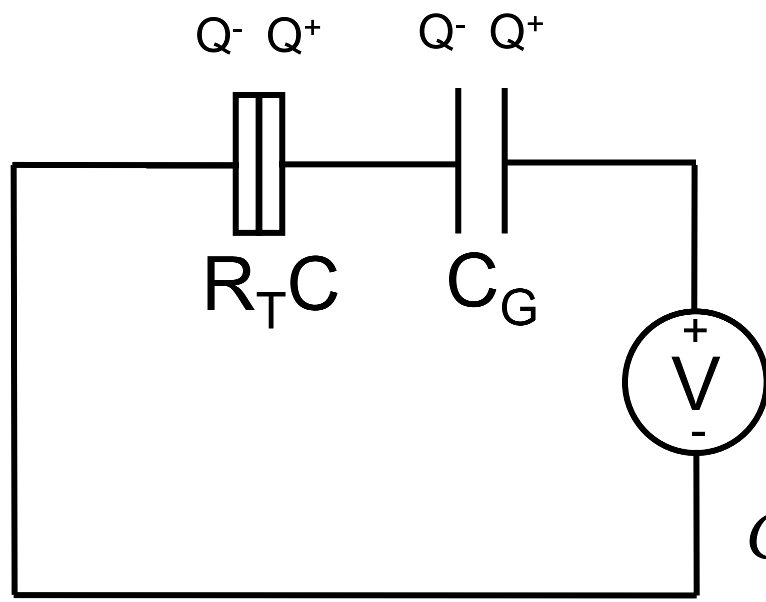
- $kT < e^2/C$ (hard)
- $R_T > R_K$ (25 kΩ) (harder)
- $Z(\omega) > R_T$ at all frequencies up to $1/R_KC$ (hardest)

Achieved by Cleland PhD thesis, Berkeley 1992.
(Congratulations, Andrew.)

Lecture 6: Single electron box



Electrostatic energy (*no tunneling*)

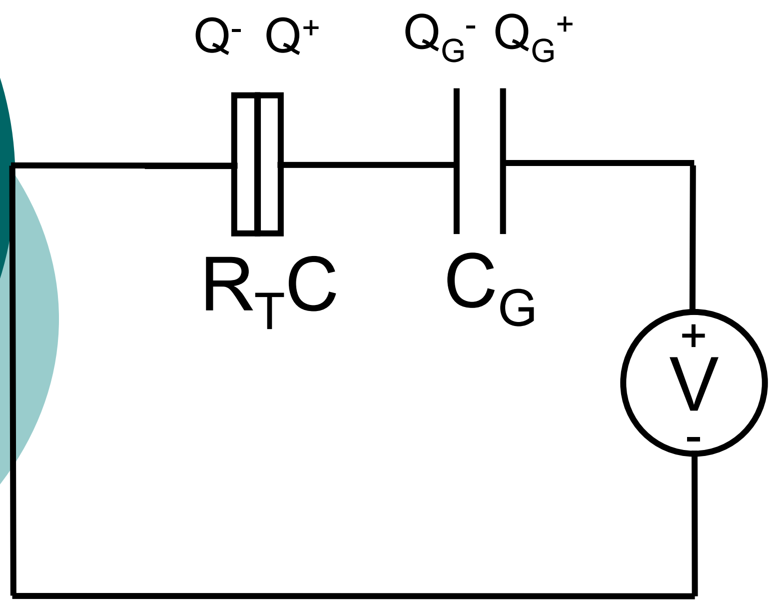


$$C_{series}{}^{-1} \equiv C^{-1} + C_G{}^{-1}$$

$$V = \frac{Q}{C} + \frac{Q}{C_G} = Q\left(\frac{1}{C} + \frac{1}{C_G}\right) = \frac{Q}{C_{series}}$$

$$E = \frac{Q^2}{2C} + \frac{Q^2}{2C_G} = \frac{Q^2}{2C_{series}} = \frac{1}{2}C_{series}V^2$$

Island charge



"Island charge":

$$Q_i = Q - Q_G$$

Kirchoff:

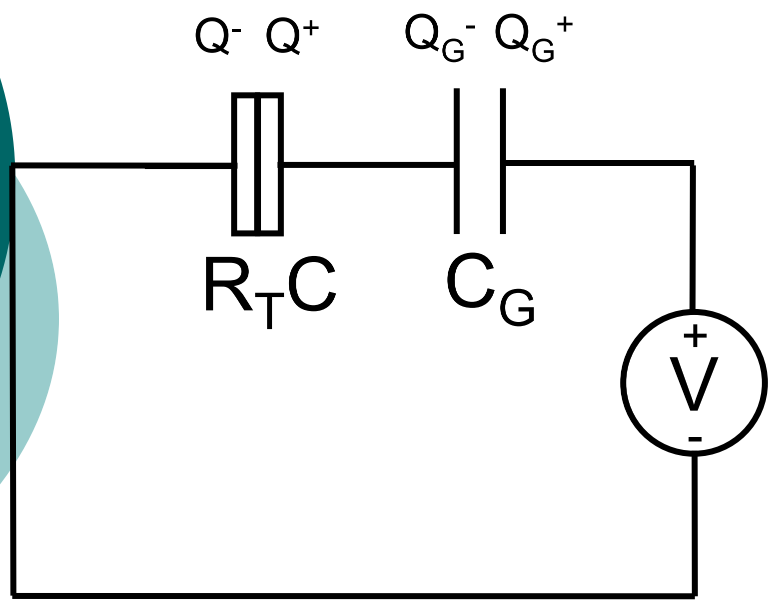
$$V = \frac{Q}{C} + \frac{Q_G}{C_G}$$

Solve for Q, Q_G:

$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

Electrostatic energy (w/tunneling)



$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$E = \frac{Q^2}{2C} + \frac{{Q_G}^2}{2C_G} = \frac{CC_G V^2 + {Q_i}^2}{2(C + C_G)}$$

Thermodynamics

Entropy:

$$S = S(E, V, N, \ldots)$$

Energy:

$$E = E(S, V, N, \ldots)$$

Entropy maximum ⇔ Energy minimum

Thermodynamic variables

Energy:

$$E = E(S, V, N, \ldots)$$

Temperature:

$$\frac{1}{T} \equiv \left.\frac{\partial E}{\partial S}\right|_{V,N,\ldots}$$

Pressure:

$$P \equiv -\left.\frac{\partial E}{\partial V}\right|_{E,N,\ldots}$$

Thermodynamic potentials

Energy:

$$E = E(S, V, N, \ldots)$$

Helmholtz potential (Helmholtz free energy):

$$F = E - TS$$

Minimized in presence of "reservoir" with temperature T.

Enthalpy:

$$H = E + PV$$

Minimized in presence of "reservoir" with pressure P.

Gibbs free energy:

$$G = E - TS + PV$$

Minimized in presence of "reservoir" with pressure P, temperature T.

Thermodynamic potentials for circuits

Energy:

$$E = E(S, V, N, Q, \ldots)$$

Gibbs free energy for electronic circuits:

$$G = E - Q_G V$$

Minimized in presence of "reservoir" with voltage V.

electrostatic energy (E)

need to calculate (Q_G)

Q= how much charge has passed through the battery onto the gate
V= voltage of the battery

Free energy of single electron box:

$$G = E - Q_G V$$

From before:

$$E = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)} \qquad Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$G = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)} - \frac{C_G(CV - Q_i)}{C + C_G} V$$

$$= \frac{1}{2}\frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

(Note: The last minus sign agrees with Lafarge thesis, but not Ferry textbook.)

Charge of island

From last slide:

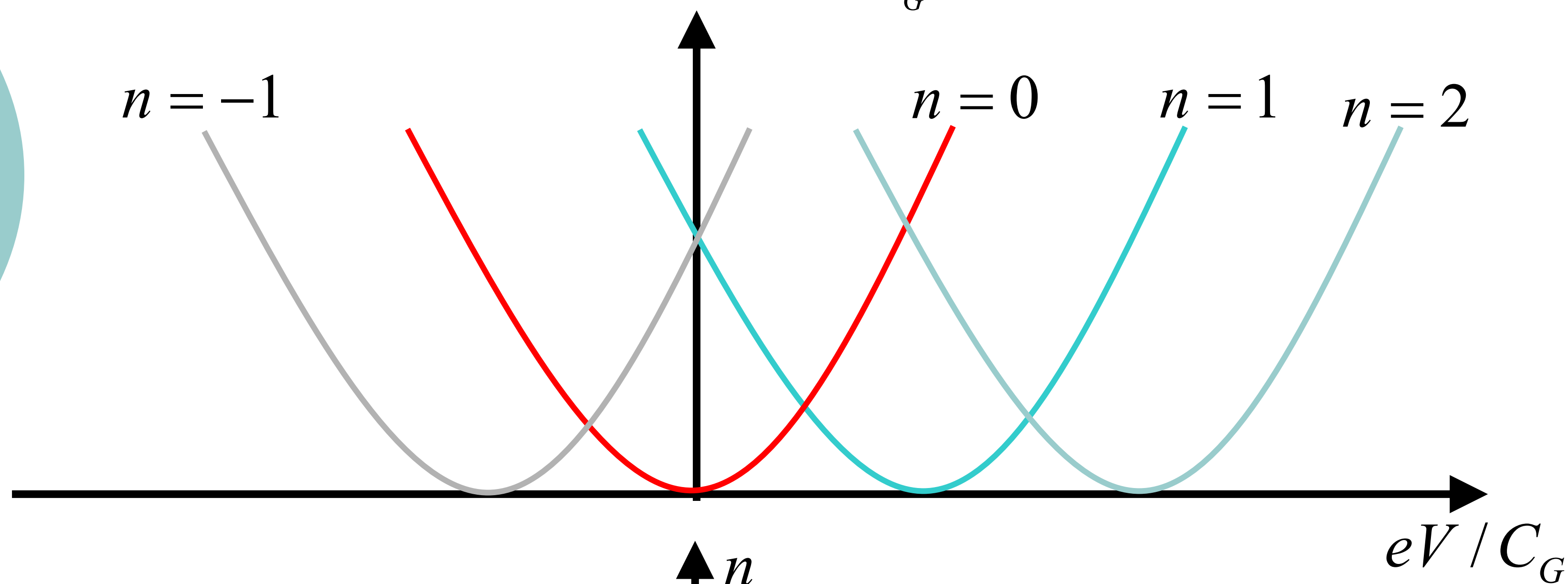
$$G = \frac{1}{2}\frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

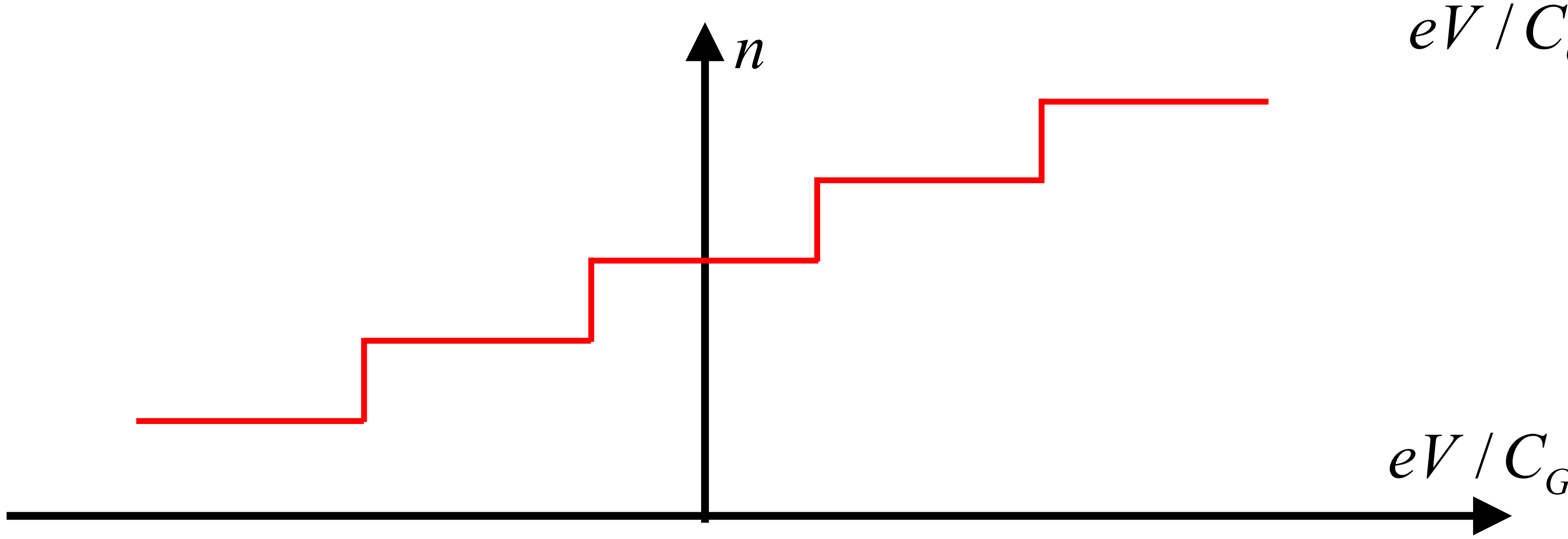
$$Q_i = -ne$$ *only if* $R_T >> R_K$

$$G = \frac{1}{2}\frac{(C_G V - ne)^2}{C + C_G} + const$$

Charge of island

$$G = \frac{1}{2}\frac{(C_G V - ne)^2}{C + C_G}$$





Finite temperatures

Need
$kT << e^2/(C+C_G)$

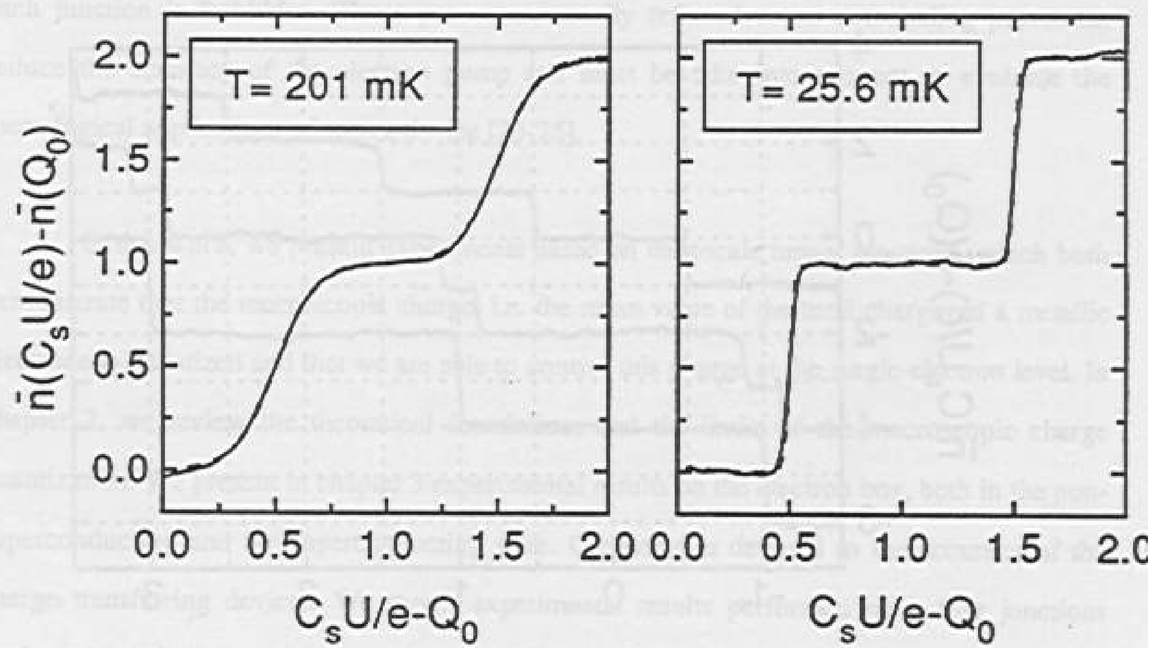


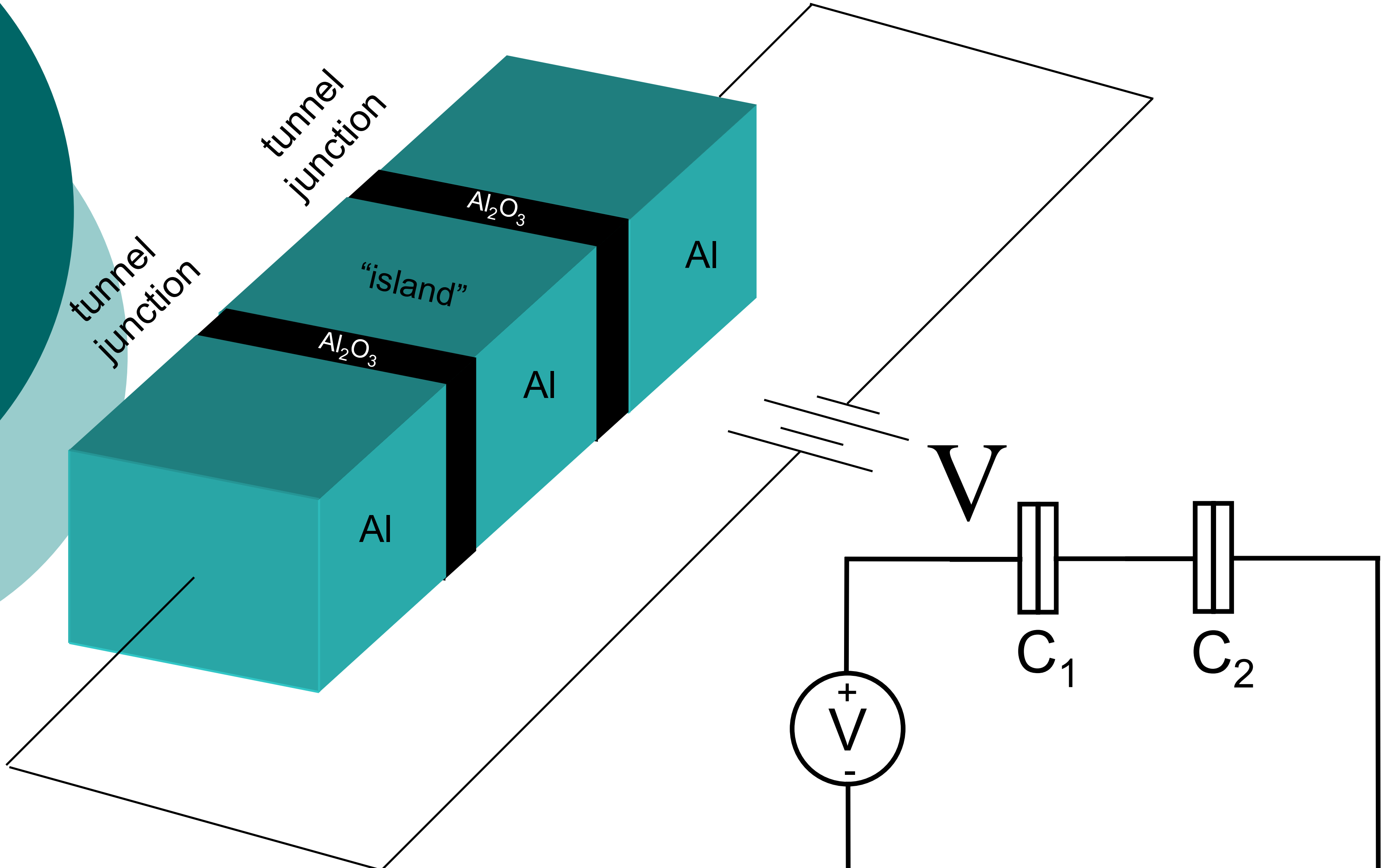
Fig. 1.5 b) *Solid lines: experimental variations of the average number $\bar{n}$ of excess electrons in in the island of an electron box. Dashed lines: theoretical calculations for an island capacitance $C_\Sigma = 0.8$ fF. The experimental parameters of the circuit are $C_s = 74$ aF and $C_c = 21$ aF. The quantity Q_0 denotes the random offset charge in the island.*

From Lafarge, PhD thesis, Universite Paris 6 (1993)

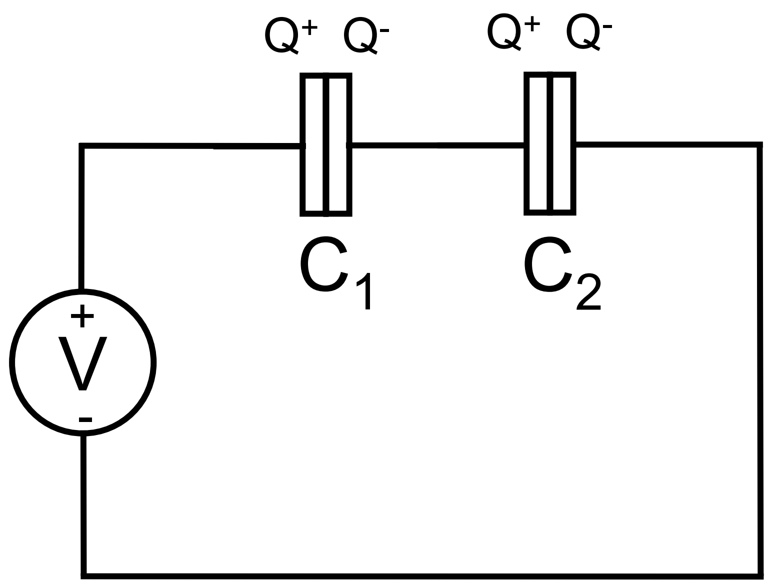
Quantum computing

- A single electron box has been proposed as a qu-bit
- $|0\rangle$ or $|1\rangle$ correspond to n or n+1 electrons
- Difficulty is fast (GHz) readout before decoherence sets in
- A superconducting box (for Cooper pairs) could have longer decoherence

Lecture 7: Double tunnel junction



Electrostatic energy (*no tunneling*)

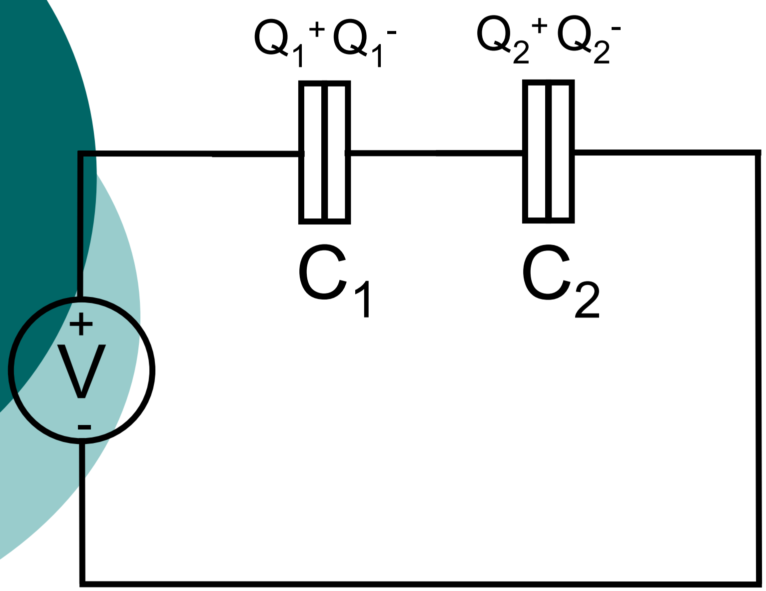


$$C_{series}^{-1} \equiv C^{-1} + C_G^{-1}$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{series}}$$

$$E = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} = \frac{Q^2}{2C_{series}} = \frac{1}{2} C_{series} V^2$$

Island charge



"Island charge":

$$Q_i = Q_2 - Q_1$$

Kirchoff:

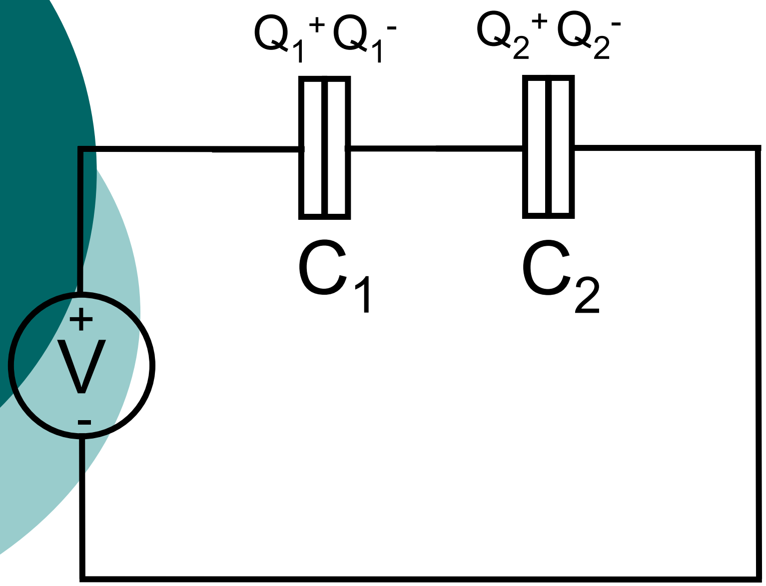
$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Solve for Q, Q_G:

$$Q_1 = \frac{C_1(C_2 V - Q_i)}{C_1 + C_2}$$

$$Q_2 = \frac{C_2(C_1 V + Q_i)}{C_1 + C_2}$$

Electrostatic energy (*with tunneling*)



$$Q_1 = \frac{C_1(C_2V - Q_i)}{C_1 + C_2}$$

$$Q_2 = \frac{C_2(C_1V + Q_i)}{C_1 + C_2}$$

$$E = \frac{Q_1^{\ 2}}{2C_1} + \frac{Q_2^{\ 2}}{2C_2} = \frac{C_1C_2V^2 + Q_i^{\ 2}}{2(C_1 + C_2)}$$

Free energy :

$$G = E - Q_1 V$$

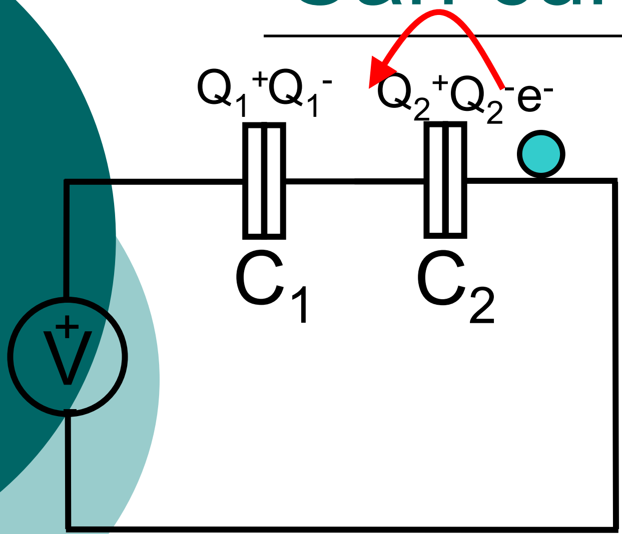
From before:

$$E = \frac{C_1 C_2 V^2 + Q_i^2}{2(C_1 + C_2)} \qquad Q_1 = \frac{C_1(C_2 V - Q_i)}{C_1 + C_2}$$

$$G = \frac{C_1 C_2 V^2 + Q_i^2}{2(C_1 + C_2)} - \frac{C_1(C_2 V - Q_i)}{C_1 + C_2} V$$

$$= \frac{1}{2}\frac{(C_1 V + Q_i)^2}{C_1 + C_2} - \frac{1}{2} C_1 V^2$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

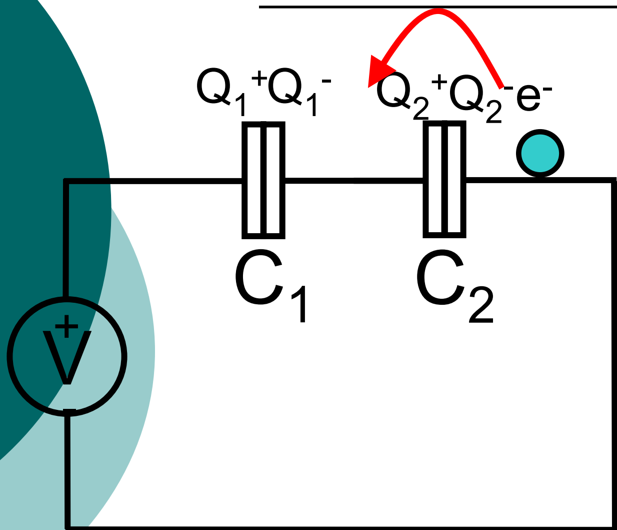
After:

$$Q_i = -n_0 e - e$$

$$\Delta E = \frac{C_1 C_2 V^2 + (-n_0 e)^2}{2(C_1 + C_2)} - \frac{C_1 C_2 V^2 + (-n_0 e - e)^2}{2(C_1 + C_2)} =$$

$$= \frac{(-n_0 e)^2}{2(C_1 + C_2)} - \frac{(-n_0 e - e)^2}{2(C_1 + C_2)} = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)}$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

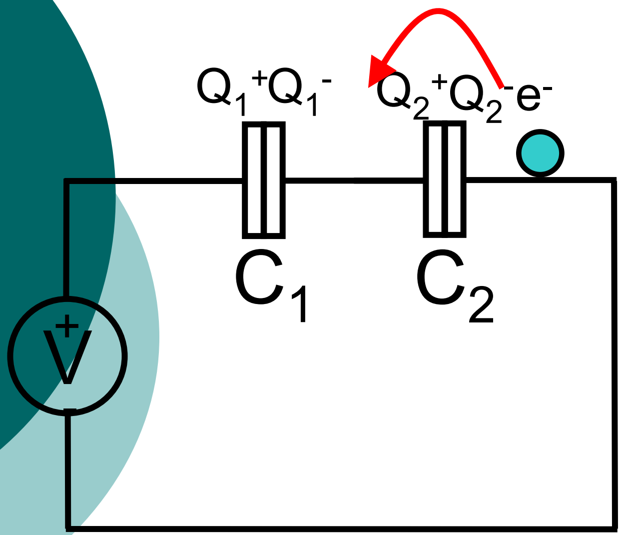
$$Q_1 = \frac{C_1(C_2V - (-n_0e))}{C_1 + C_2}$$

After:

$$Q_i = -n_0 e - e \quad Q_1 = \frac{C_1(C_2V - (-n_0e - e))}{C_1 + C_2}$$

$$\Delta Q_1 = -\frac{C_1 e}{C_1 + C_2}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1$$

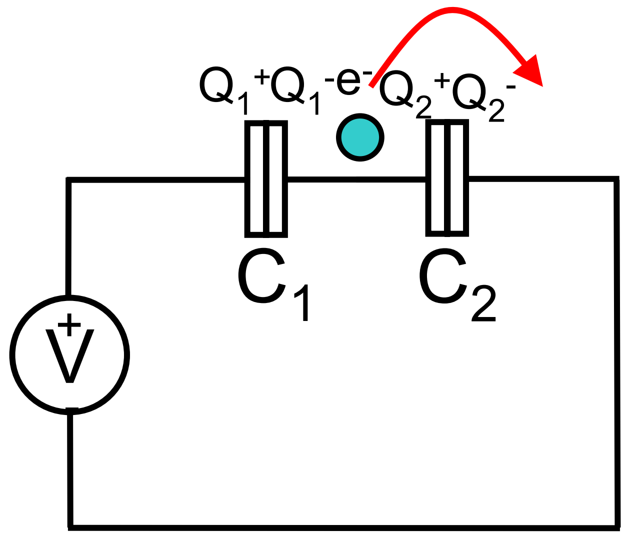
$$\Delta E = \frac{-2n_0e^2 - e^2}{2(C_1 + C_2)}$$

$$\Delta Q_1 = -\frac{C_1 e}{C_1 + C_2}$$

$$\Delta G = \frac{-2n_0e^2 - e^2}{2(C_1 + C_2)} + V\frac{C_1 e}{C_1 + C_2} = \frac{e}{C_1 + C_2}\left[-n_0 e - \frac{e}{2} + C_1 V\right] > 0$$

$$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$$

Similarly:

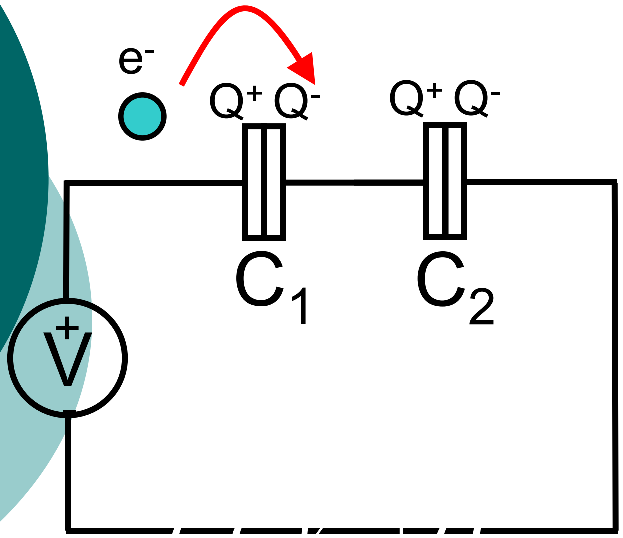


Allowed only if:

$$V < \frac{e}{C_1}\left(n_0 - \frac{1}{2}\right)$$

n_0 is the number of electrons on the island *before* the tunnel event.

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1$$

Before:

$$Q_i = -n_0 e$$

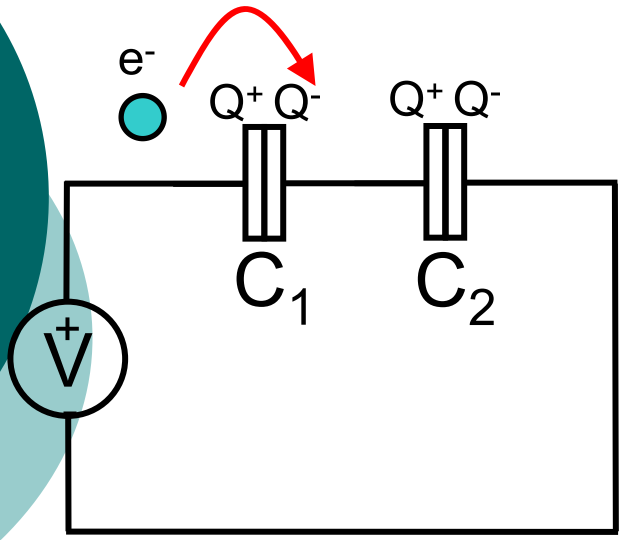
After:

$$Q_i = -n_0 e - e$$

$$\Delta E = \frac{C_1 C_2 V^2 + (-n_0 e)^2}{2(C_1 + C_2)} - \frac{C_1 C_2 V^2 + (-n_0 e - e)^2}{2(C_1 + C_2)} =$$

$$= \frac{(-n_0 e)^2}{2(C_1 + C_2)} - \frac{(-n_0 e - e)^2}{2(C_1 + C_2)} = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)}$$

Can current flow?



$$\Delta G = \Delta E - V \Delta Q_1$$

Before:

$$Q_i = -n_0 e \qquad Q_1 = \frac{C_1(C_2 V - (-n_0 e))}{C_1 + C_2}$$

After:

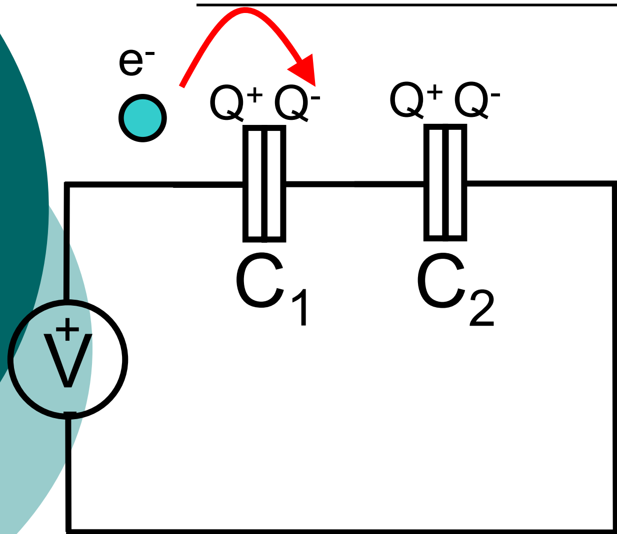
$$Q_i = -n_0 e - e \qquad Q_1 = \frac{C_1(C_2 V - (-n_0 e - e))}{C_1 + C_2}$$

$$\Delta Q_{1,polarization} = -\frac{C_1 e}{C_1 + C_2}$$

"But " $\Delta Q_{1,tunnel} = e$

$$\Delta Q_{1,total} = e - \frac{C_1 e}{C_1 + C_2} = \frac{C_1 + C_2}{C_1 + C_2} e - \frac{C_1 e}{C_1 + C_2} = \frac{C_2 e}{C_1 + C_2}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1$$

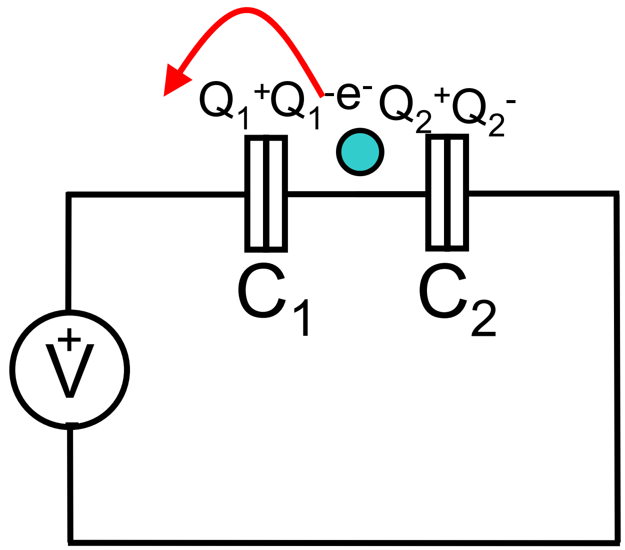
$$\Delta Q_{1,total} = \frac{C_2 e}{C_1 + C_2}$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2(C_1 + C_2)} + V\frac{C_2 e}{C_1 + C_2} = \frac{e}{C_1 + C_2}\left[-n_0 e - \frac{e}{2} - C_2 V\right] > 0$$

$$V < -\frac{e}{C_2}\left(n_0 + \frac{1}{2}\right)$$

Similarly:

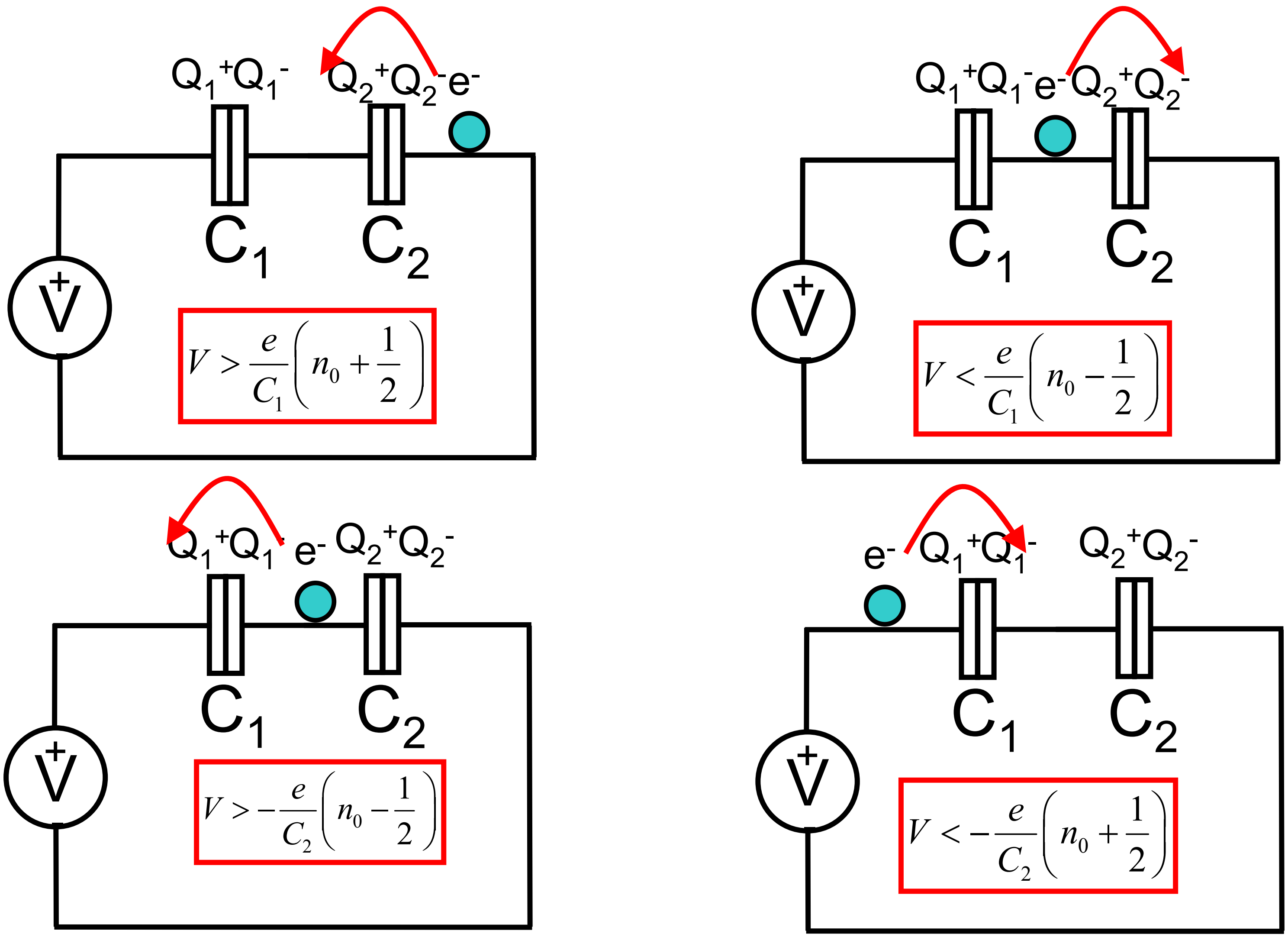


Allowed only if:

$$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$$

n_0 is the number of electrons on the island *before* the tunnel event.

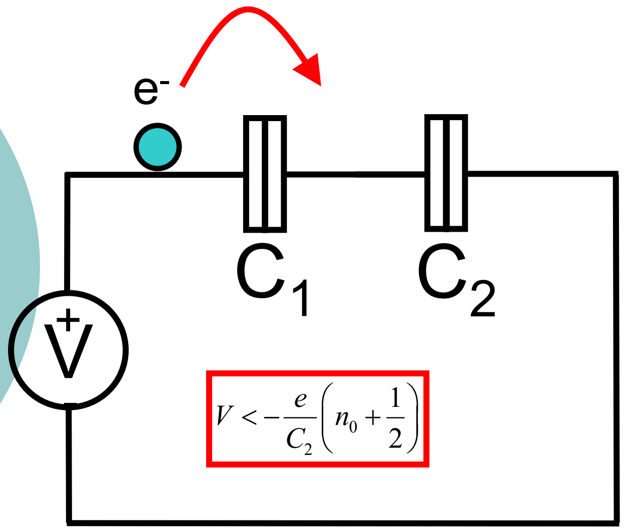
Summary



$Q_1^+ Q_1^-$ $Q_2^+ Q_2^-$ e^-
C_1 C_2
V
$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$
$Q_1^+ Q_1^-$ e^- $Q_2^+ Q_2^-$
C_1 C_2
V
$V < \frac{e}{C_1}\left(n_0 - \frac{1}{2}\right)$
$Q_1^+ Q_1^-$ e^- $Q_2^+ Q_2^-$
C_1 C_2
V
$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$
e^- $Q_1^+ Q_1^-$ $Q_2^+ Q_2^-$
C_1 C_2
V
$V < -\frac{e}{C_2}\left(n_0 + \frac{1}{2}\right)$

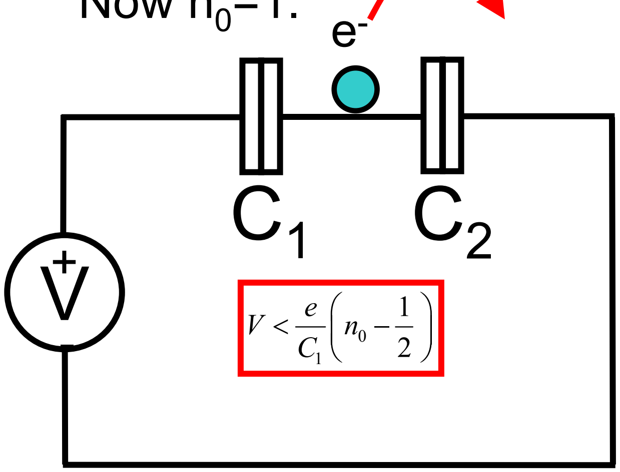
Current

Let $n_0=0$. Let $C_1= C_2$



$$V < -\frac{e}{2C_2}$$

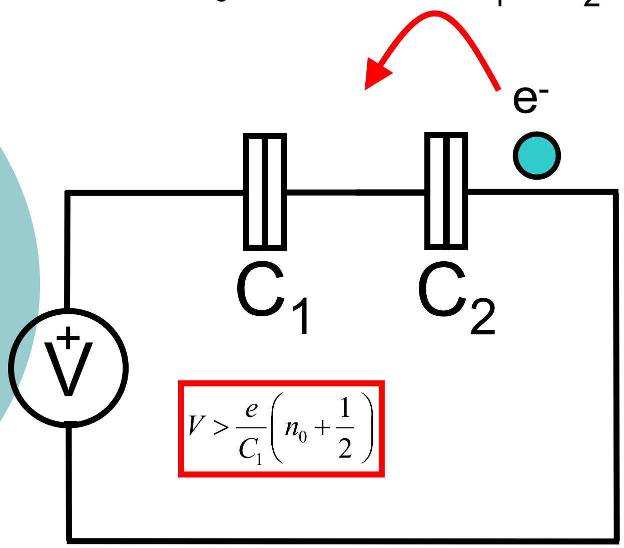
Now $n_0=1$.



$$V < \frac{e}{2C_2}$$

Current

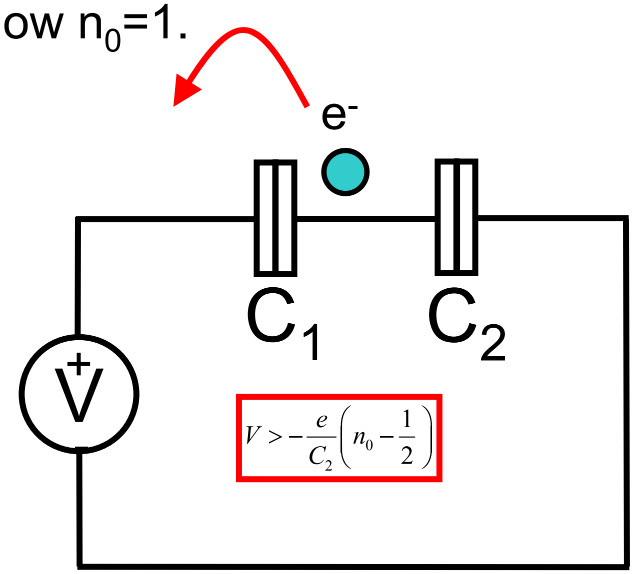
Let n_0=0. Let C_1= C_2



$$V > \frac{e}{C_1}\left(n_0 + \frac{1}{2}\right)$$

$$V > \frac{e}{2C_2}$$

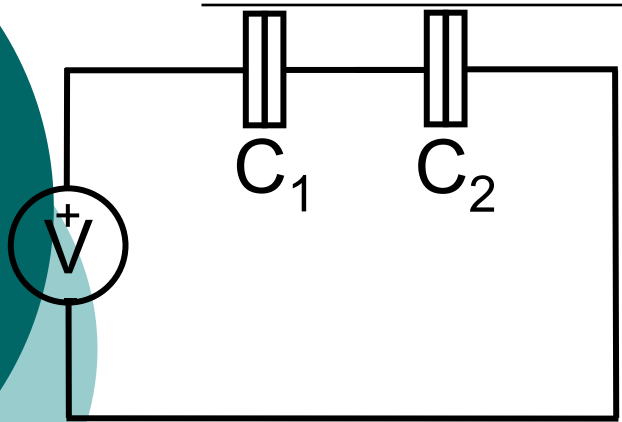
Now n_0=1.



$$V > -\frac{e}{C_2}\left(n_0 - \frac{1}{2}\right)$$

$$V > -\frac{e}{2C_2}$$

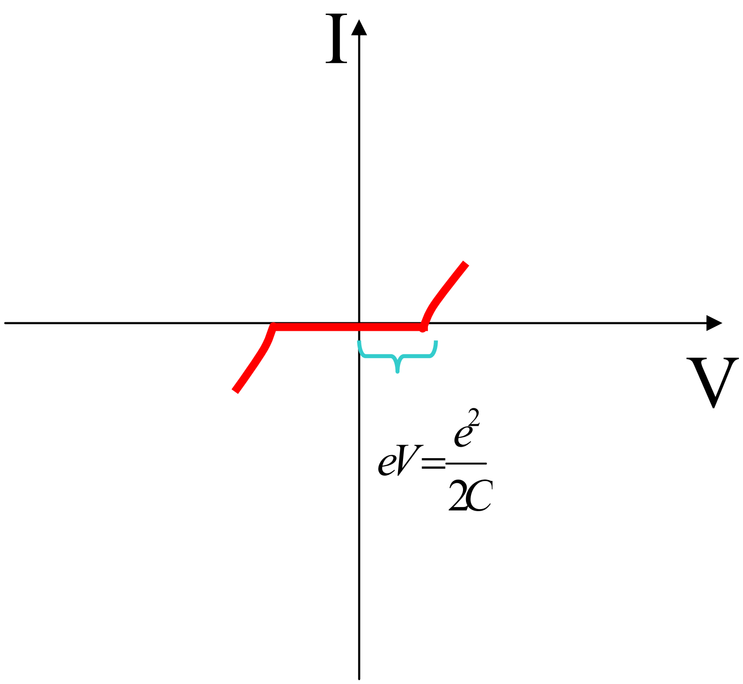
Coulomb blockade



Let $C_1 = C_2$

No current:

$$-\frac{e}{2C_2} < V < \frac{e}{2C_2}$$



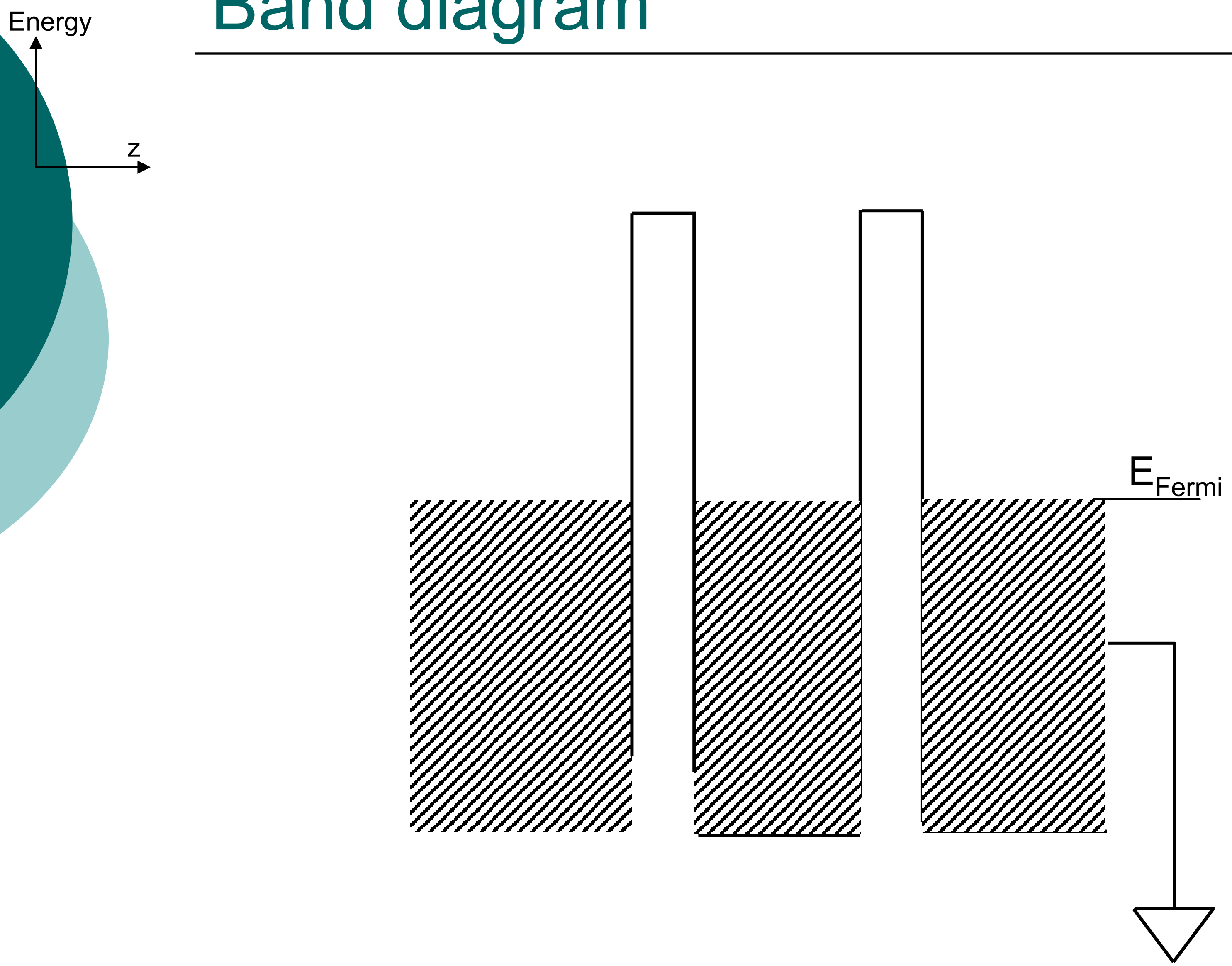
$$eV = \frac{e^2}{2C}$$

From quantum circuit theory, works even when voltage biased.

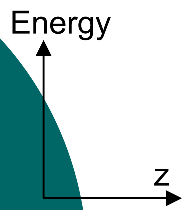
In single junction, Coulomb blockade *hard* to observe.

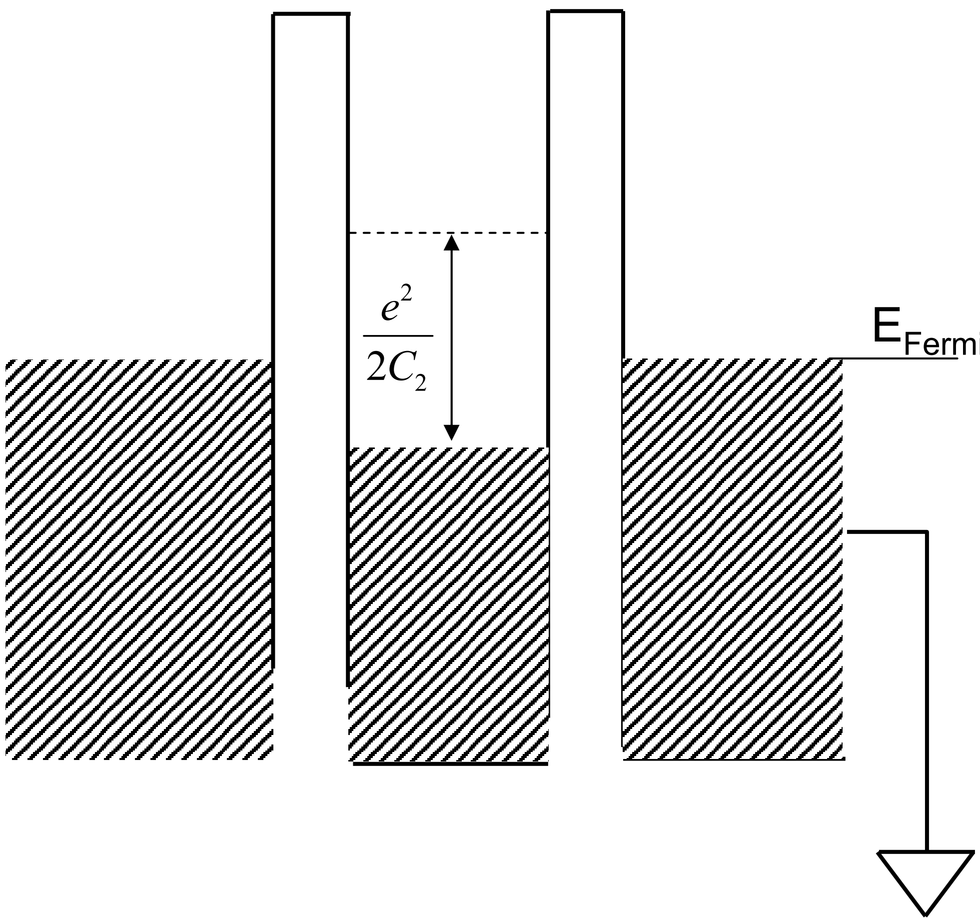
In double junction, Coulomb blockade *easy* to observe.

Band diagram

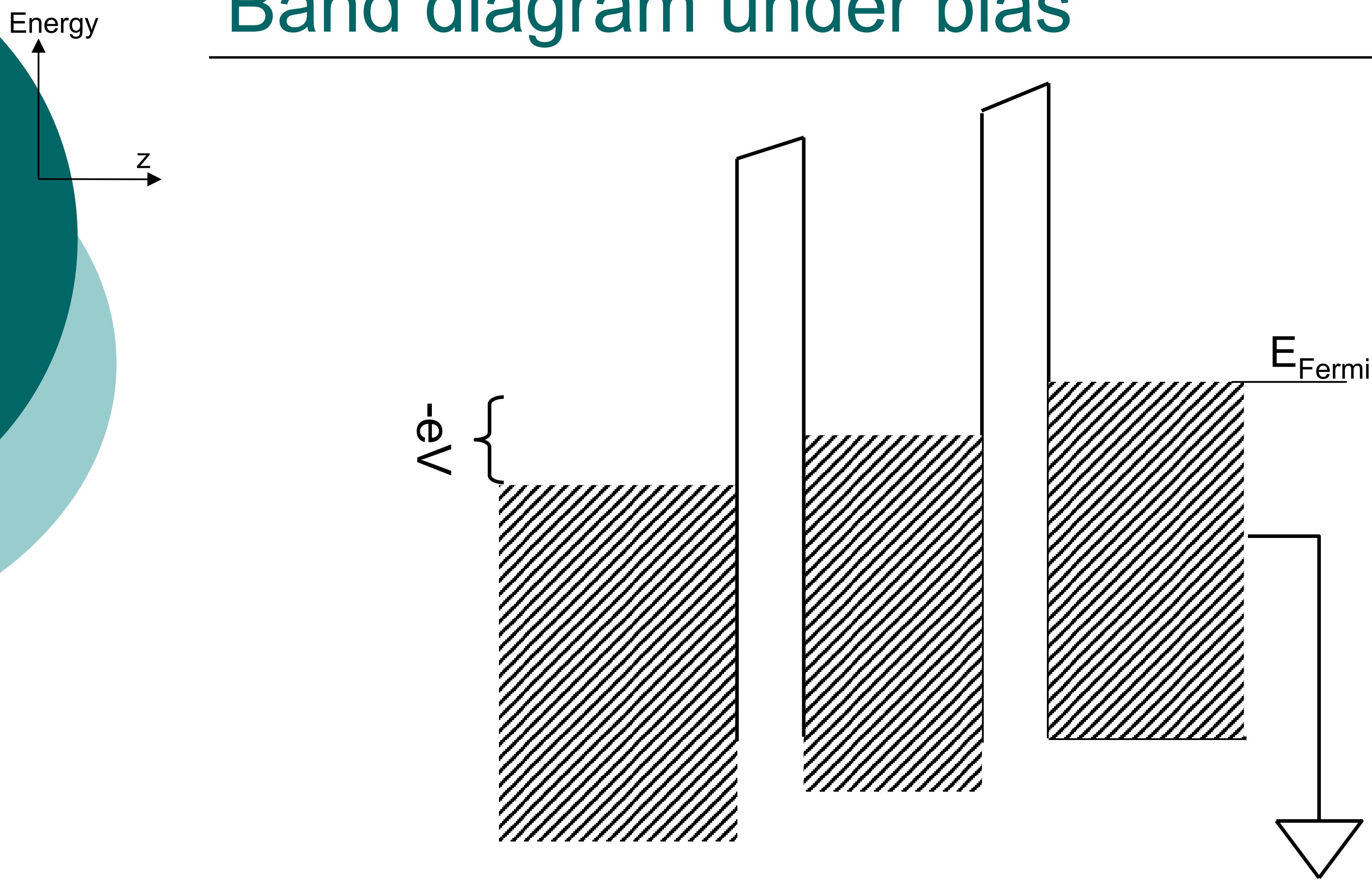


Band diagram with Coulomb "gap"

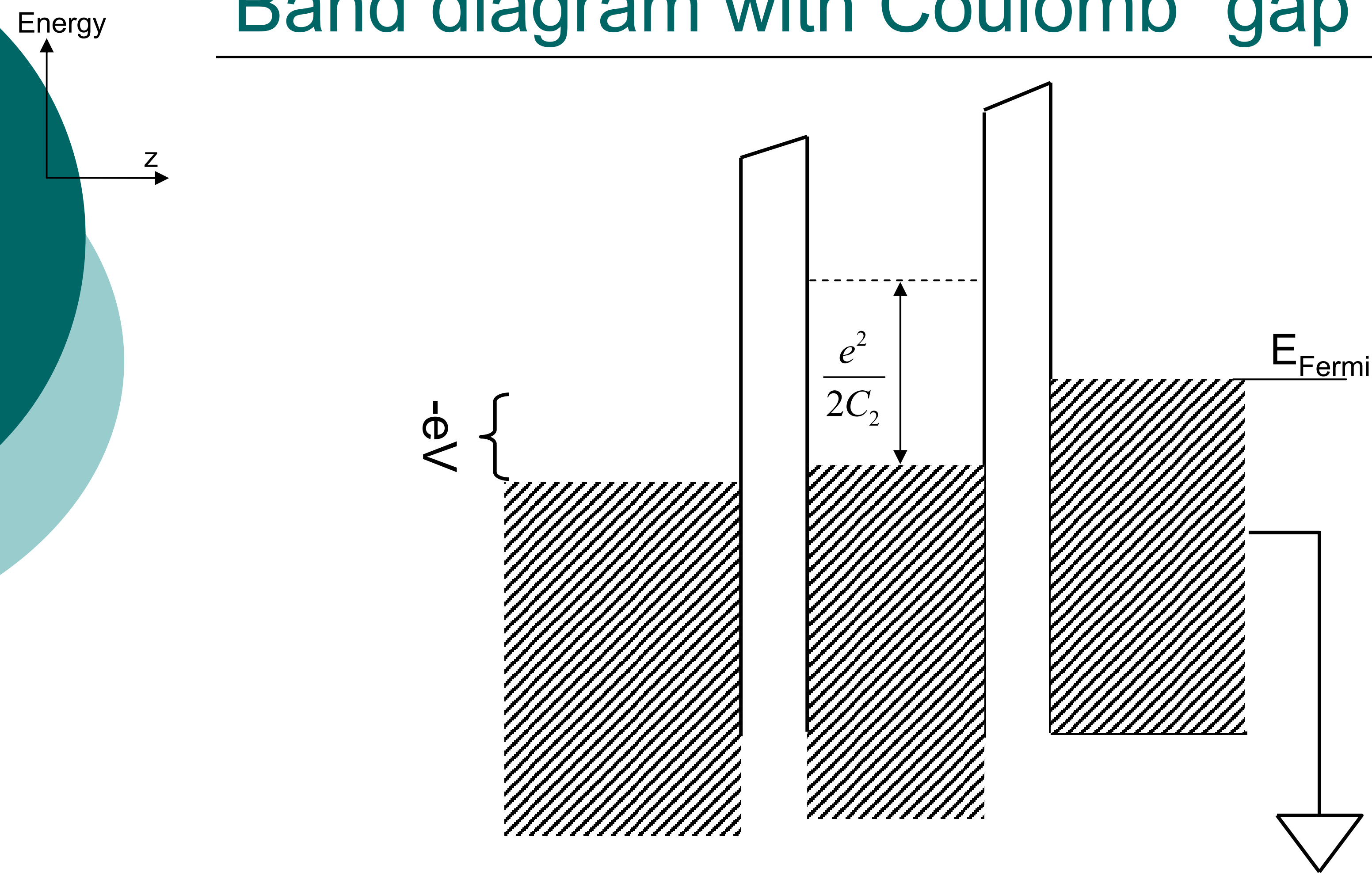




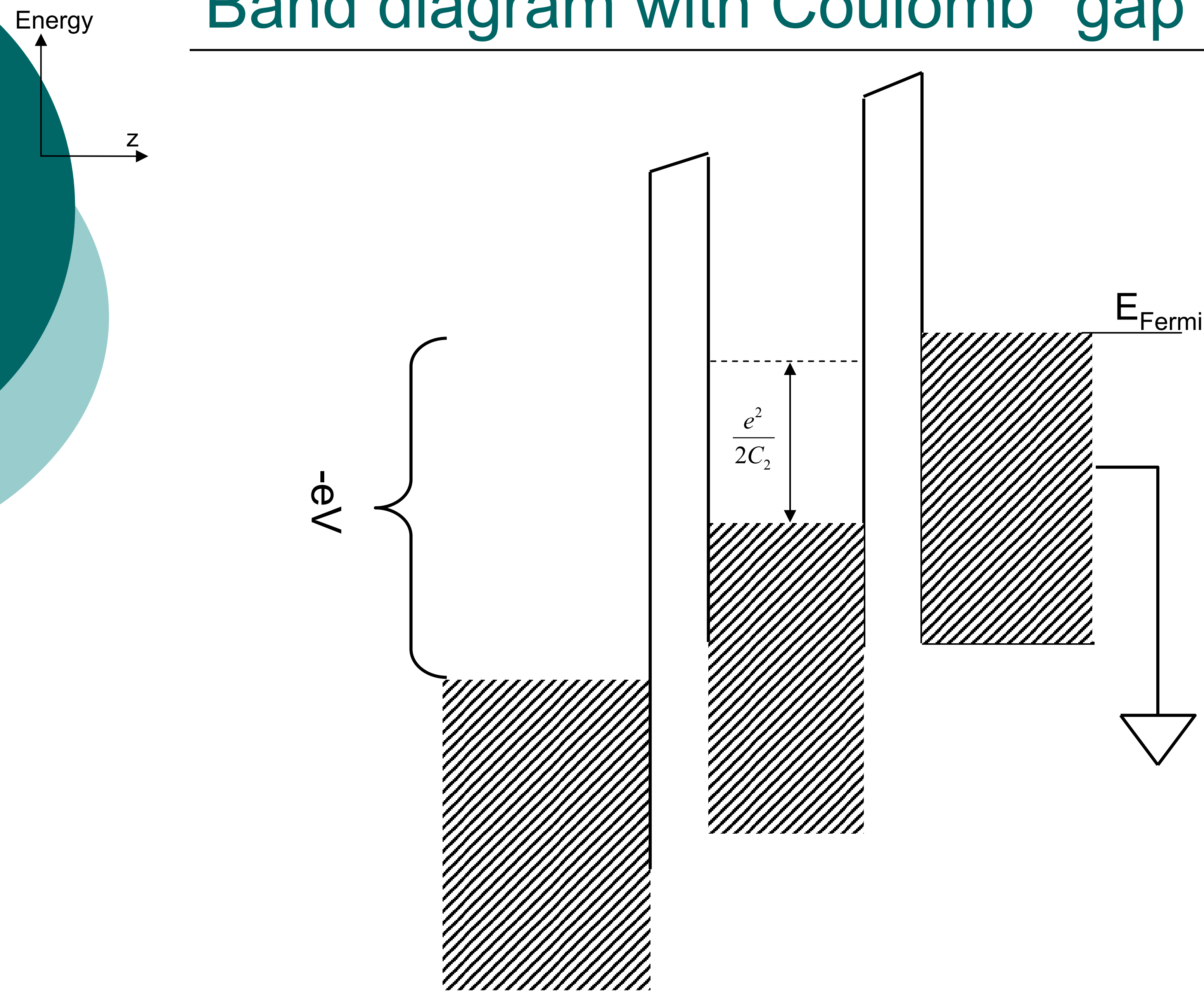
Band diagram under bias



Band diagram with Coulomb "gap"

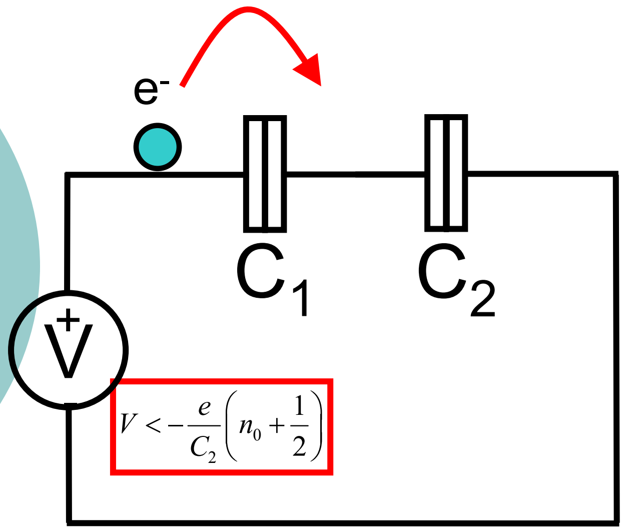


Band diagram with Coulomb "gap"



Higher voltages

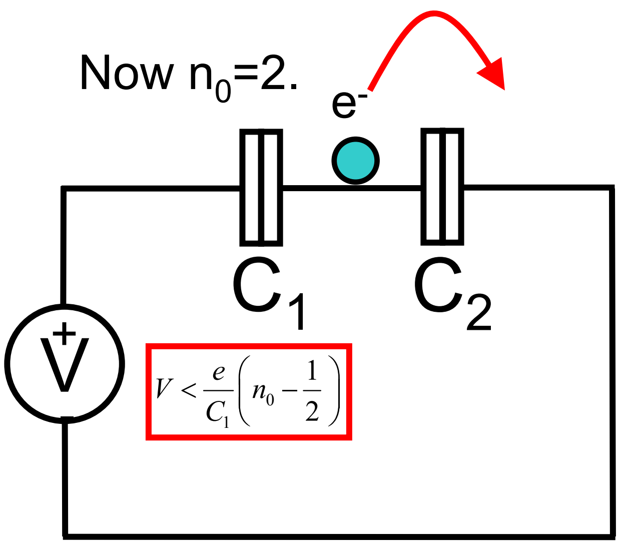
Let n_0=1. Let C_1= C_2



$$V < -\frac{e}{C_2}\left(n_0 + \frac{1}{2}\right)$$

$$V < -\frac{3e}{2C_2}$$

Now n_0=2.

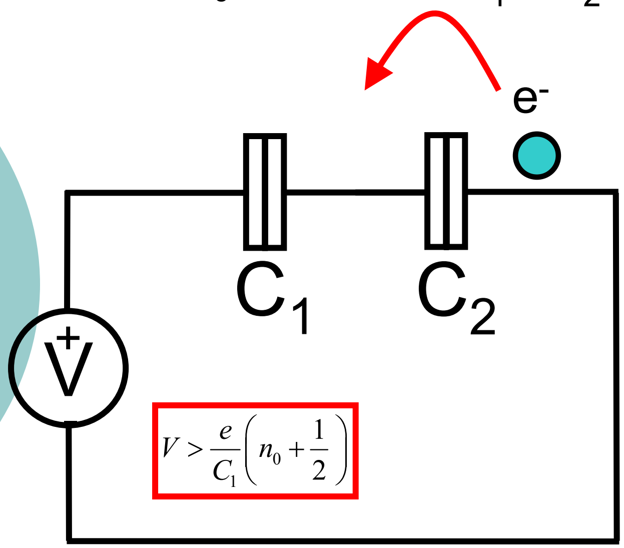


$$V < \frac{e}{C_1}\left(n_0 - \frac{1}{2}\right)$$

$$V < -\frac{3e}{2C_2}$$

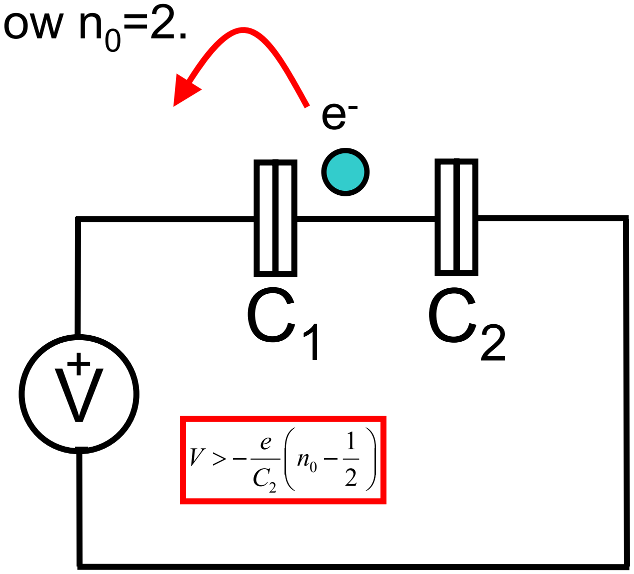
Higher voltages

Let n_0=1. Let C_1= C_2



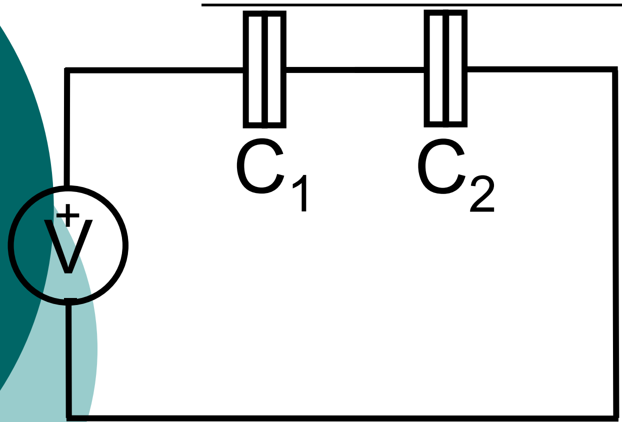
$$V > \frac{3e}{2C_2}$$

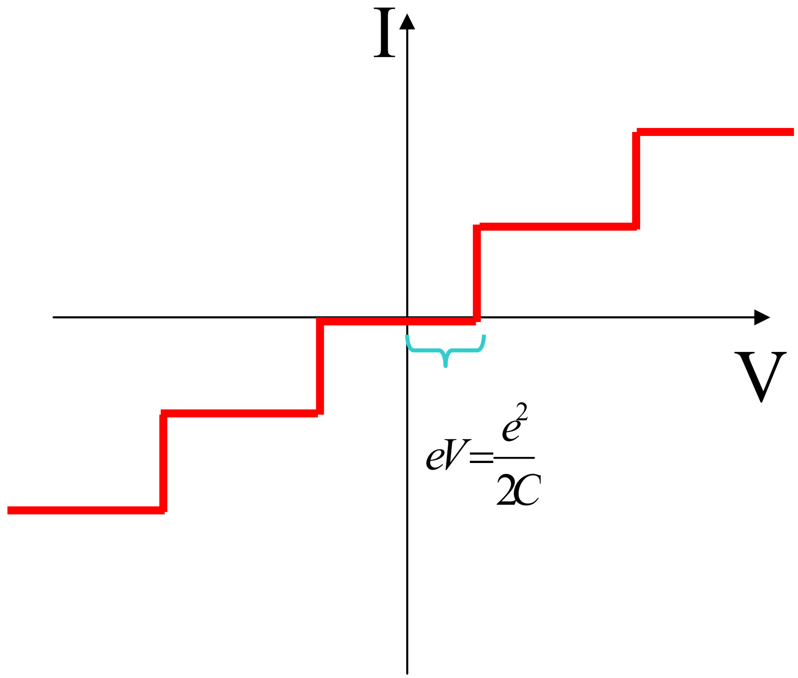
Now n_0=2.



$$V > \frac{3e}{2C_2}$$

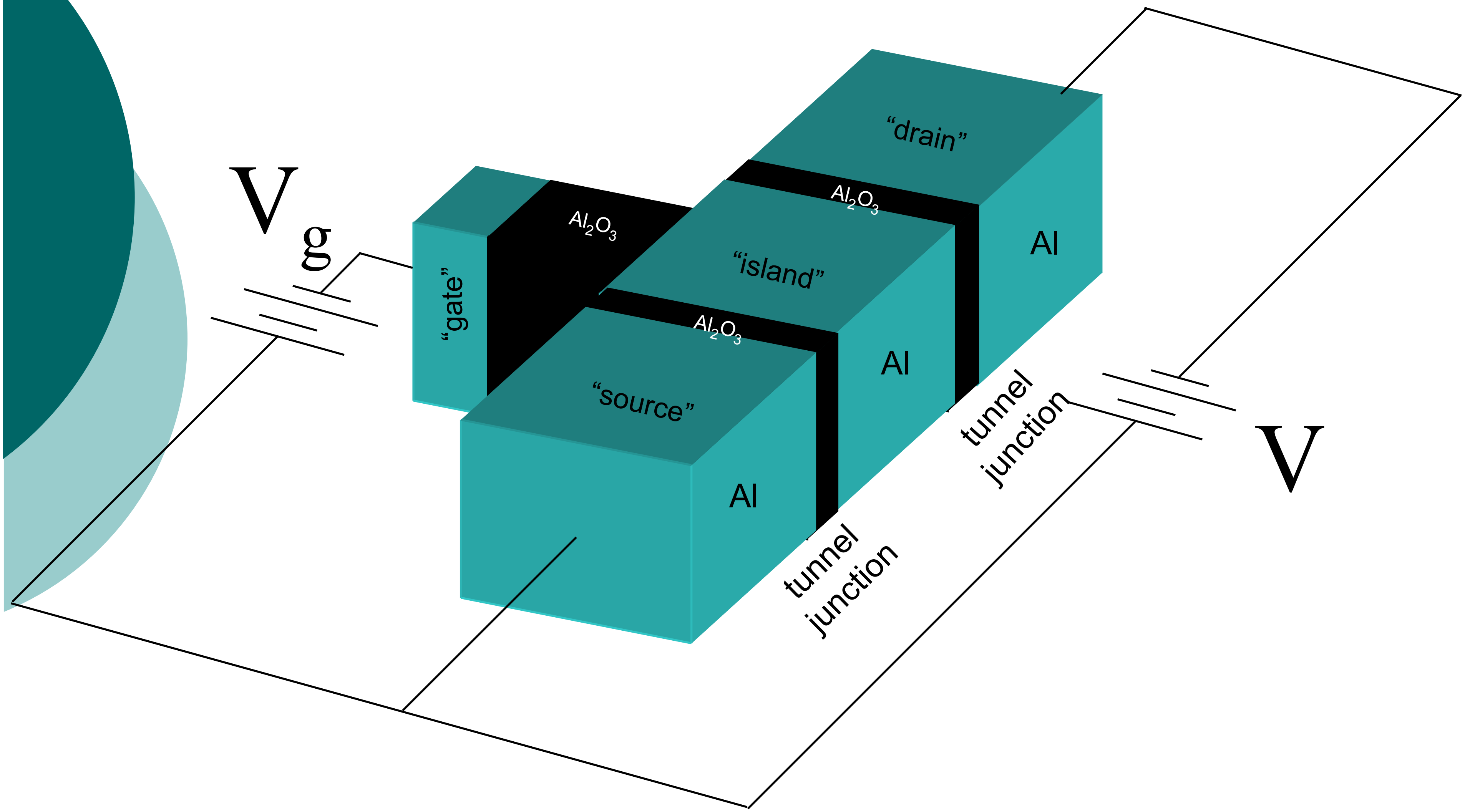
Coulomb staircase



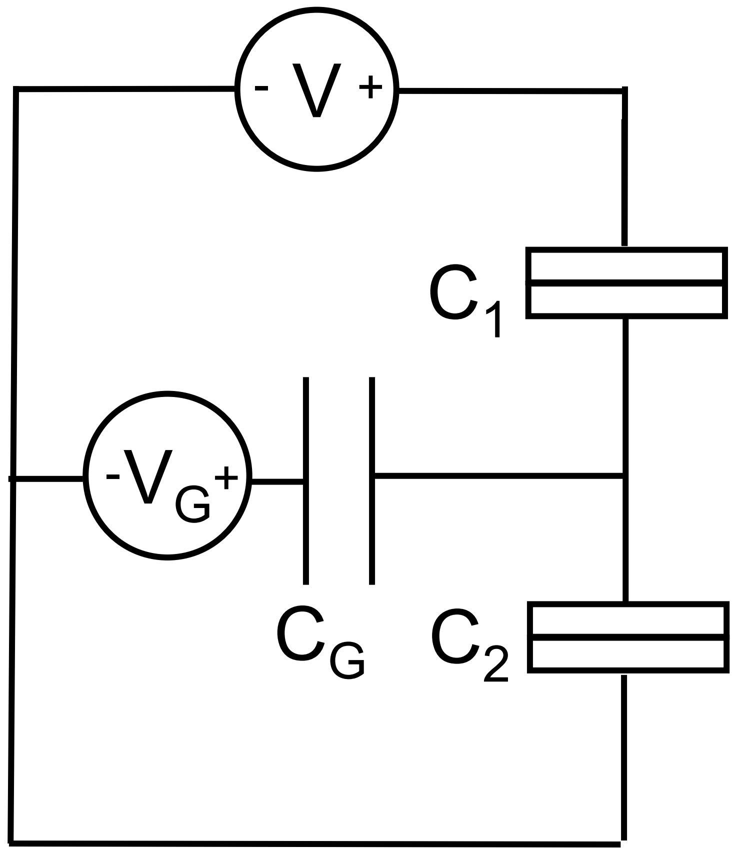


Overall slope is R_{tunnel}, which is *large*.

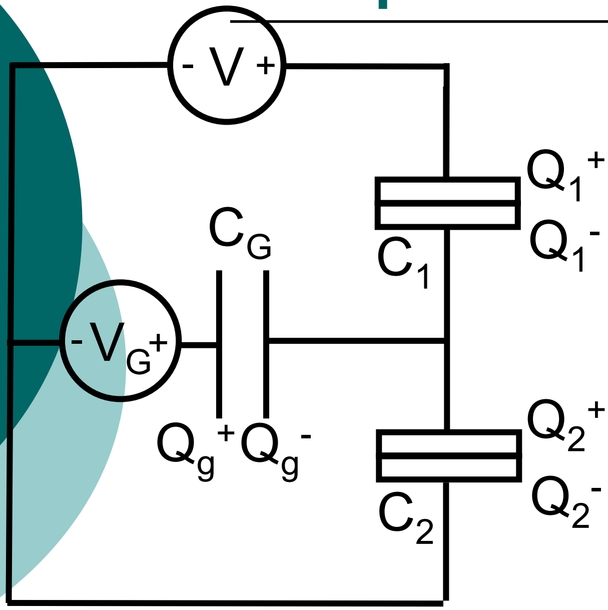
Lecture 8: Single electron transistor



Single electron transistor circuit



Capacitor charges



Kirchoff:

$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V_g = \frac{Q_g}{C_g} + \frac{Q_2}{C_2}$$

Island charge:

$$Q_i = Q_2 - Q_1 - Q_g$$

3 equations, 3 unknowns, solve:

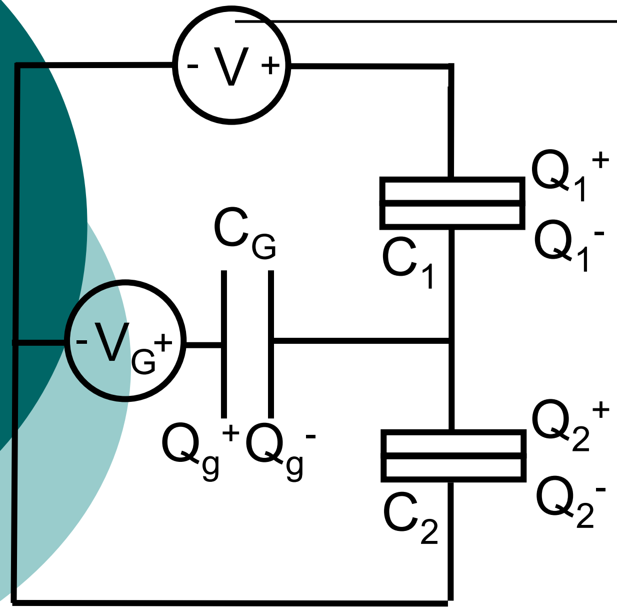
$$Q_1 = \frac{C_1 V (C_2 + C_g) - C_1 C_g (Q_i + V_g)}{C_\Sigma}$$

$$C_\Sigma \equiv C_1 + C_2 + C_g$$

$$Q_2 = \frac{C_2 Q_i + C_1 C_2 V + C_g C_2 V_g}{C_\Sigma}$$

$$Q_2 = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

Electrostatic energy



$$E = \frac{Q_1^{\ 2}}{2C_1} + \frac{Q_2^{\ 2}}{2C_2} + \frac{Q_G^{\ 2}}{2C_G}$$

$$Q_1 = \frac{C_1 V (C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$Q_2 = \frac{C_2 Q_i + C_1 C_2 V + C_g C_2 V_g}{C_\Sigma}$$

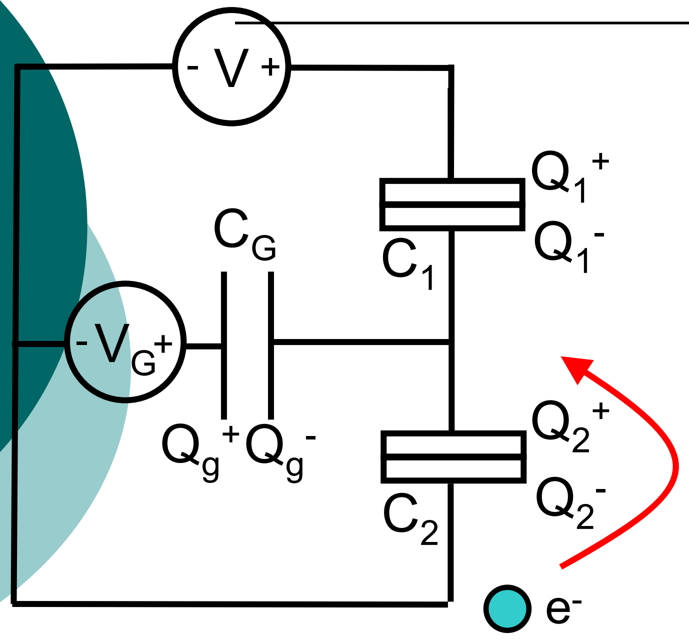
$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + \left(V - V_G\right)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2\right]$$

Free energy :

$$G = E - Q_1V - Q_GV_G$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

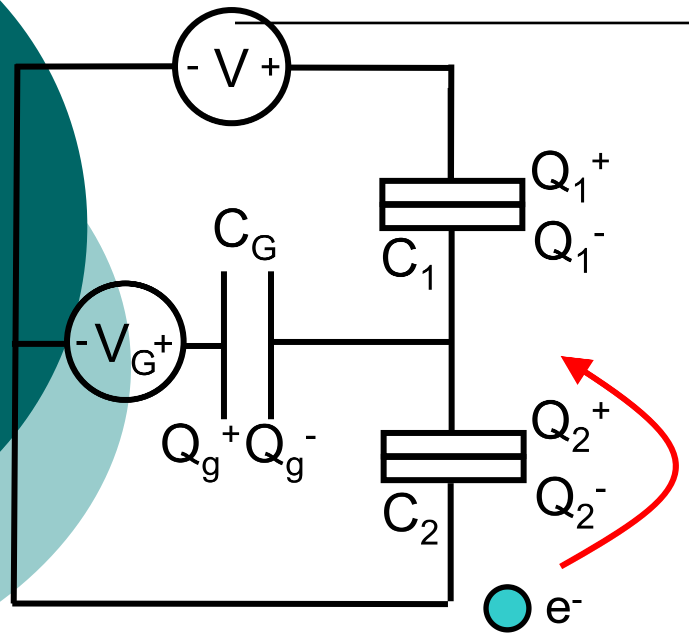
After:

$$Q_i = -n_0 e - e$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + (V - V_G)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2\right]$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before: $Q_i = -n_0 e$

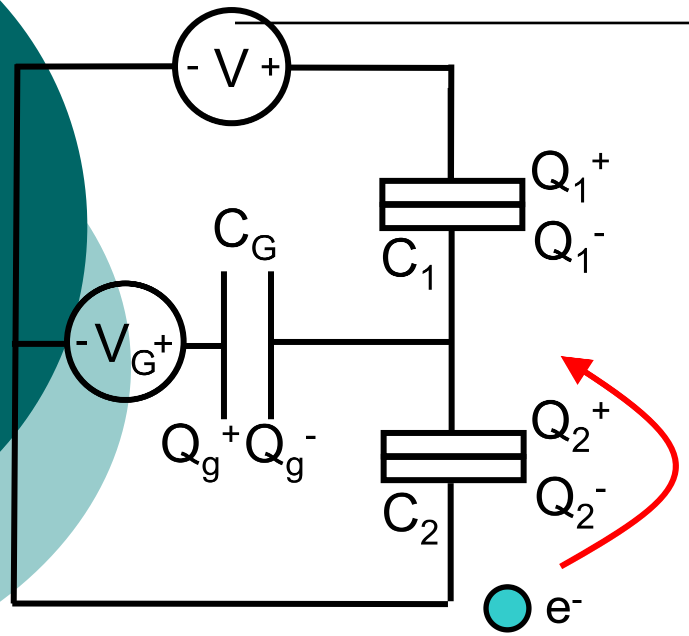
After: $Q_i = -n_0 e - e$

$$Q_1 = \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$\Delta Q_1 = \Delta \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma} = \frac{-C_1\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_1\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_1 = \frac{-eC_1}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before: $$Q_i = -n_0 e$$

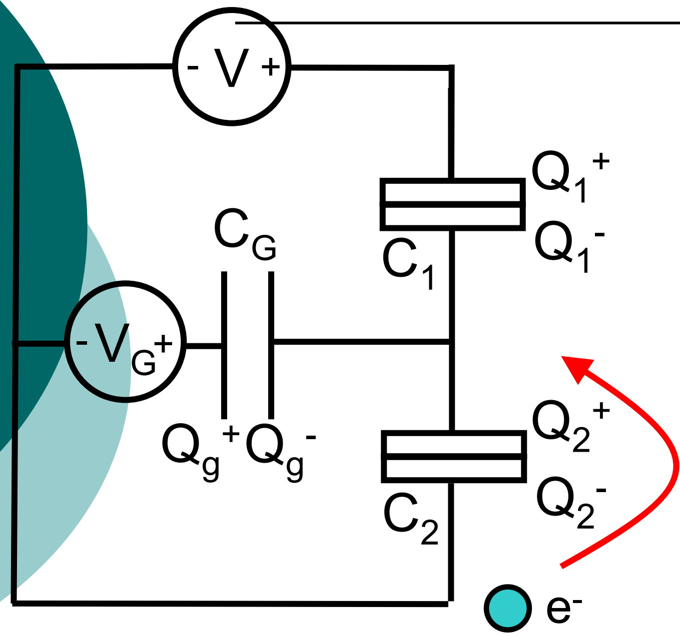
After: $$Q_i = -n_0 e - e$$

$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$\Delta Q_G = \Delta \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma} = \frac{-C_g\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_g\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

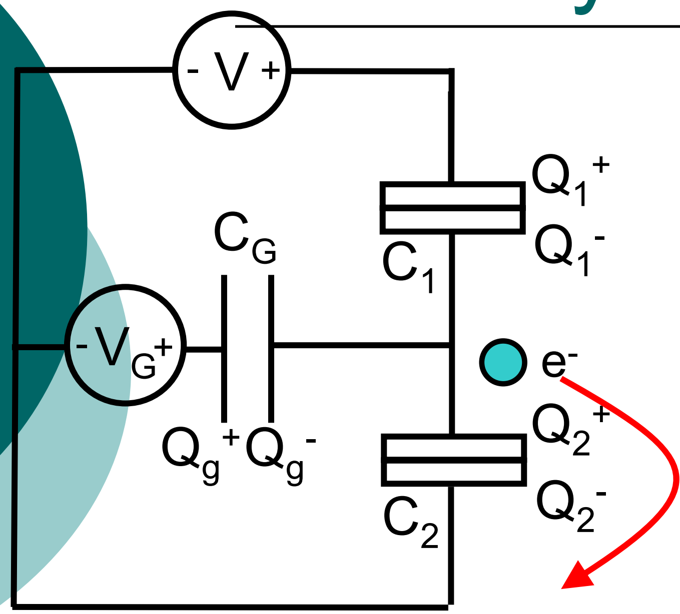
$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

$$\Delta Q_1 = \frac{-eC_1}{C_\Sigma} \qquad \Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2C_\Sigma} + V\frac{eC_1 C_g}{C_\Sigma} + V_G \frac{eC_g}{C_\Sigma}$$

$$\boxed{\Delta G = \frac{e}{C_\Sigma}\left[-n_0 e - \frac{e}{2} + C_1 V + C_g V_G\right] > 0}$$

Similarly

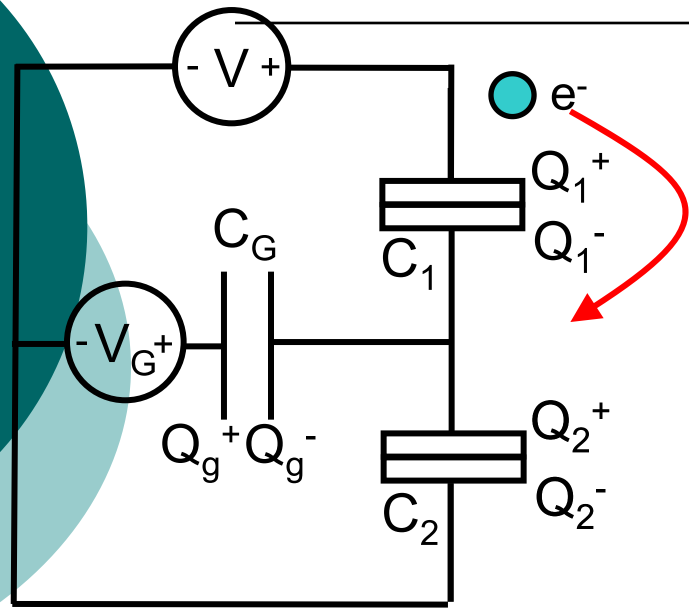


Allowed only if:

$$\Delta G = \frac{e}{C_\Sigma}\left[+n_0 e - \frac{e}{2} - C_1 V - C_g V_G \right] > 0$$

n_0 is the number of electrons on the island *before* the tunnel event.

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

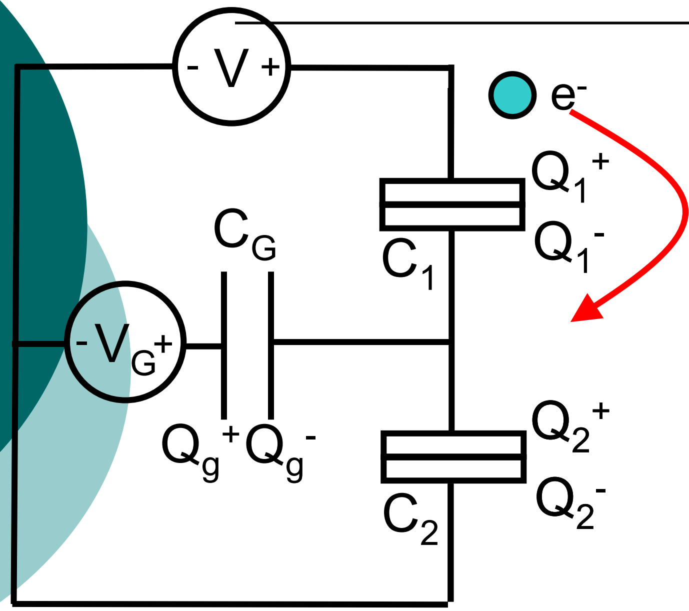
After:

$$Q_i = -n_0 e + e$$

$$E = \frac{1}{2C_\Sigma}\left[C_G C_1 + \left(V - V_G\right)^2 + C_1 C_2 V^2 + C_G C_2 V_G^2 + Q_i^2\right]$$

$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

After:

$$Q_i = -n_0 e + e$$

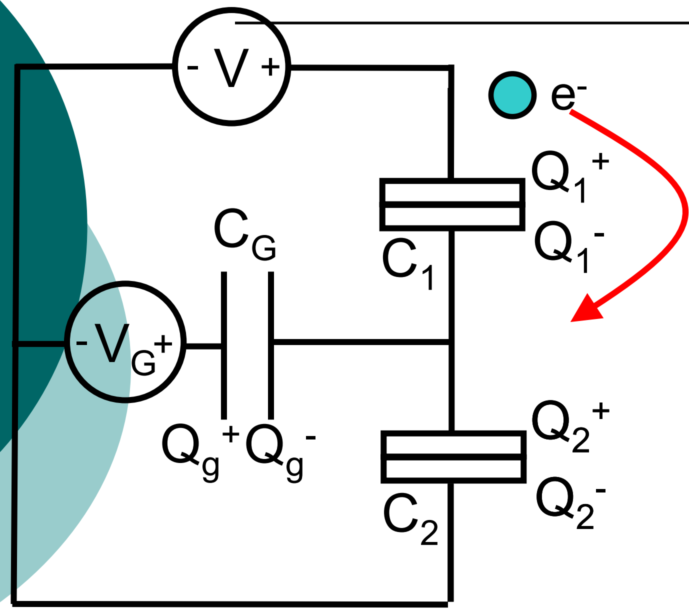
$$Q_1 = \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma}$$

$$\Delta Q_1 = \Delta \frac{C_1 V(C_2 + C_g) - C_1 C_G V_G - C_1 Q_i}{C_\Sigma} = \frac{-C_1\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_1\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_{1,polarization} = \frac{-eC_1}{C_\Sigma} \qquad \Delta Q_{1,tunnel} = e$$

$$\Delta Q_{1,total} = \Delta Q_{1,polarization} + \Delta Q_{1,tunnel} = \frac{-eC_1}{C_\Sigma} + e = \frac{-eC_1}{C_1 + C_2 + C_G} + \frac{eC_1 + eC_2 + eC_G}{C_1 + C_2 + C_G} = e\frac{C_2 + C_G}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

Before:

$$Q_i = -n_0 e$$

After:

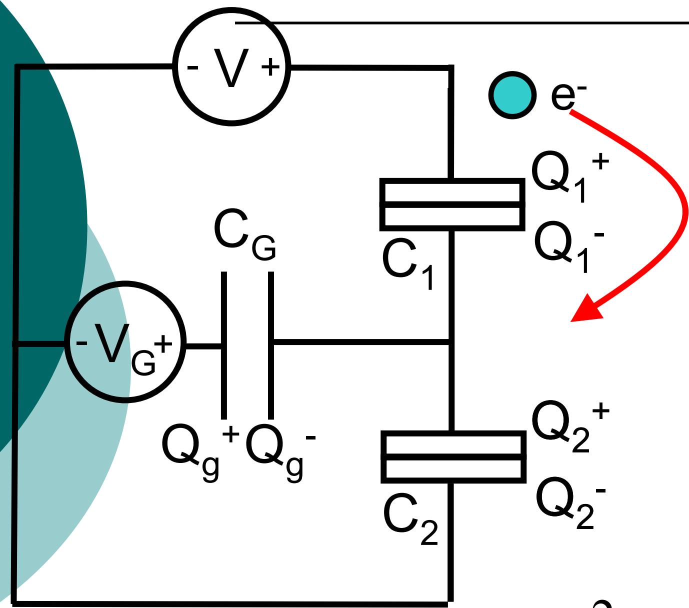
$$Q_i = -n_0 e + e$$

$$Q_G = \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma}$$

$$\Delta Q_G = \Delta \frac{-C_g Q_i - C_1 C_g V + C_g V_g (C_1 + C_2)}{C_\Sigma} = \frac{-C_g\left(-n_0 e\right)}{C_\Sigma} - \frac{-C_g\left(-n_0 e - e\right)}{C_\Sigma}$$

$$\Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

Can current flow?



$$\Delta G = \Delta E - V\Delta Q_1 - V_G \Delta Q_G$$

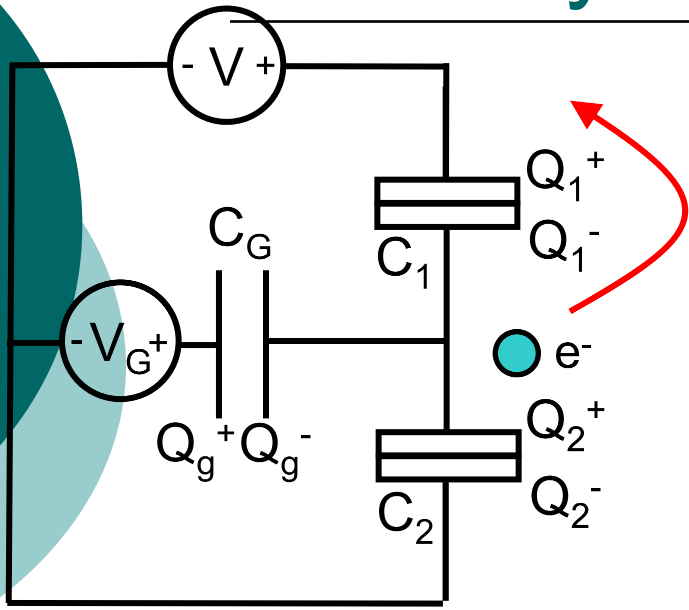
$$\Delta E = \frac{-2n_0 e^2 - e^2}{2C_\Sigma}$$

$$\Delta Q_{1,total} = e\frac{C_2 + C_G}{C_\Sigma} \qquad \Delta Q_G = \frac{-eC_g}{C_\Sigma}$$

$$\Delta G = \frac{-2n_0 e^2 - e^2}{2C_\Sigma} - Ve\frac{C_2 + C_G}{C_\Sigma} + V_G \frac{eC_g}{C_\Sigma}$$

$$\Delta G = \frac{e}{C_\Sigma}\left[-n_0 e - \frac{e}{2} - V\left(C_2 + C_G\right) + V_G C_g\right] > 0$$

Similarly:



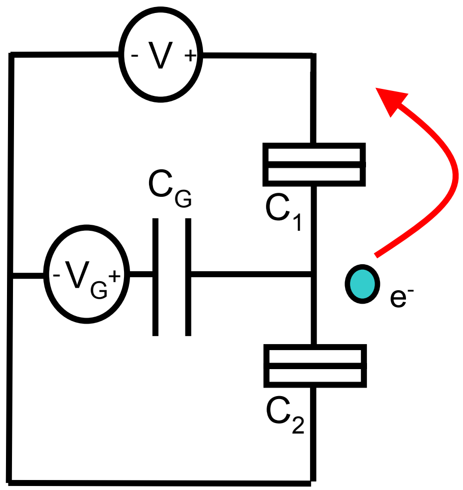
Allowed only if:

$$\Delta G = \frac{e}{C_\Sigma}\left[+n_0 e - \frac{e}{2} + V\left(C_2 + C_G\right) - V_G C_g \right] > 0$$

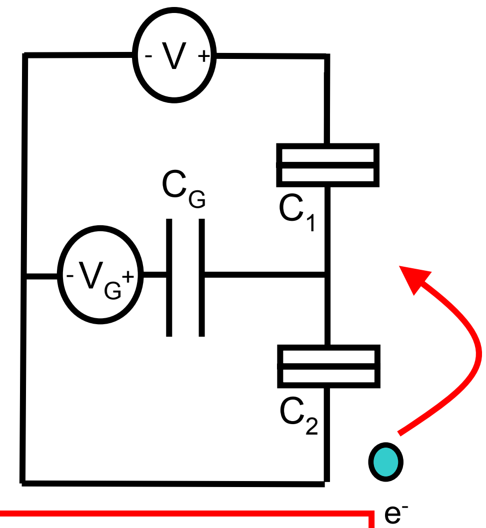
n_0 is the number of electrons on the island *before* the tunnel event.

Summary:

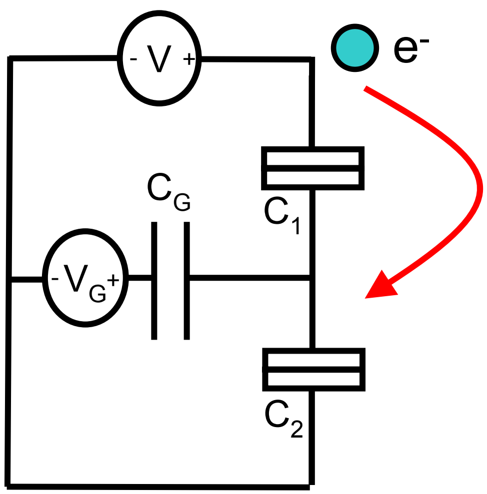
$$+n_0e-\frac{e}{2}+V\left(C_2+C_G\right)-V_GC_g>0$$



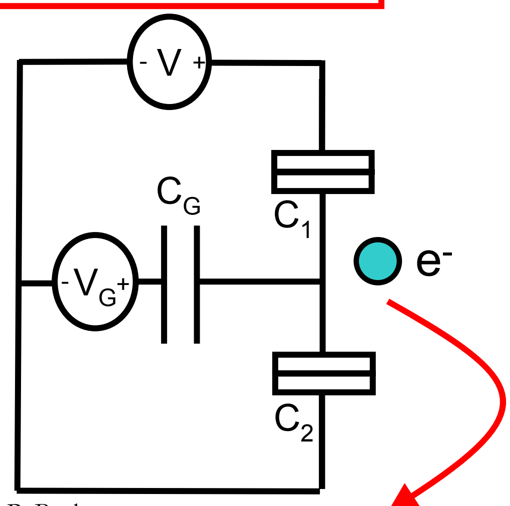
$$-n_0e-\frac{e}{2}+C_1V+C_gV_G>0$$



$$-n_0e-\frac{e}{2}-V\left(C_2+C_G\right)+V_GC_g>0$$

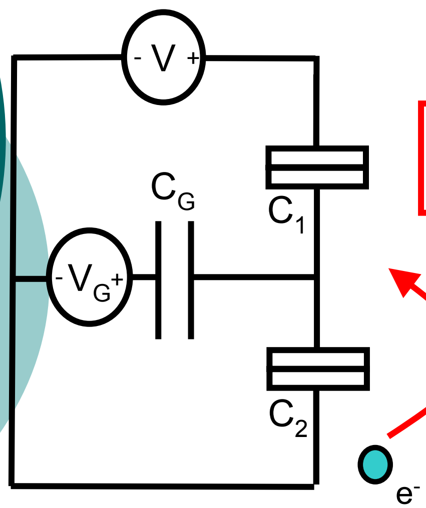


$$+n_0e-\frac{e}{2}-C_1V-C_gV_G>0$$



Current?

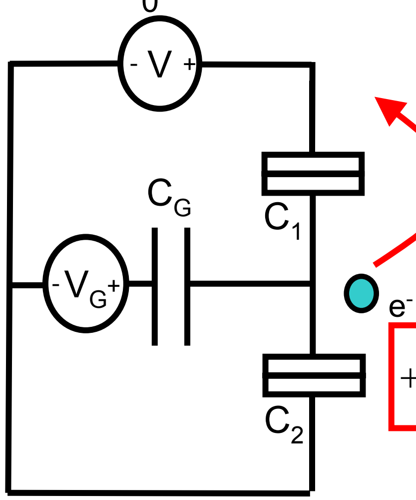
Let $n_0=0$.



$$-\frac{e}{2}+C_1V+C_gV_G>0$$

$$V>\frac{e}{2C_1}-\frac{C_g}{C_1}V_G$$

Now $n_0=1$.

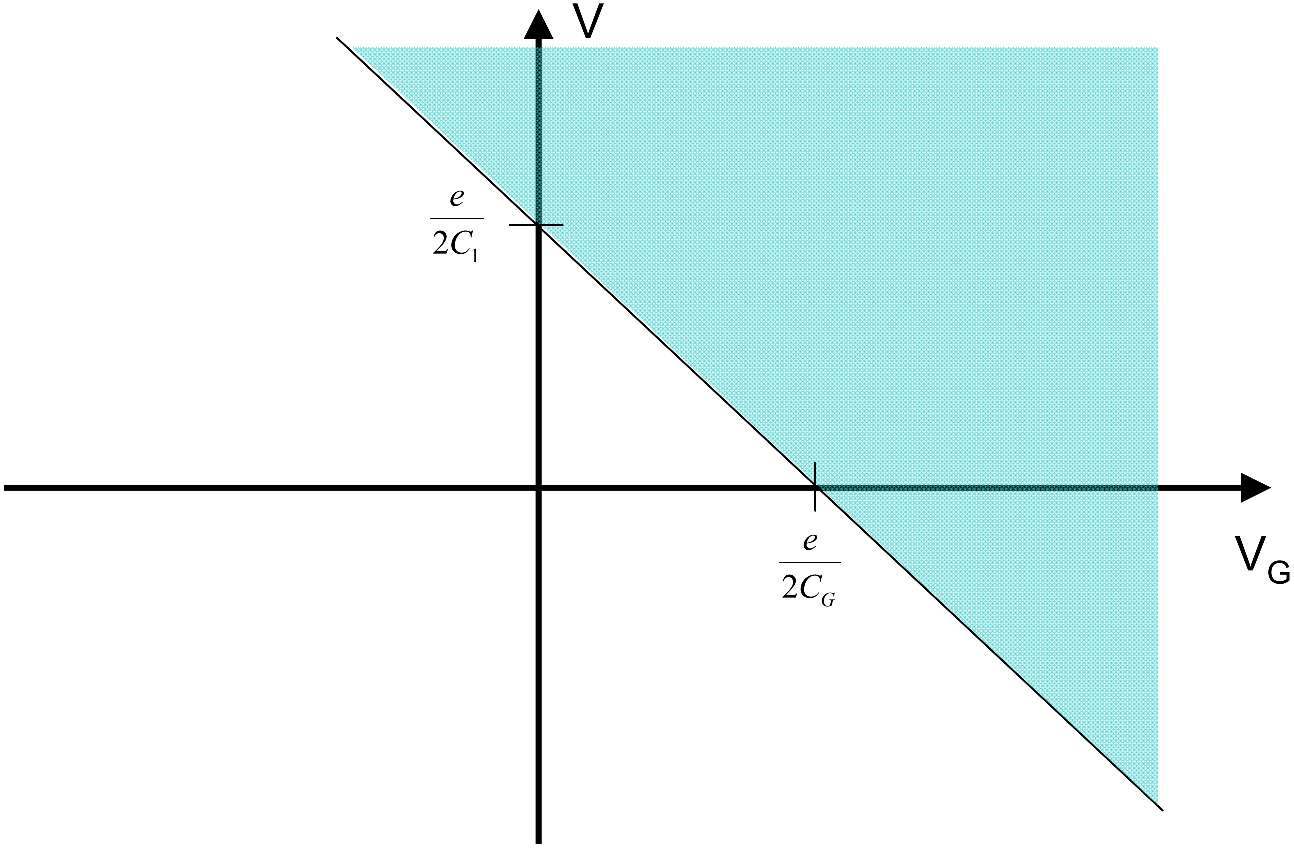


$$+\frac{e}{2}+V\left(C_2+C_G\right)-V_GC_g>0$$

$$V>\frac{C_g}{\left(C_2+C_G\right)}V_G-\frac{e}{2\left(C_2+C_G\right)}$$

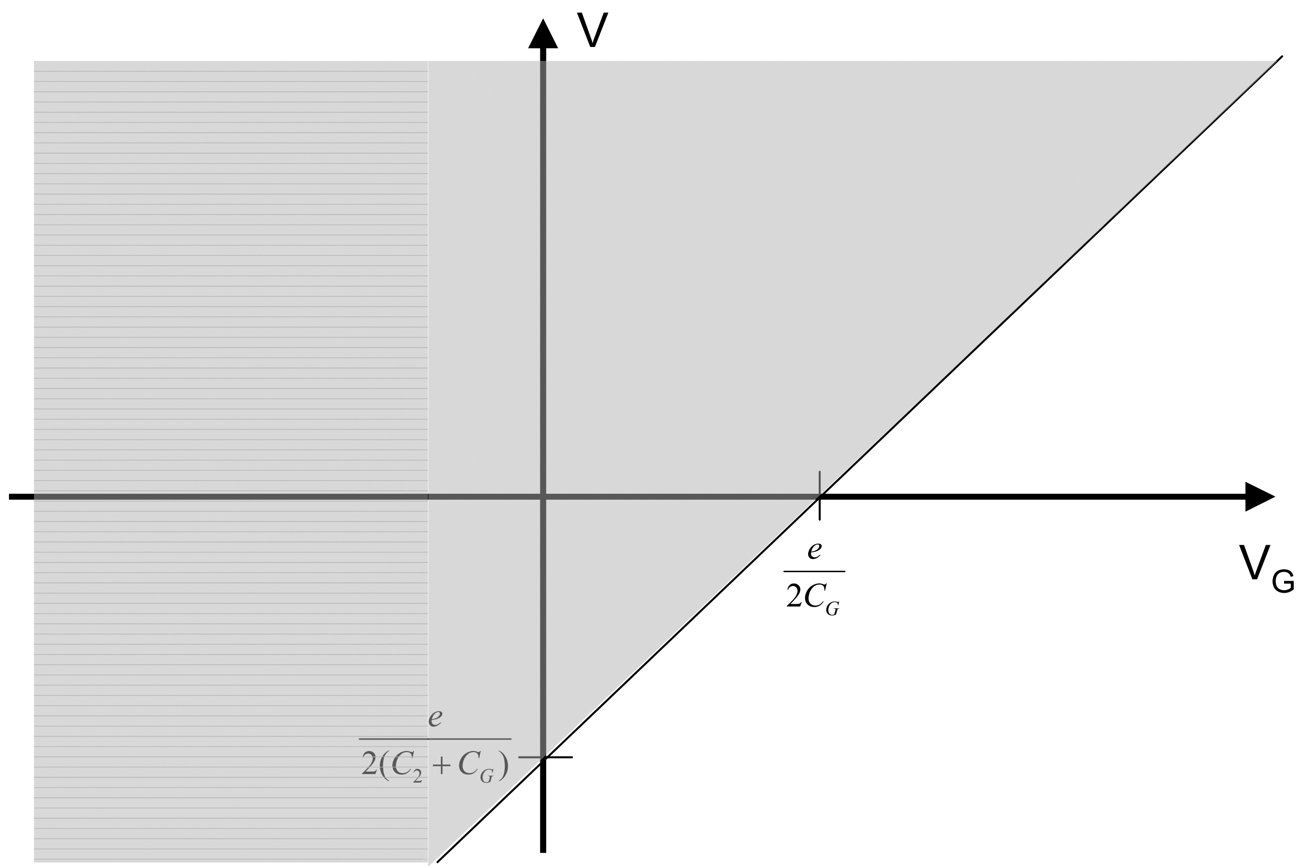
Current?

$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$



Current?

$$V > \frac{C_g}{(C_2 + C_G)} V_G - \frac{e}{2(C_2 + C_G)}$$

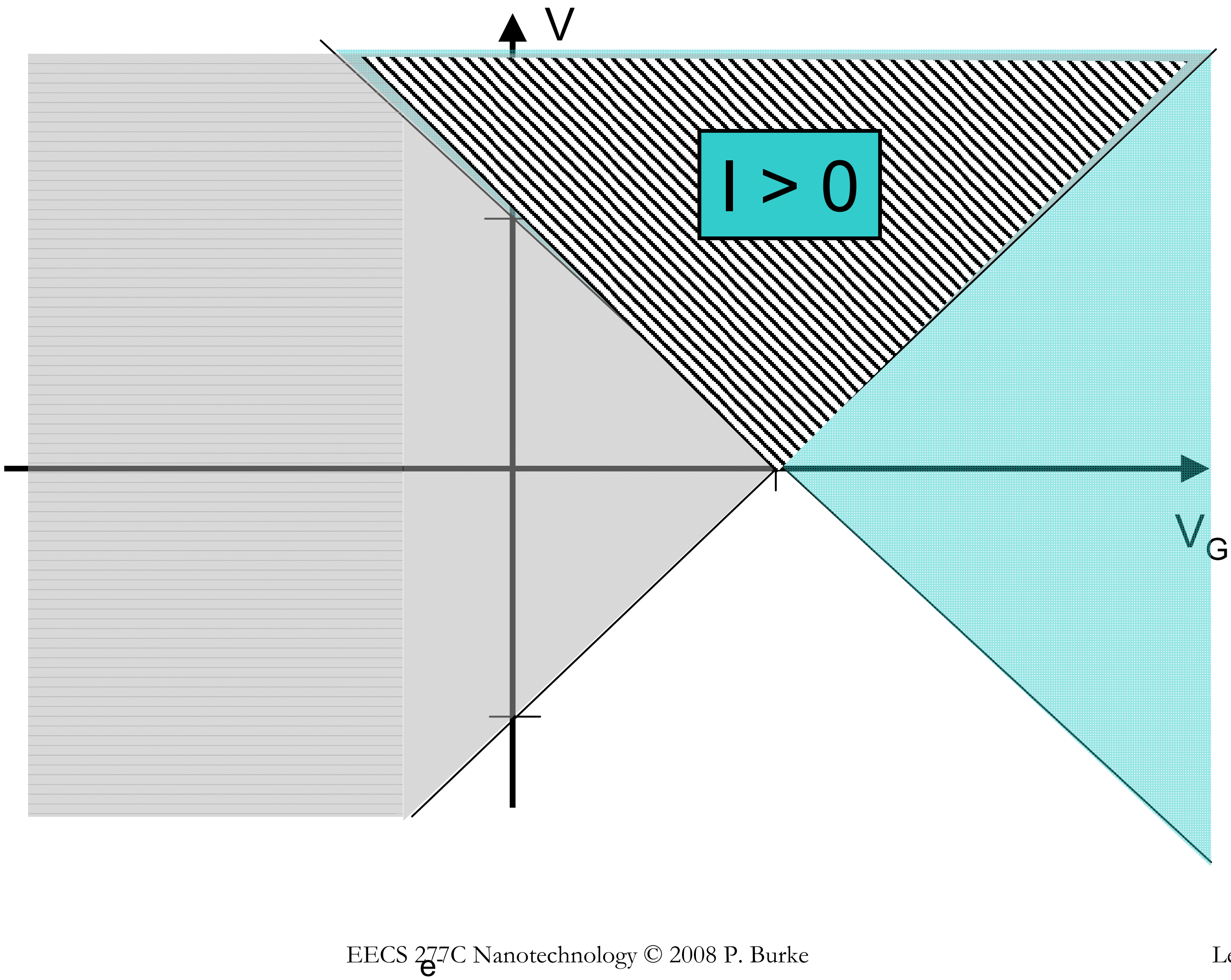


e

Current?

$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$

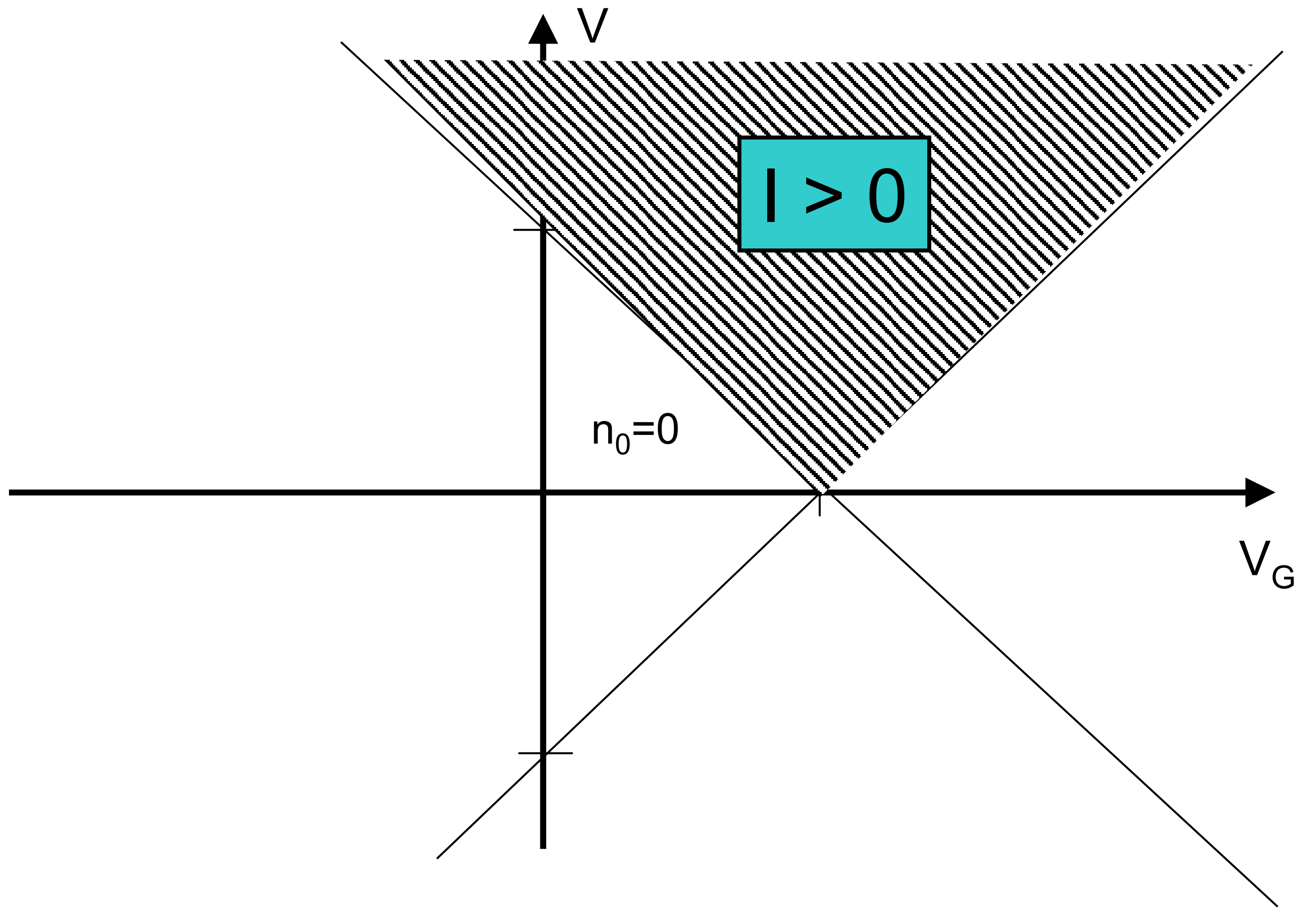
$$V > \frac{C_g}{(C_2 + C_G)} V_G - \frac{e}{2(C_2 + C_G)}$$



Current?

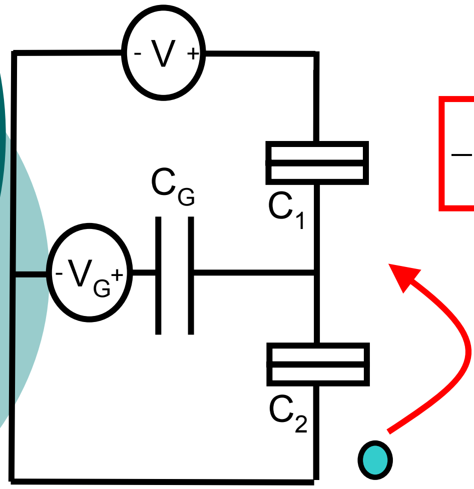
$$V > \frac{e}{2C_1} - \frac{C_g}{C_1} V_G$$

$$V > \frac{C_g}{\left(C_2 + C_G\right)} V_G - \frac{e}{2\left(C_2 + C_G\right)}$$



Current for 1 electrons on island:

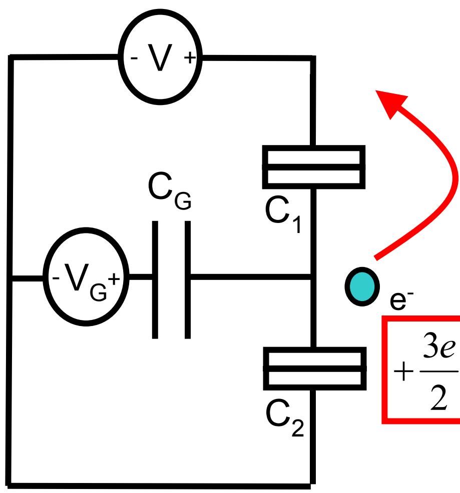
Let $n_0=1$.



$$-\frac{3e}{2}+C_1V+C_gV_G>0$$

$$V>\frac{3e}{2C_1}-\frac{C_g}{C_1}V_G$$

Now $n_0=2$.

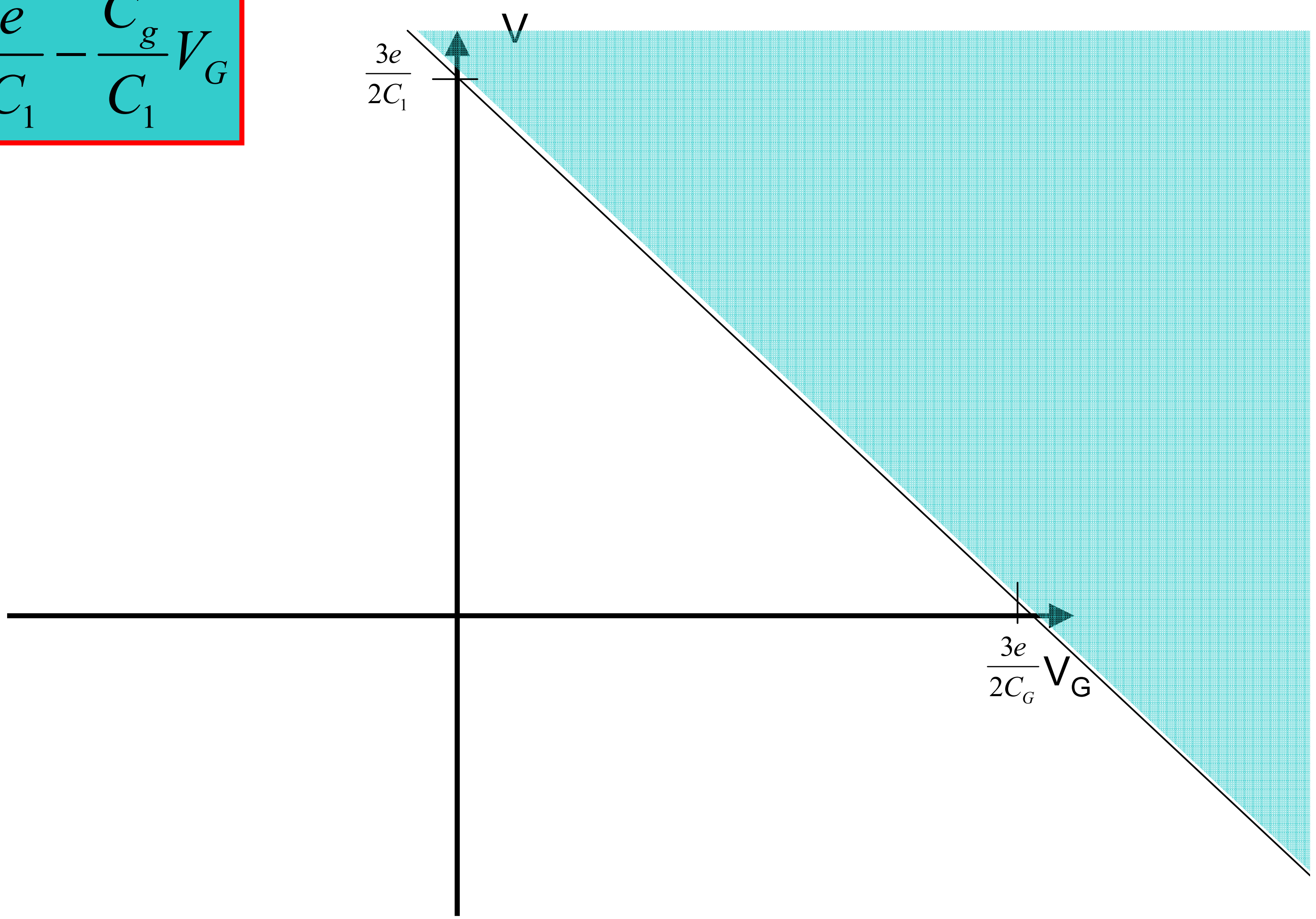


$$+\frac{3e}{2}+V\left(C_2+C_G\right)-V_GC_g>0$$

$$V>\frac{C_g}{\left(C_2+C_G\right)}V_G-\frac{3e}{2\left(C_2+C_G\right)}$$

Current?

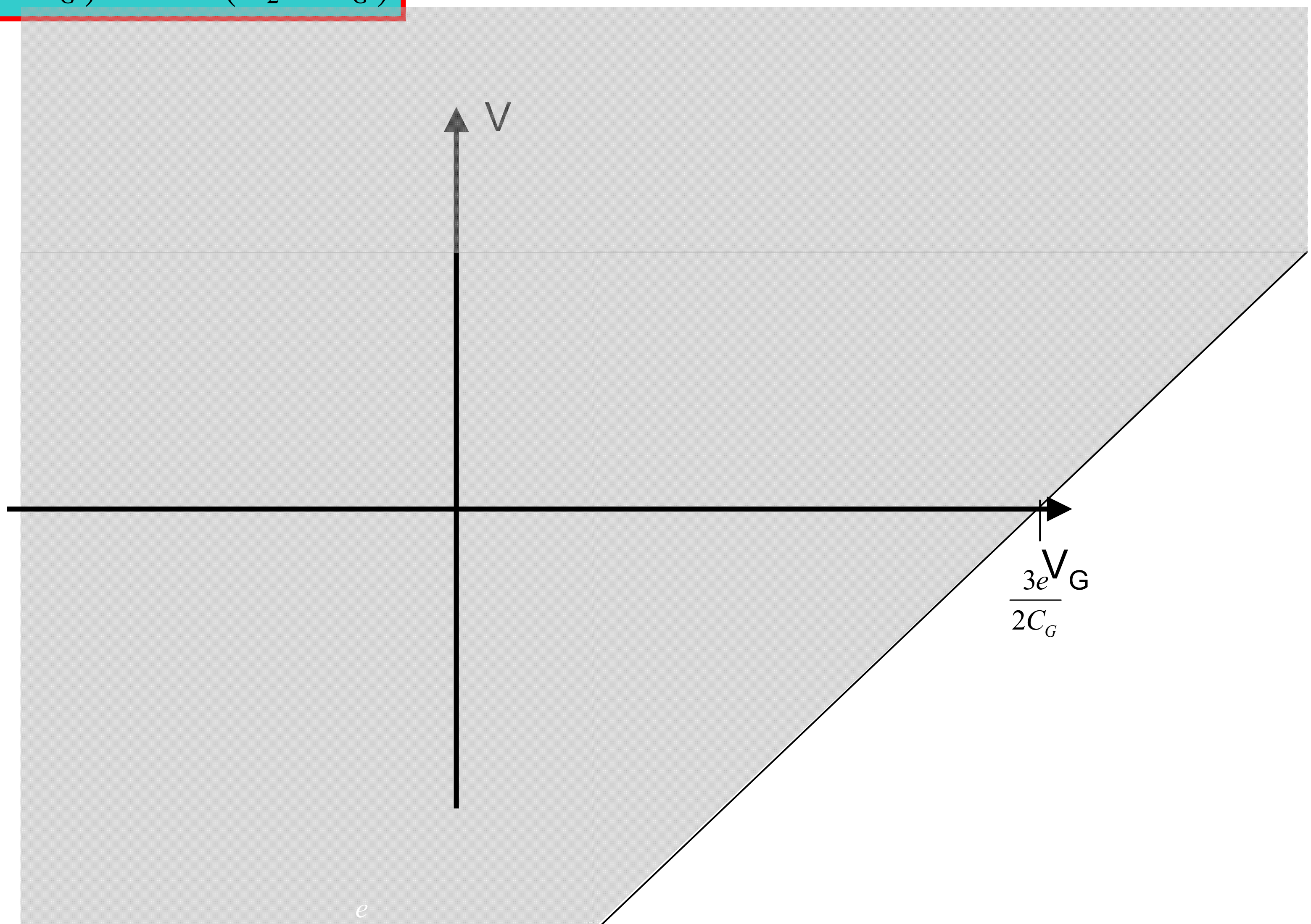
$$V > \frac{3e}{2C_1} - \frac{C_g}{C_1} V_G$$



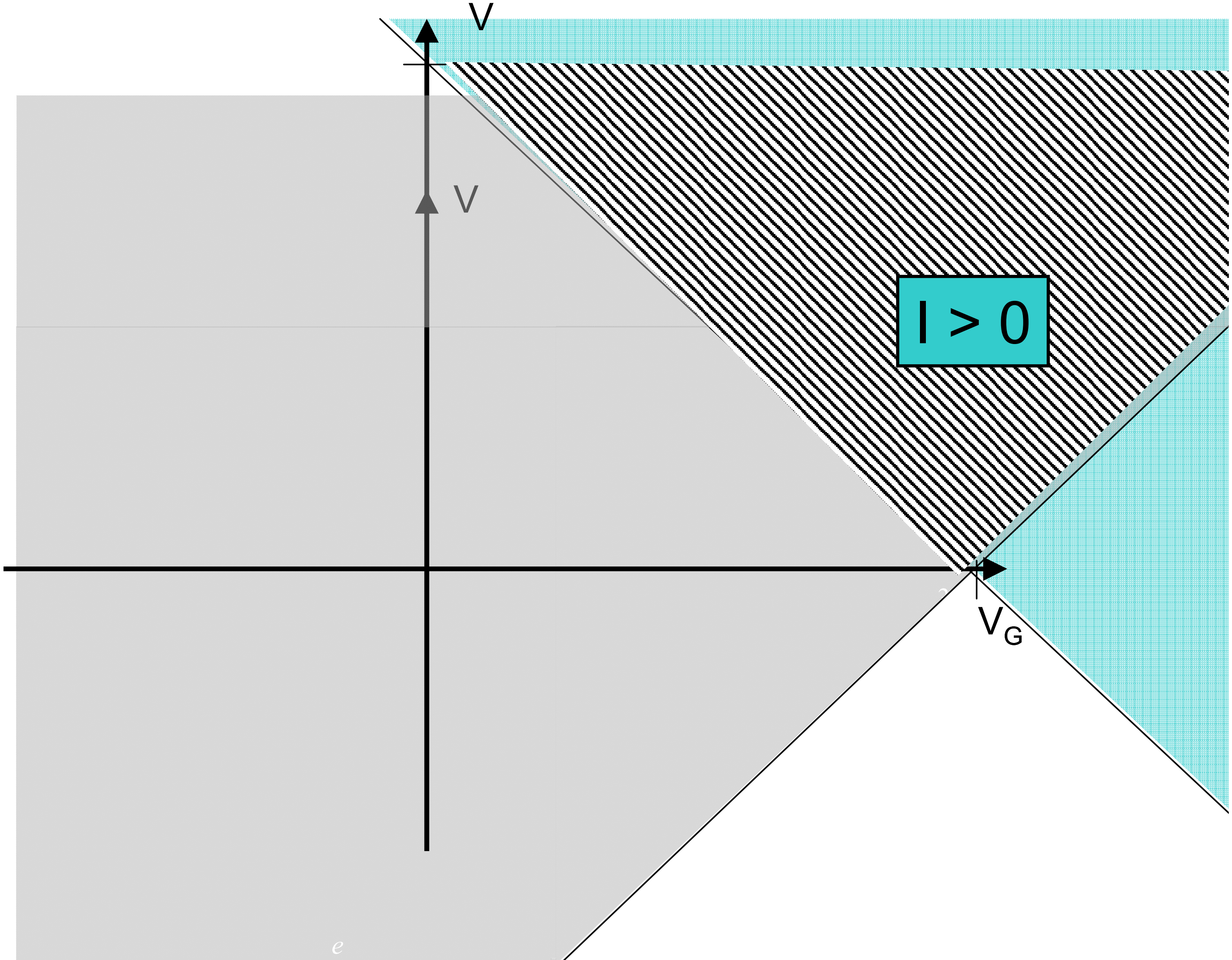
e-

Current?

$$V > \frac{C_g}{(C_2 + C_G)} V_G - \frac{3e}{2(C_2 + C_G)}$$

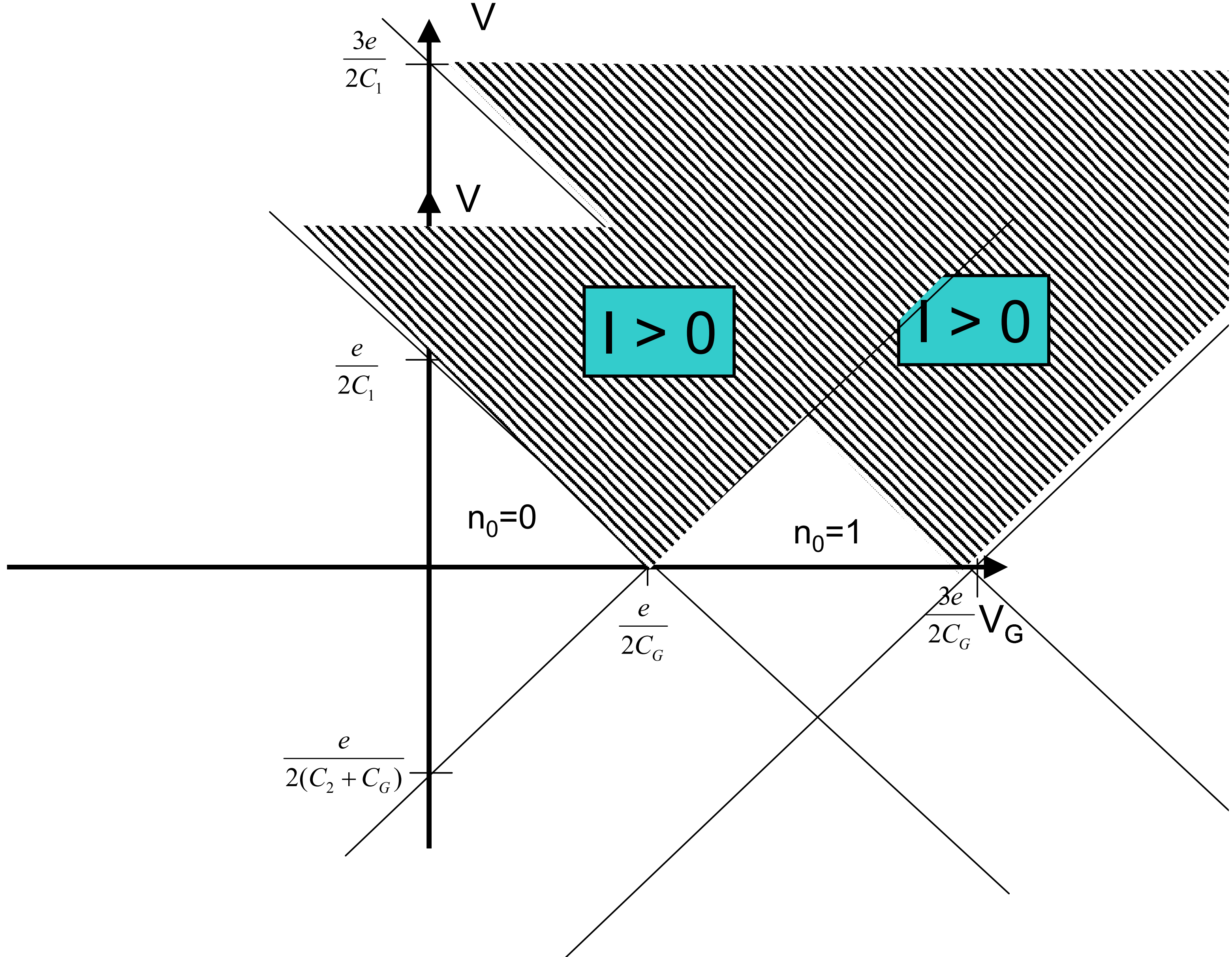


Current?

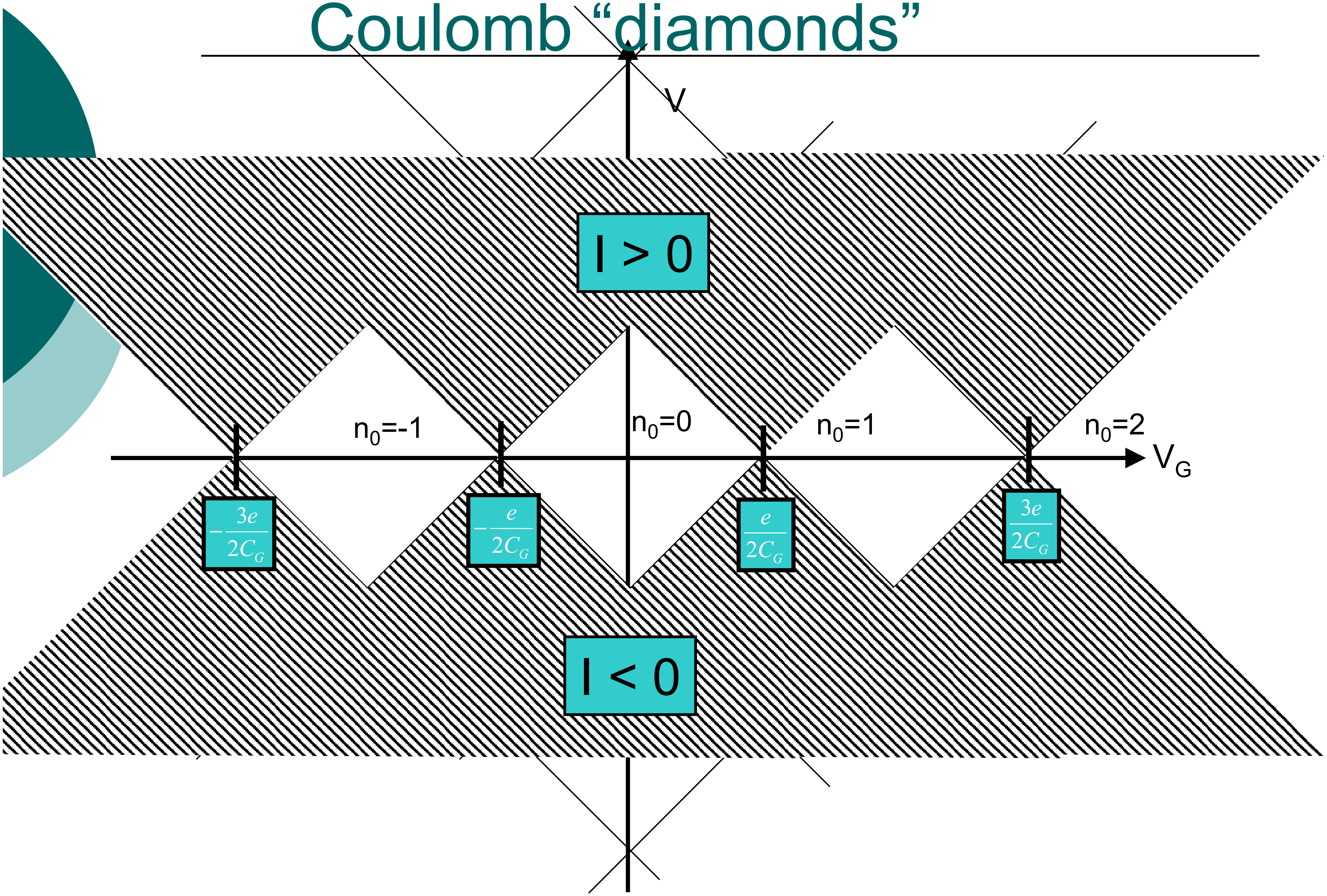


Current?

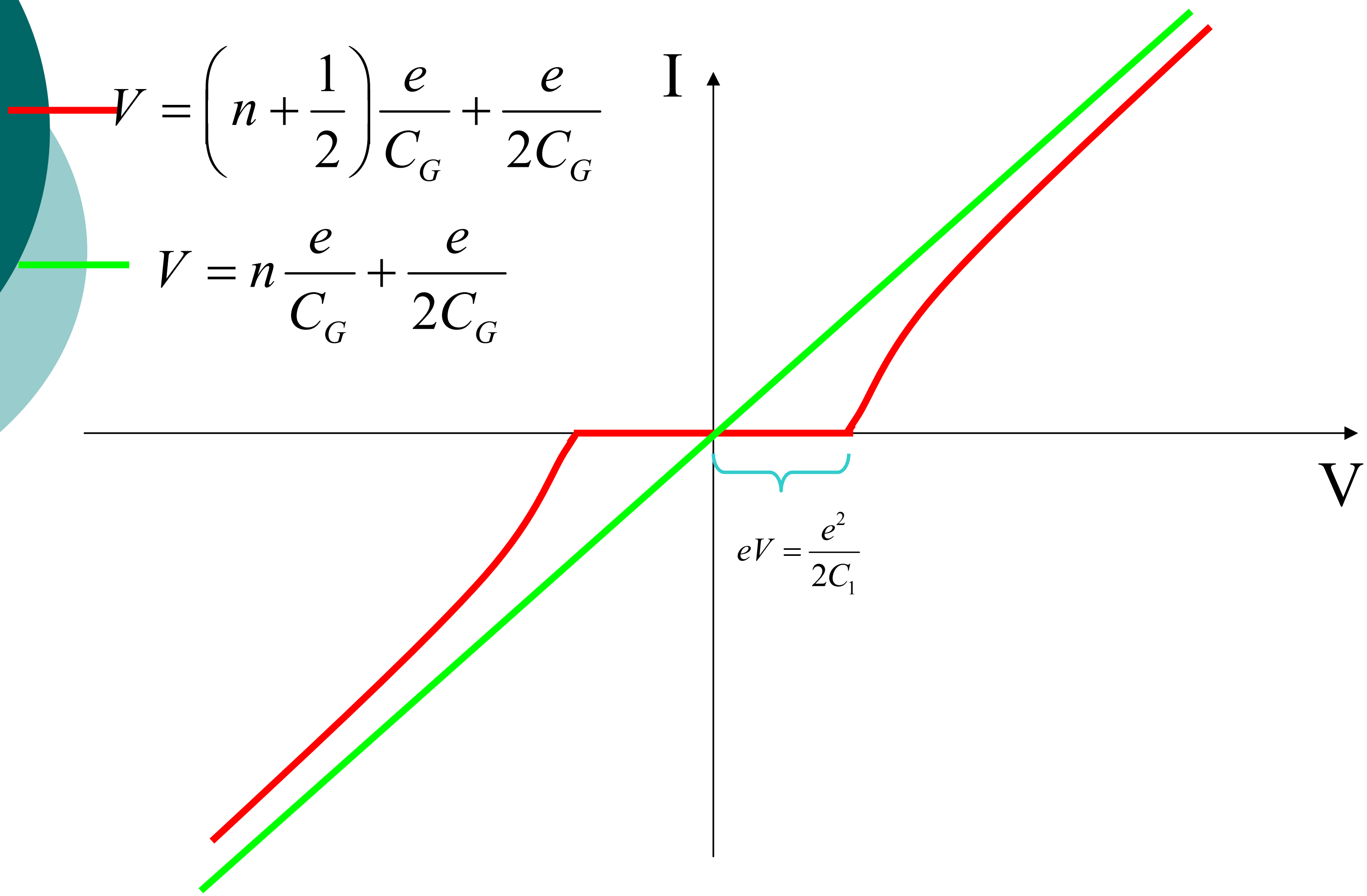
Current?



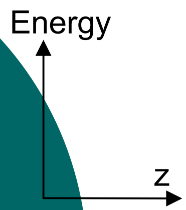
Coulomb "diamonds"

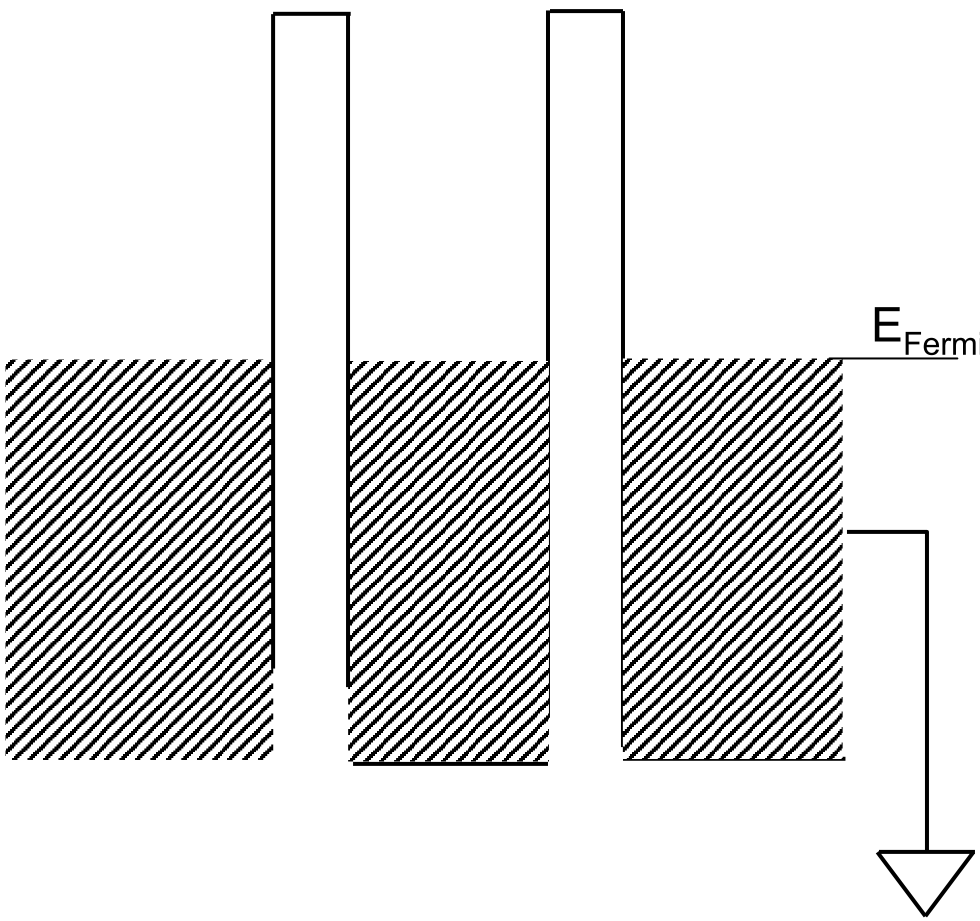


SET I-V curve vs. gate voltage

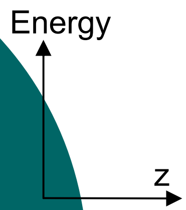


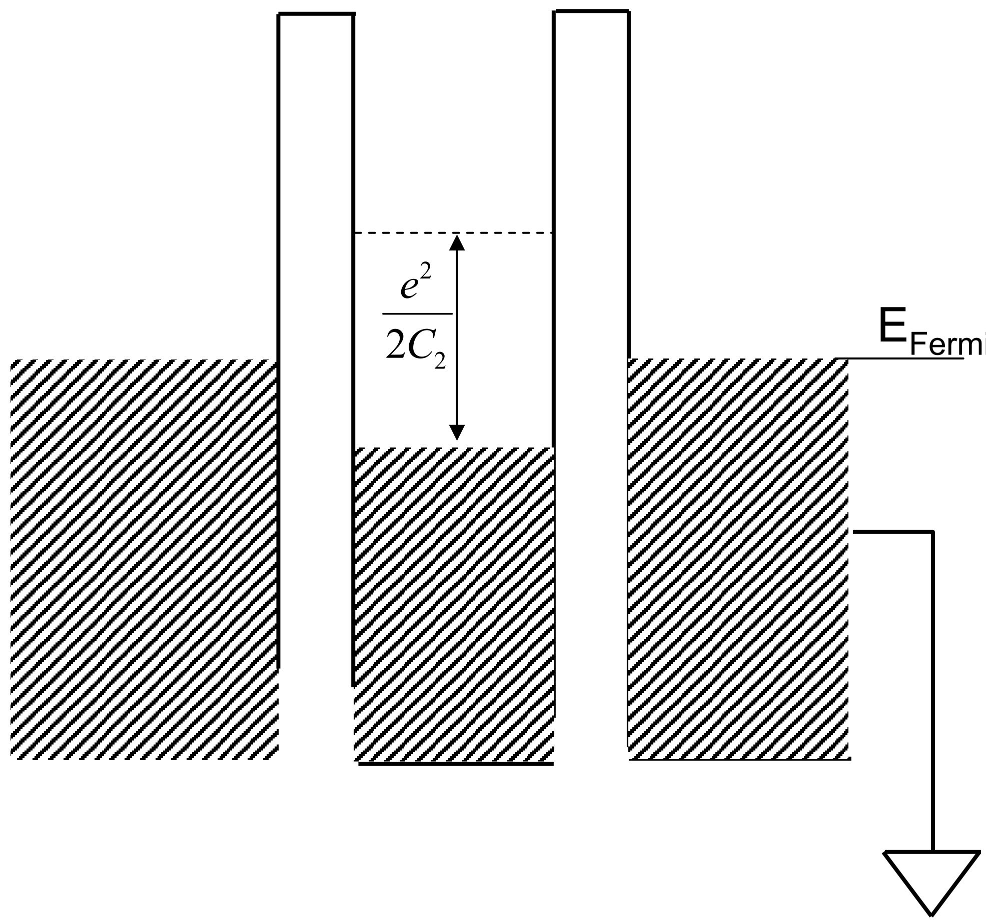
Band diagram



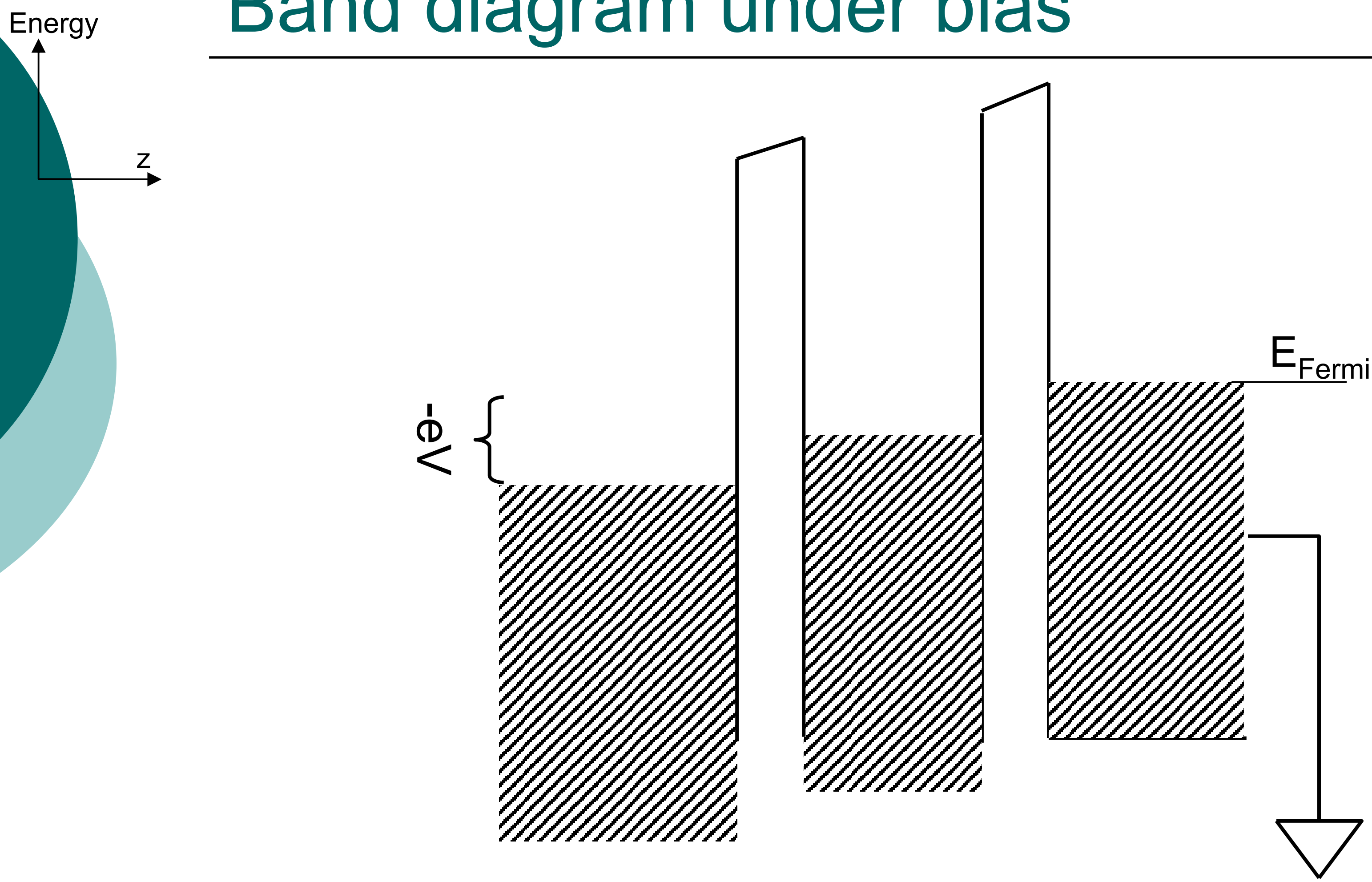


Band diagram with Coulomb "gap"

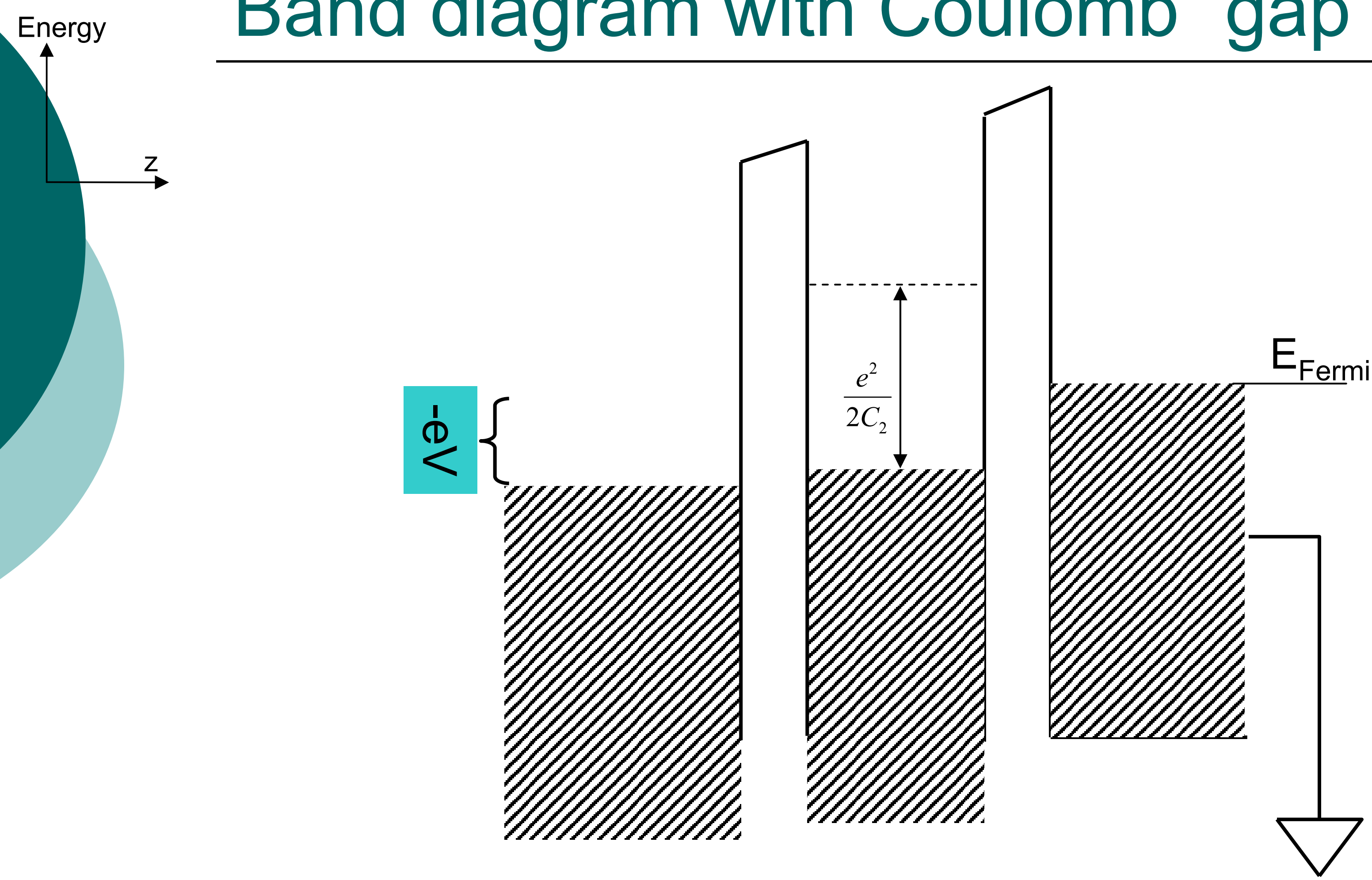




Band diagram under bias



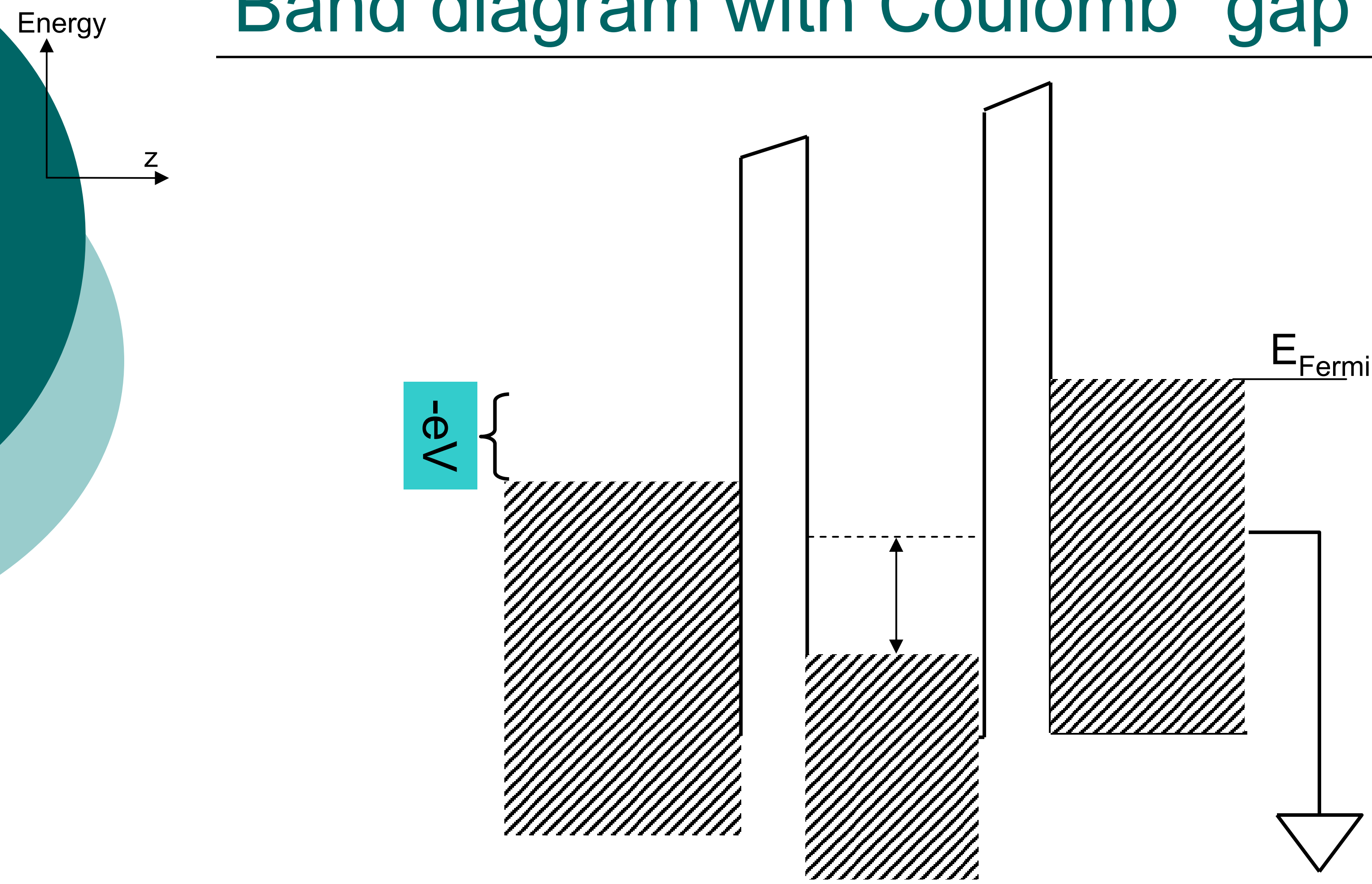
Band diagram with Coulomb “gap”



Gate voltage like a “plunger”

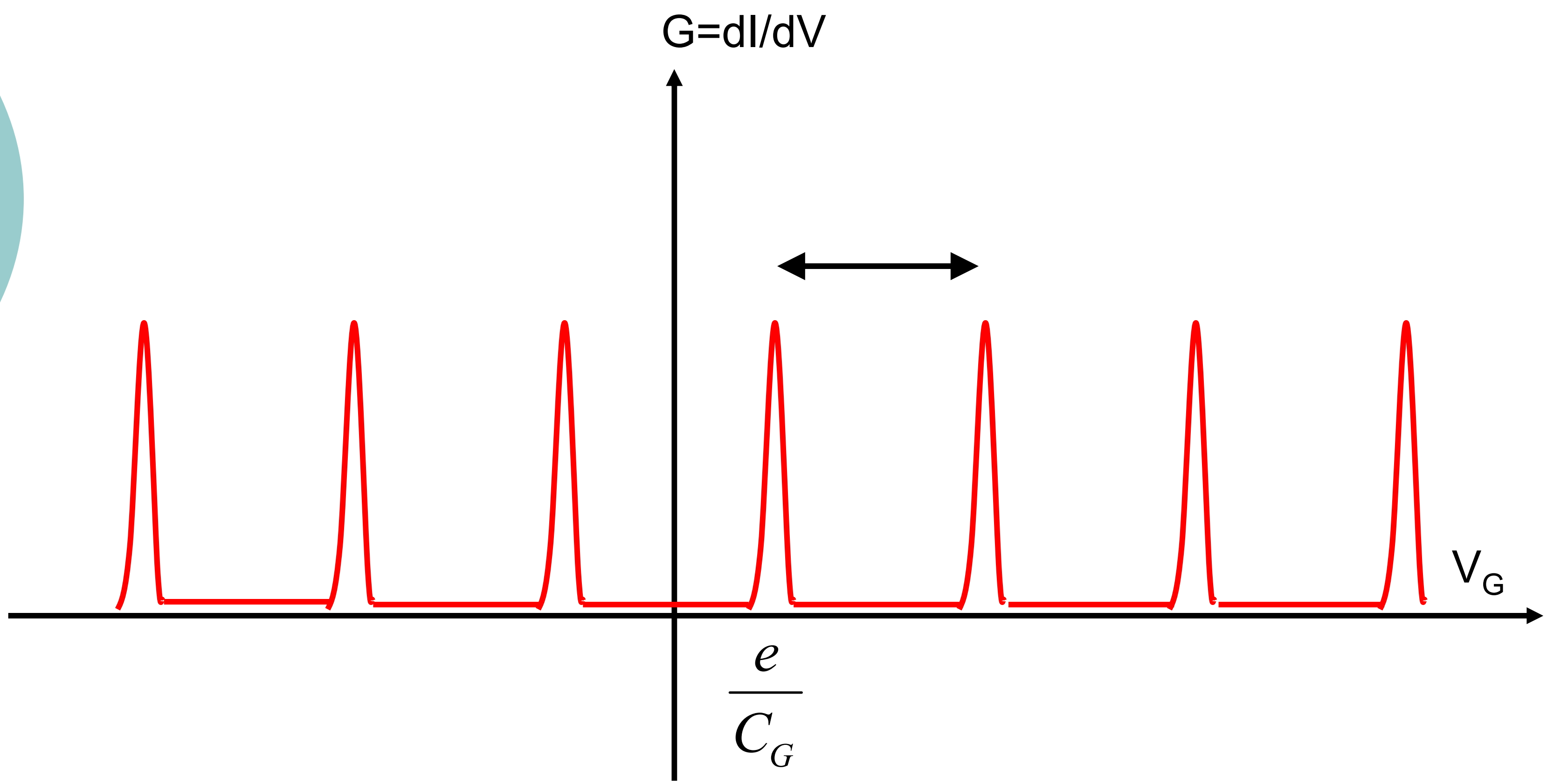
Moves island up/down by e^2/V_G

Band diagram with Coulomb "gap"



Gate voltage like a "plunger"
Moves island up/down by e^2/V_G

Zero bias conductance



Width of peaks set by temperature.

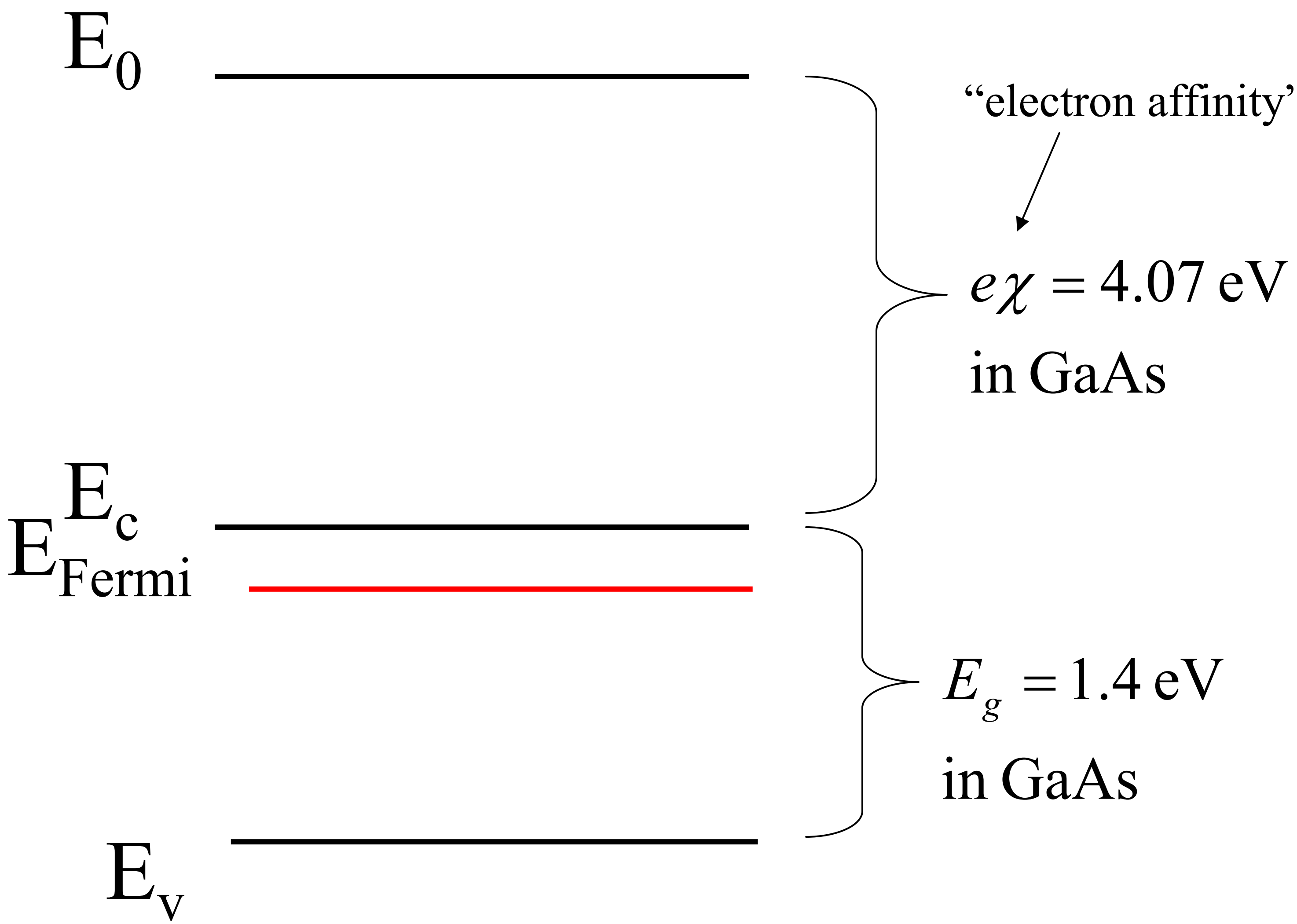
Lecture 9

2 dimensional electron gas (2DEG)

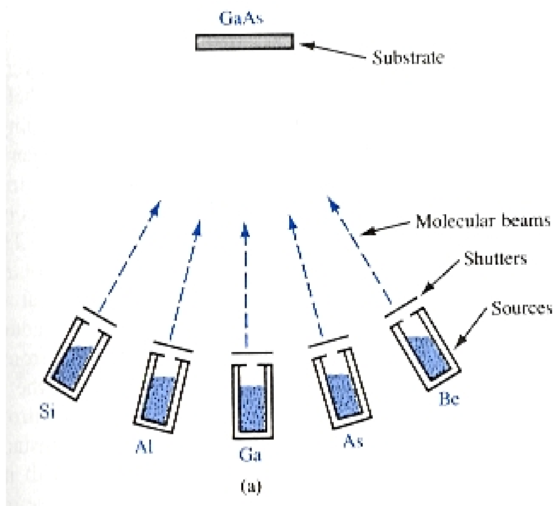
Readings this lecture covers

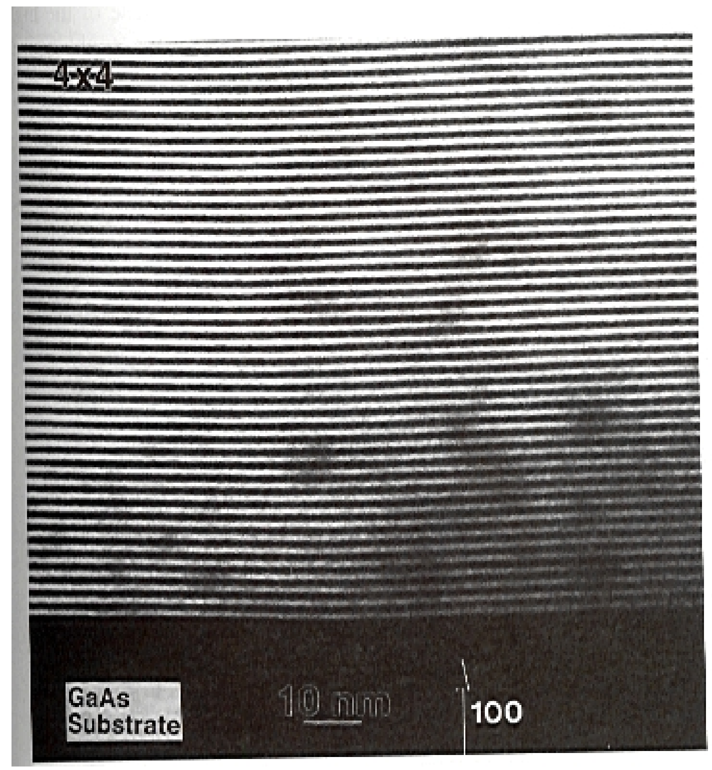
- Ferry, pp. 23-39
- Hanson, pp. 118-123

Vacuum level



MBE

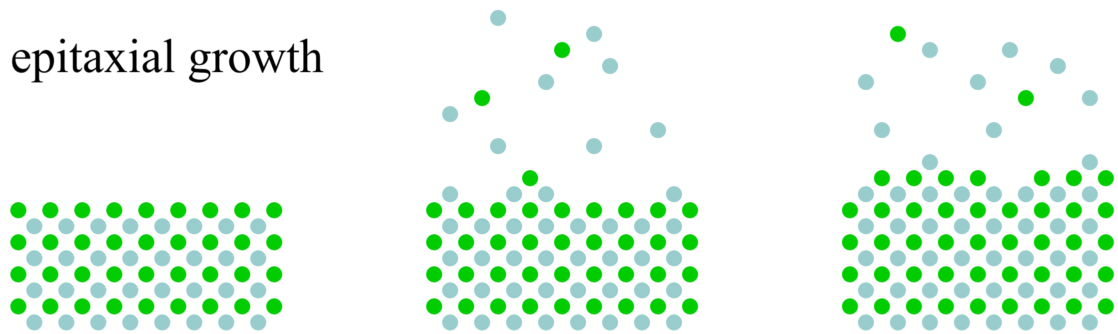


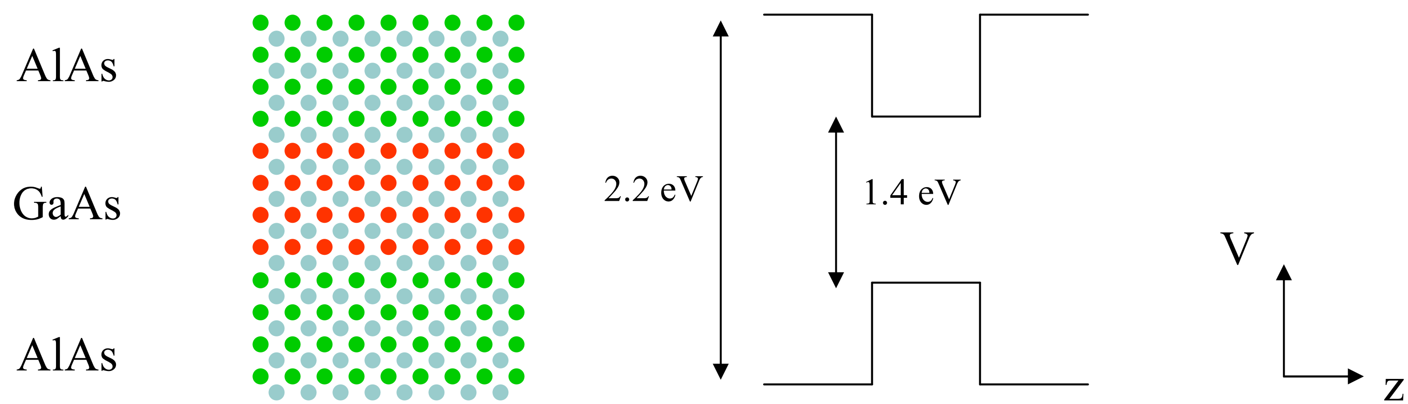


4 atom per layer!

(From Streetman, Solid State Electronic Devices)

MBE





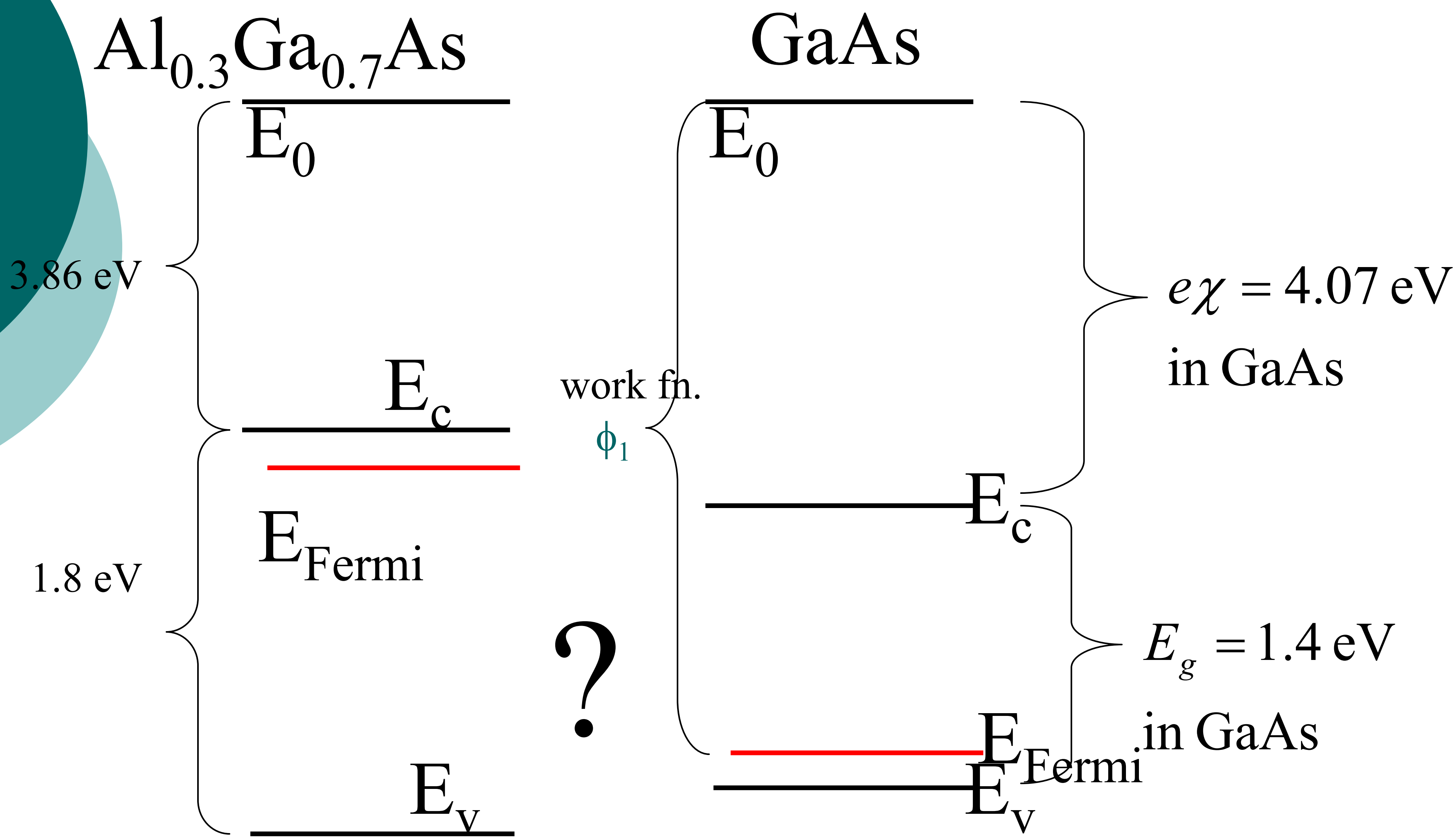
Also InP, InGaAs, InAlAs, InGaAsP …

Picture adapted from M. Lilly.

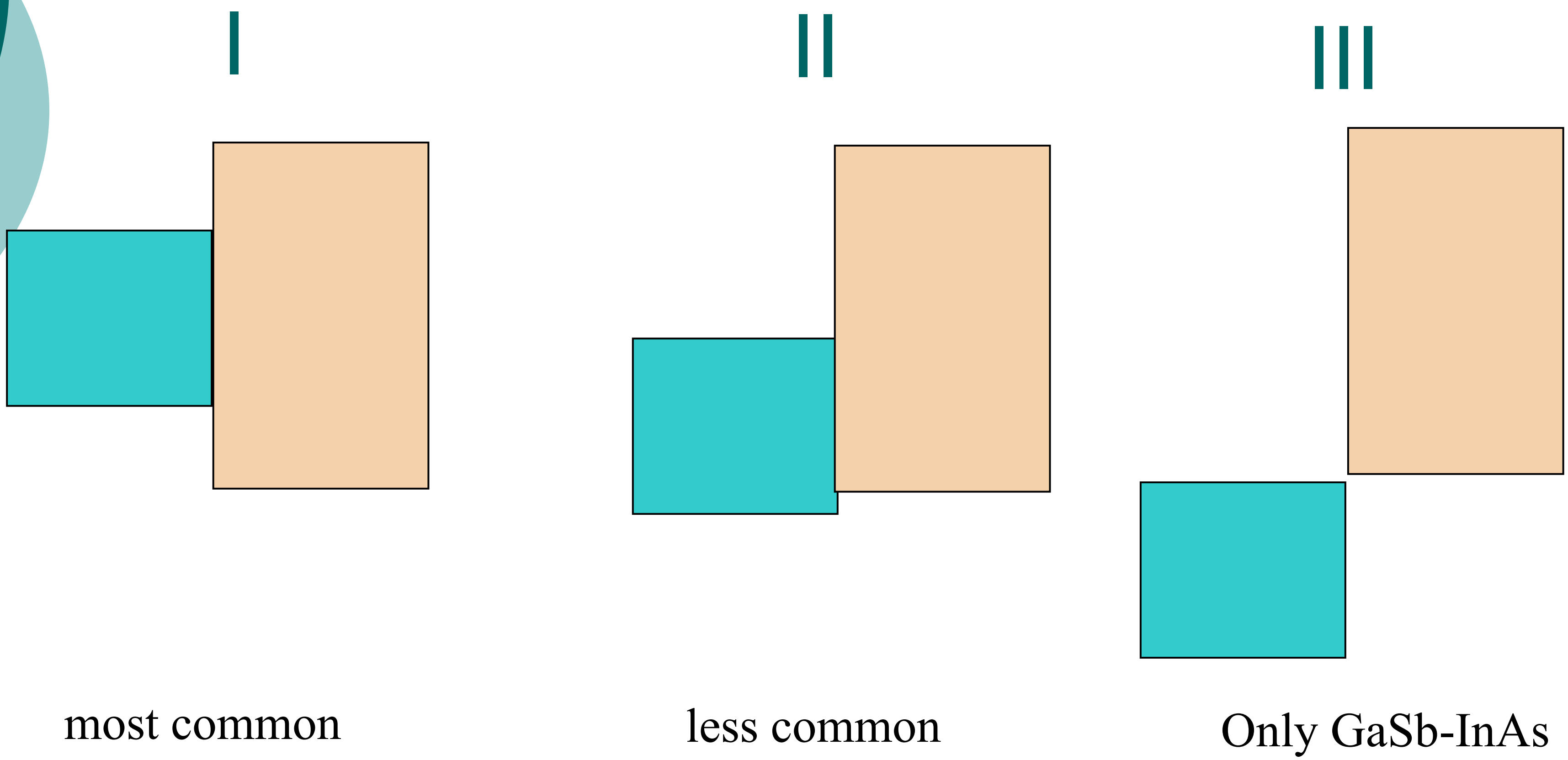
Heterojunction band diagrams

- Determine $\Delta E_c = \chi_1 - \chi_2$
  (Vacuum levels line up)
- Determine $\Delta E_v = \Delta E_g - \Delta E_c$
- There will be some charge transfer and built-in electric field/voltage as in p-n homojunction
- Built in voltage $\phi = \phi_1 - \phi_2$
- Draw the diagram (You will in HW#2)

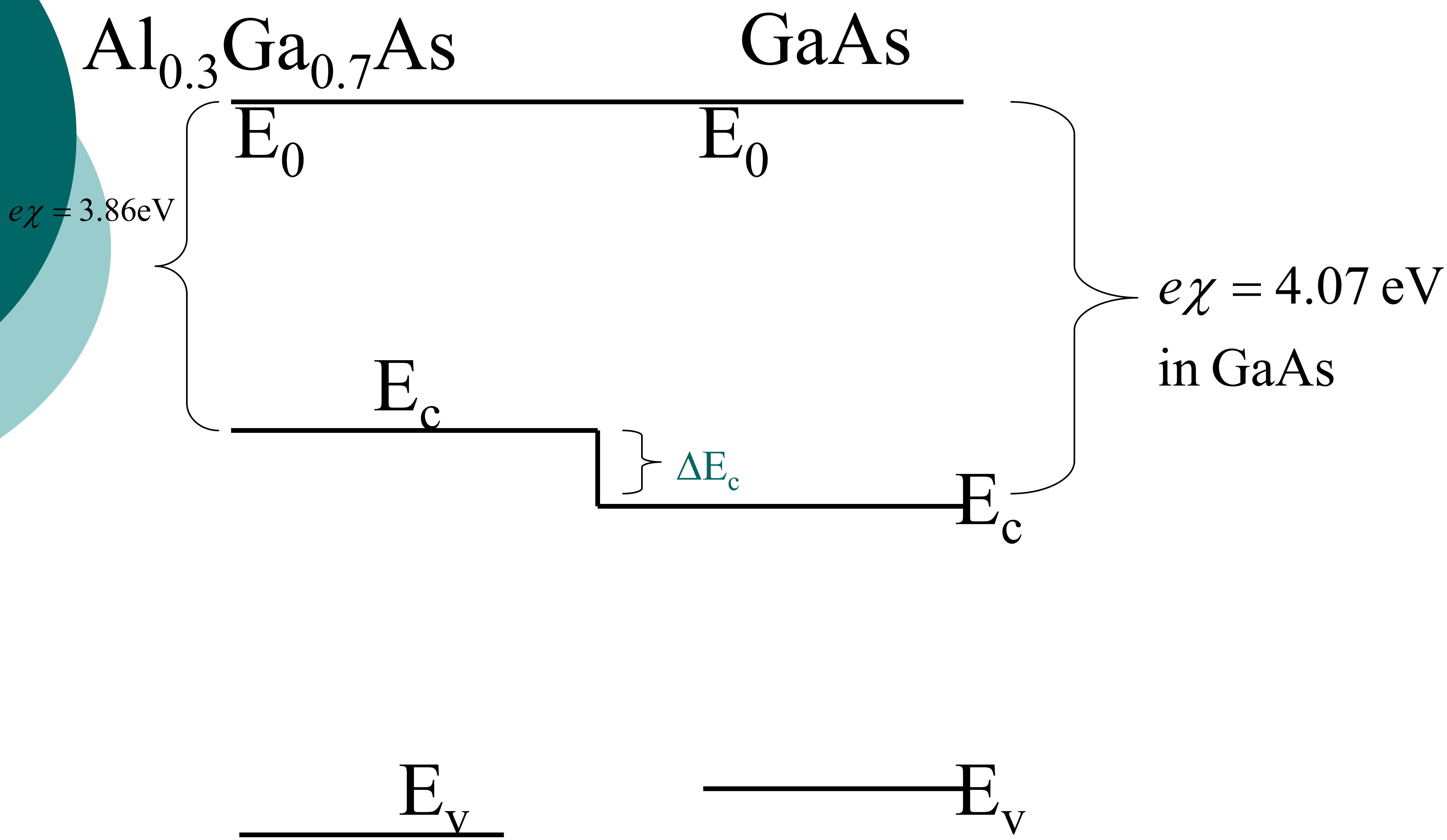
Intrinsic heterojunction



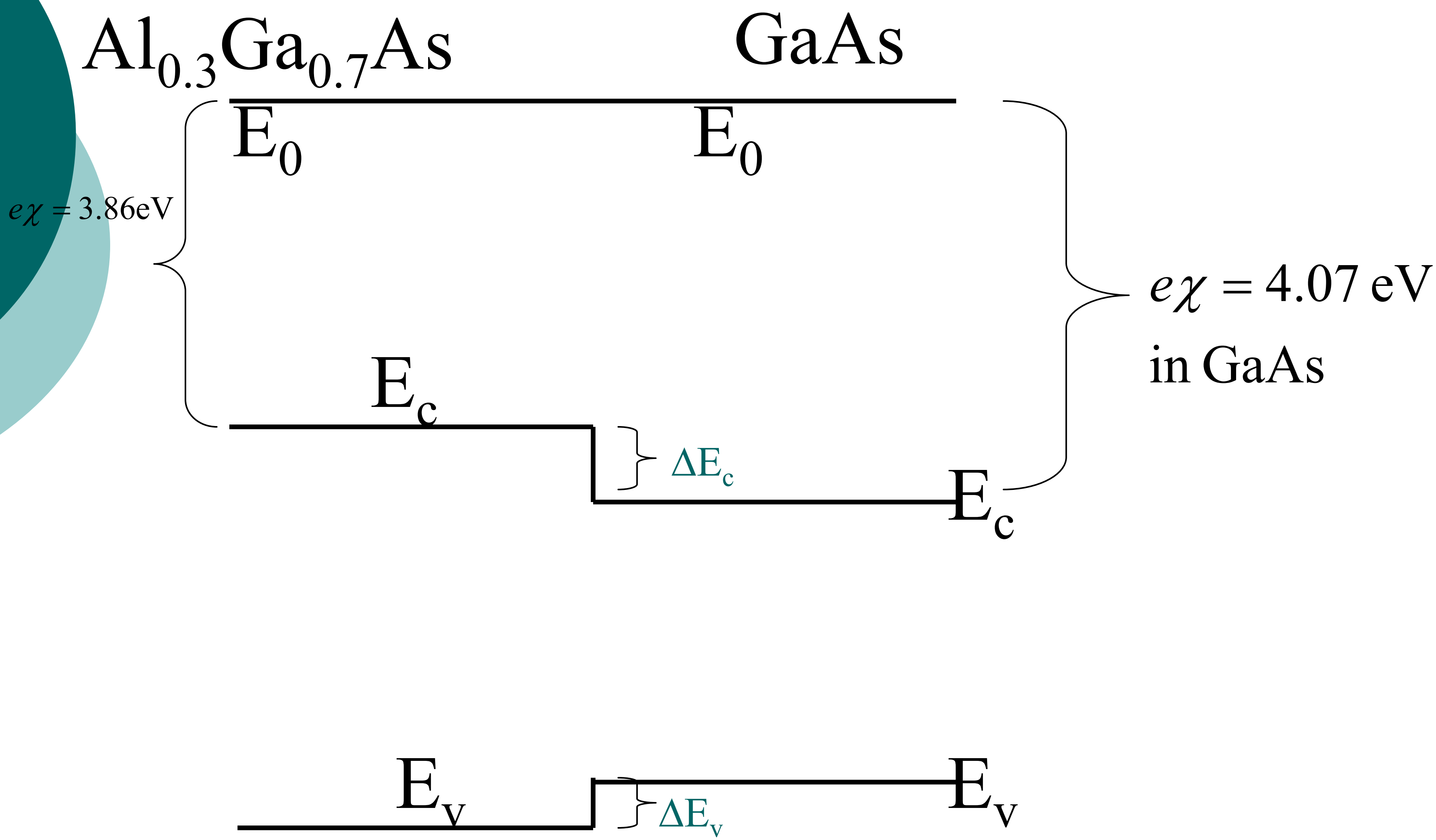
Types



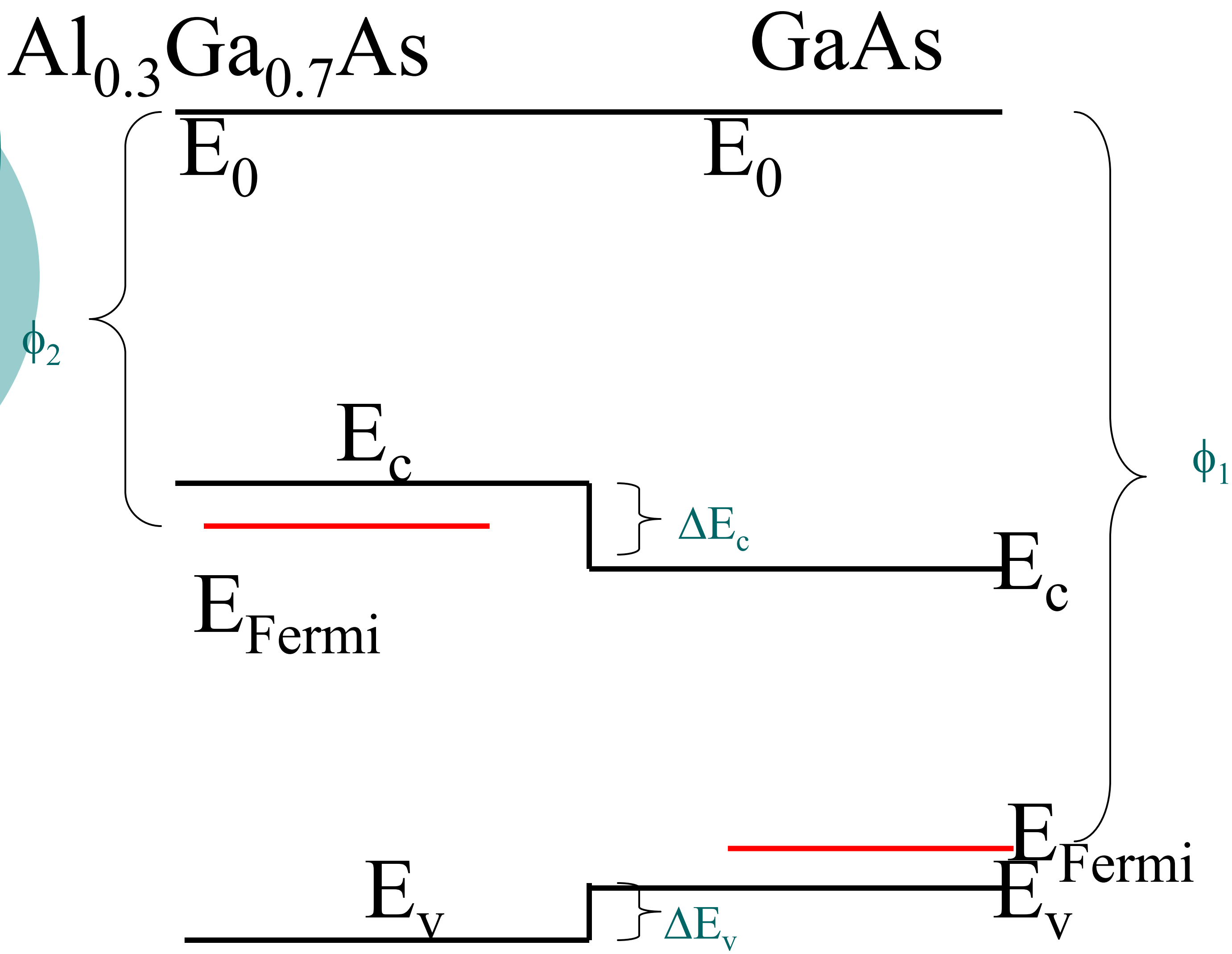
Determine $\Delta E_c = \chi_1 - \chi_2$



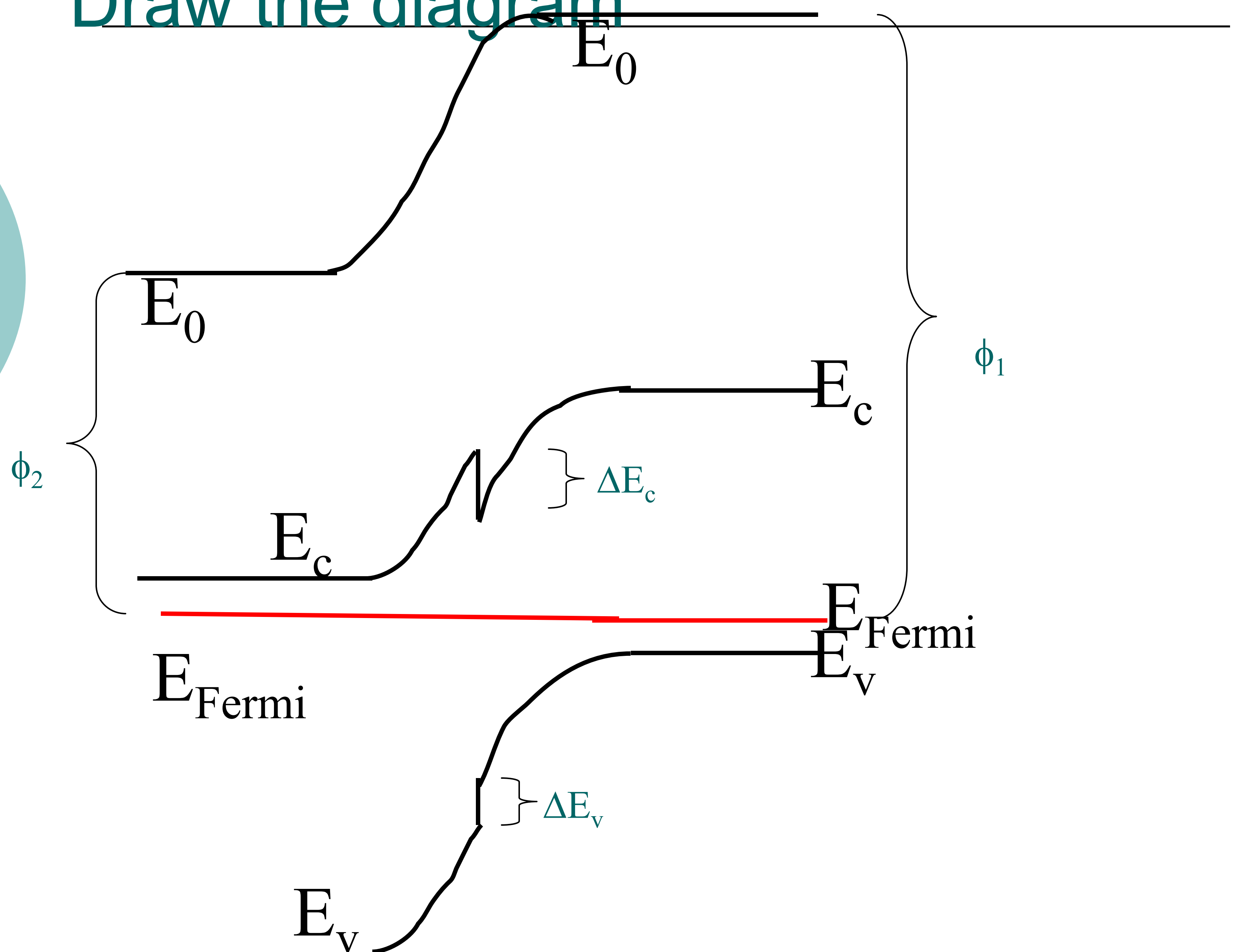
Determine $\Delta E_v = \Delta E_g - \Delta E_c$



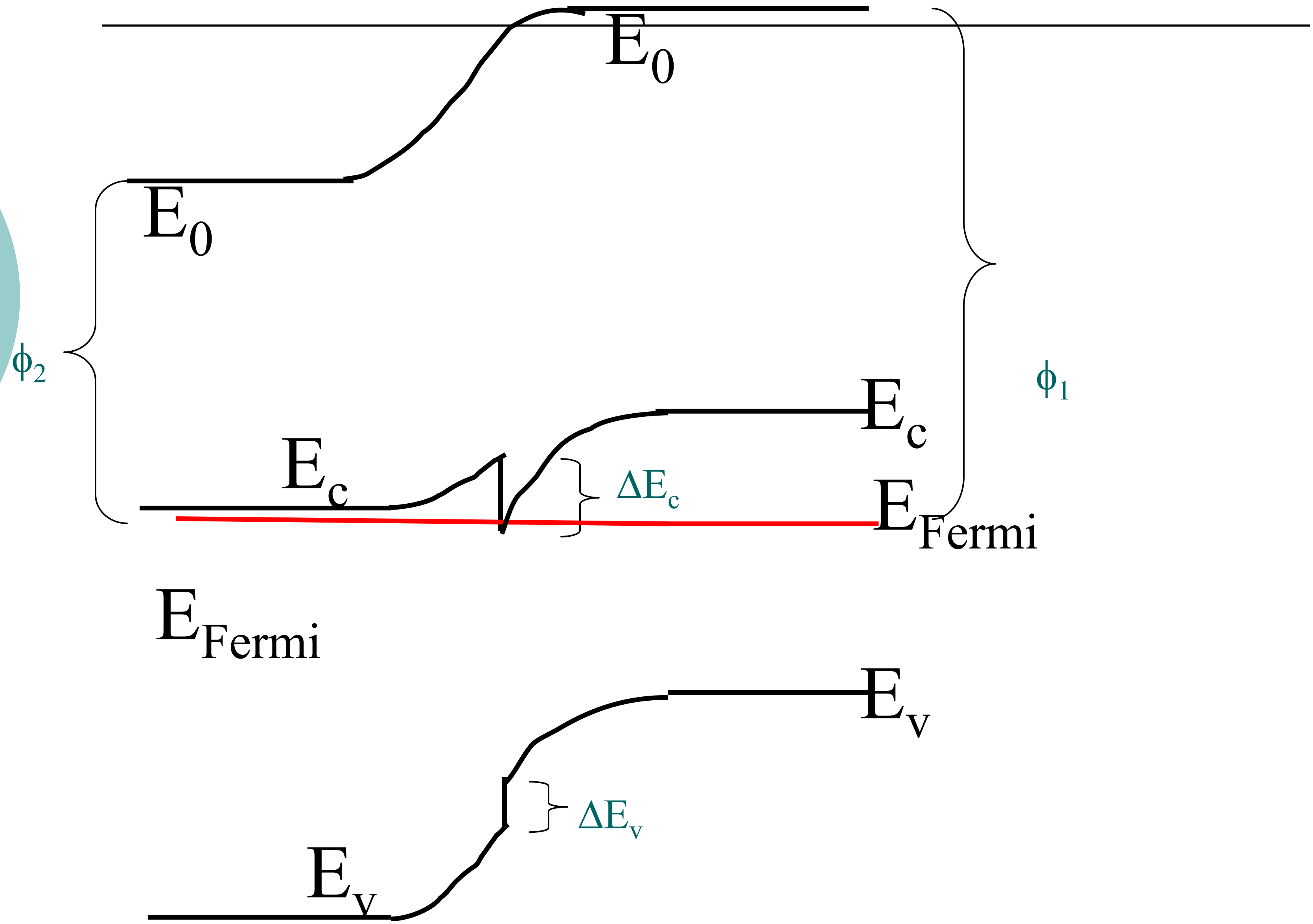
Built in voltage $\phi = \phi_1 - \phi_2$



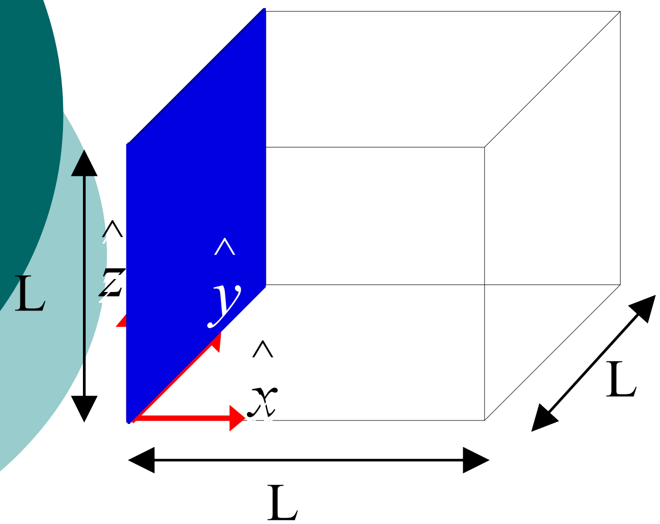
Draw the diagram



2DEG



Particle in a box:



$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or "quantum states"

Particle in a box

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$$k_{n_x} = n_x \pi / L$$

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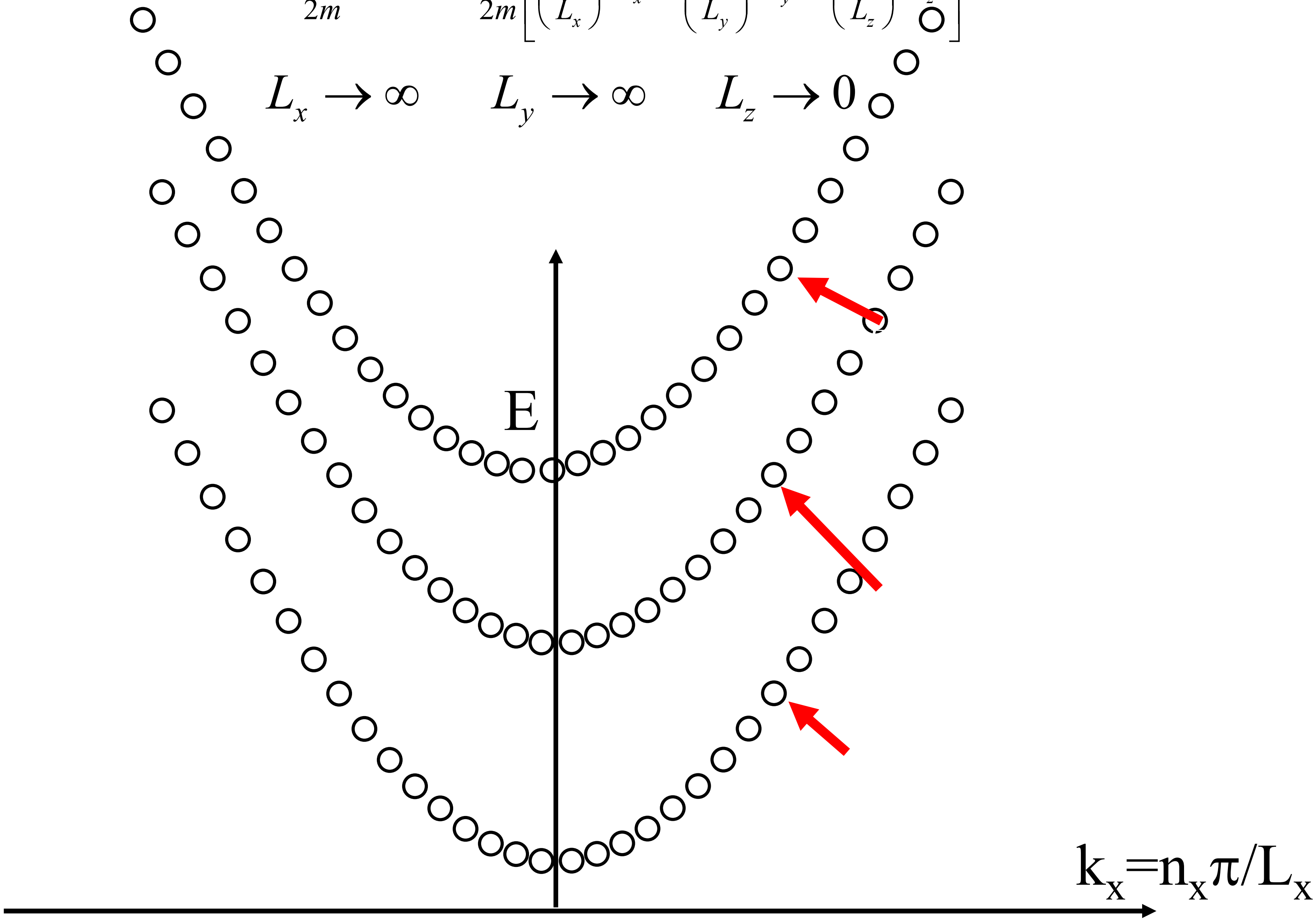
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

These are the allowed energy levels, or “quantum states”

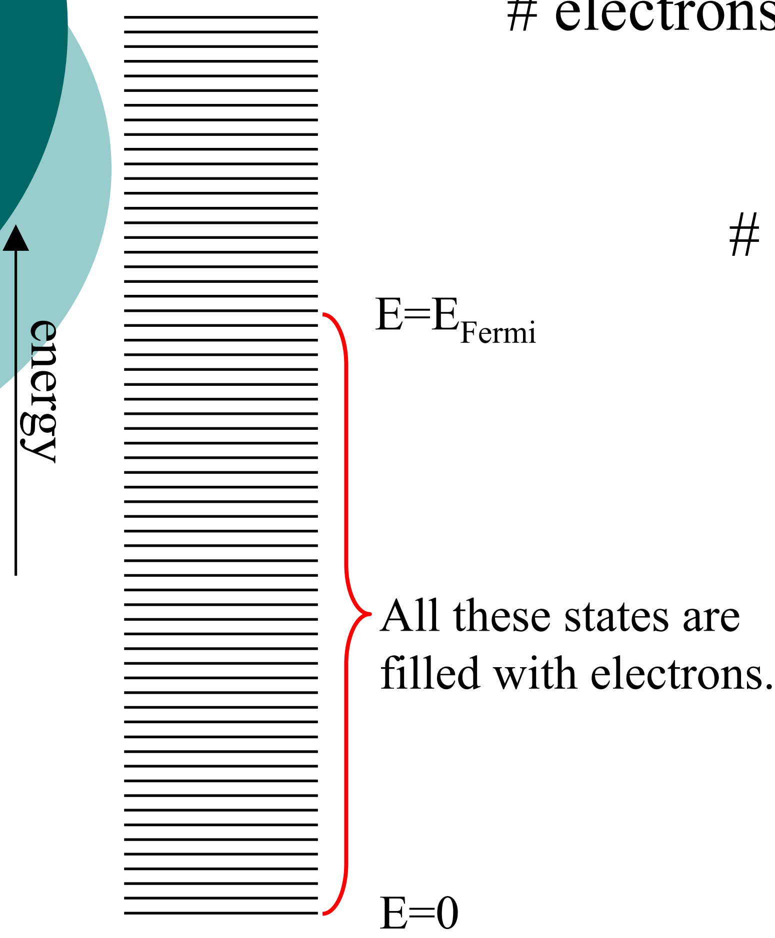
Limit:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$L_x \to \infty \quad L_y \to \infty \quad L_z \to 0$



Fermi energy in 3 dimensions



$$\#\text{ electrons} = \int_0^{E_f} N_{\mathrm{E}}\,\mathrm{dE} = \int_0^{E_f} L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \cdot E^{1/2} dE$$

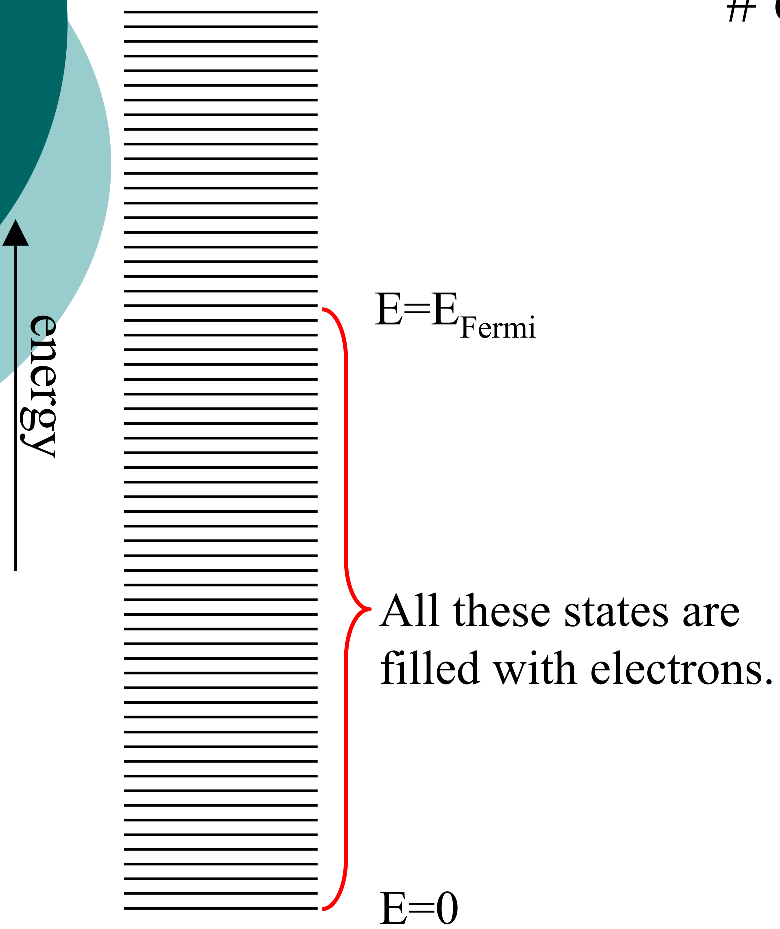
$$\#\text{ electrons} = L^3 \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \frac{2}{3} E_f^{3/2}$$

$$\Rightarrow E_f = \frac{\hbar^2 3^{2/3} \pi^{4/3}}{2m} \left(\frac{\#\text{ electrons}}{L^3} \right)^{2/3}$$

In a typical metal, L ~ 0.1 nm.
E_f ~ 10 eV

Fermi energy in 2 dimensions

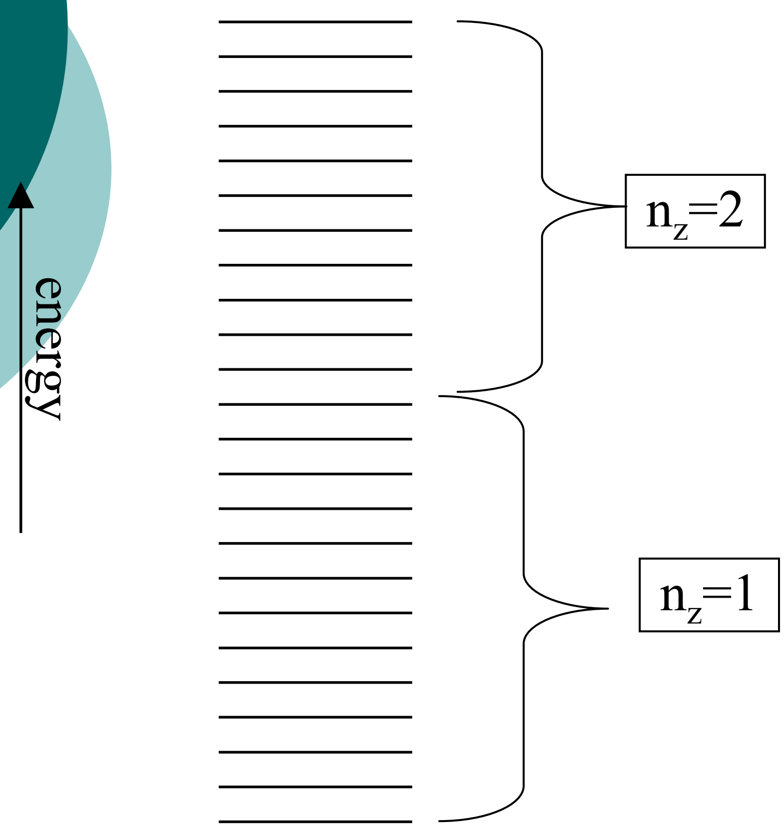
$$\#\text{ electrons} = \int_0^{E_f} N_E \, f(E)dE = ?$$



Need to evaluate integral numerically,
just as in 3d.

Energy spectrum of free particles

2 dimensions

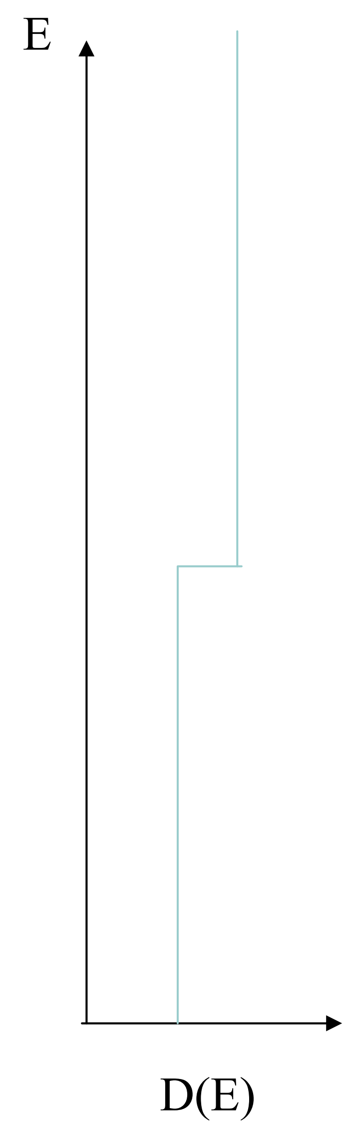


In typical HEMT,
filled states up to here.

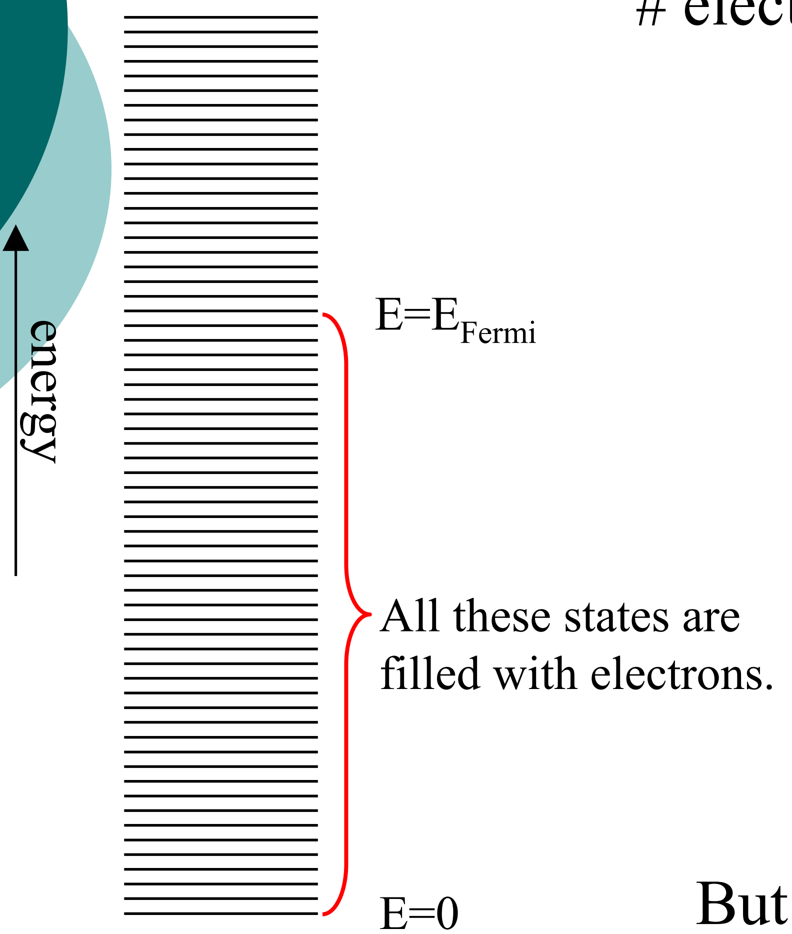
Etc.

$n_x=2, n_y=1$ $n_x=1, n_y=2$

$n_x=1, n_y=1$



Fermi energy in 2d



$$\#\ \text{electrons} = \int_0^{E_f} N_{\mathrm{E}}\, \mathrm{dE} = \int_0^{E_f} L^2 \frac{m}{\pi\hbar} dE$$

$$\#\ \text{electrons} = L^2 \frac{m}{\pi\hbar} E_f$$

$$\Rightarrow E_f = \frac{\hbar\pi}{m}\left(\frac{\#\ \text{electrons}}{L^2}\right)$$

In GaAs, 10^{11}cm^{-2} gives

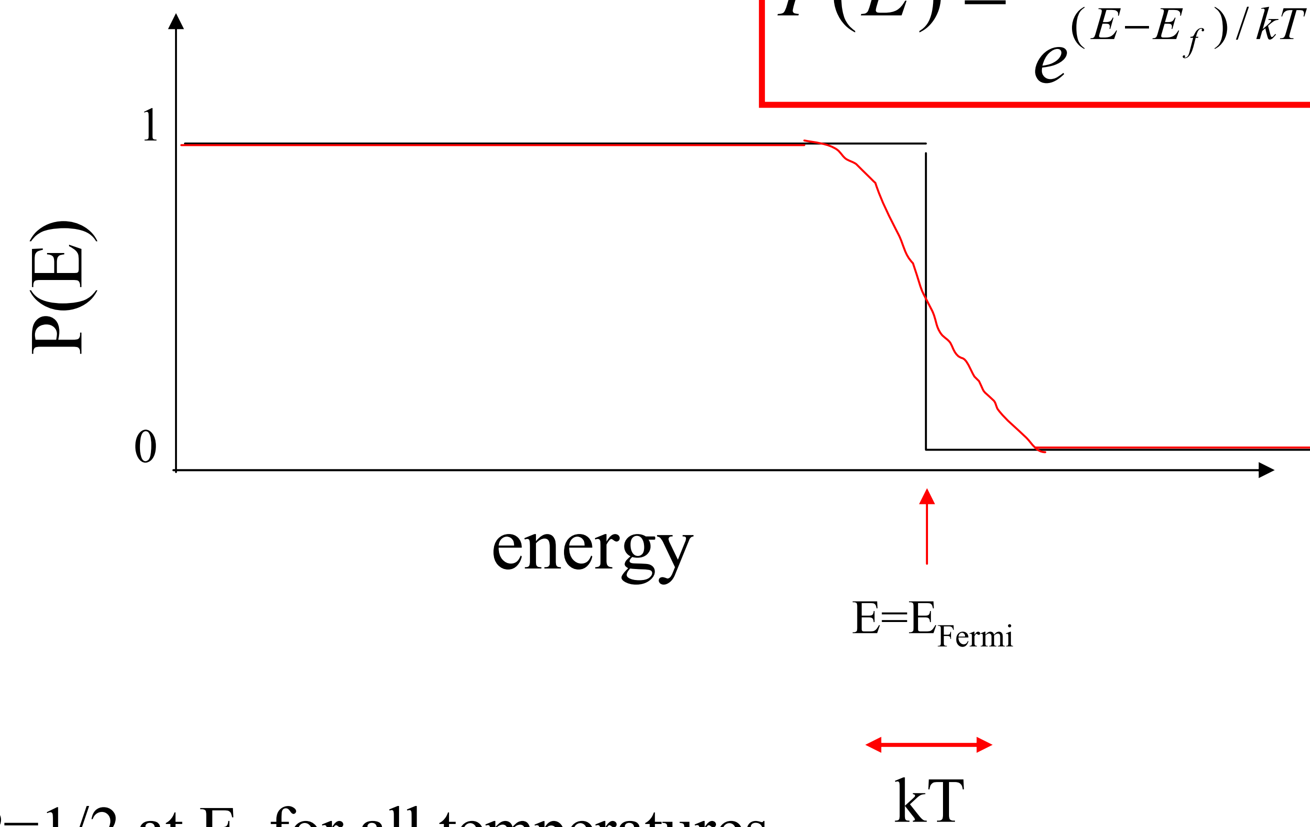
$E_f \sim$ meV

But 10^{12}cm^{-2} gives more than first subband.

Discuss "subband", how above integral gets modified.

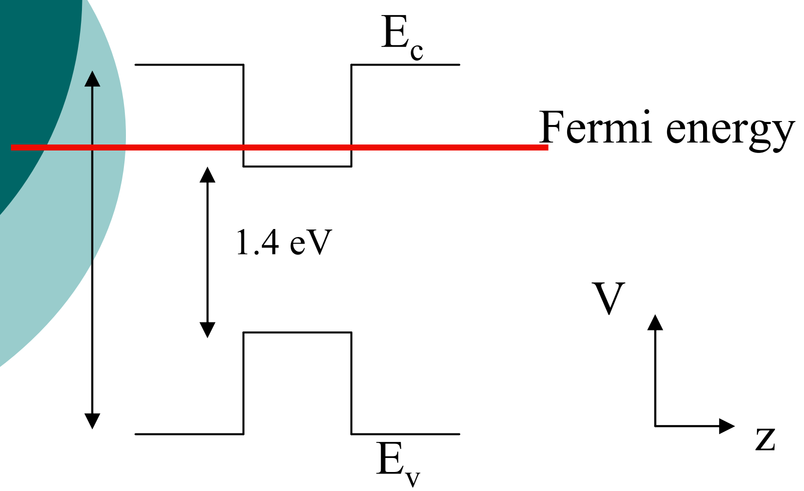
Fermi-Dirac

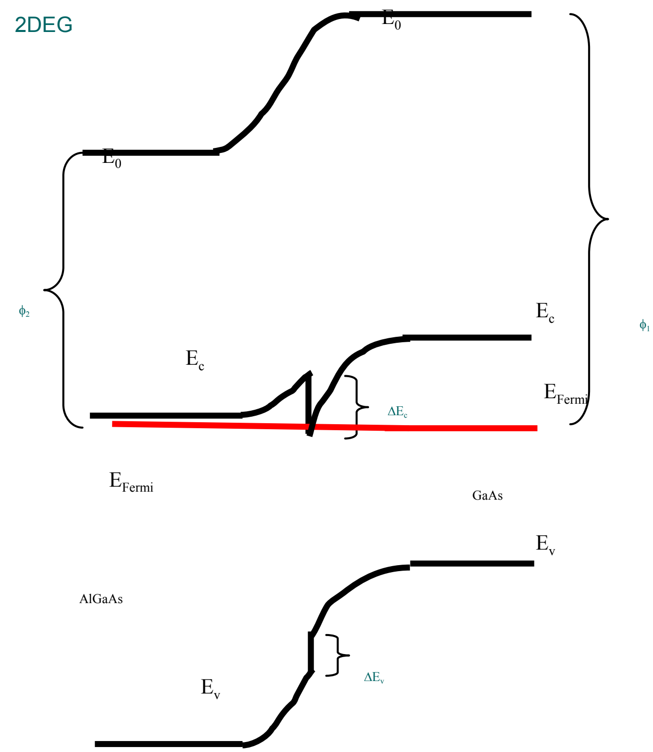
$$P(E) = \frac{1}{e^{(E-E_f)/kT} + 1}$$



P=1/2 at E_f for all temperatures.

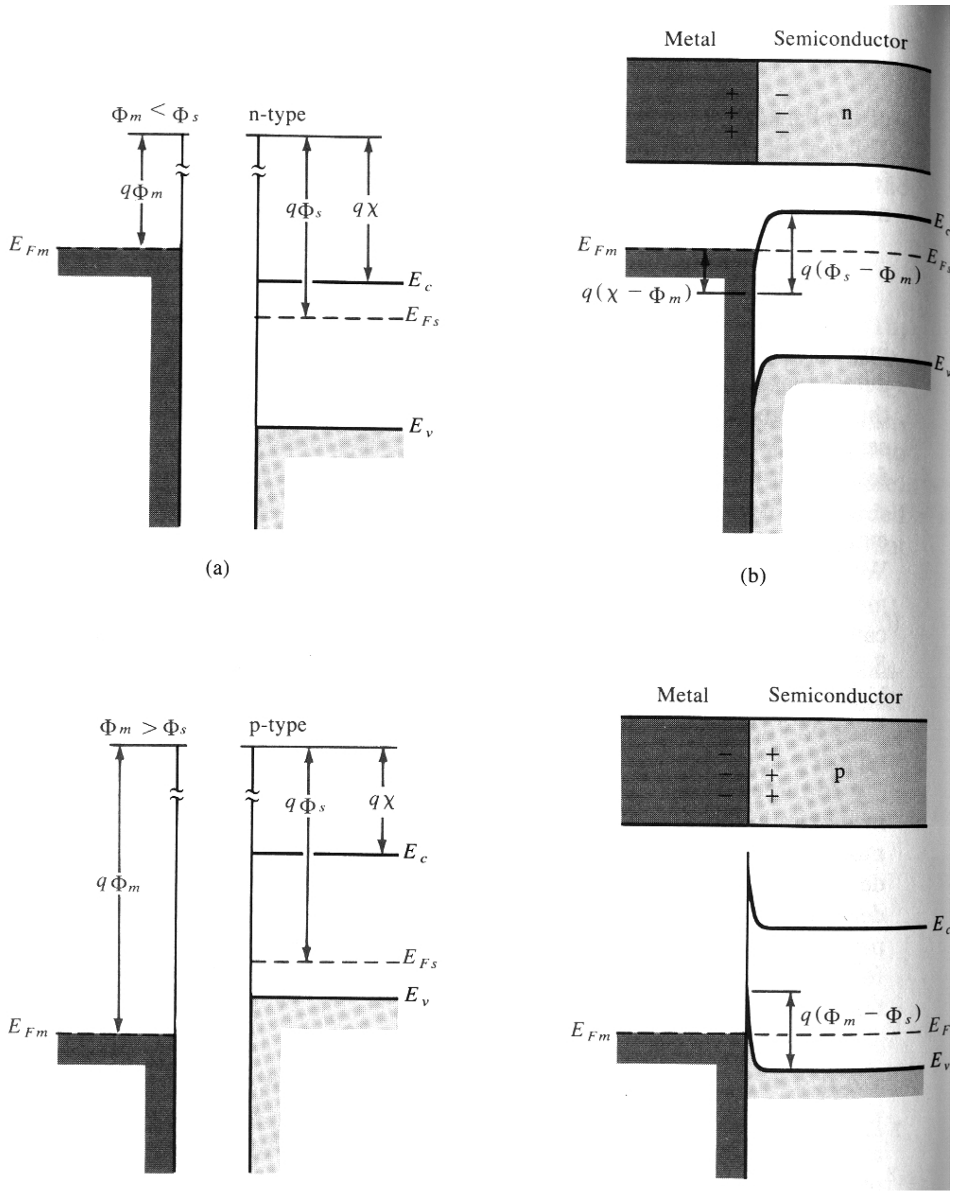
Triangle vs. square well:





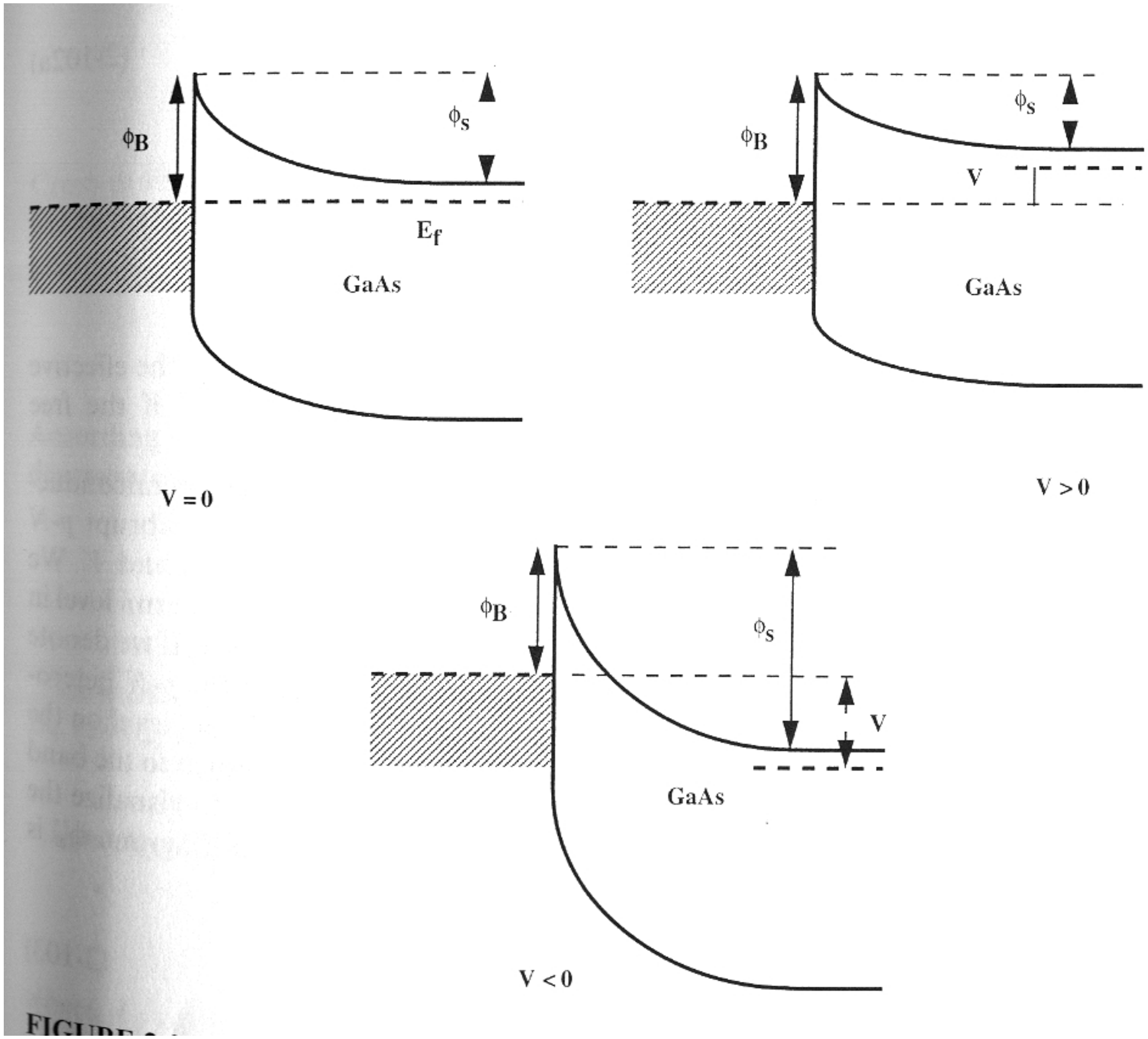
(Draw both bound states on board.
In particular discuss figure 5.21 from Liu.)
Also discuss shallow vs. wide wells on board.
(Typically 100 angstroms works.)
Discuss setback doping, mobility (time permitting).

Schottky barriers



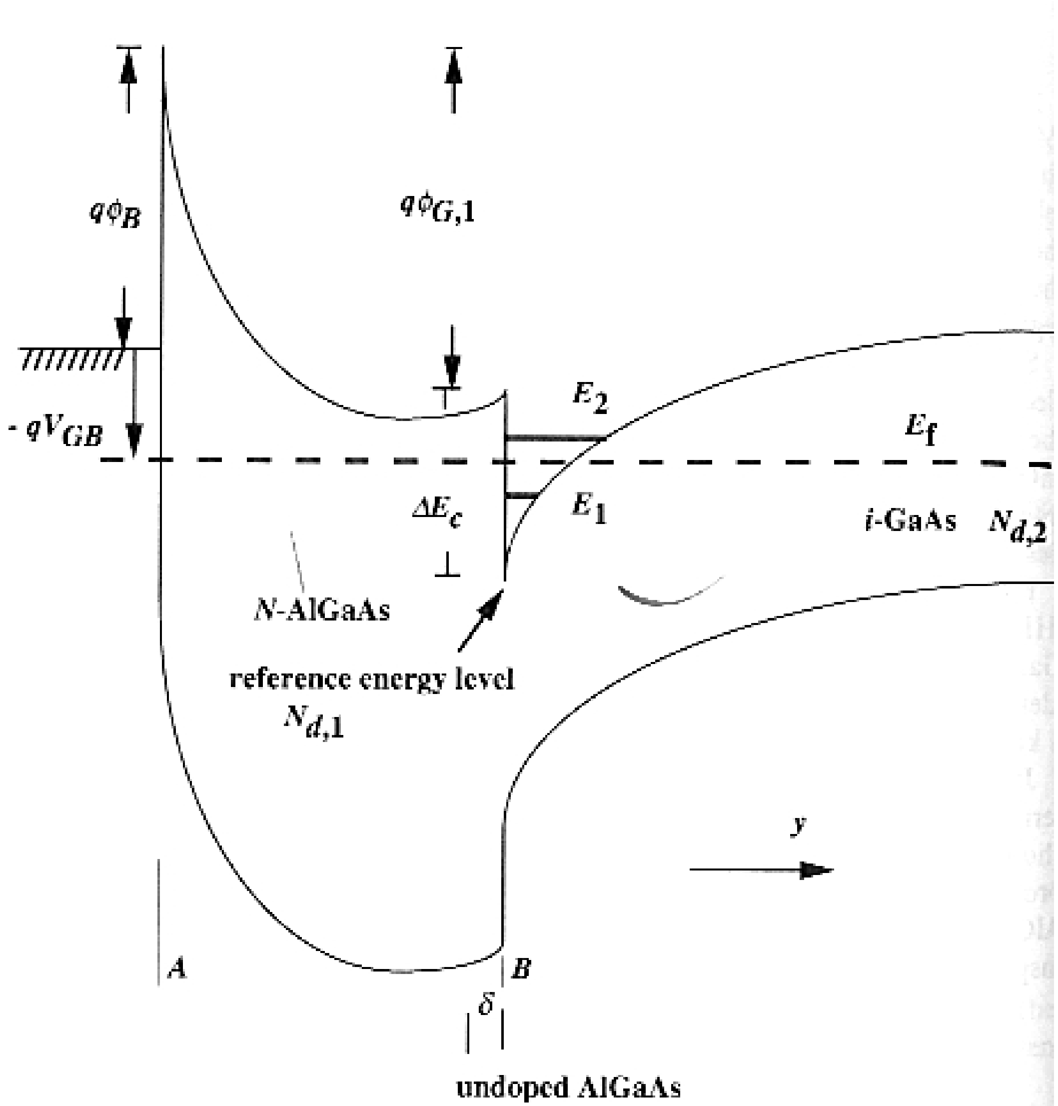
From Streetman

Schottky barriers



From Liu

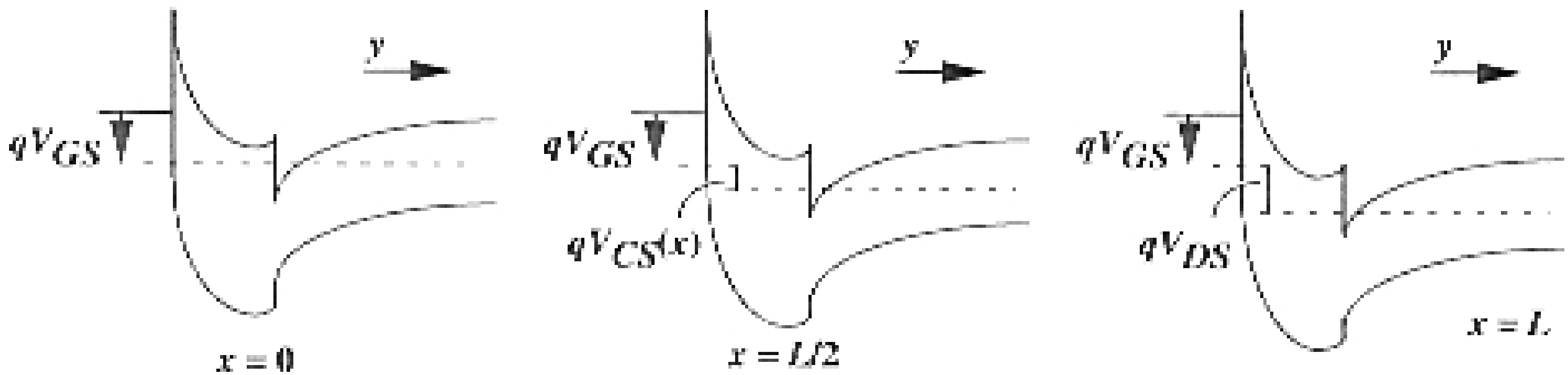
Band diagram



- Bias changes Fermi level
- hence density.
- Can "pinchoff"

From Liu.

Vary gate voltage



Changes Fermi energy which changes density.

(Draw better pictures on board.)

From Liu.

n_s vs E_f

After all that mumbo-jumbo, we know it is complicated.
We approximate it many times as:

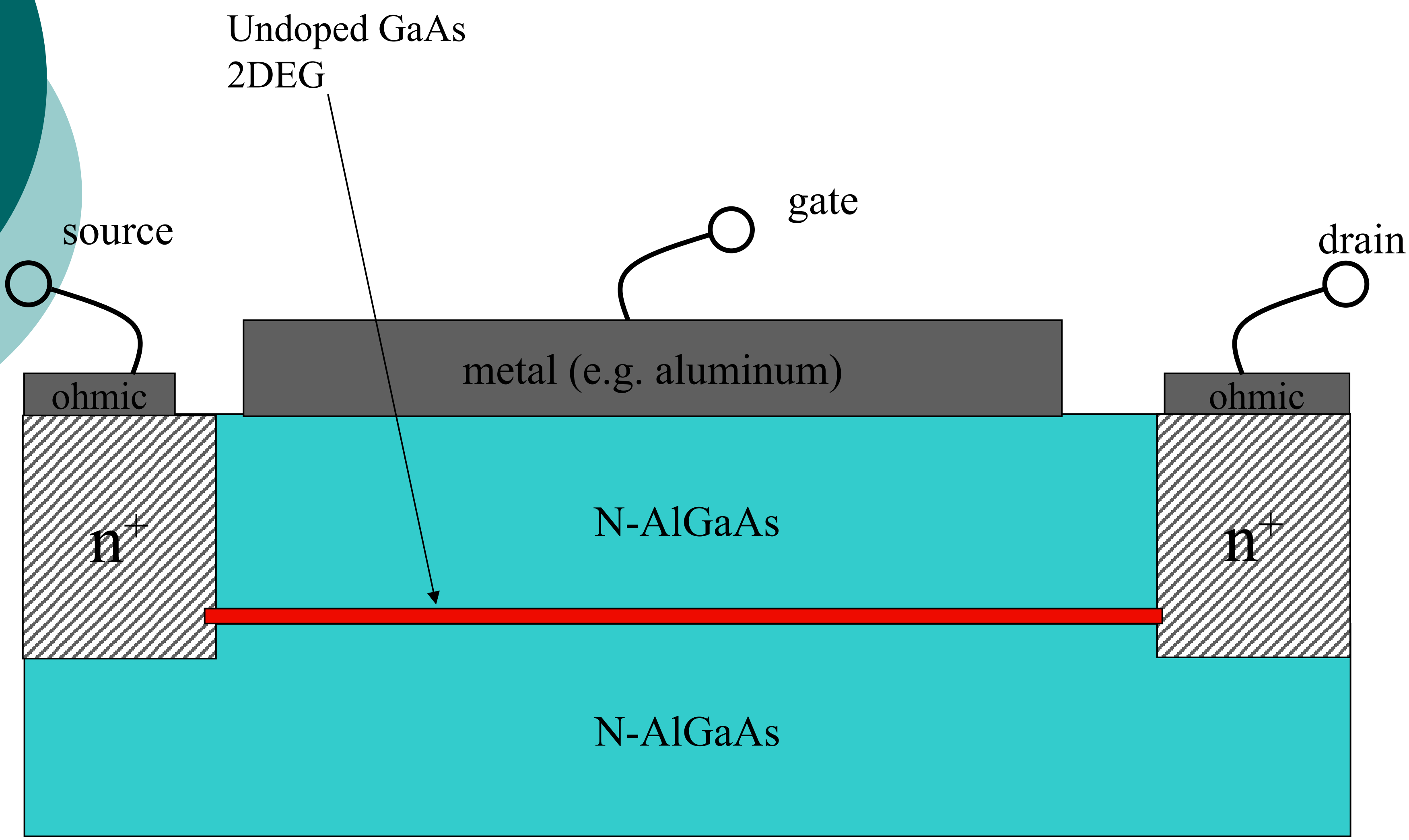
$$E_f(n_s) = E_{f,0} + a \cdot n_s$$

Density

$$en_s = \frac{\varepsilon}{t_b + \varepsilon a / e^2}\left(V_{GB} - V_T\right)$$

$$V_T \equiv \phi_B + \frac{E_{f,0}}{e} - \frac{eN_{d,1}}{2\varepsilon}(t_b - \delta)^2 - \frac{\Delta E_c}{e}$$

HEMT:

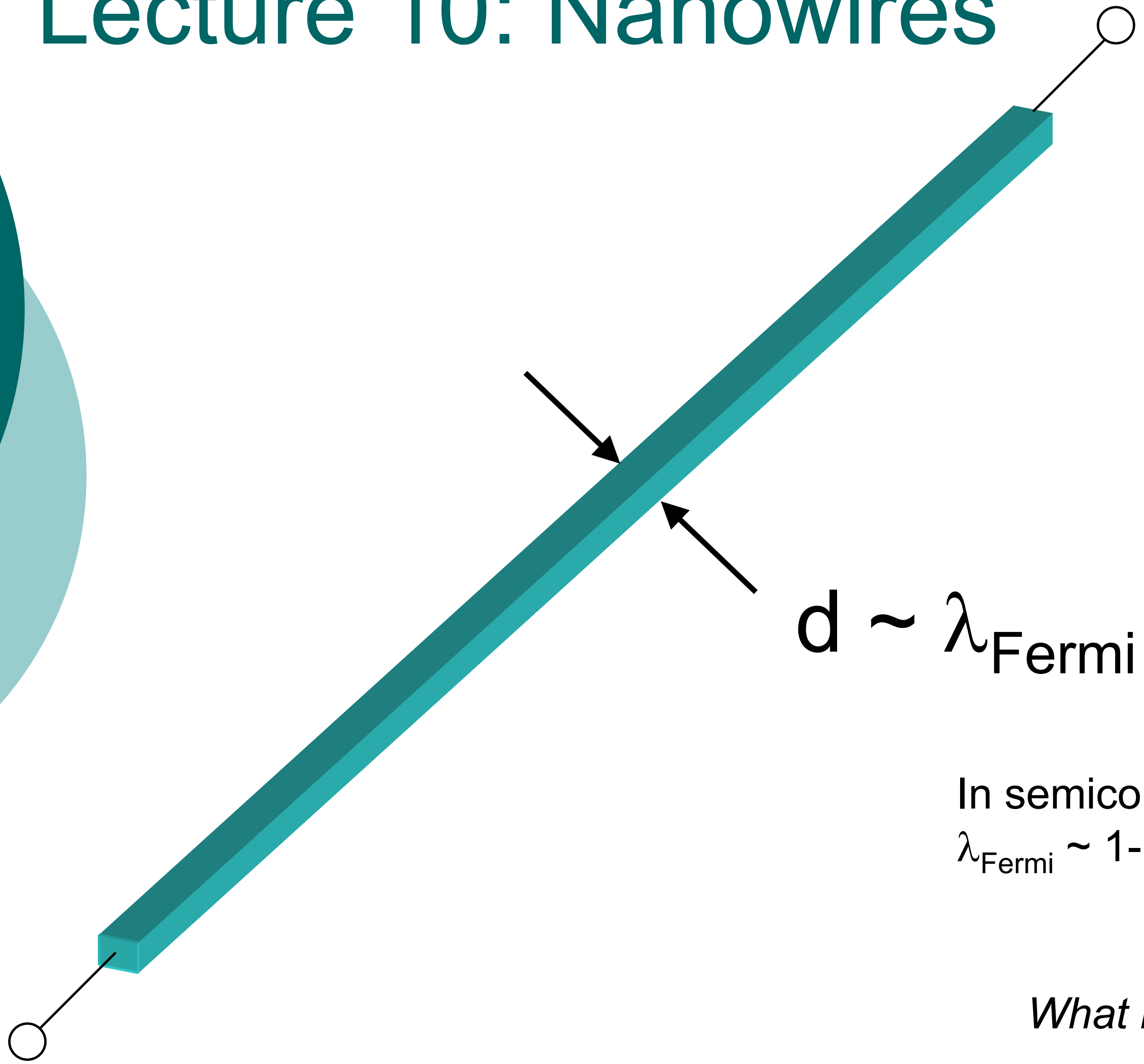


Undoped GaAs
2DEG
gate
source
drain
metal (e.g. aluminum)
ohmic
ohmic
n+
n+
N-AlGaAs
N-AlGaAs

Tunneling

- Resonant tunnel diodes
  - Draw band diagram, I-V on board
  - Fast (> 700 GHz)
- Optical/IR detectors
  - Like photoelectric effect
- Quantum cascade lasers
  - Levels within quantum wells lase

Lecture 10: Nanowires



In semiconductors,
λ_{Fermi} ~ 1-10 nm

What is the resistance?

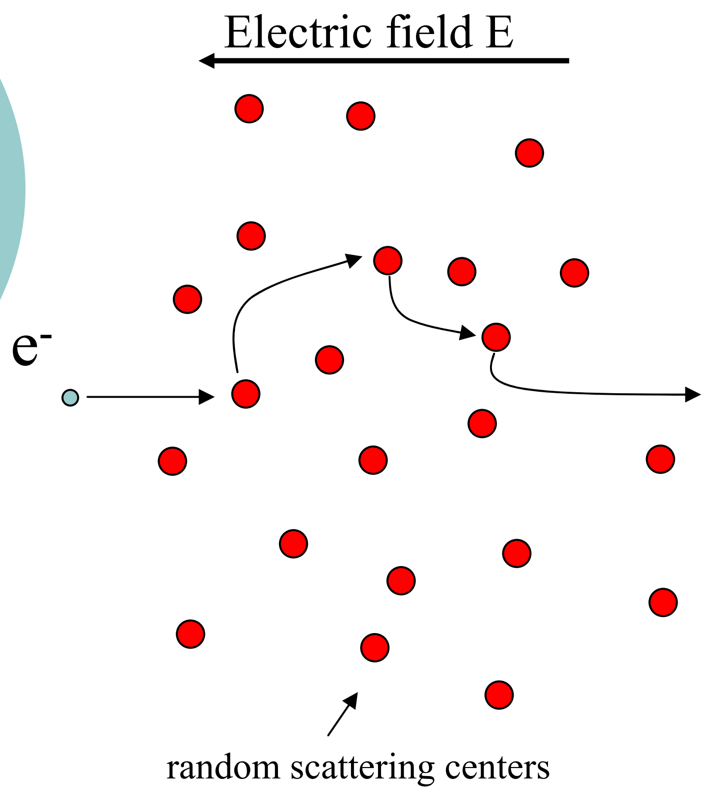
Readings this lecture covers

- Ferry pp. 39-46
- Hanson, pp. 124-125, 317-344

Drift current

- Caused by electric field
- Electron density constant
- Analogy: swarm of mosquitoes in the wind

Drift: Drude model



$$F = ma$$

$$eE = m \frac{\partial v}{\partial t}$$

$$v_{avg} = \underbrace{\frac{e\tau}{m}}_{\mu} E$$

$$j = ne\, v_{avg} = \underbrace{\frac{ne^2 \tau}{m}}_{\sigma} E$$

Types of scattering

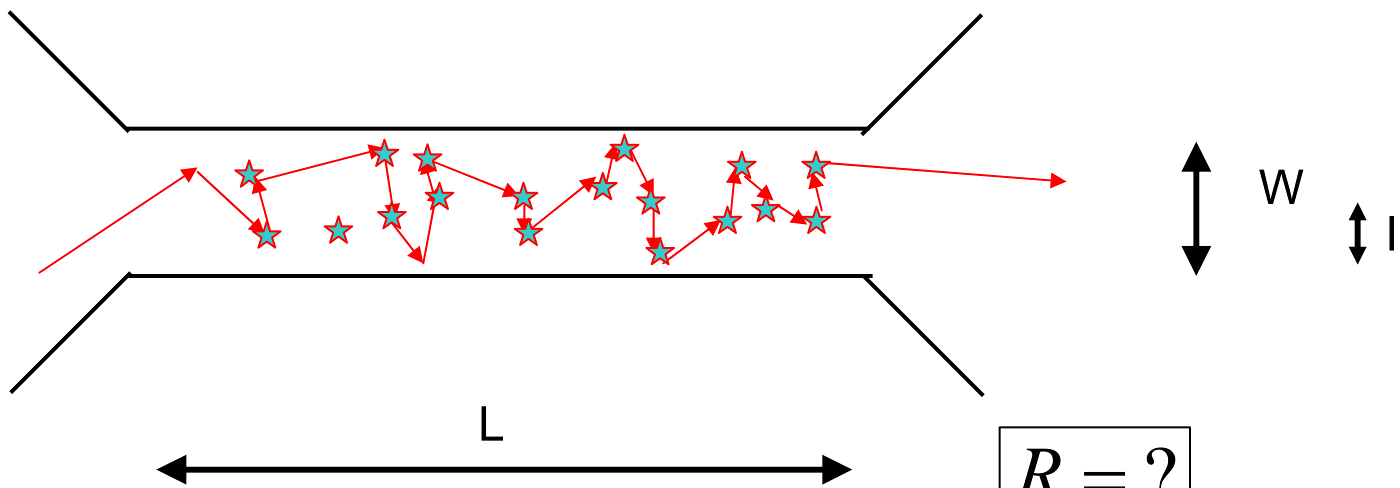
- Electron-phonon:
  - Very temperature dependent
  - Phonons are lattice vibrations
  - At low temperatures, lattice is “perfectly still”
- Impurity scattering
  - Temperature independent
  - Depends on impurity concentration

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_{electron-phonon}} + \frac{1}{\tau_{impurity}}$$

Ballistic vs. diffusive transport

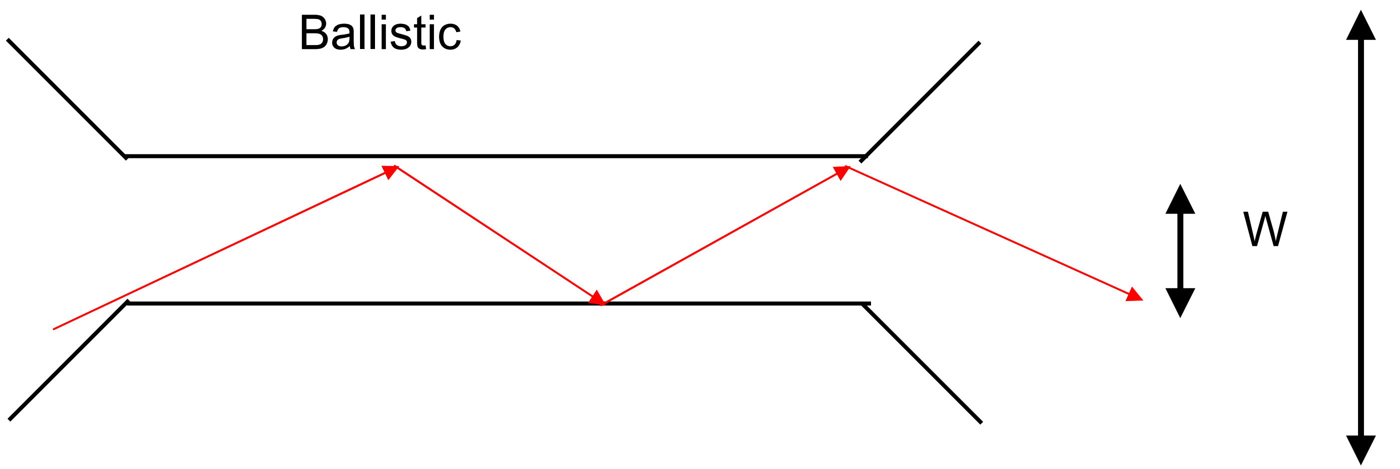
Diffusive

$$R = \frac{L}{W^2}\rho$$

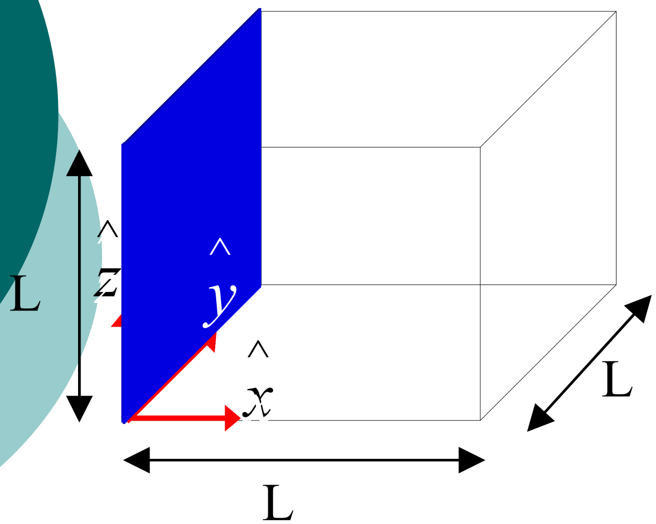


$$R = ?$$

Ballistic



Particle in a box



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$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or “quantum states”

Particle in a nanowire

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$
$$k_{n_y} = n_y \pi / L$$
$$k_{n_z} = n_z \pi / L$$

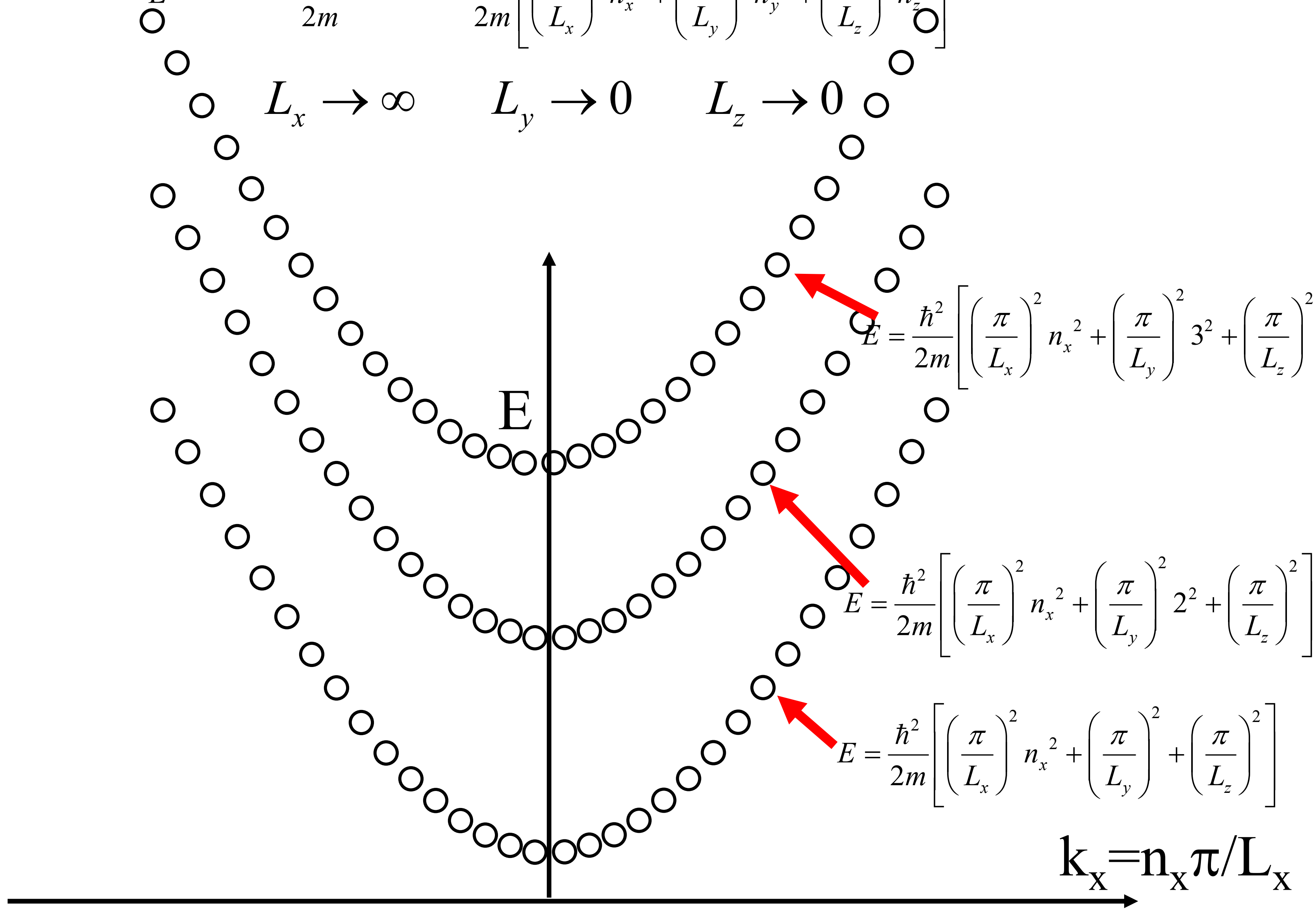
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

These are the allowed energy levels, or “quantum states”

Limits:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

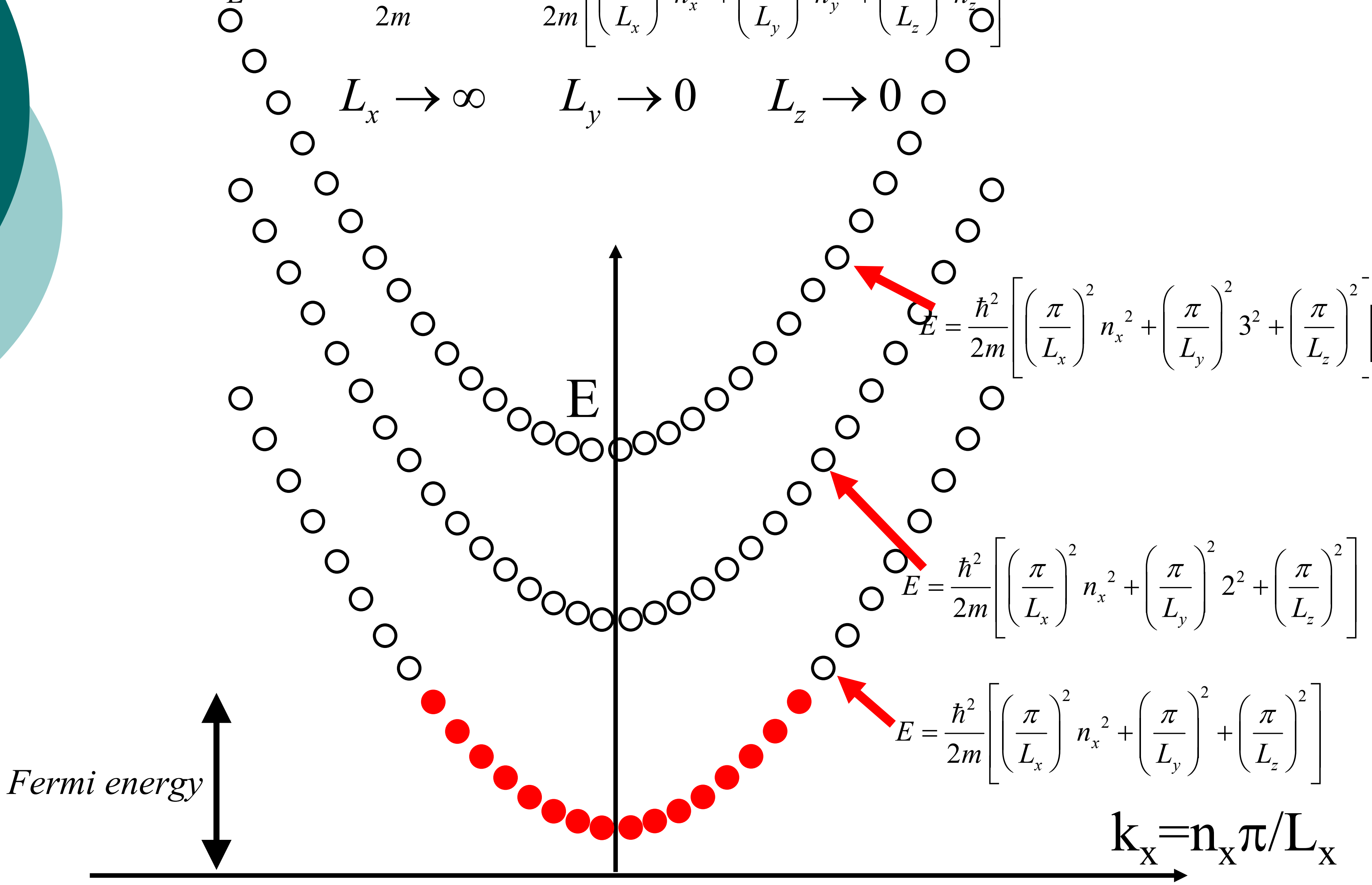
$$L_x \to \infty \qquad L_y \to 0 \qquad L_z \to 0$$



1d system:

$$E=\frac{\hbar^2(k_{n_x}{}^2+k_{n_y}{}^2+k_{n_z}{}^2)}{2m}=\frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2+\left(\frac{\pi}{L_y}\right)^2 n_y{}^2+\left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$$L_x \to \infty \qquad L_y \to 0 \qquad L_z \to 0$$



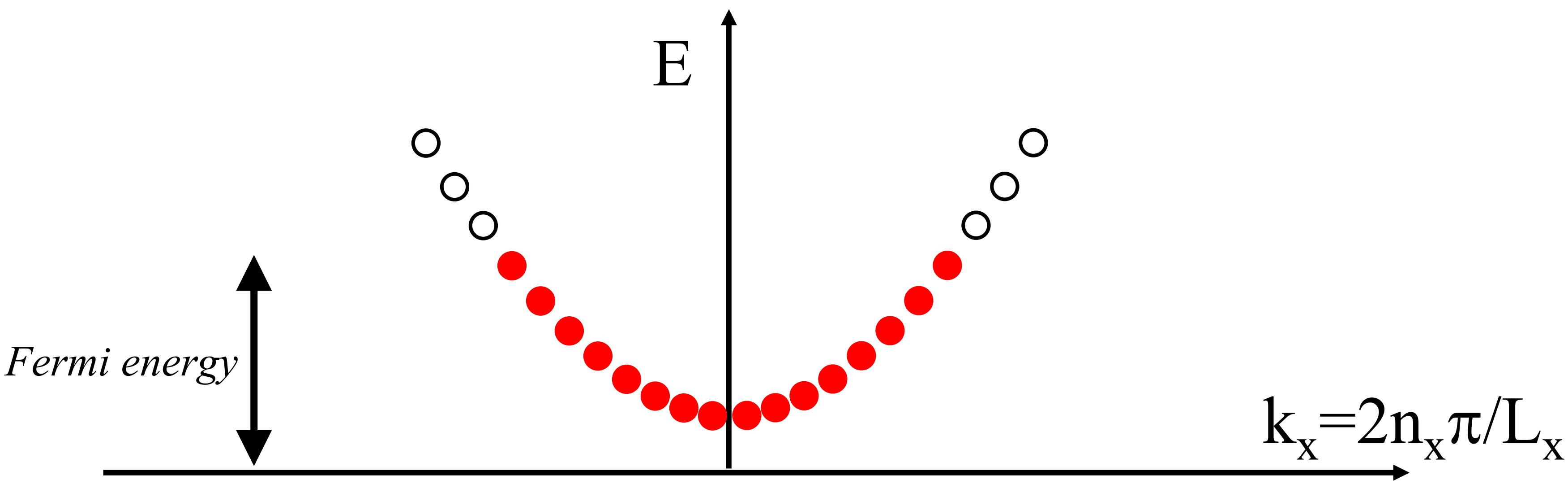
Positive and negative k-vectors:

Particle in a box: (positive k-vectors only)

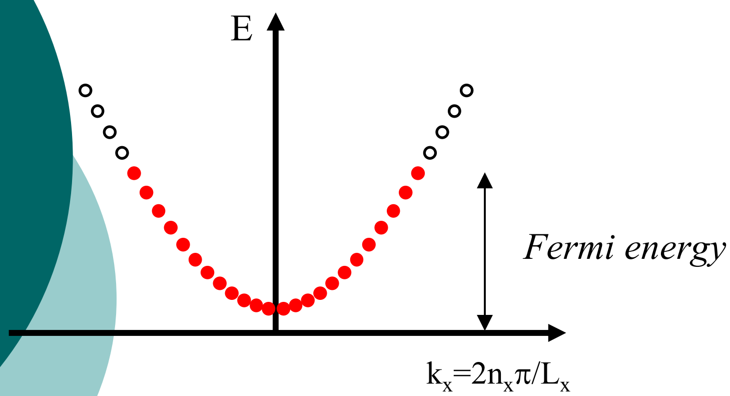
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

"Born-Von Karman" boundary conditions: (positive *and* negative k-vectors)

$$E = \frac{\hbar^2}{2m}\left[\left(\frac{2\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{2\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{2\pi}{L_z}\right)^2 n_z{}^2\right]$$



Single sub-band:



$$I=\frac{\text{charge}}{\text{time}}=e\cdot\frac{\#\text{elec}}{\text{time}}=e\cdot \text{v}\frac{\#\text{elec}}{\text{length}}$$

Different electrons have different velocities.

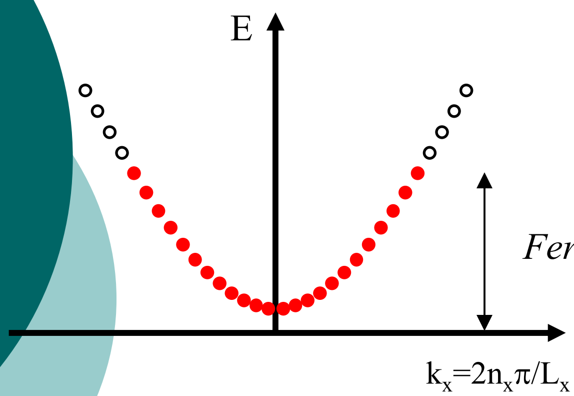
$$\text{v}=\frac{\text{momentum}}{\text{mass}}=\frac{\text{p}}{\text{m}}=\frac{\hbar k}{\text{m}}$$

$$I=I_{right\ goers}-I_{left\ goers}$$

$$I_{right\ goers}=\sum_{right\ going\ electrons} e\,\text{v}\,\frac{1}{\text{length}}=\sum_{\substack{occupied\ states\\(right\ goers)}} e\,\text{v}\,\frac{1}{\text{length}}$$

$$I_{right\ goers}=\frac{e}{L_x}\sum_{k_x=0}^{k_F}\frac{\hbar k_x}{m}=\frac{e}{L_x}\sum_{n_x=0}^{n_F}\frac{\hbar\left(n_x 2\pi/L_x\right)}{m}=\frac{e2\pi\hbar}{mL_x^{\ 2}}\sum_{n_x=0}^{n_F}n_x$$

Single sub-band:



$$I_{right\ goers} = \frac{e2\pi\hbar}{mL_x^{\ 2}} \sum_{n_x=0}^{n_F} n_x \rightarrow \frac{e2\pi\hbar}{mL_x^{\ 2}} \int_0^{n_F} n_x dn_x$$

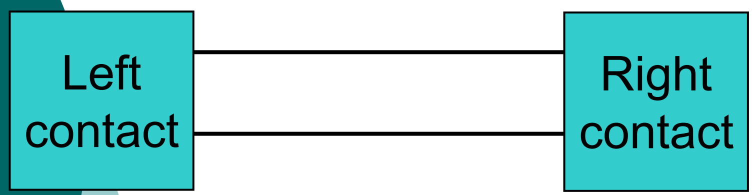
Change of variables:

$$E = \frac{\hbar^2 k_x^{\ 2}}{2m} = \frac{\hbar^2 (2n_x \pi / L_x)^2}{2m} \Rightarrow dE = \frac{4\hbar^2 (\pi / L_x)^2}{m} n_x dn_x$$

$$\Rightarrow n_x dn_x = \frac{m}{4\hbar^2 (\pi / L_x)^2} dE$$

$$I_{right\ goers} = \frac{e\pi\hbar}{2mL_x^{\ 2}} \int_0^{n_F} n_x dn_x \rightarrow \frac{e\pi\hbar}{2mL_x^{\ 2}} \frac{m}{\hbar^2 (\pi / L_x)^2} \int dE = \frac{e}{h} \int dE$$

Resistance quantum



Ballistic conductor

$$I_{right\ goers} = \frac{e}{h}\int dE \quad I_{right\ goers} = \frac{e}{h}\int dE$$

$$I = \frac{e}{h}\left[\int dE_{\text{right goers}} - \int dE_{\text{left goers}}\right]$$

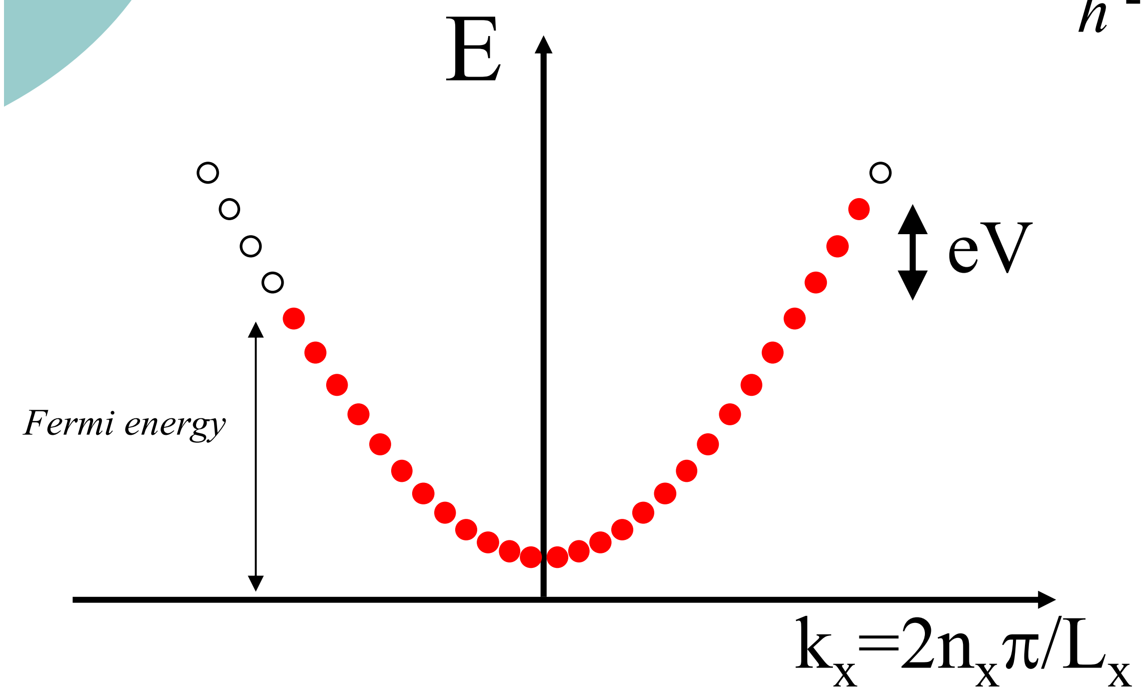
$$I = \frac{e}{h}\left[(E_F + eV) - E_F\right] = \frac{e^2}{h}V$$

$$V = I\frac{h}{e^2} = IR_{quantum}$$

$$R_{quantum} = \frac{h}{e^2} = 25\ k\Omega$$

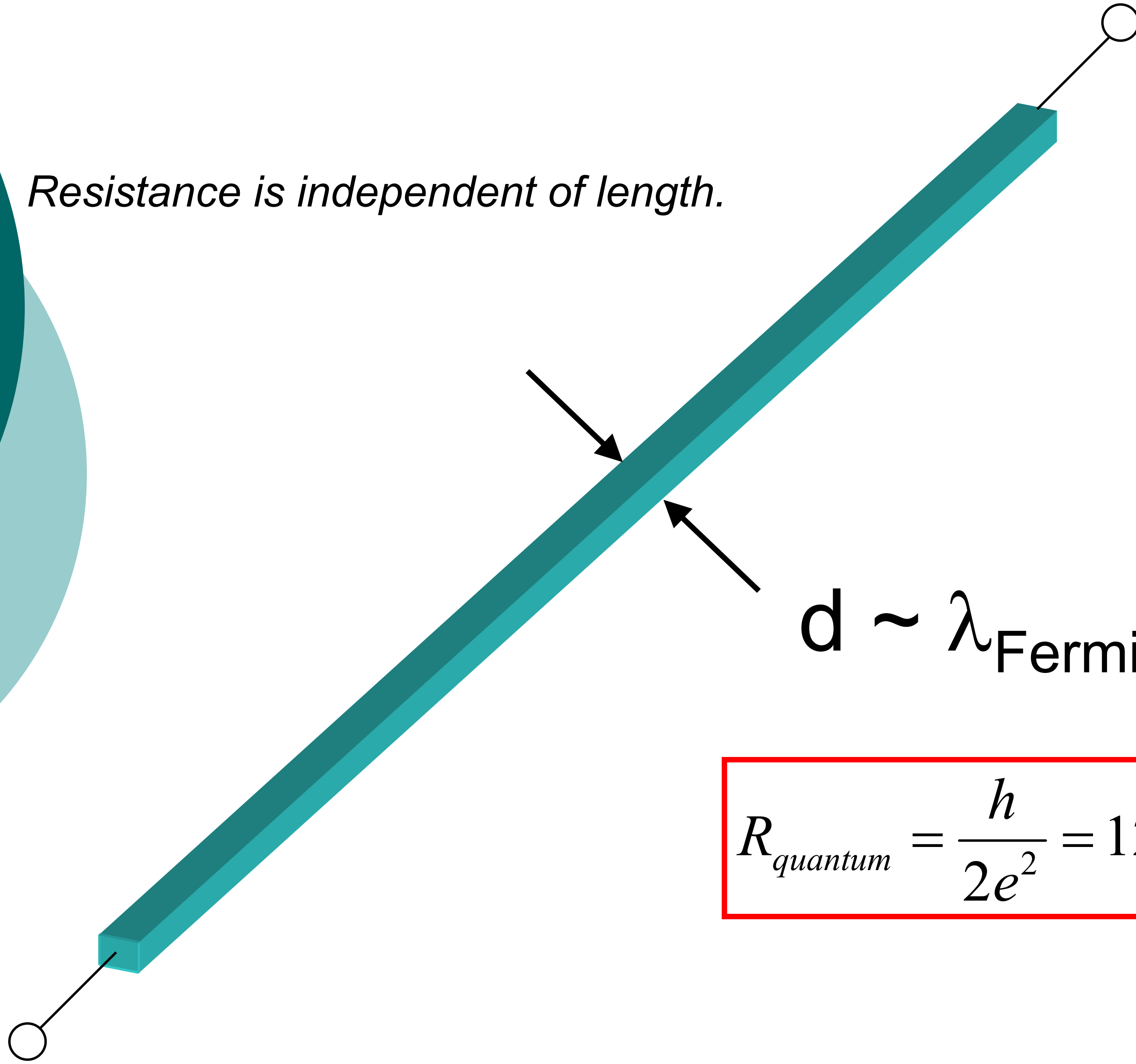
With spin:

$$R_{quantum} = \frac{h}{2e^2} = 12.5\ k\Omega$$



Lecture 11: Quantum point contact

Resistance is independent of length.



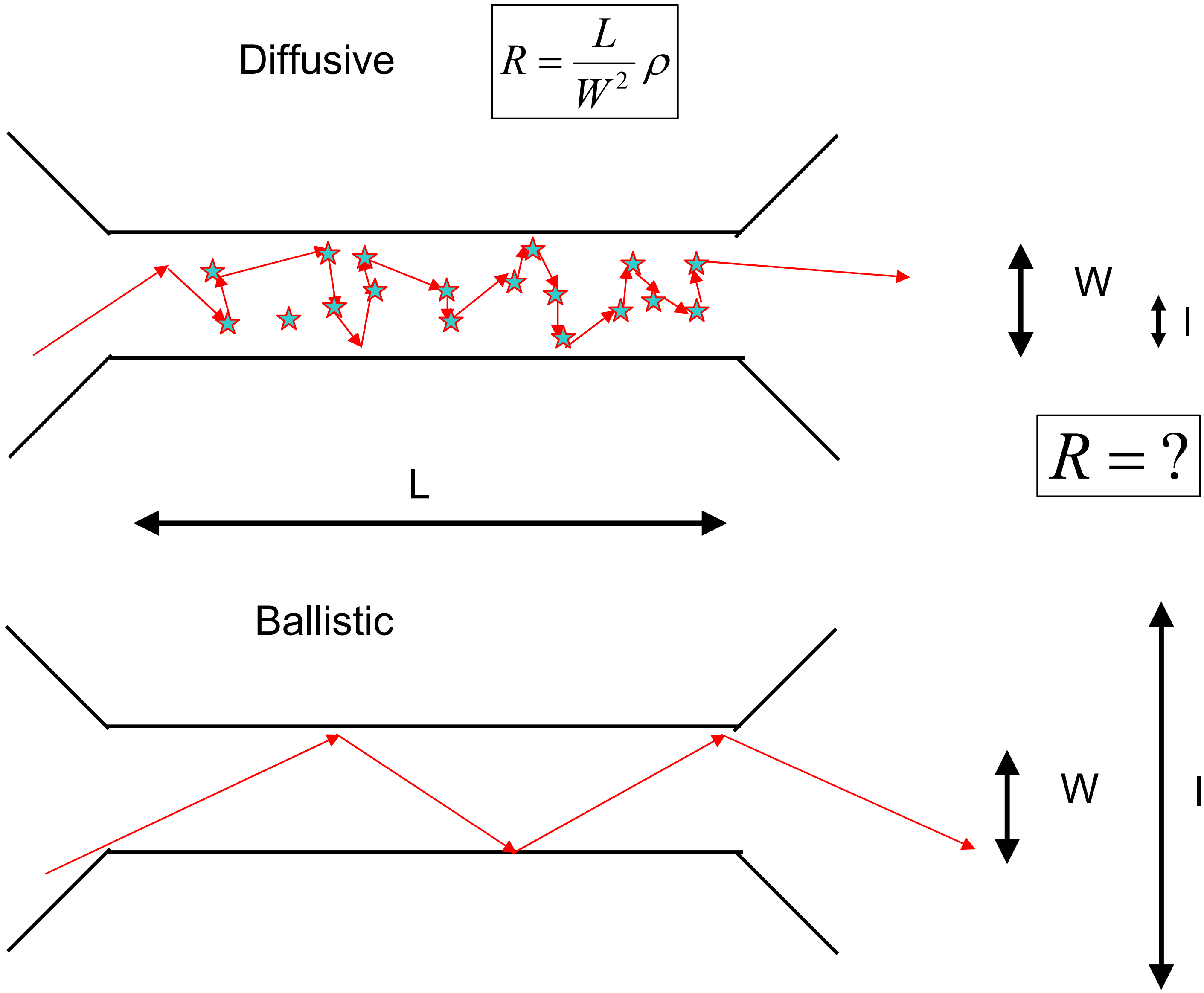
$d \sim \lambda_{Fermi}$

$$R_{quantum} = \frac{h}{2e^2} = 12.5\ k\Omega$$

Readings this lecture covers

- Ferry pp. 124-139
- Van Wees PRL (reading packet)
- Marcus APL (reading packet)
- Zhou APL (reading packet)

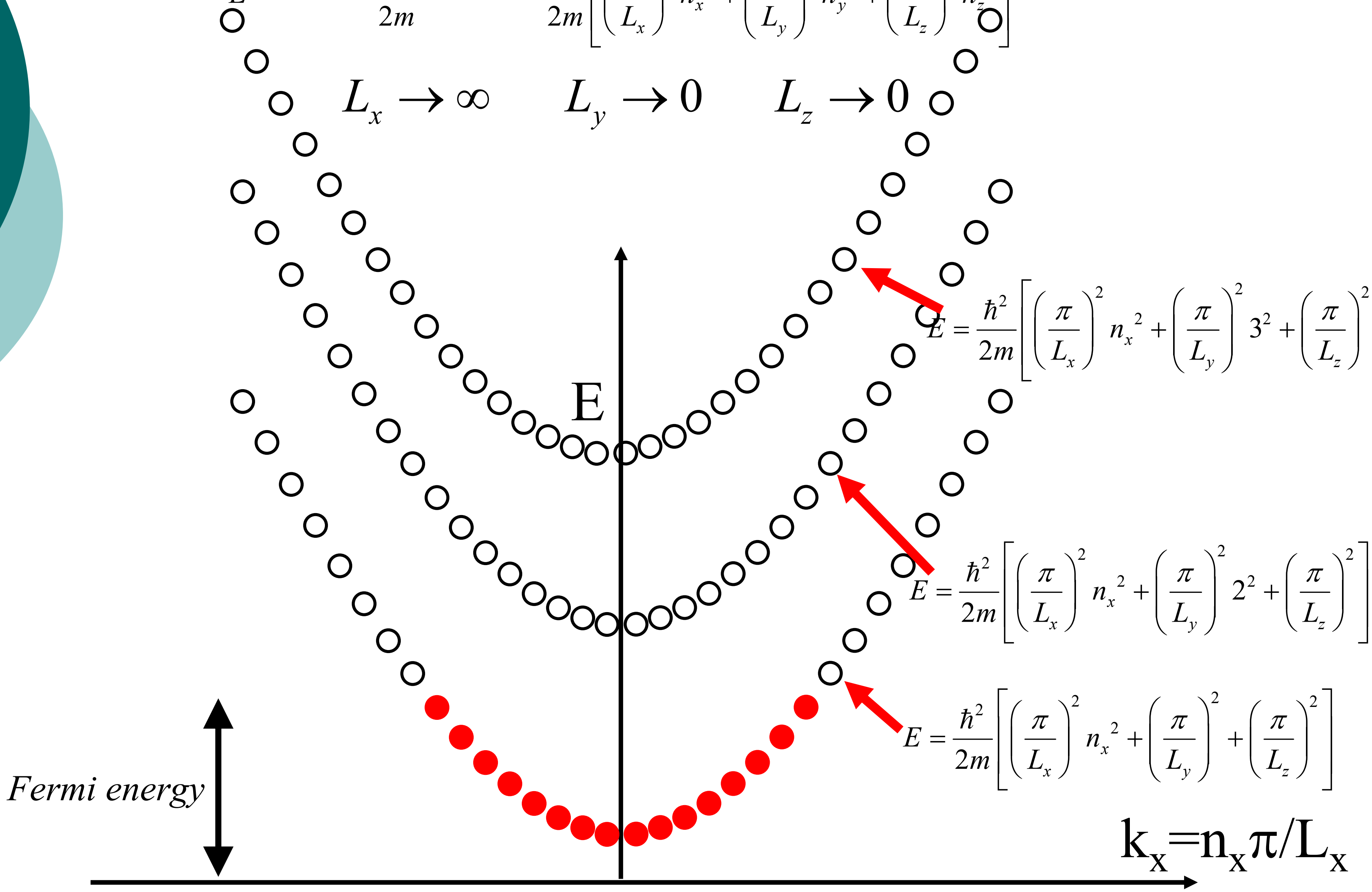
Ballistic vs. diffusive transport



1d system:

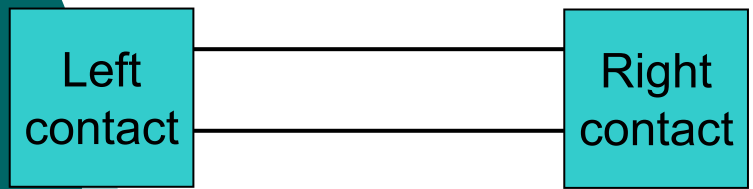
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$$L_x \to \infty \qquad L_y \to 0 \qquad L_z \to 0$$



Resistance quantum

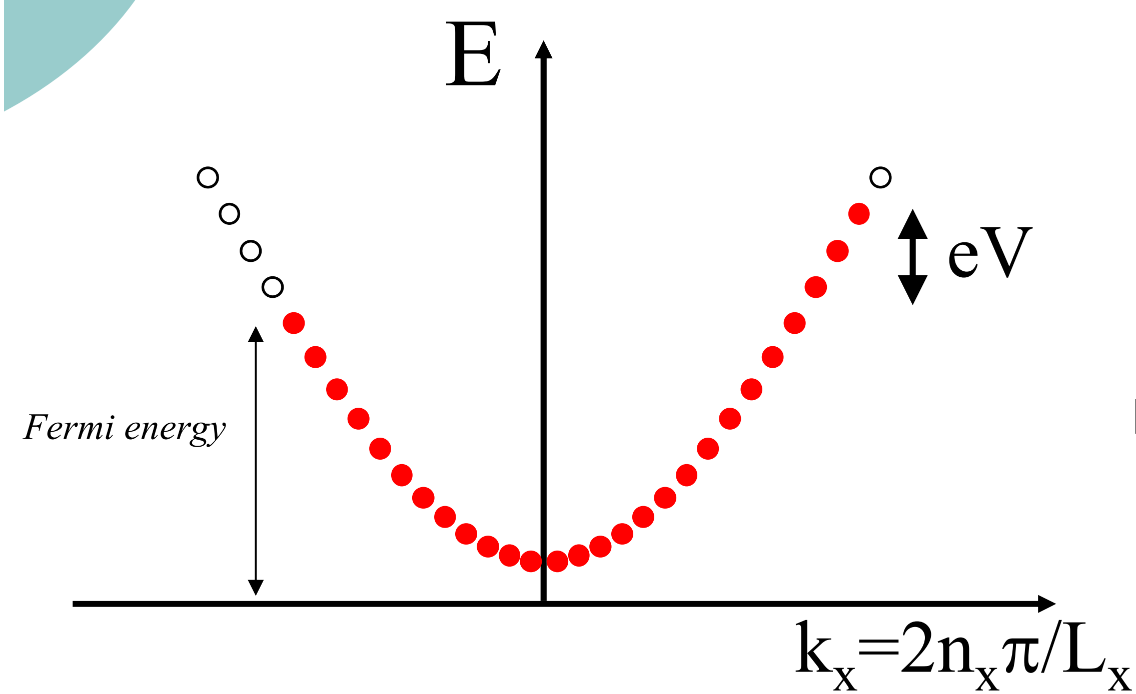
Ballistic conductor



$$R_{quantum} = \frac{h}{e^2} = 25\ k\Omega$$

With spin:

$$R_{quantum} = \frac{h}{2e^2} = 12.5\ k\Omega$$



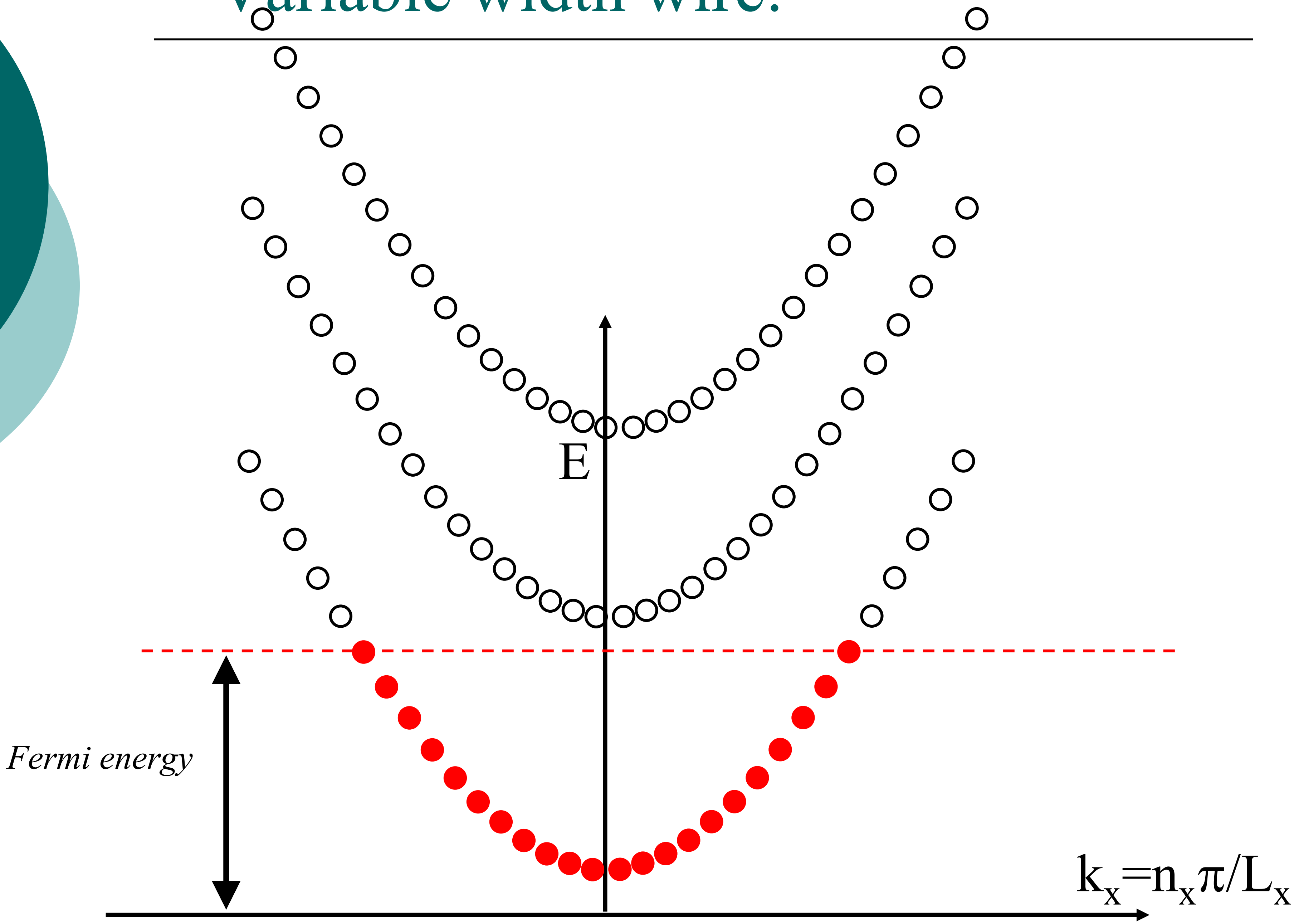
$$G_{quantum} = \frac{2e^2}{h}$$

If injection from leads is not perfect:

$$G = T\frac{2e^2}{h}$$

T is the transmission probability.

Variable width wire:



Landauer formula:

$$G = n\frac{2e^2}{h}$$

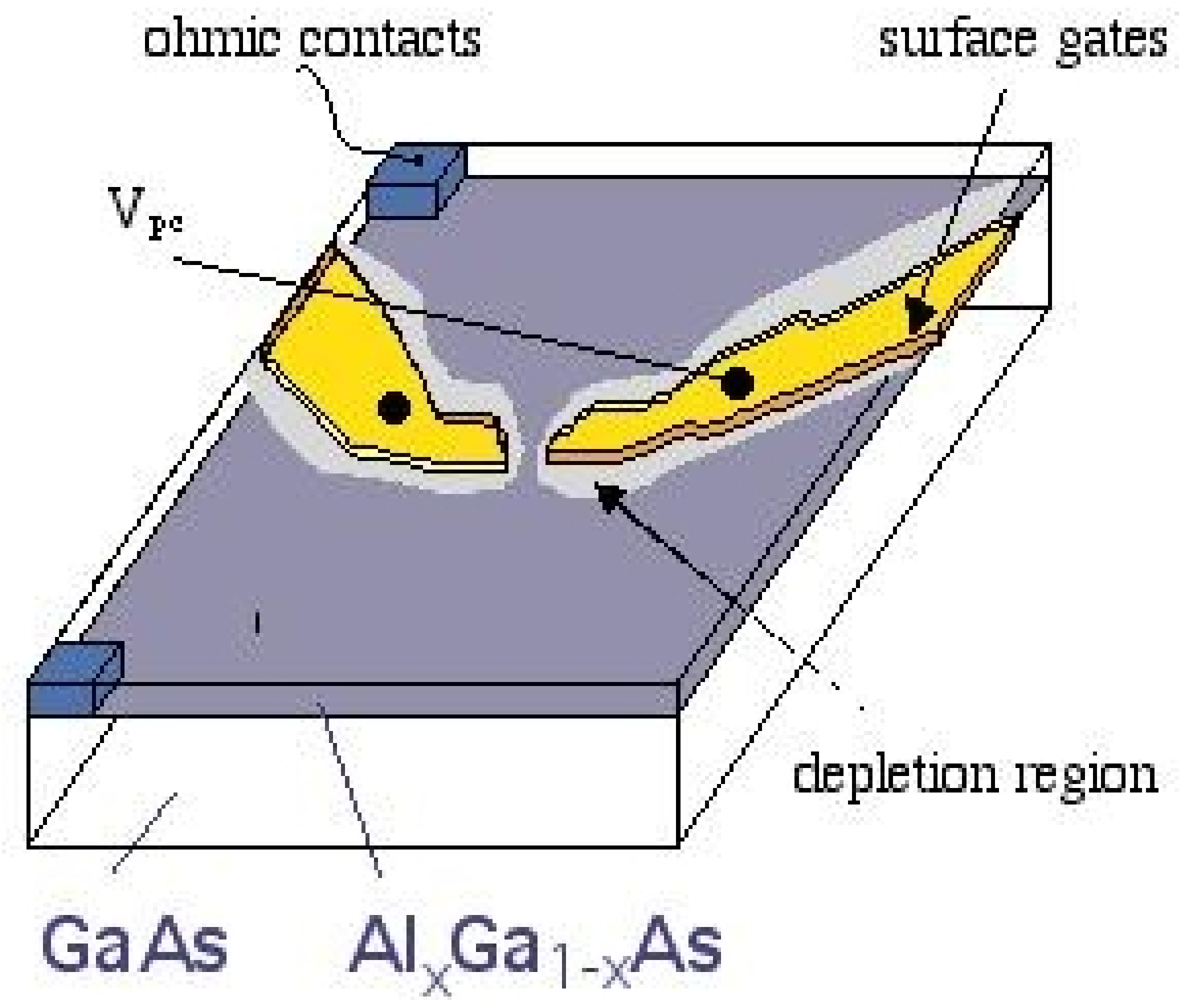
If the leads are not perfect injectors into each "channel" then:

$$G = \frac{2e^2}{h}\sum T_n$$

Experimental realizations:

- Pinch-off gate in semiconductor 2DEG (QPC)
- Break junction
- Electrochemical addition of atoms
- Scanning tunneling microscope

Quantum point contact



Quantum point contact

VOLUME 60, NUMBER 9 PHYSICAL REVIEW LETTERS 29 FEBRUARY 1988

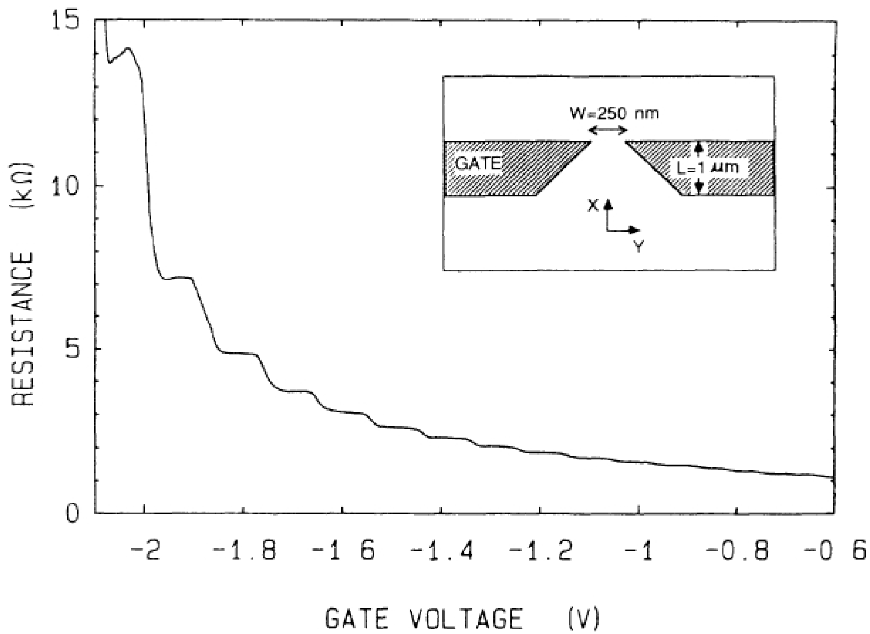


FIG. 1. Point-contact resistance as a function of gate voltage at 0.6 K. Inset: Point-contact layout.

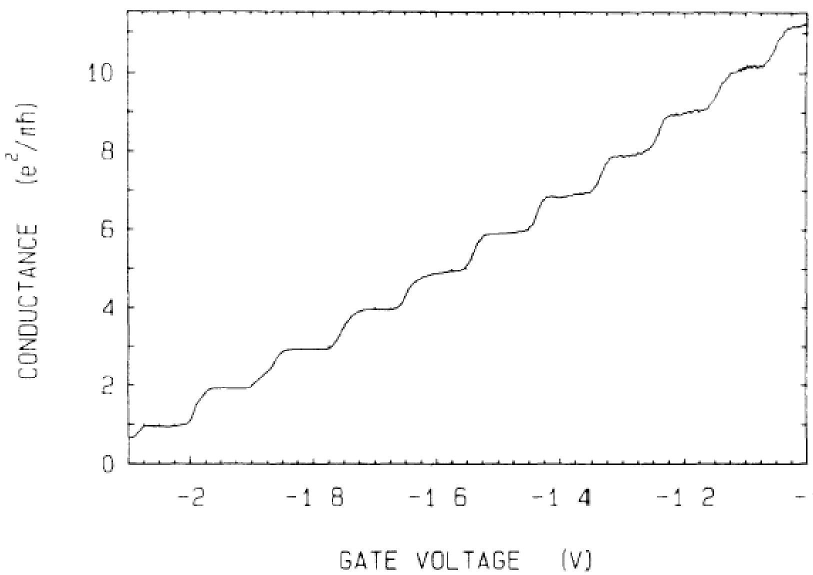


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.

B.J. van Wees et al. (1988), Phys. Rev. Lett., **60**, 848.

0.7 anomaly

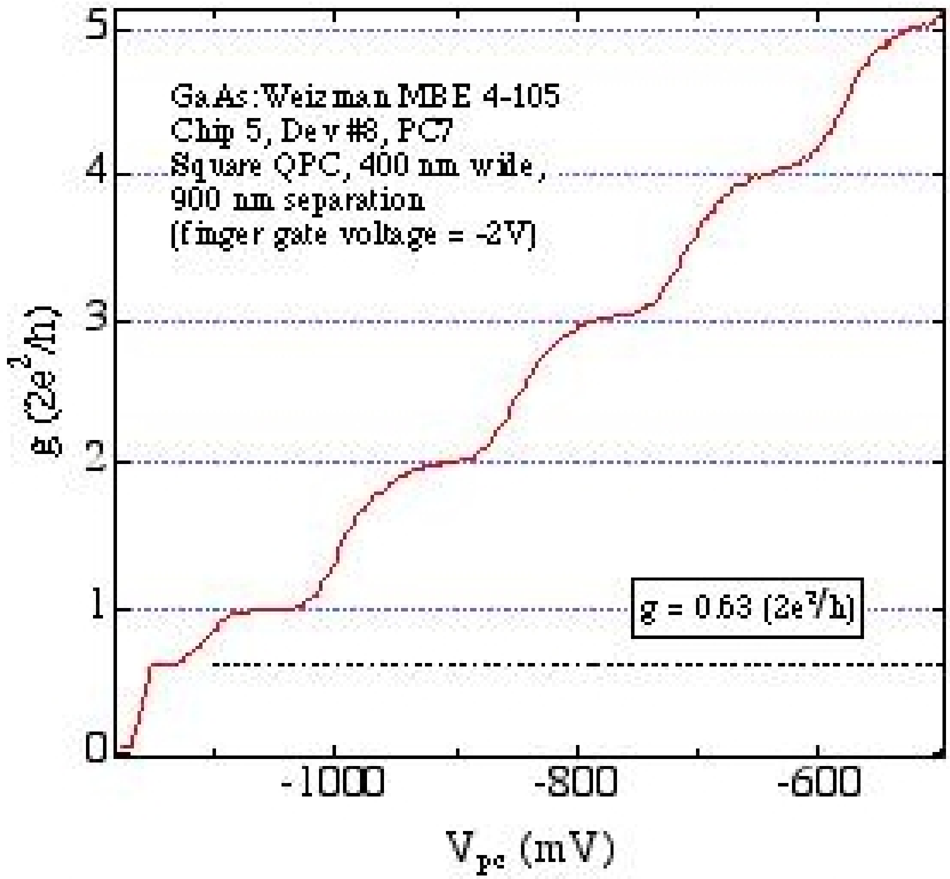
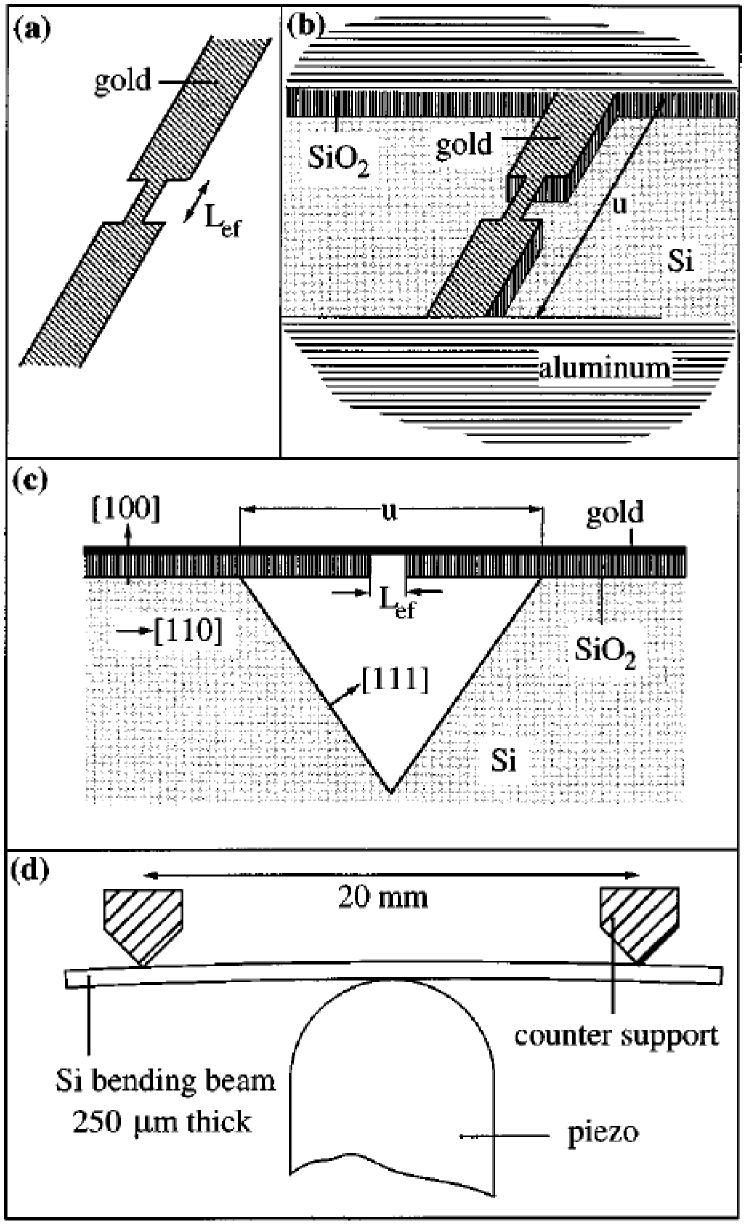
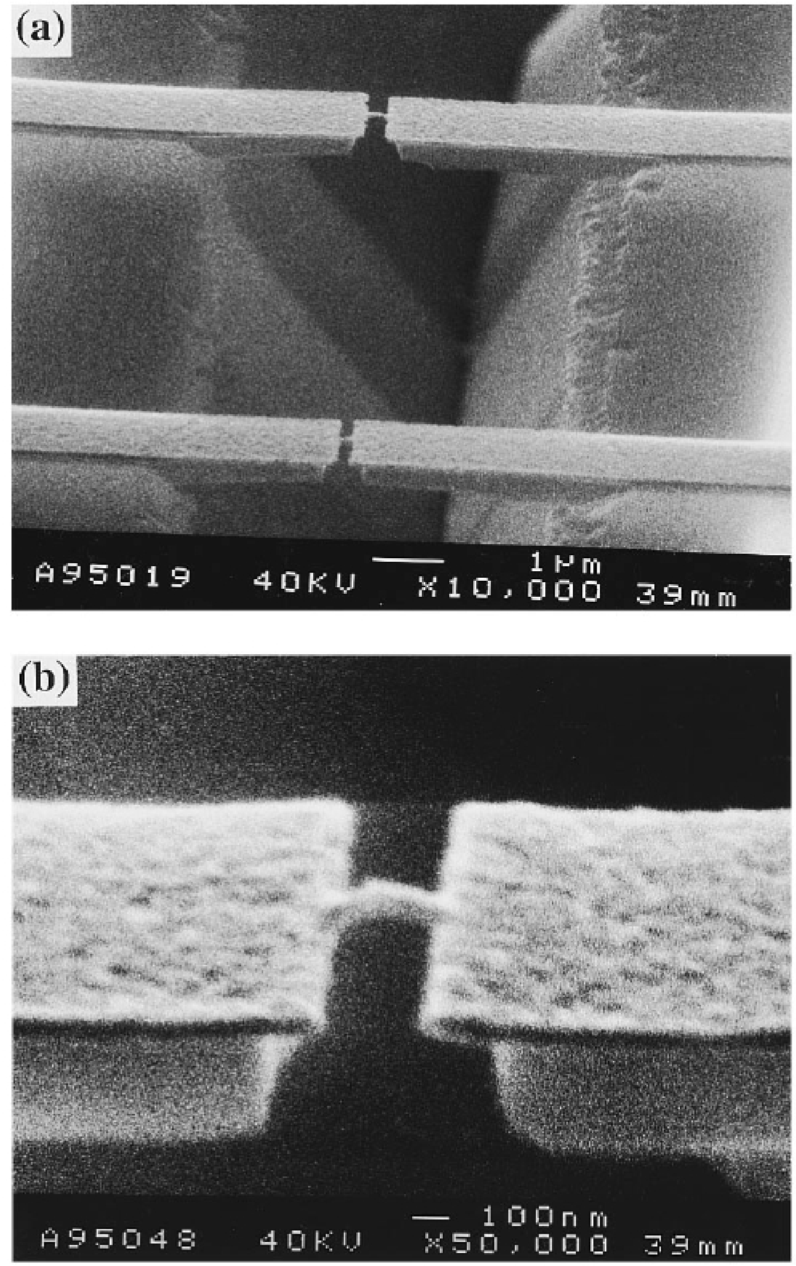


Figure 2: The conductance g through a point contact shows quantized plateaus at integer values of $2e^2/h$ with applied gate voltage, V_{pc} . This QPC shows a very prominent structure at ~0.6 ($2e^2/h$). The gates of this QPC are 400 nm wide and 900 nm apart.

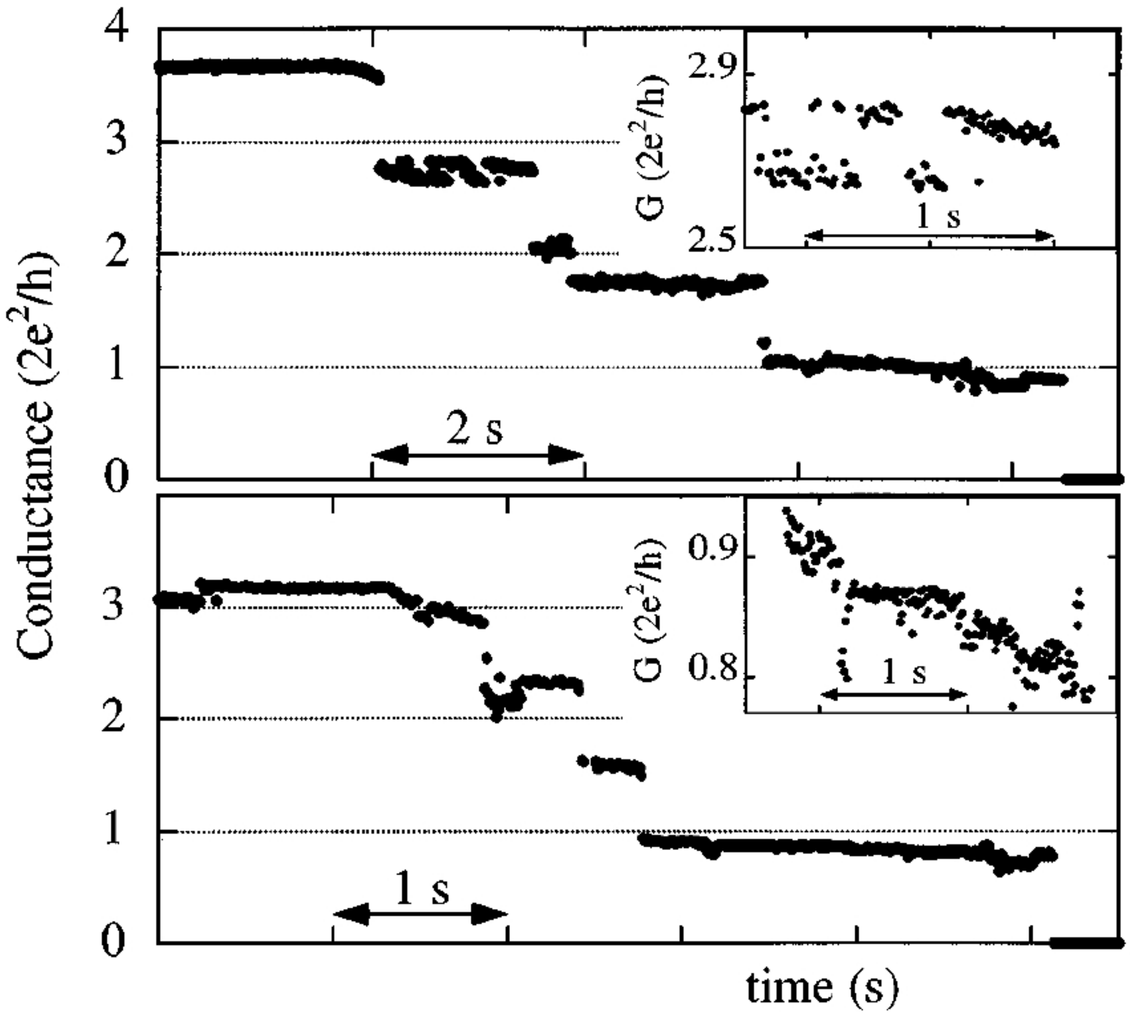
Break junction





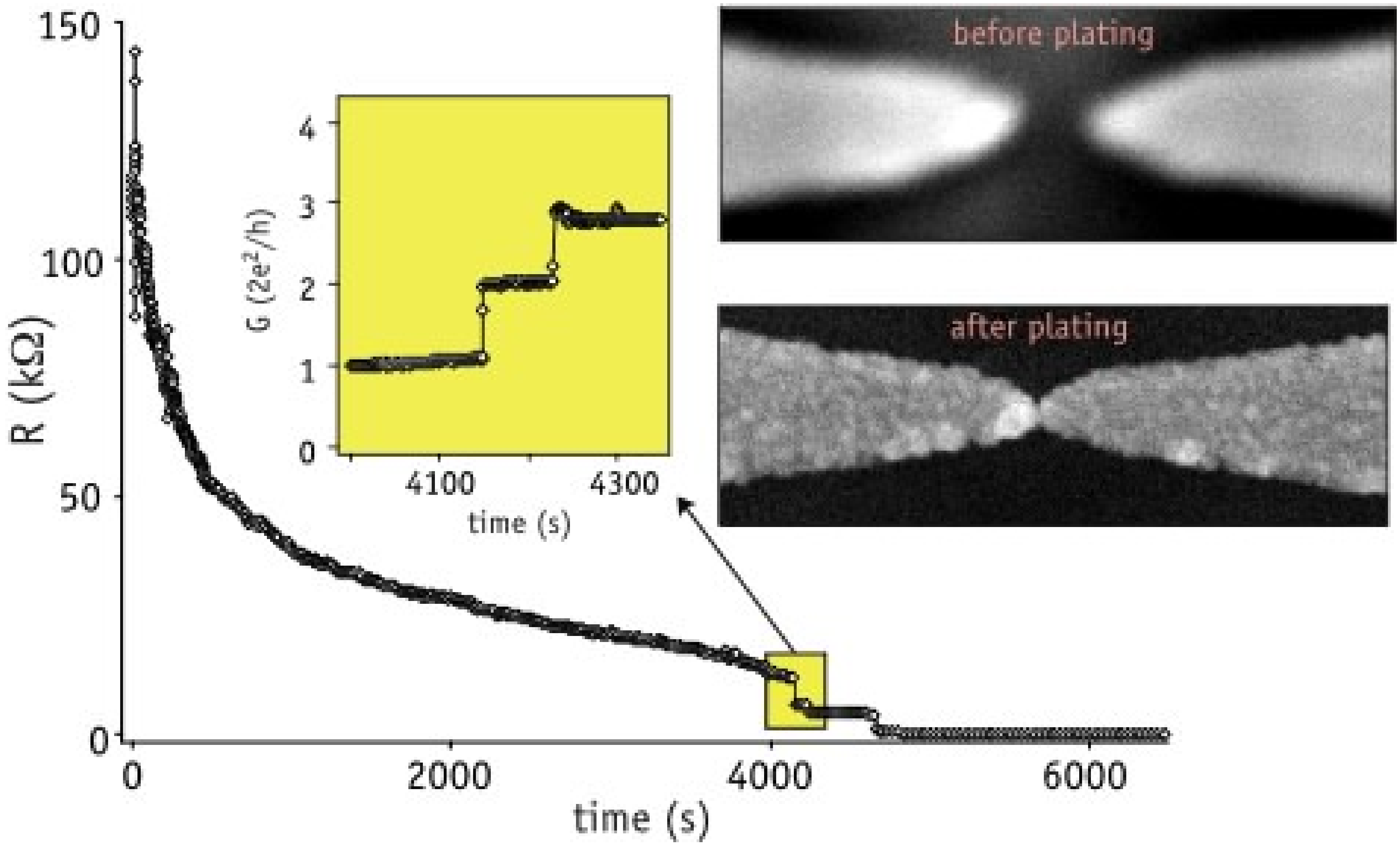
Microfabrication of a mechanically controllable break junction in silicon

C. Zhou, C. J. Muller, M. R. Deshpande, J. W. Sleight, and M. A. Reed
Center for Microelectronic Materials and Structures, Yale University, P.O. Box 208284, New Haven, Connecticut 06520-8284



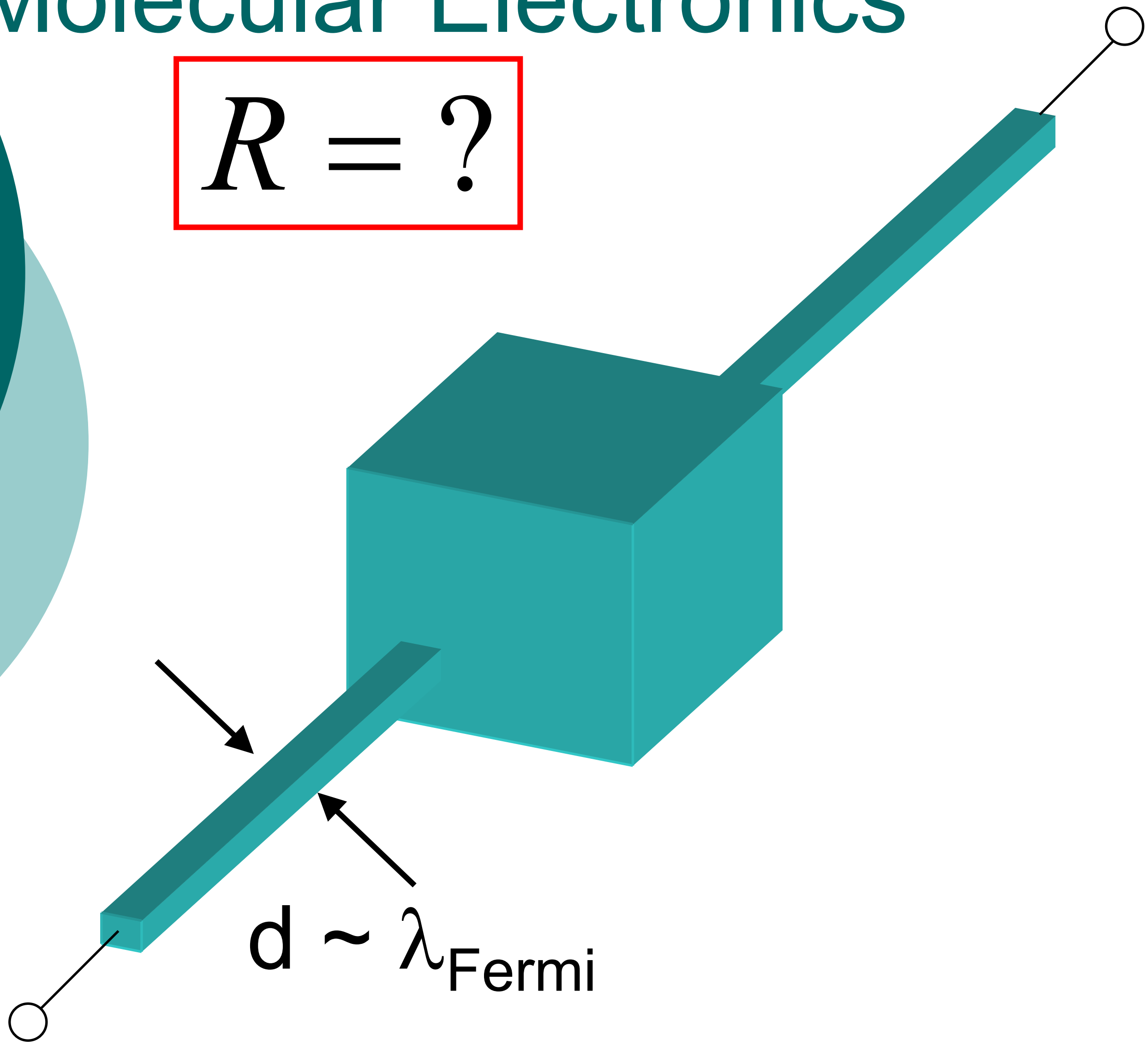
Zhou, et al, Applied Physics Letters **67**, 8 (1995) p. 1160.

Electroplating



A.F.Morpurgo, C.M.Marcus and D. B. Robinson,
Controlled Fabrication of Metallic Electrodes with Atomic Separation, Appl. Phys. Lett. **74**, 2084 (1999).

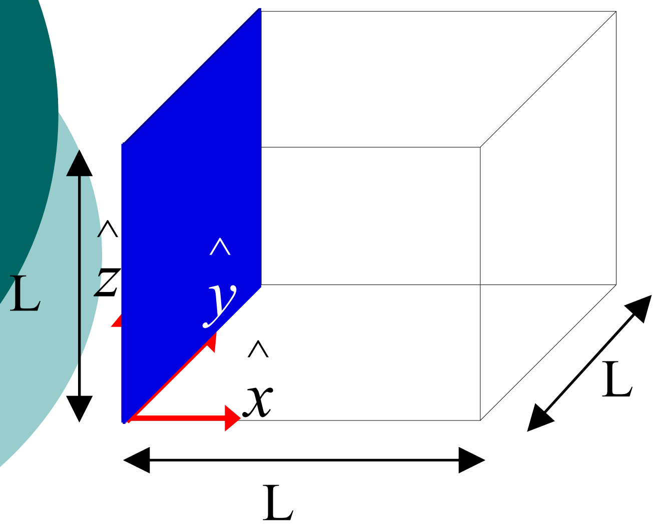
Lecture 12: Quantum dots and Molecular Electronics



Readings that cover this lecture

- Ferry, pp. 209-226
- Hanson, pp. 125-127

Particle in a box



We can do the same for y, z:

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

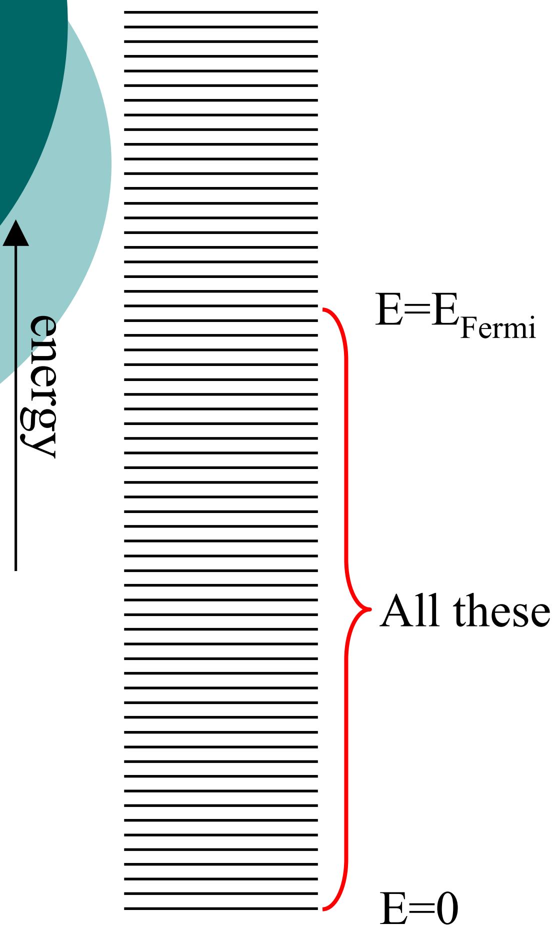
$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or "quantum states"

Fermi gas

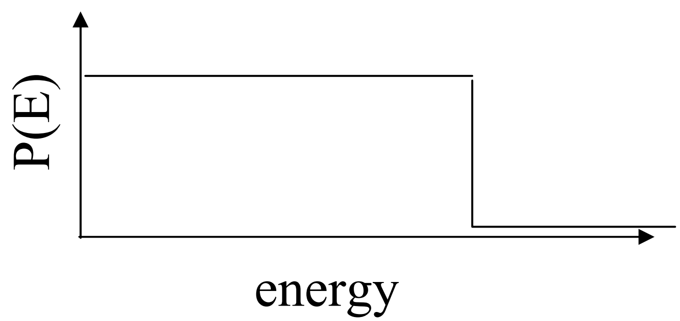


At zero temperature, as we add electrons to the box, we gradually fill up all the states. (DISCUSS PAULI EXCLUSION PRINCIPLE -IMPORTANT!)

When we are done filling the box, the energy of the last electron is called the "Fermi energy."

"Gas" means we neglect electron-electron interactions.

All these states are filled with electrons.



Particle in a box

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

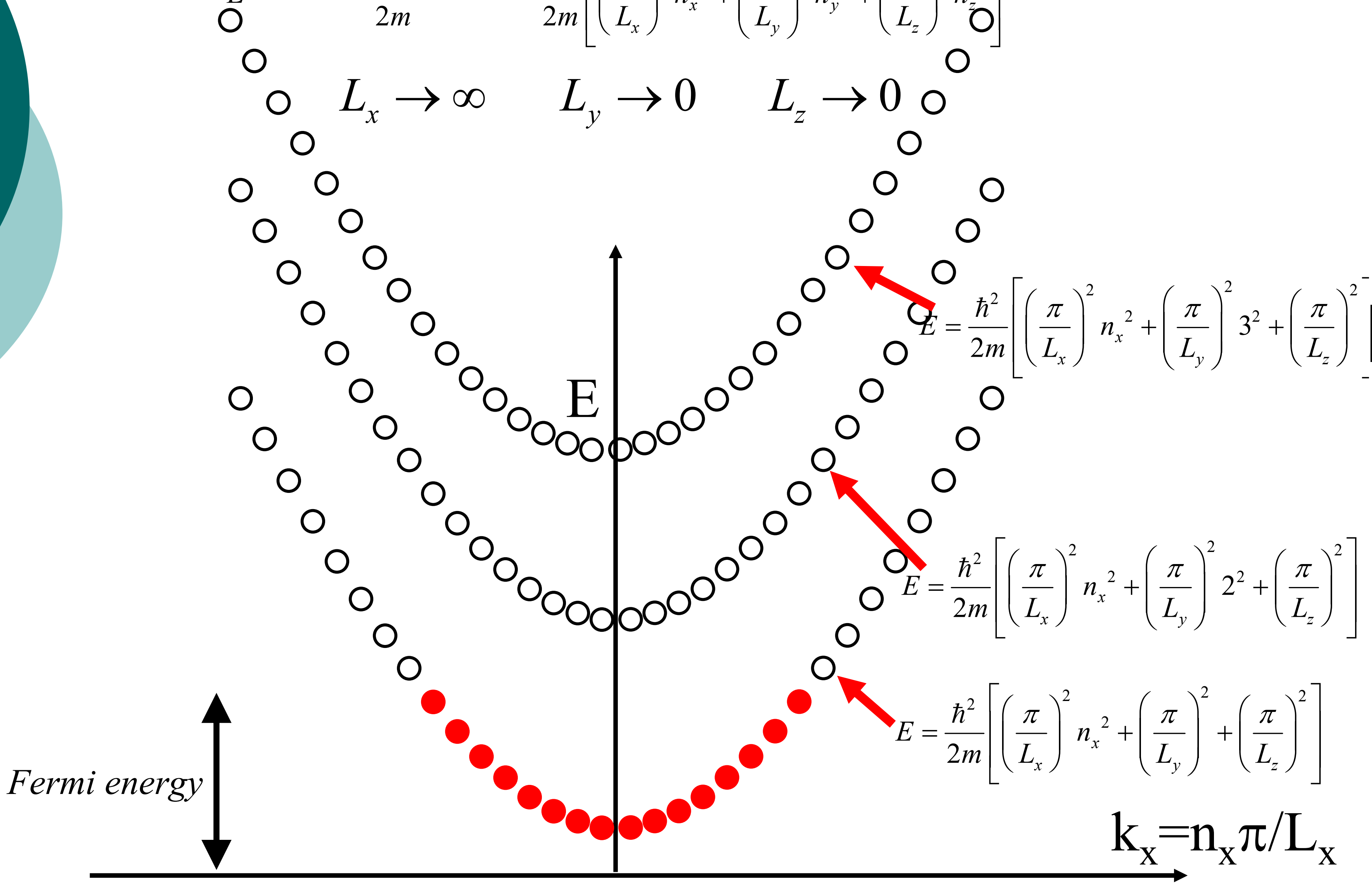
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

These are the allowed energy levels, or "quantum states"

1d system:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

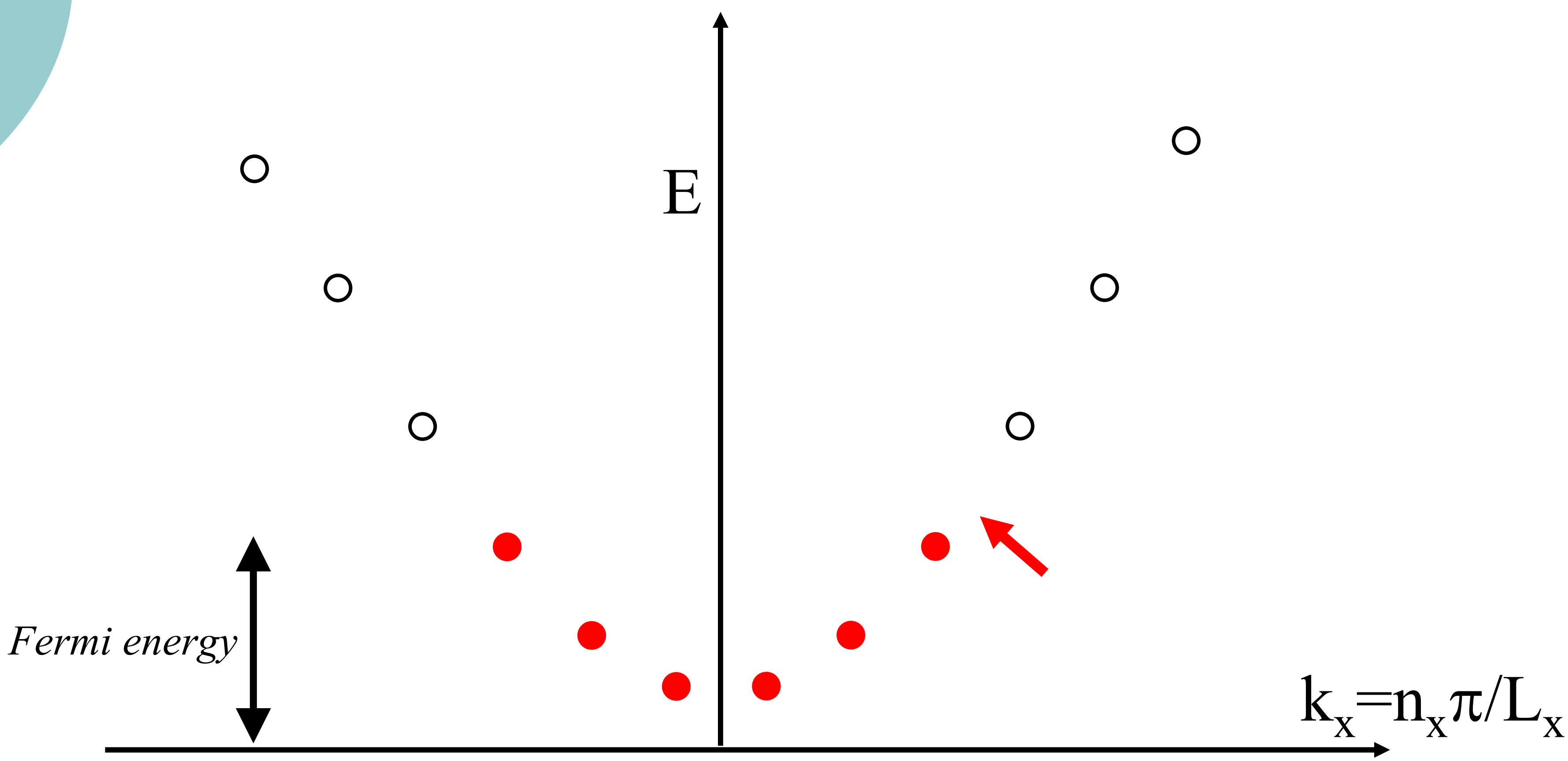
$$L_x \to \infty \qquad L_y \to 0 \qquad L_z \to 0$$



0d system

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

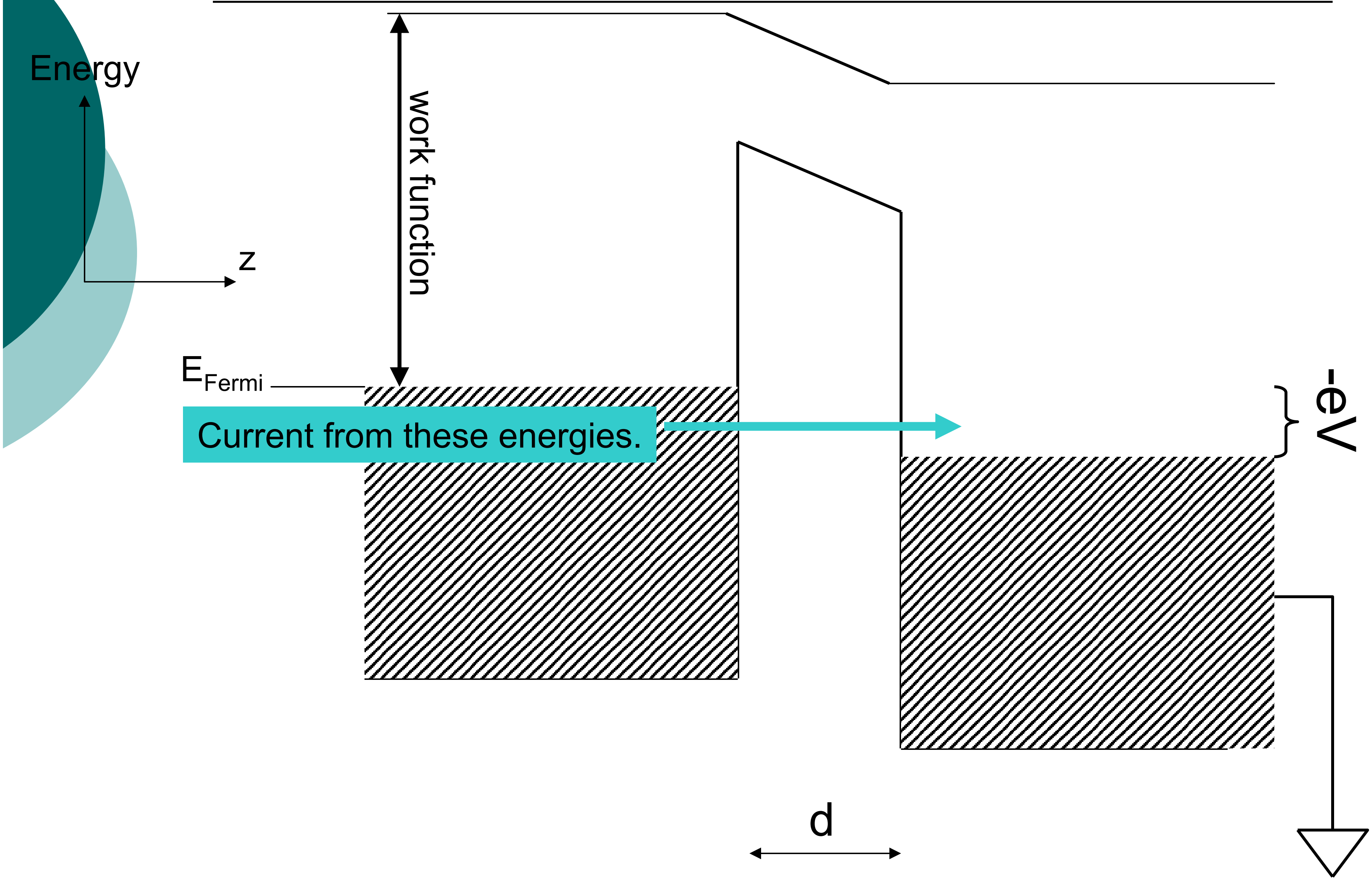
$$L_x \to 0 \qquad L_y \to 0 \qquad L_z \to 0$$



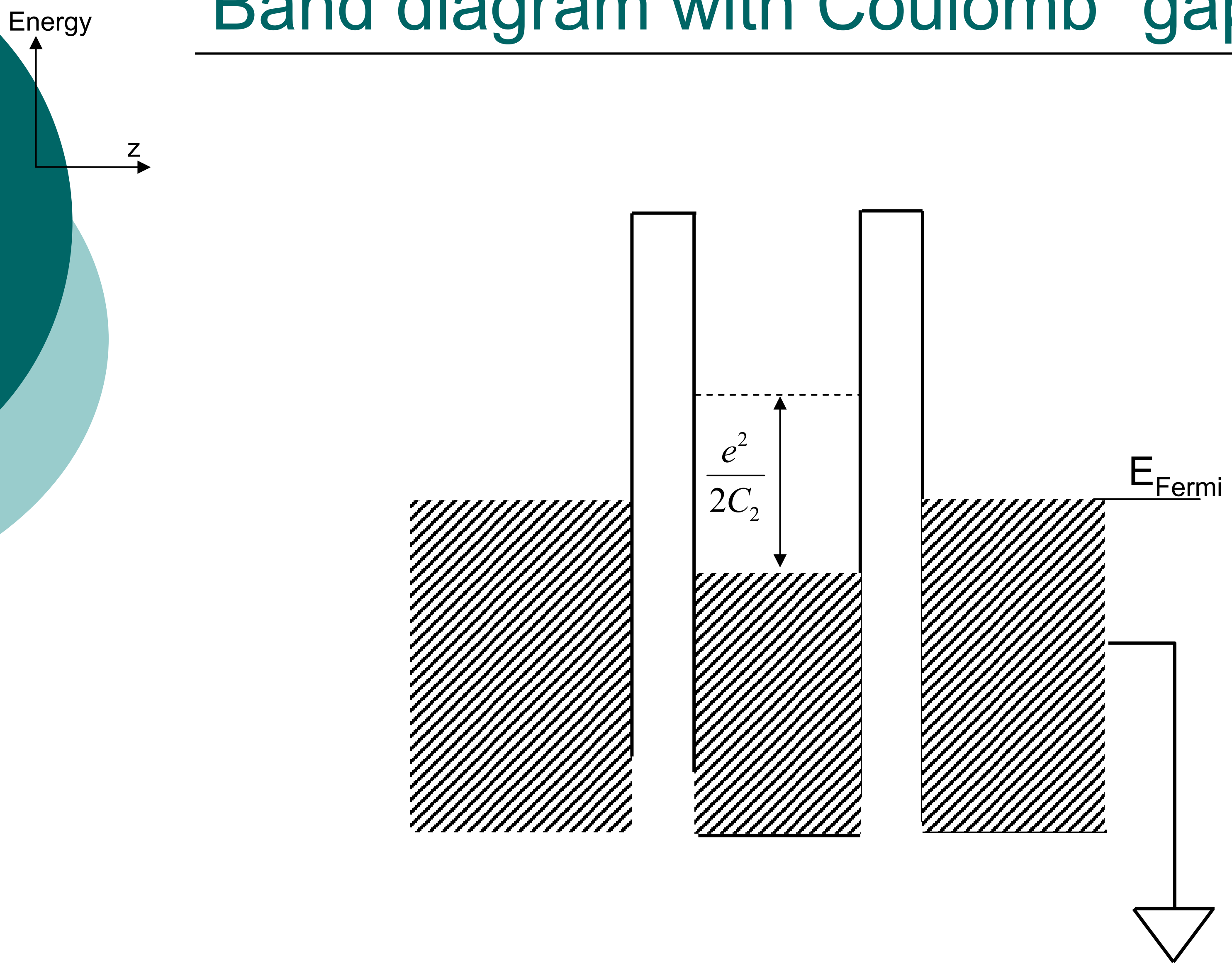
Energy scales

- Charging energy
- Single electron energy level spacing
- Temperature

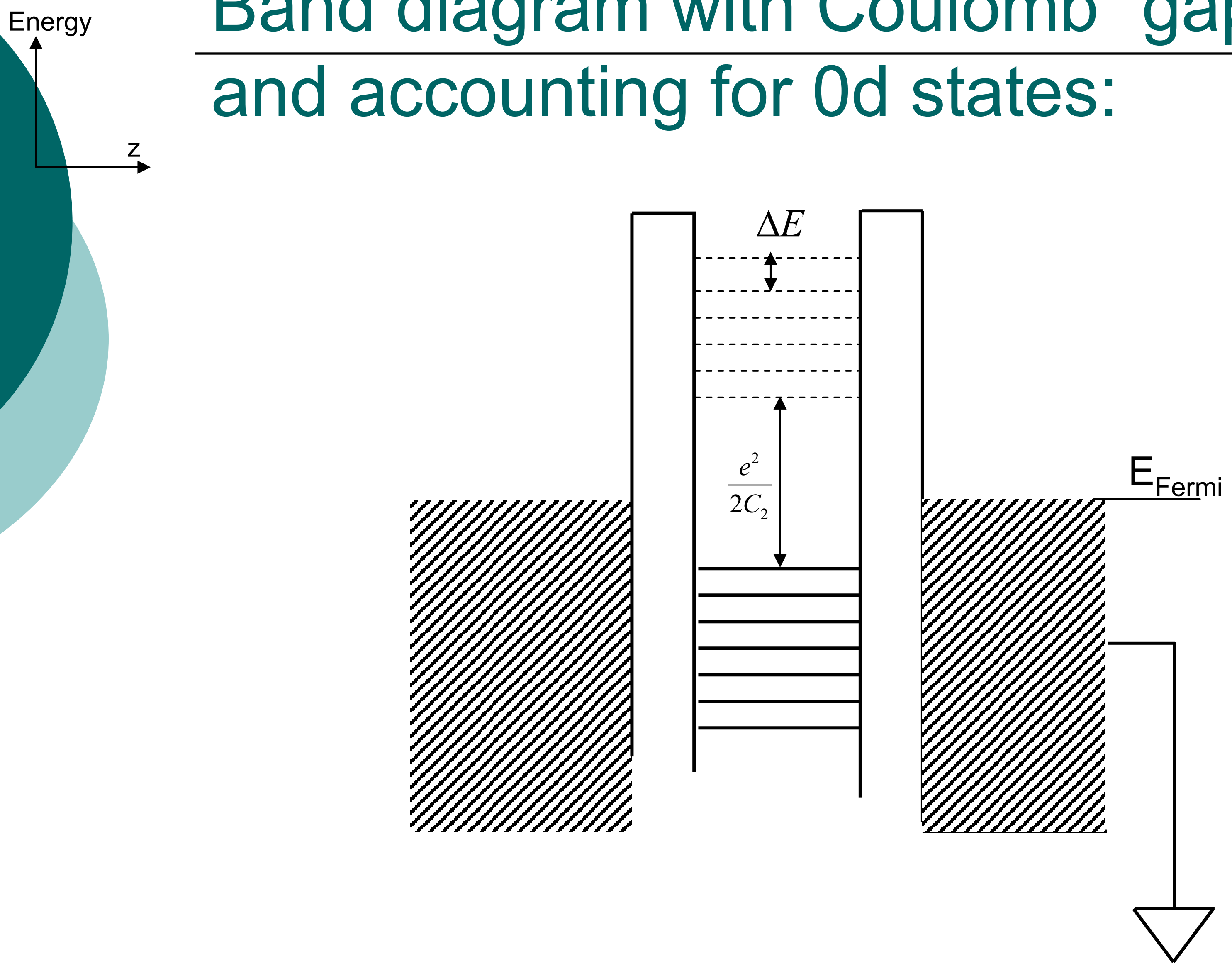
Continuum band diagrams like this no longer apply



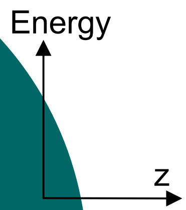
Band diagram with Coulomb “gap”

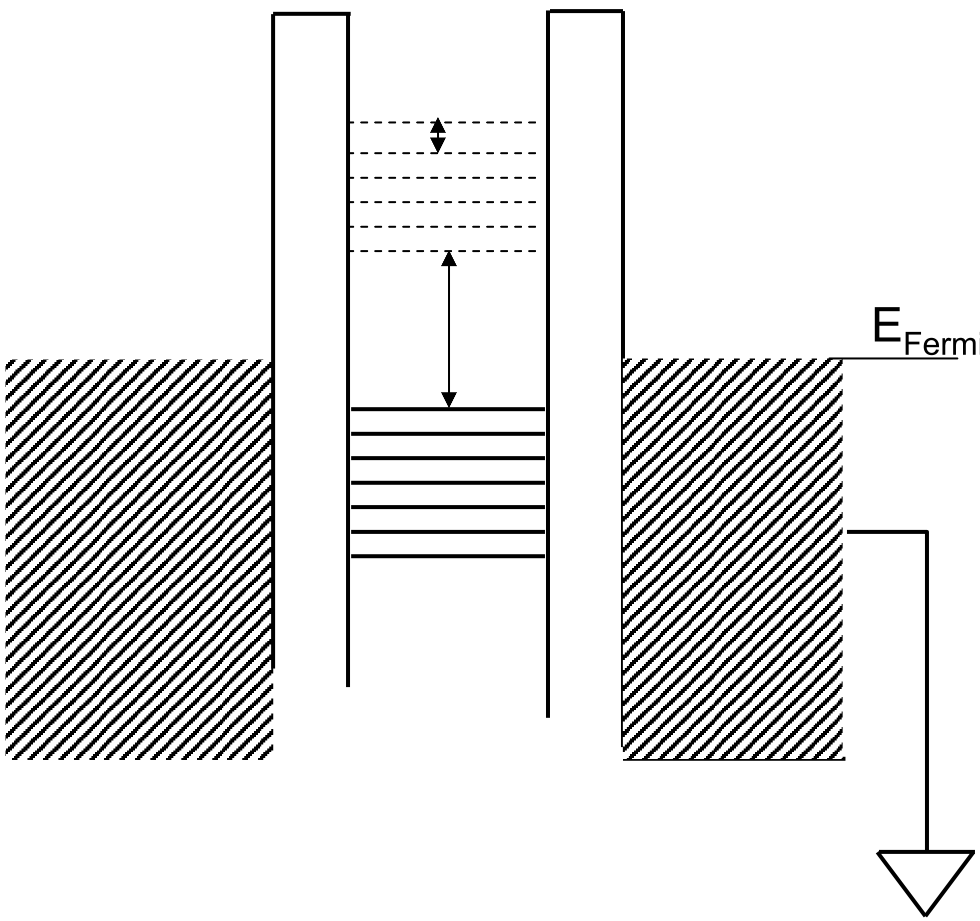


Band diagram with Coulomb "gap" and accounting for 0d states:

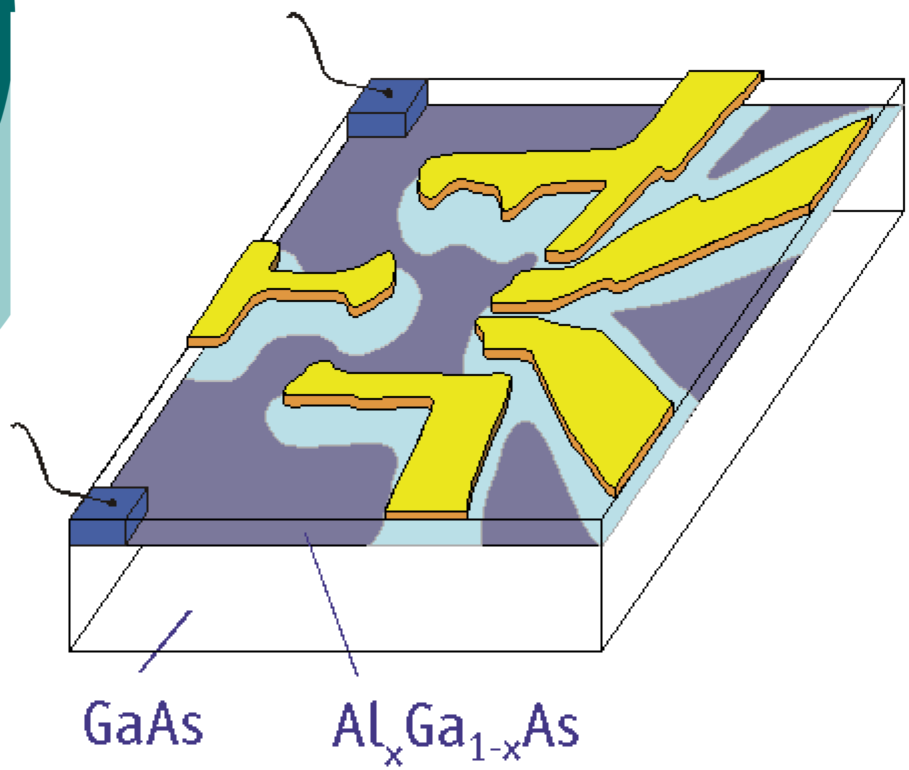


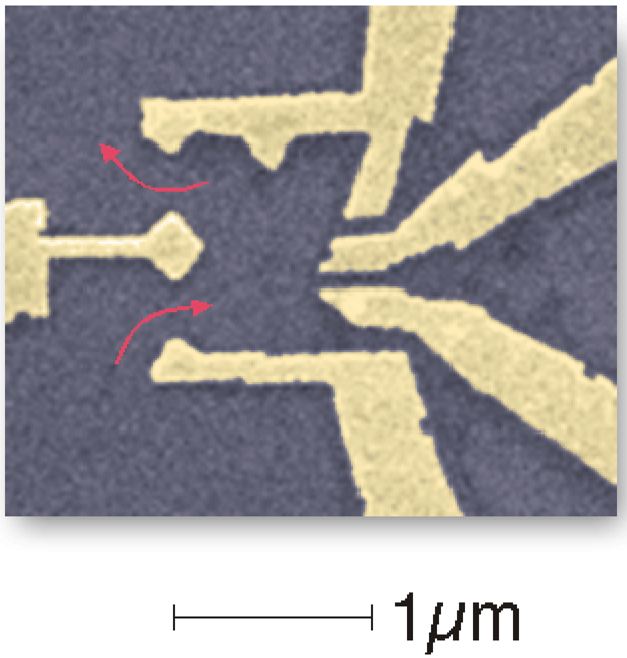
Band diagram with Coulomb "gap" and accounting for 0d states:





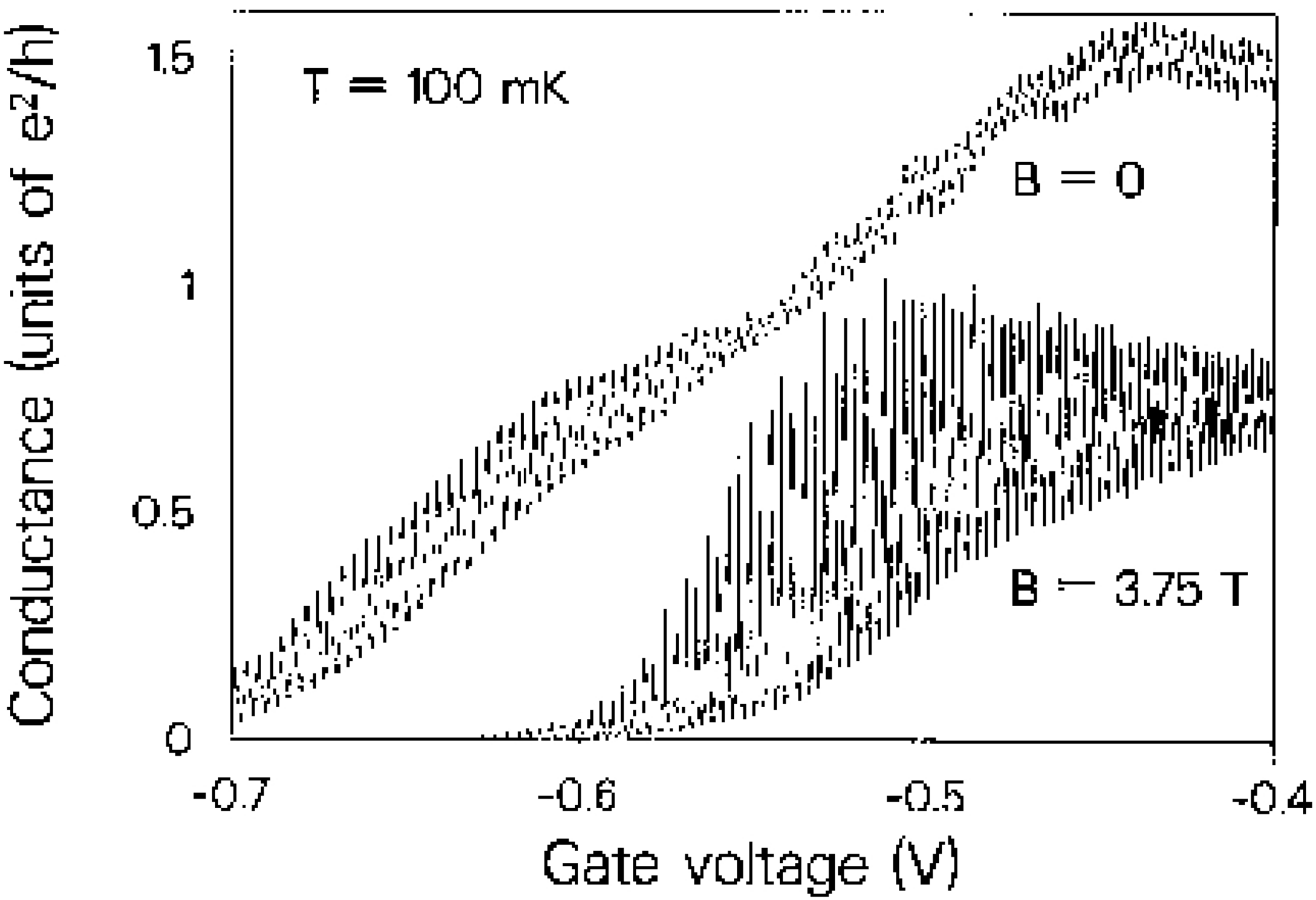
Experimental realization w/2DEGS





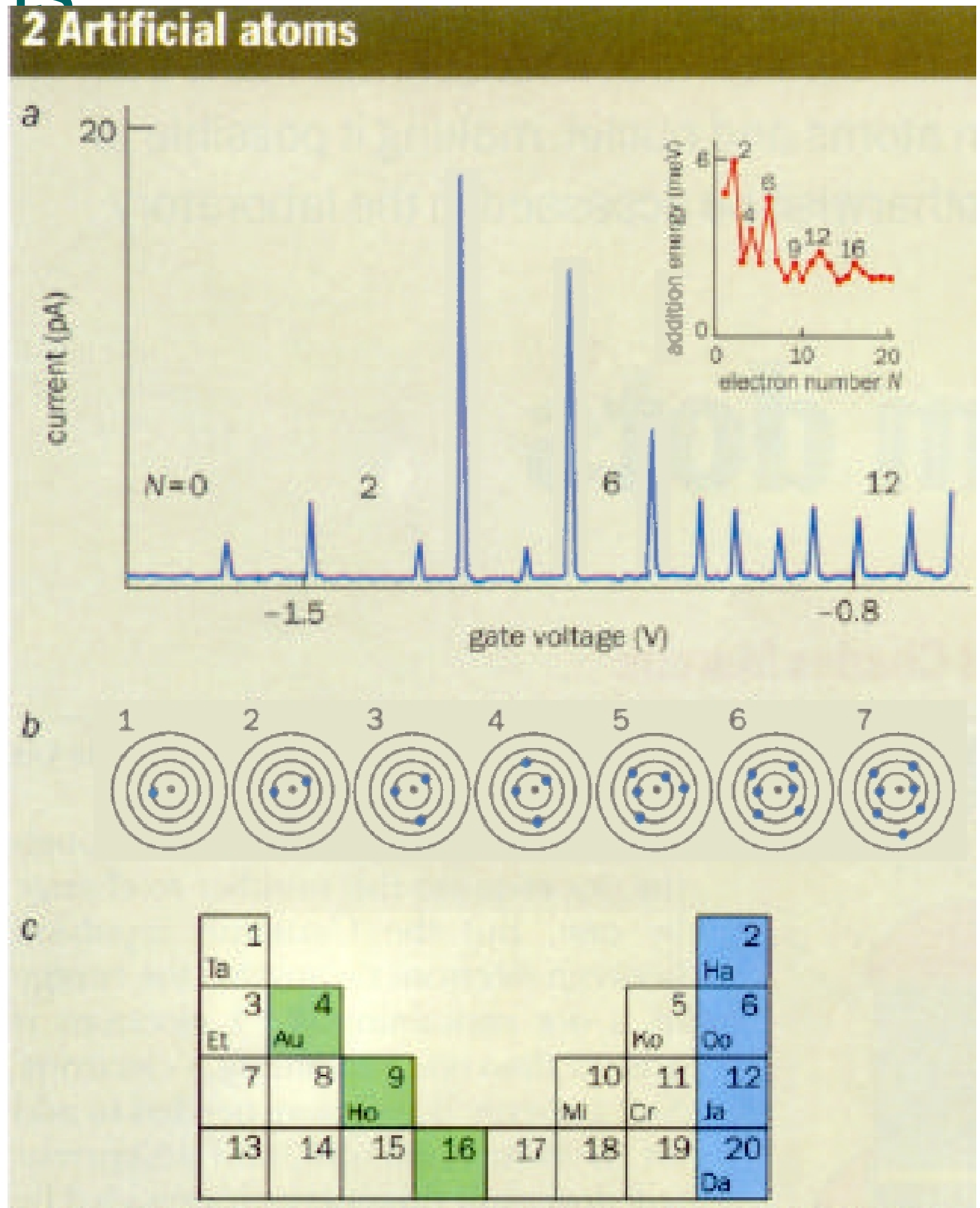
- http://marcuslab.harvard.edu/res.php

Results



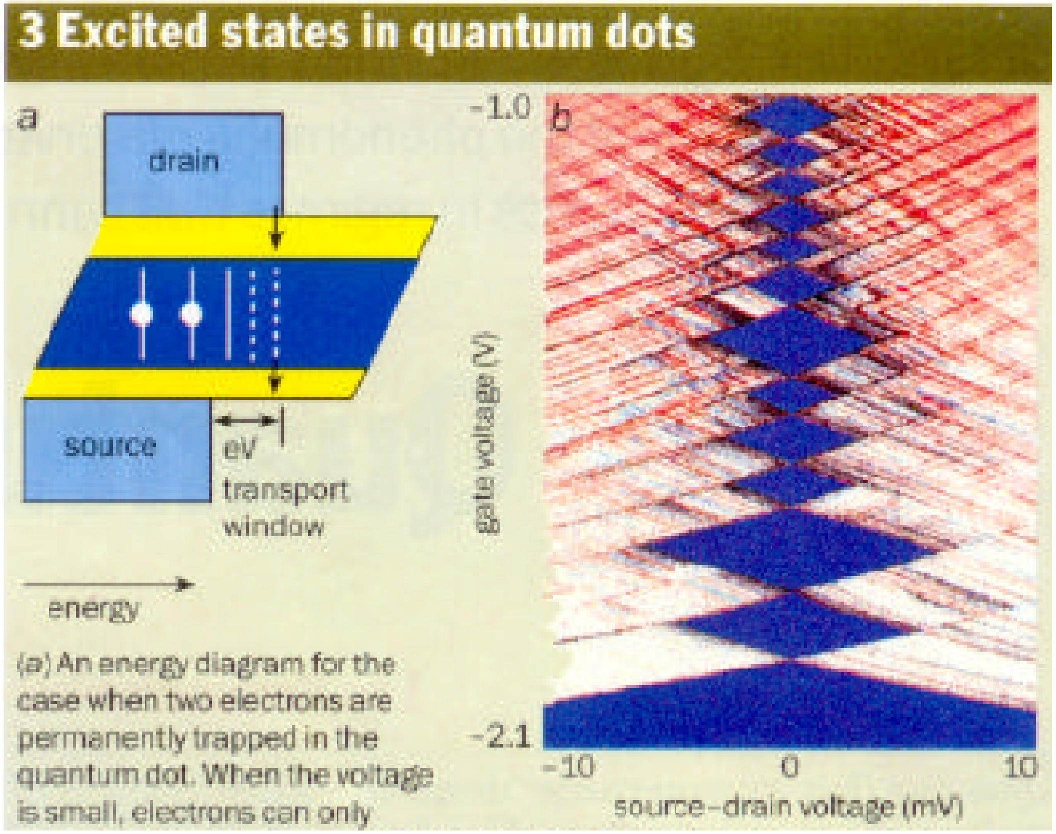
From .P. Kouwenhoven, C.M. Marcus, P.L. McEuen, S. Tarucha, R.M. Westervelt, and N.S. Wingreen
Electron Transport in Quantum Dots
Nato ASI conference proceedings, ed. By L. P. Kouwenhoven, G. Schön, L.L. Sohn (Kluwer, Dordrecht, 1997

Artificial atoms



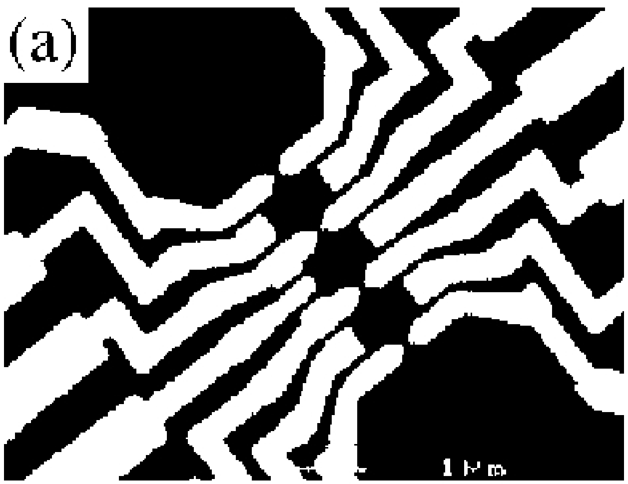
Leo Kouwenhoven and Charles Marcus
Quantum Dots
Physics World, June 1998

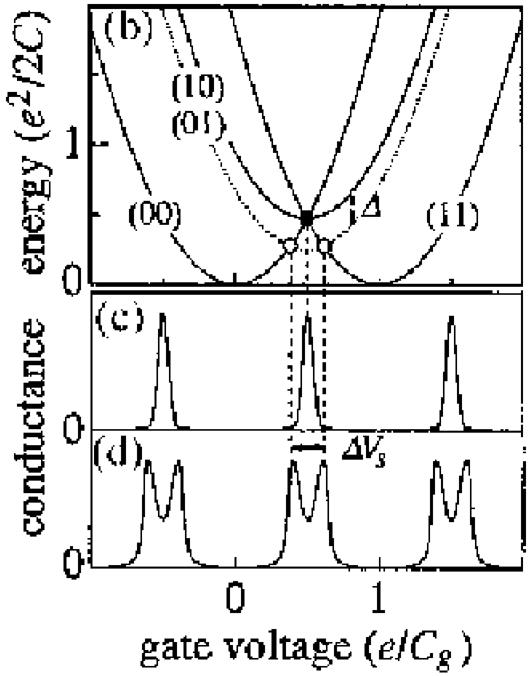
Coulomb diamonds



Leo Kouwenhoven and Charles Marcus
Quantum Dots
Physics World, June 1998

Artificial molecules





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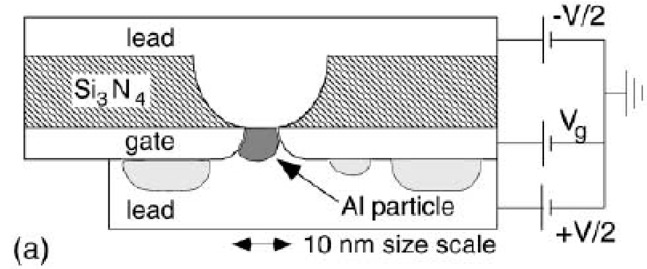
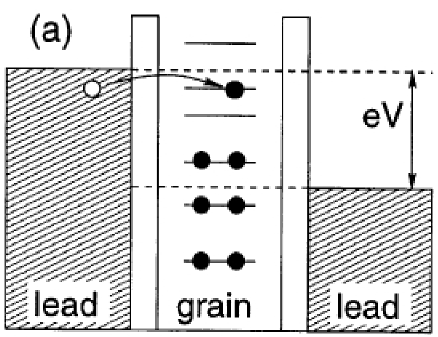
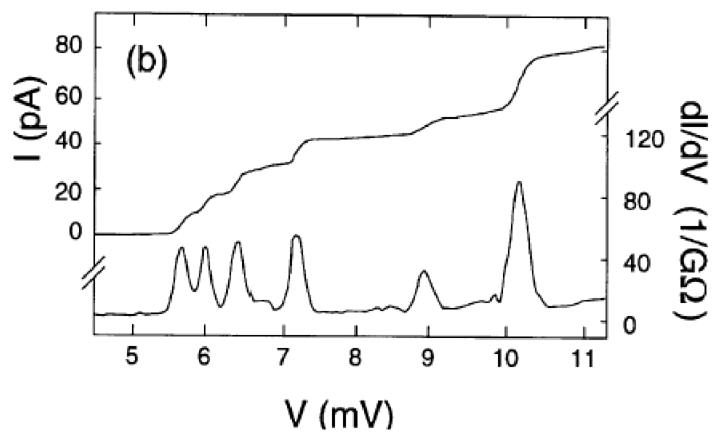
Electron Transport in Quantum Dots
Nato ASI conference proceedings, ed. By L. P. Kouwenhoven, G. Schön, L.L. Sohn (Kluwer, Dordrecht, 1997

Metallic quantum dots

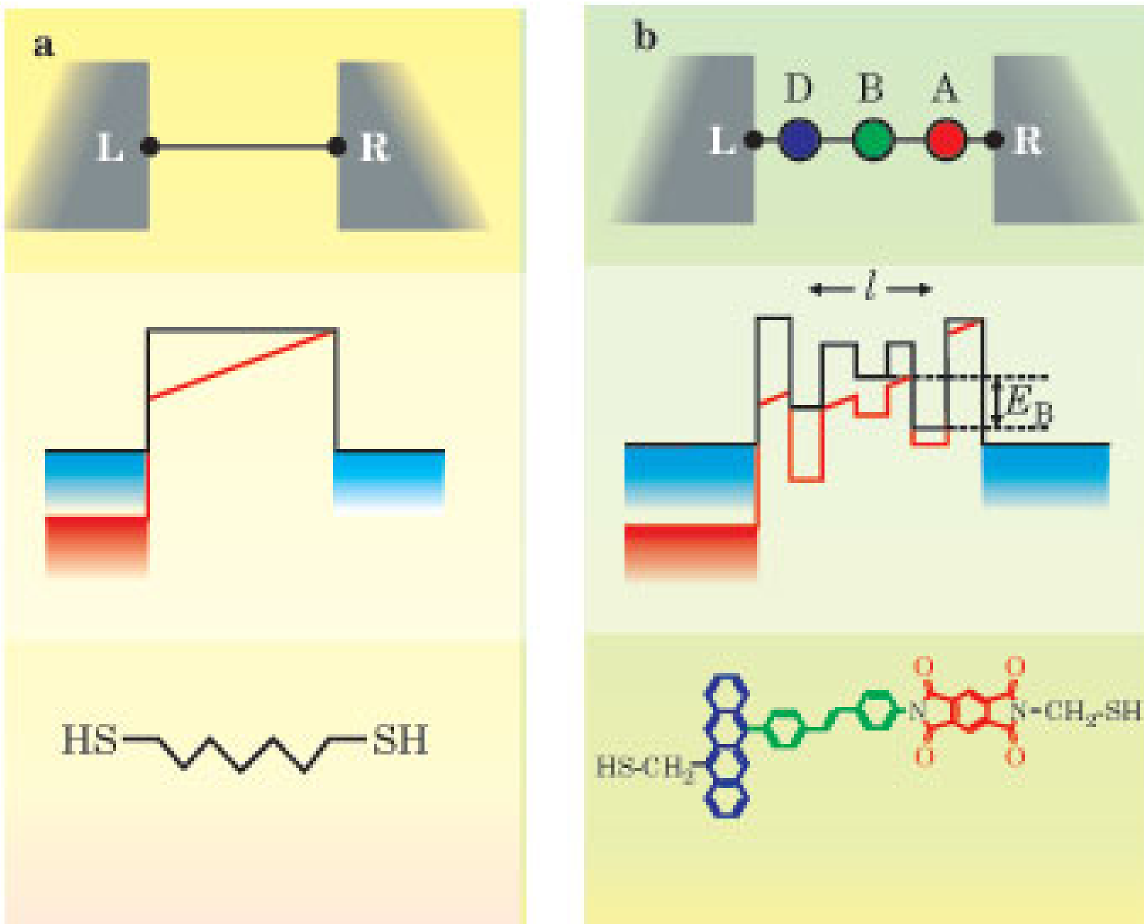






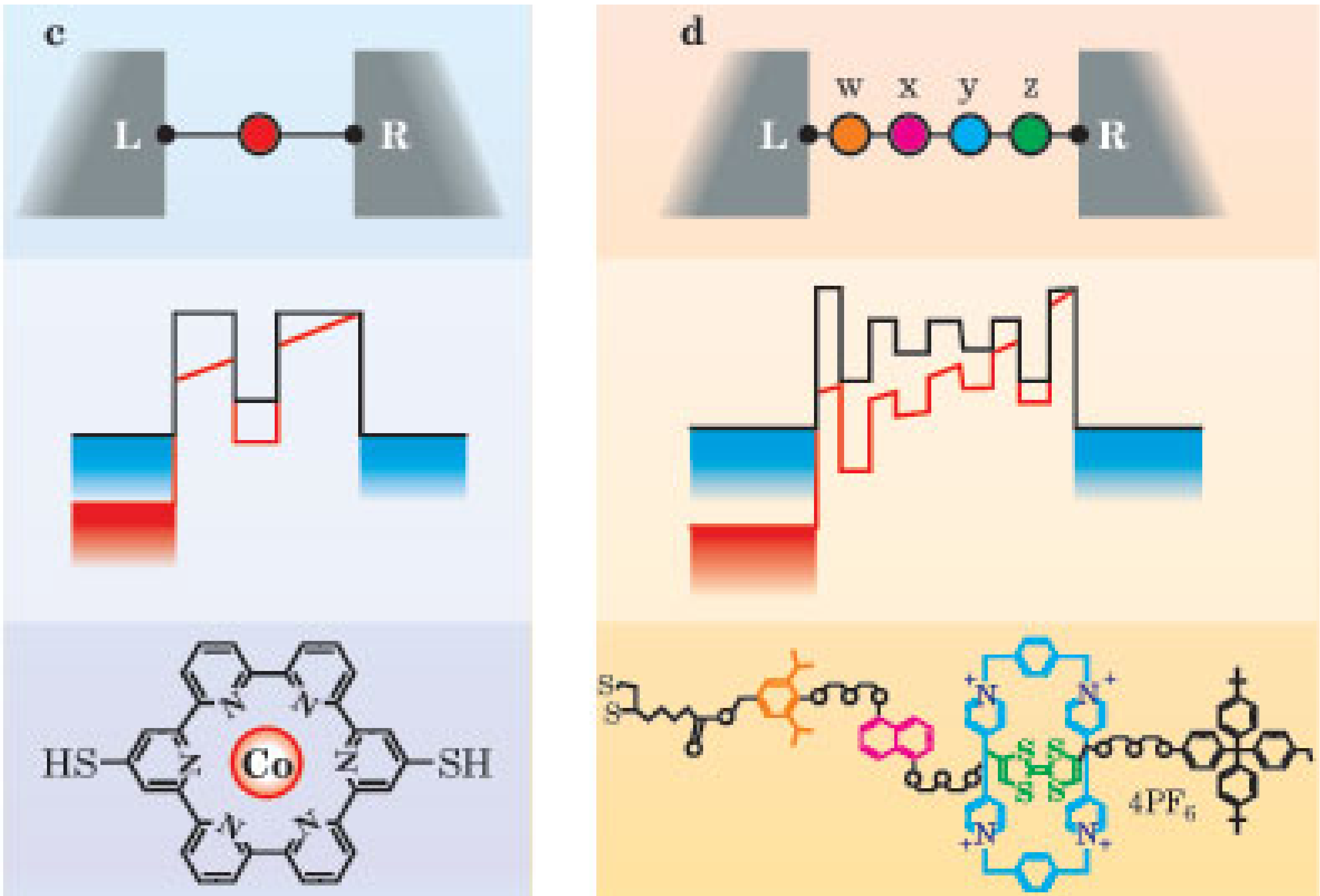
Spectroscopy of Discrete Energy Levels in Ultrasmall Metallic Grains, Jan von Delft and D. C. Ralph, *Physics Reports* **345**, 61 (2001).

Molecular electronics



J. Heath, Physics Today, May 2003

Molecular electronics



J. Heath, Physics Today, May 2003

Molecular electronics

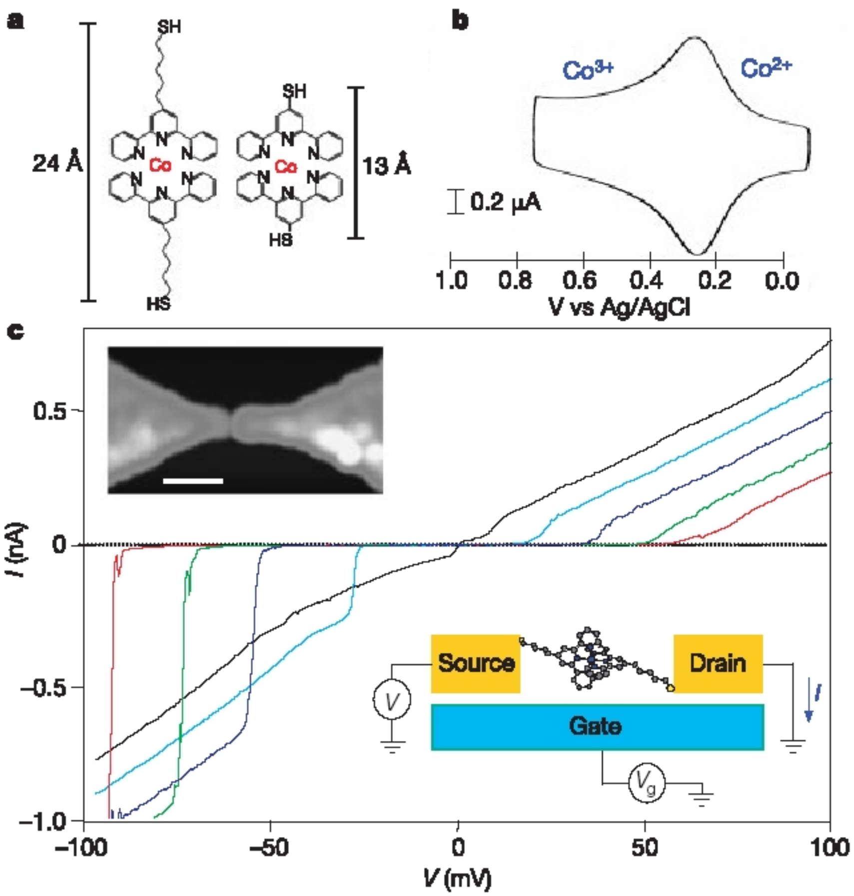
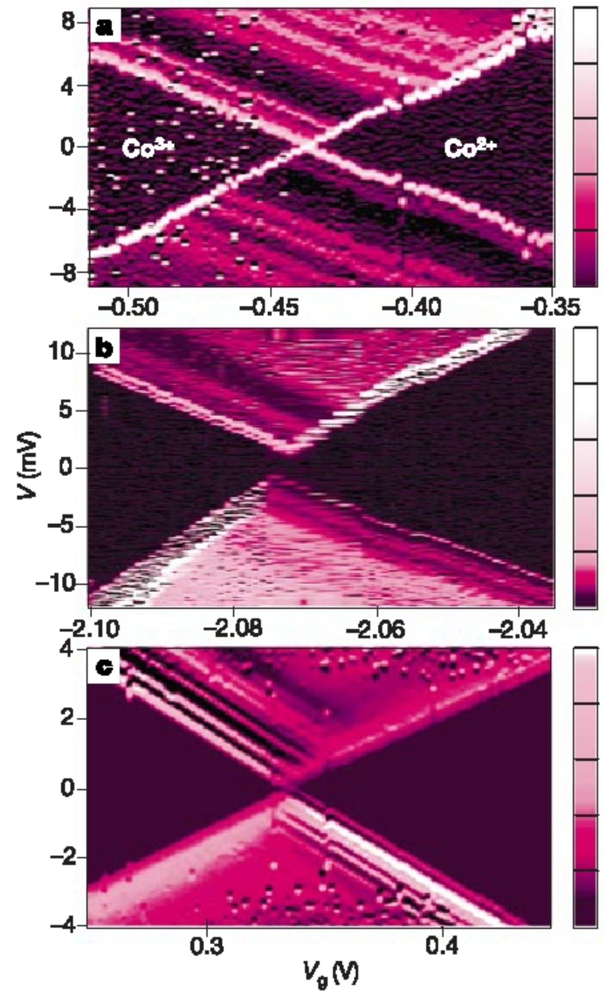


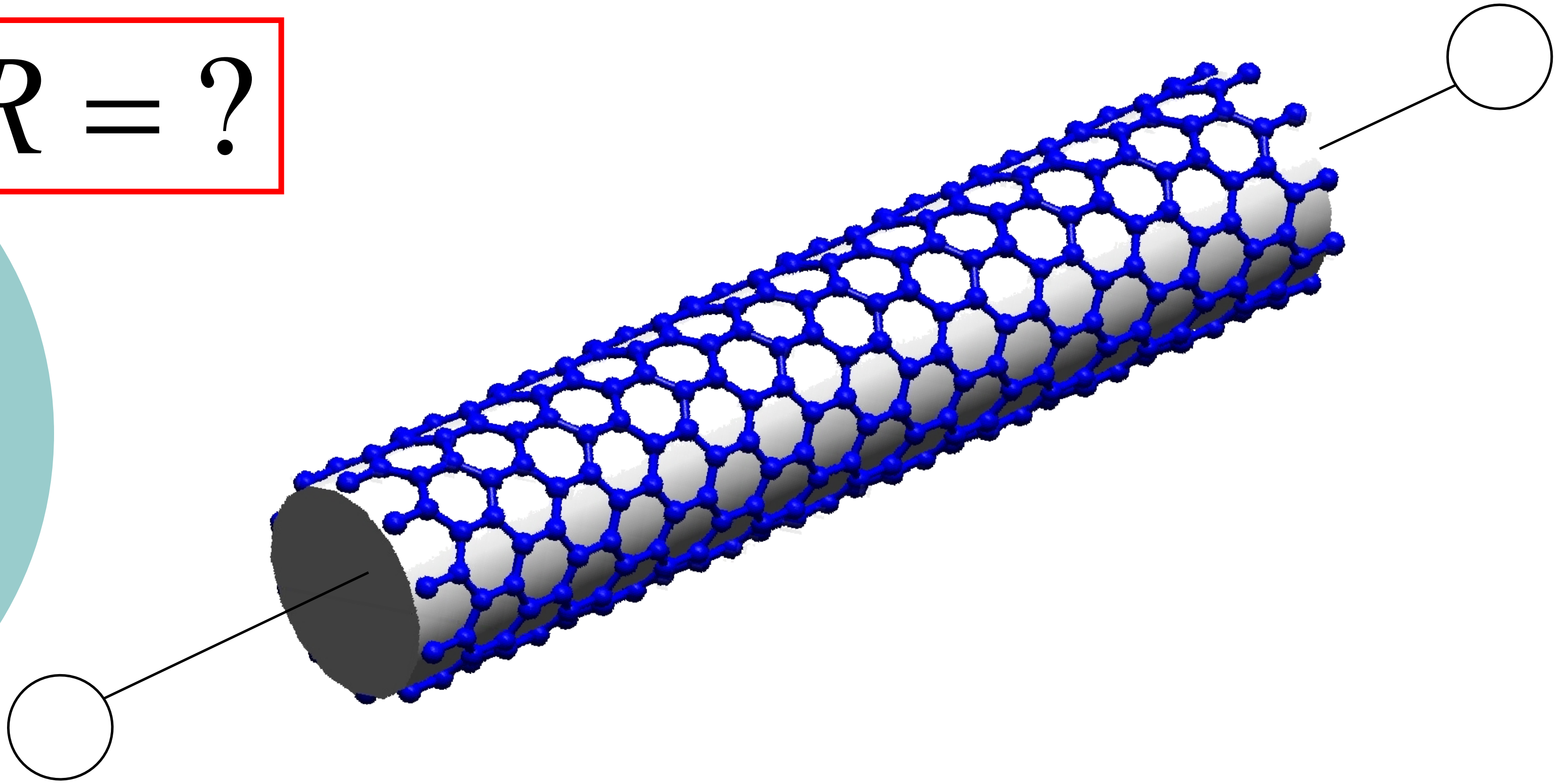
Figure 1 The molecules used in this study and their electronic properties. **a**, Structure of $[Co(tpy-(CH_2)_5-SH)_2]^{2+}$ (where tpy-$(CH_2)_5$-SH is 4′-(5-mercaptopentyl)-2,2′:6′,2″-terpyridinyl) and $[Co(tpy-SH)_2]^{2+}$ (where tpy-SH is 4′-(mercapto)-2,2′:6′,2″-terpyridinyl). The scale bars show the lengths of the molecules as calculated by energy minimization.



"Coulomb blockade and the Kondo effect in single-atom transistors," Jiwoong Park, Abhay N. Pasupathy, Jonas I. Goldsmith, Connie Chang, Yuval Yaish, Jason R. Petta, Marie Rinkoski, James P. Sethna, Hector D. Abruna, Paul L. McEuen & Daniel C. Ralph, Nature, 417, 722-725 (2002).

Lecture 13: Carbon nanotubes

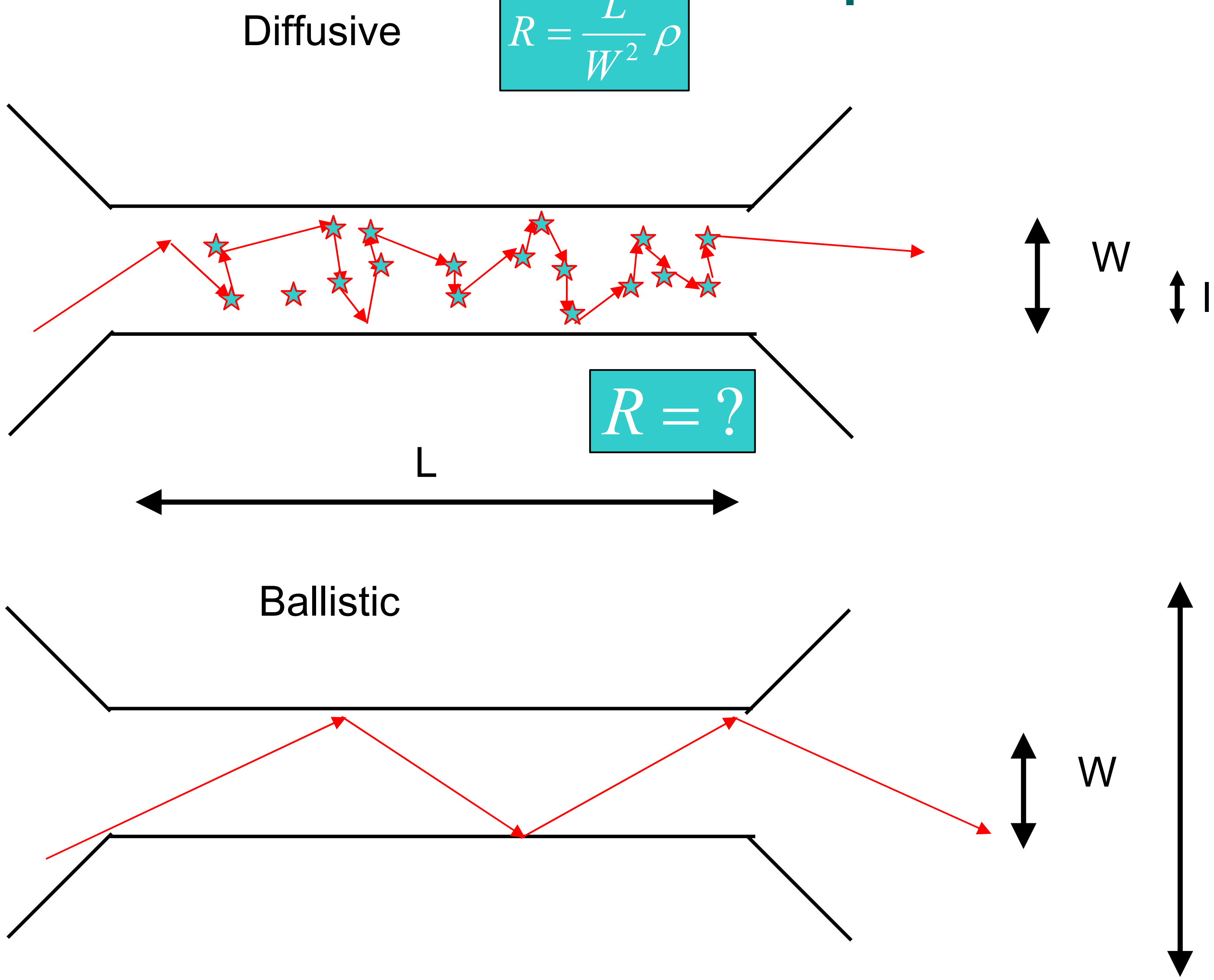
$$R = ?$$



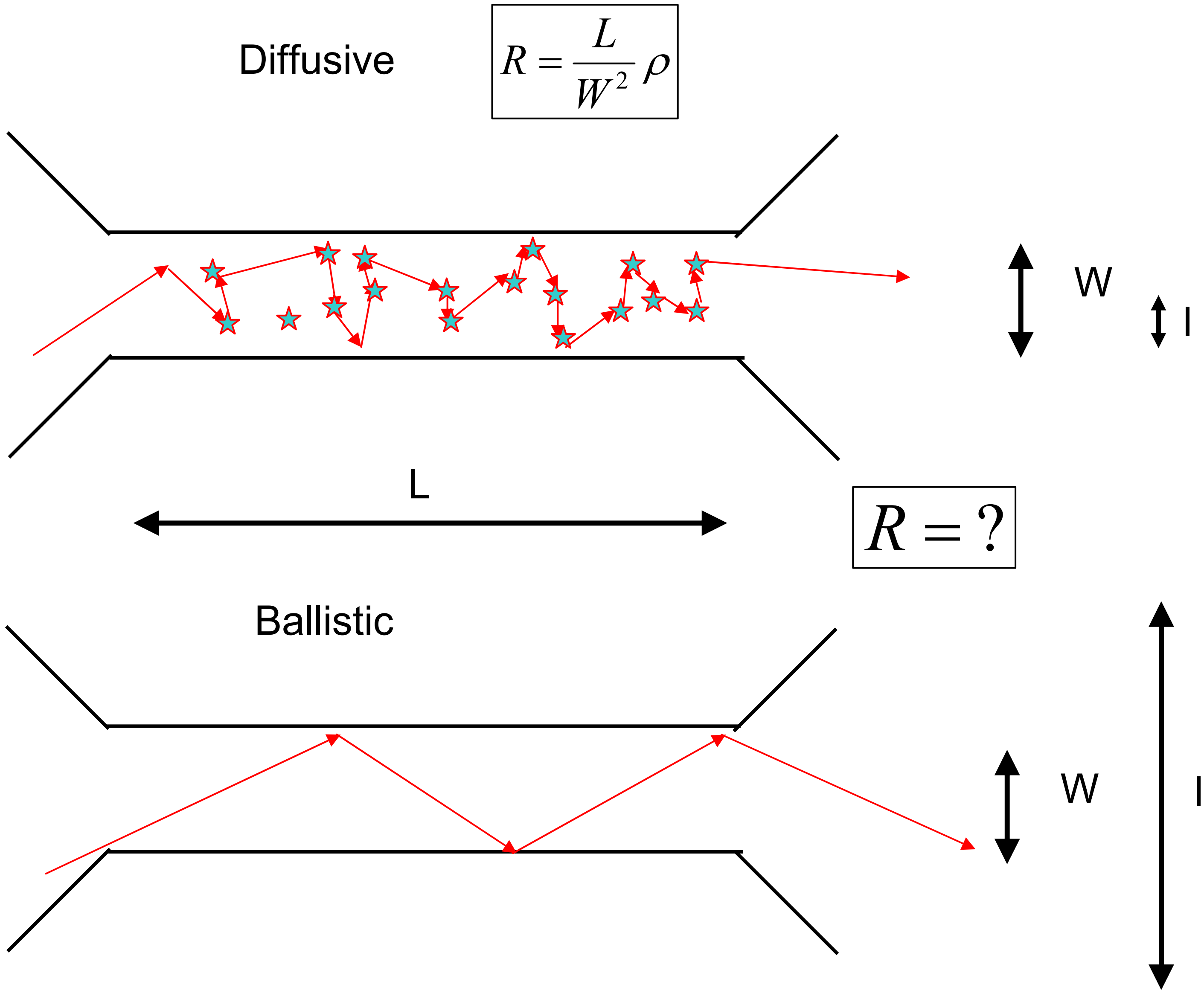
Readings this lecture covers

- Hanson, pp. 170-176
- McEuen review, *IEEE Transactions on Nanotechnology*, reading packet

Ballistic vs. diffusive transport



Ballistic vs. diffusive transport



Landauer formula:

$$G = n\frac{2e^2}{h}$$

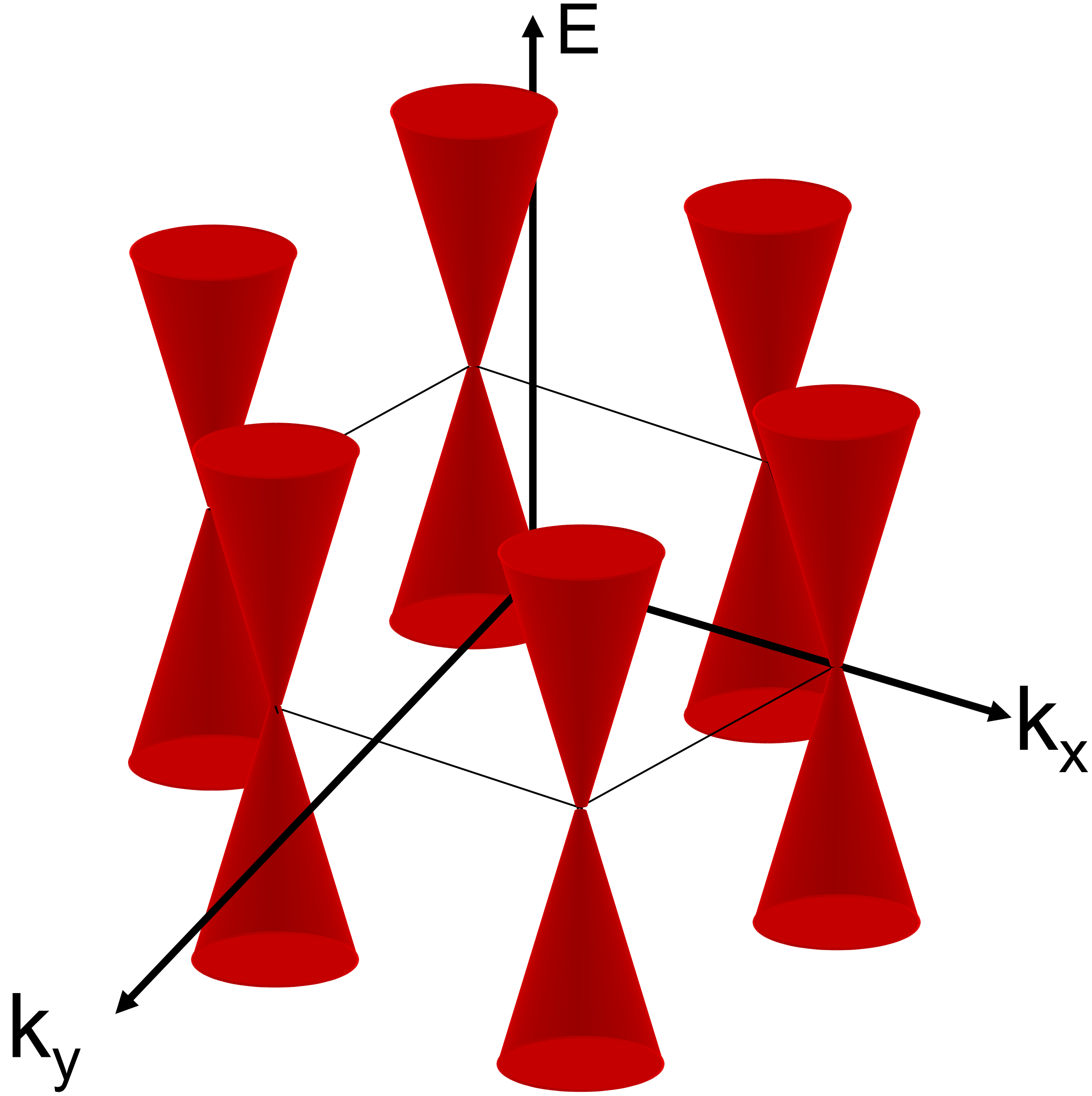
If the leads are not perfect injectors into each "channel" then:

$$G = \frac{2e^2}{h}\sum T_n$$

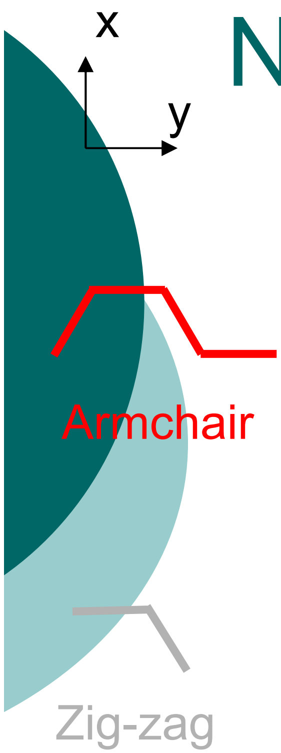
Graphite: 2d semiconductor

Bond length = 0.142 nm

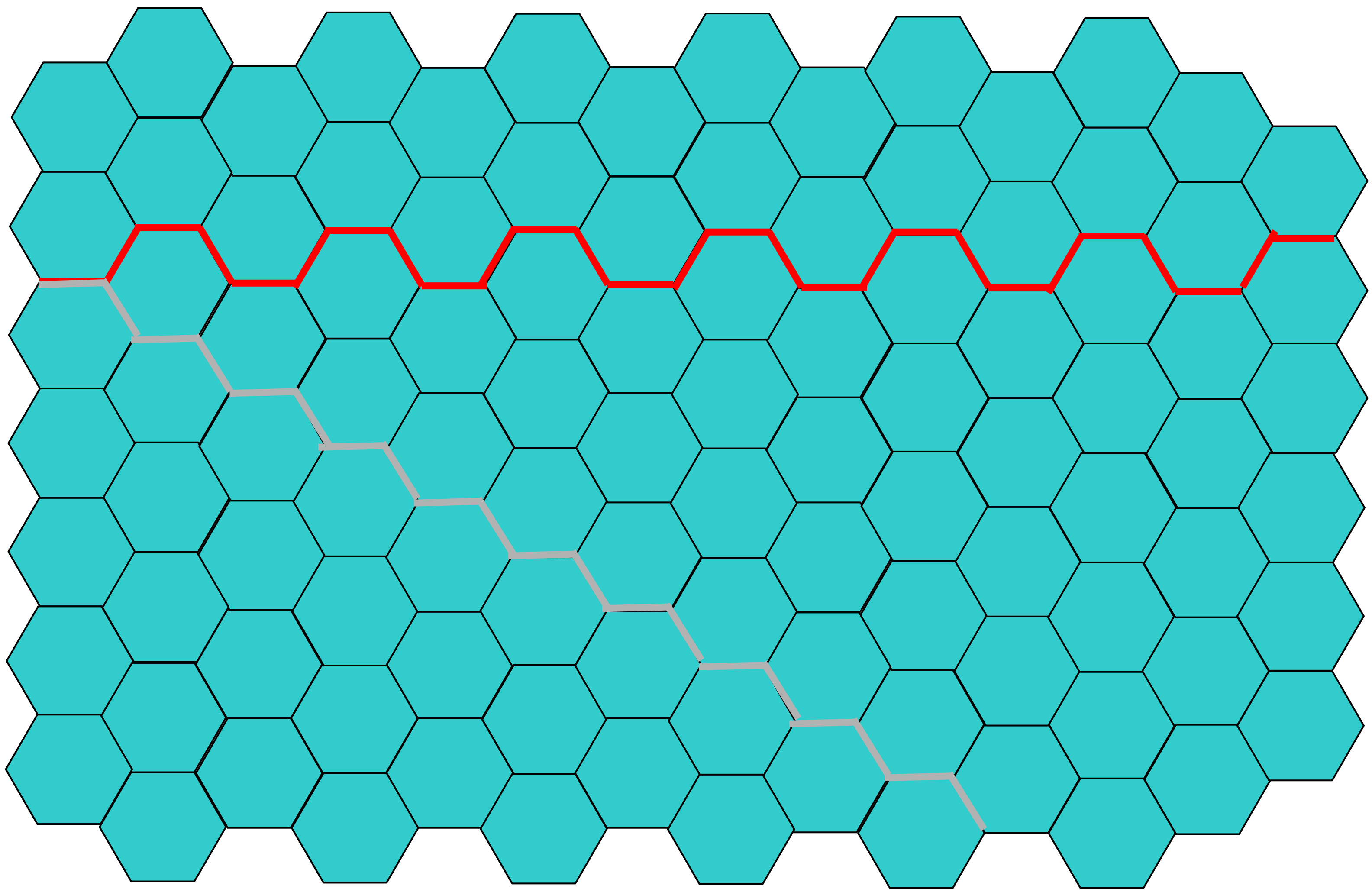
Graphite band structure



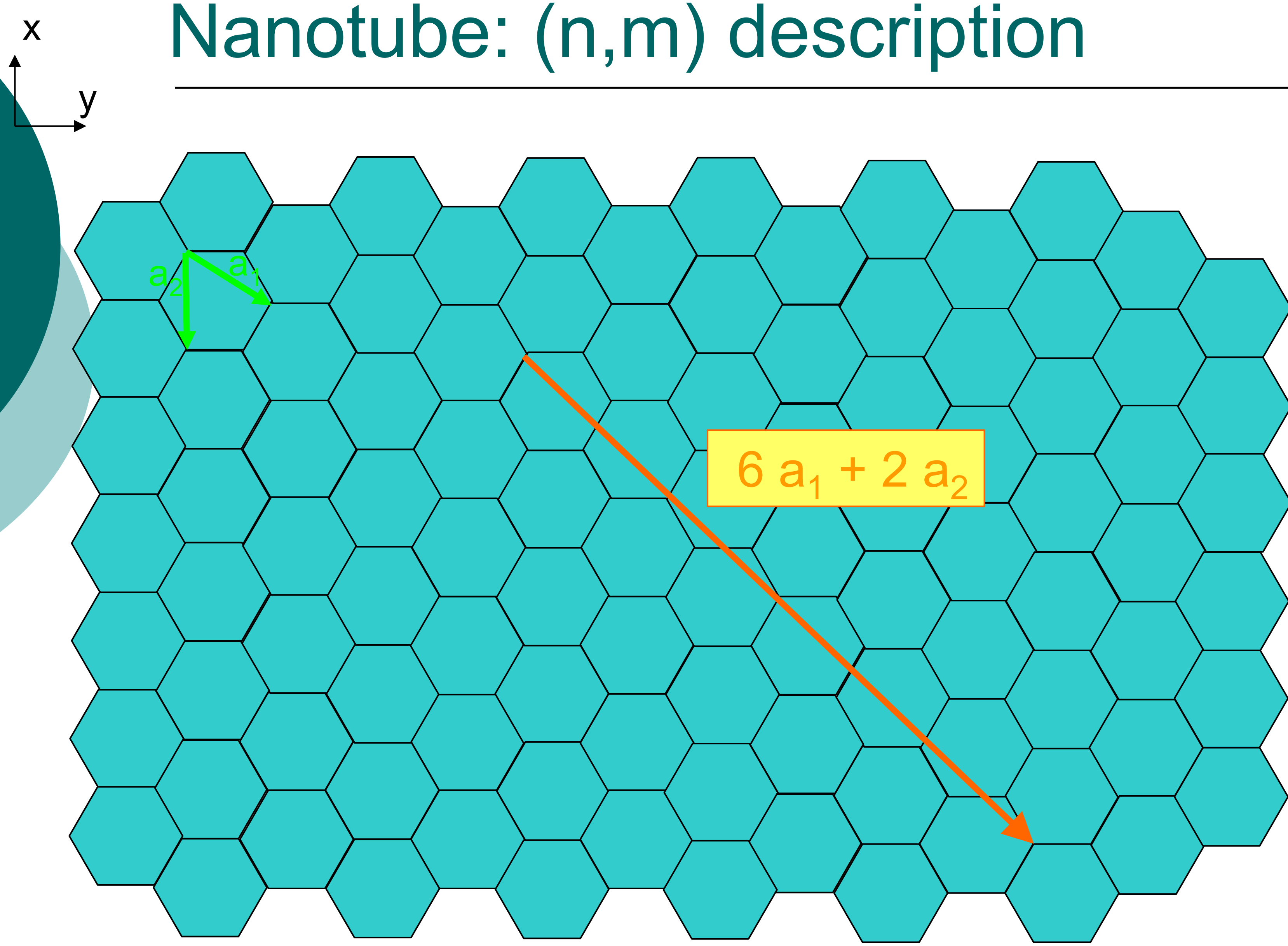
Nanotube: Zig-zag vs. armchair



In between:
Chiral



Nanotube: (n,m) description



In this example: (n,m) = (6,2)

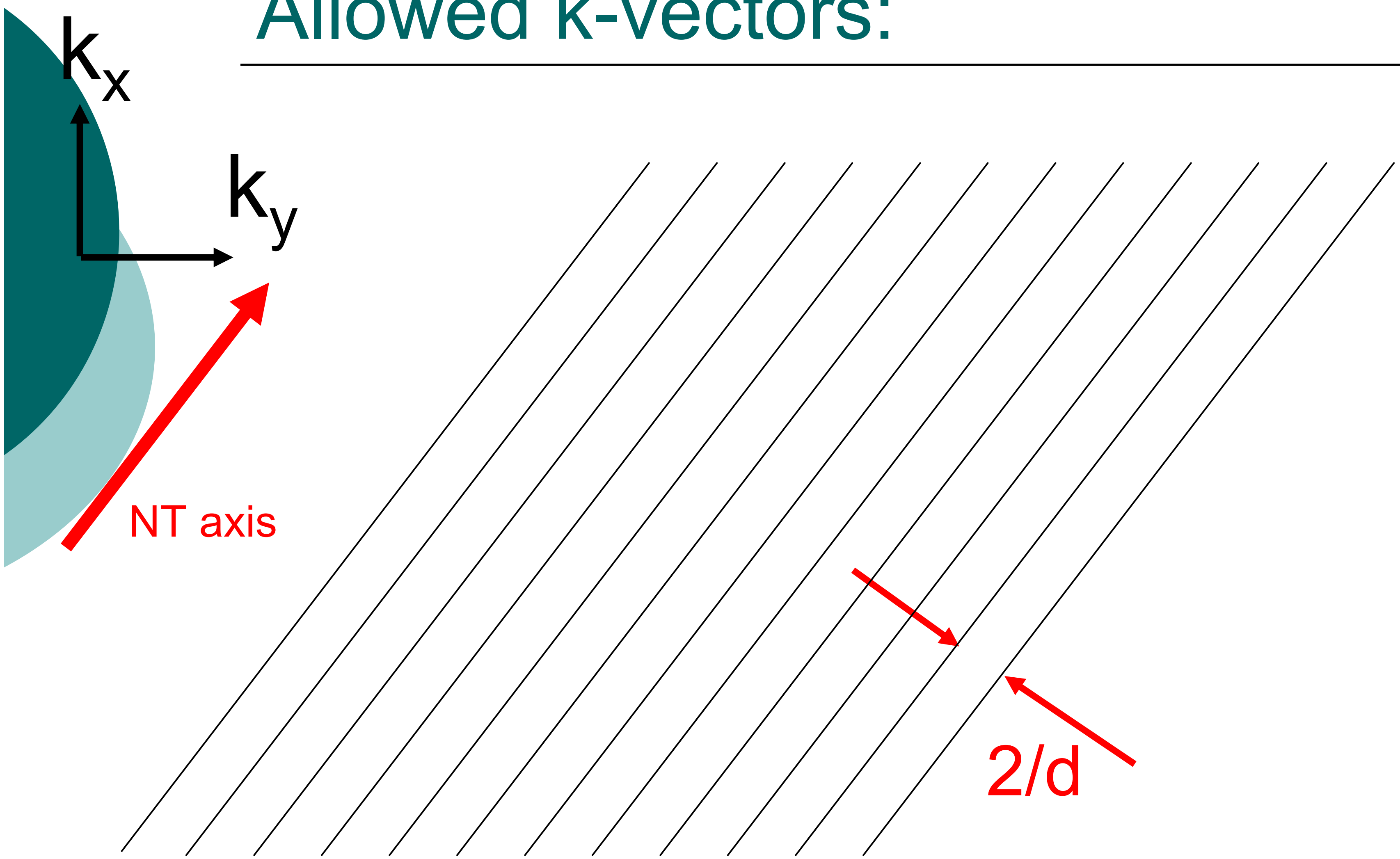
k-vector

- Graphite:
  - Arbitratrary k_x, k_y allowed
- Nanotube:

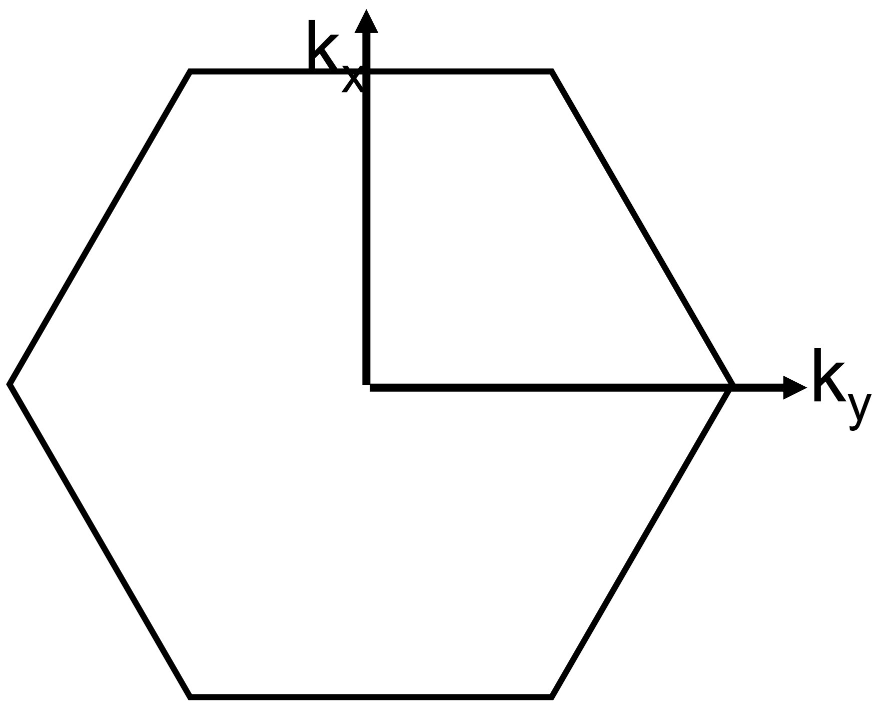
  $\psi(\phi)=\psi(\phi+2\pi)$

  - k_{perp} spaced by 2/d

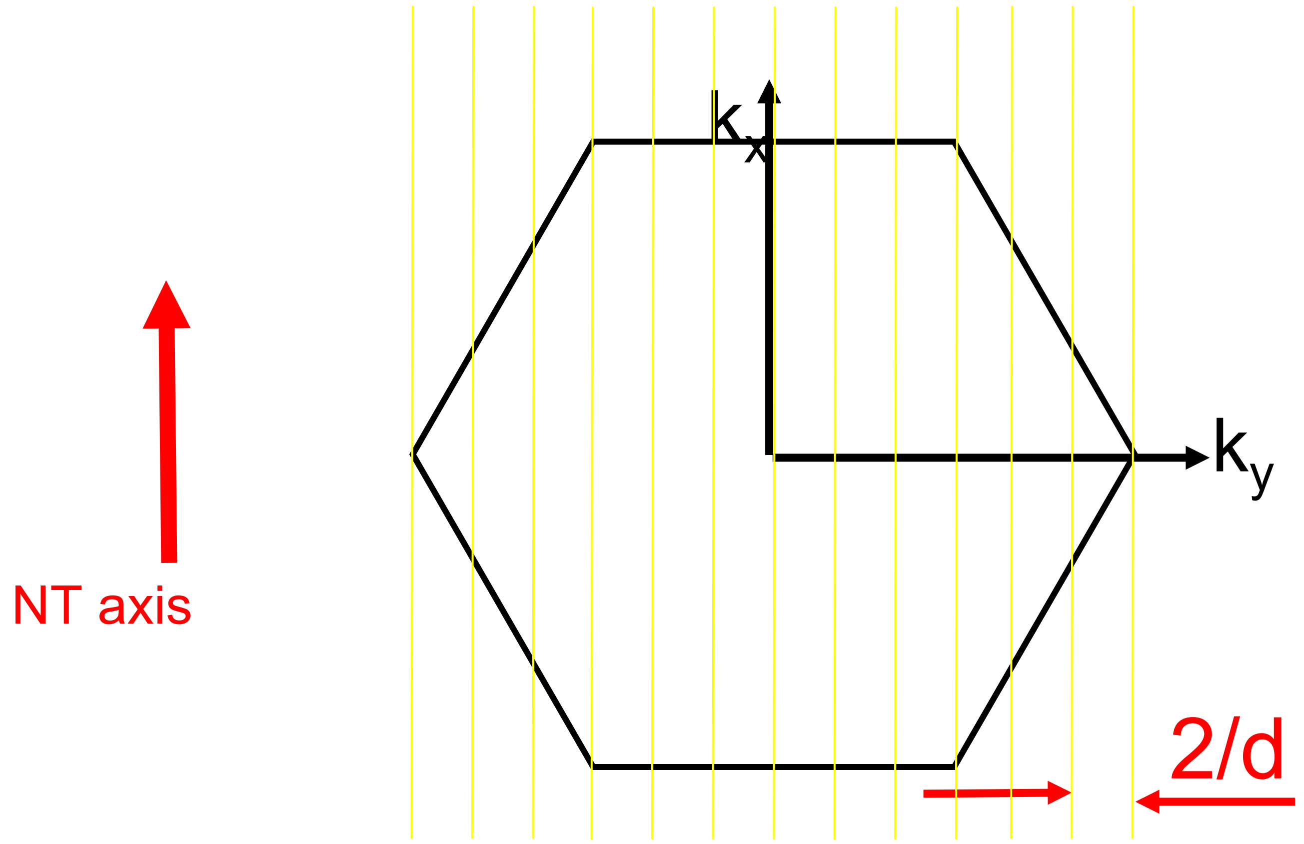
Allowed k-vectors:



k-space



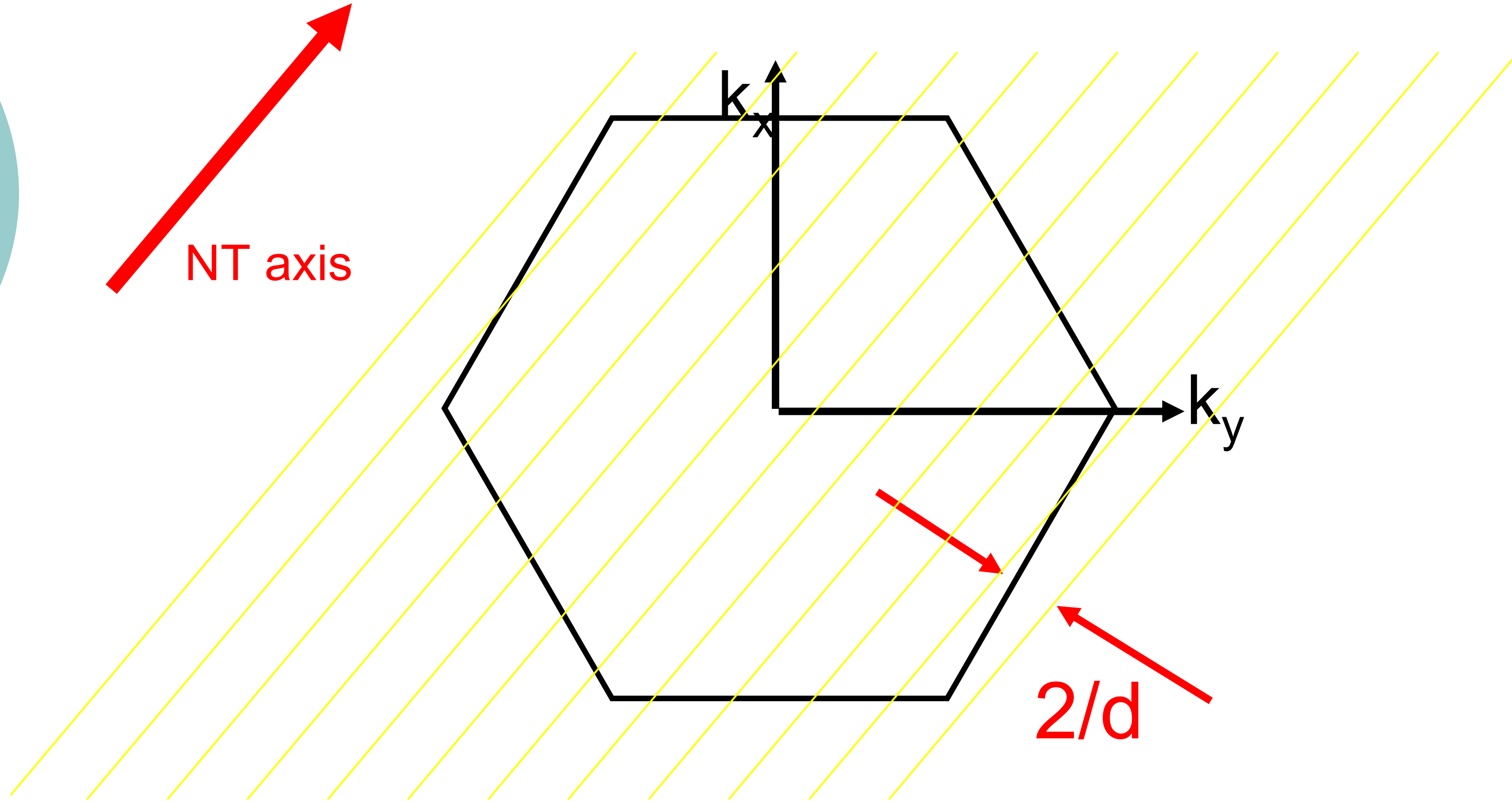
(9,0) armchair nanotube



All armchairs are metallic.

$$G = \frac{2e^2}{h}\sum T_n = \frac{4e^2}{h}$$

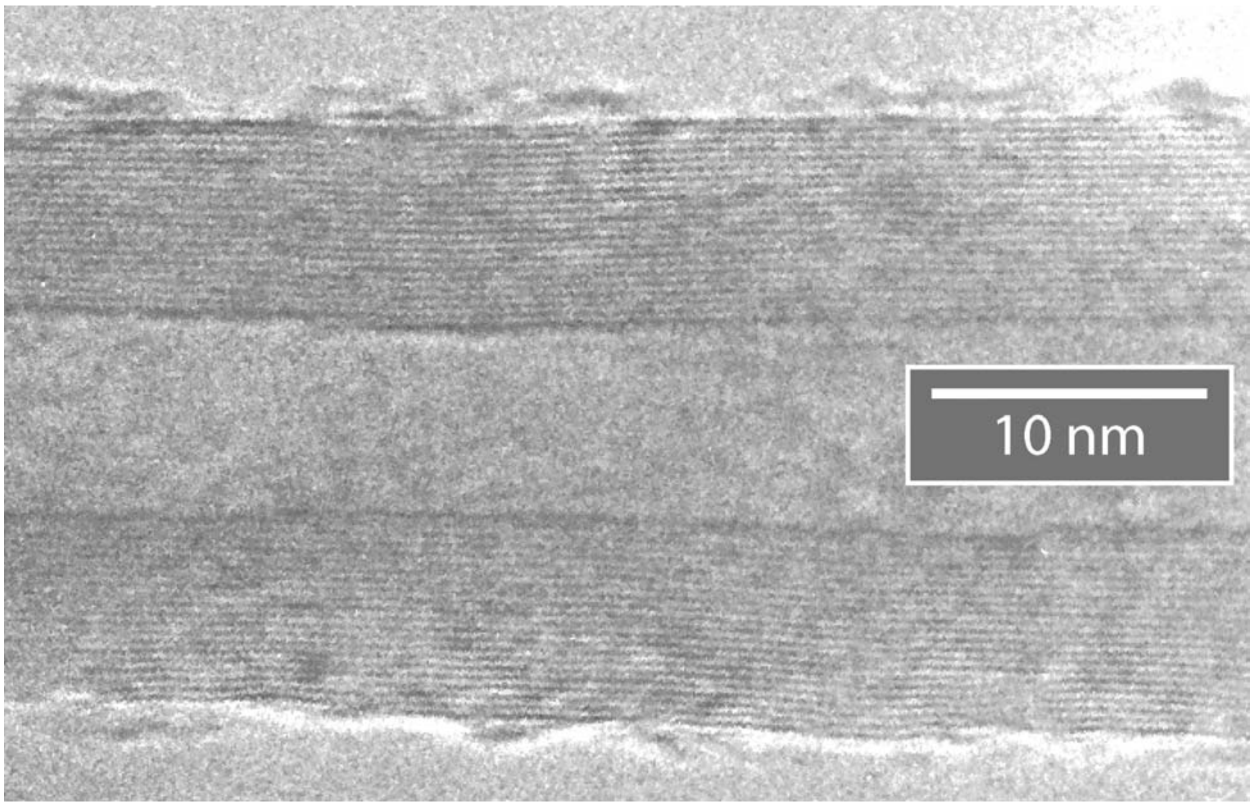
Semiconducting nanotube



Electrical properties

- All armchair metallic
- 33% of zig-zag metallic
- Semiconducting tubes:
  - Gap = 0.9 eV/d[nm]

Multi-walled nanotube (MWNT)

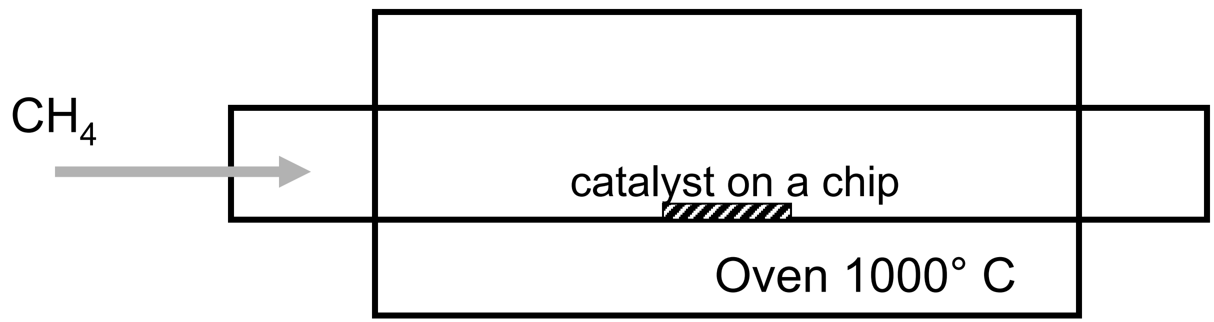


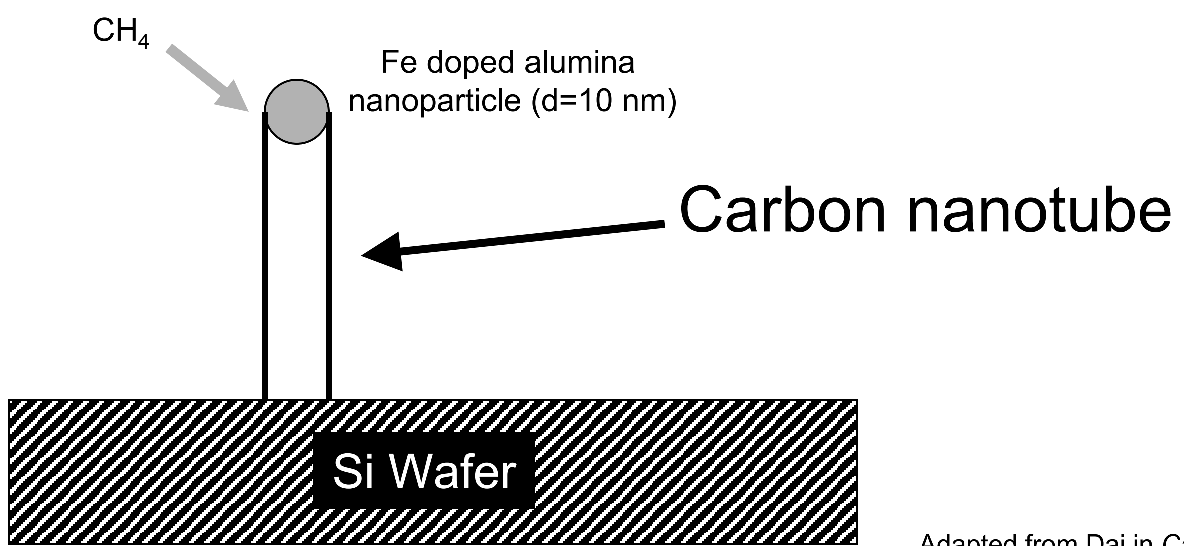
Shengdong Li, unpublished

Growth technologies

- Arc discharge
- Laser ablation
- Chemical vapor deposition (CVD)

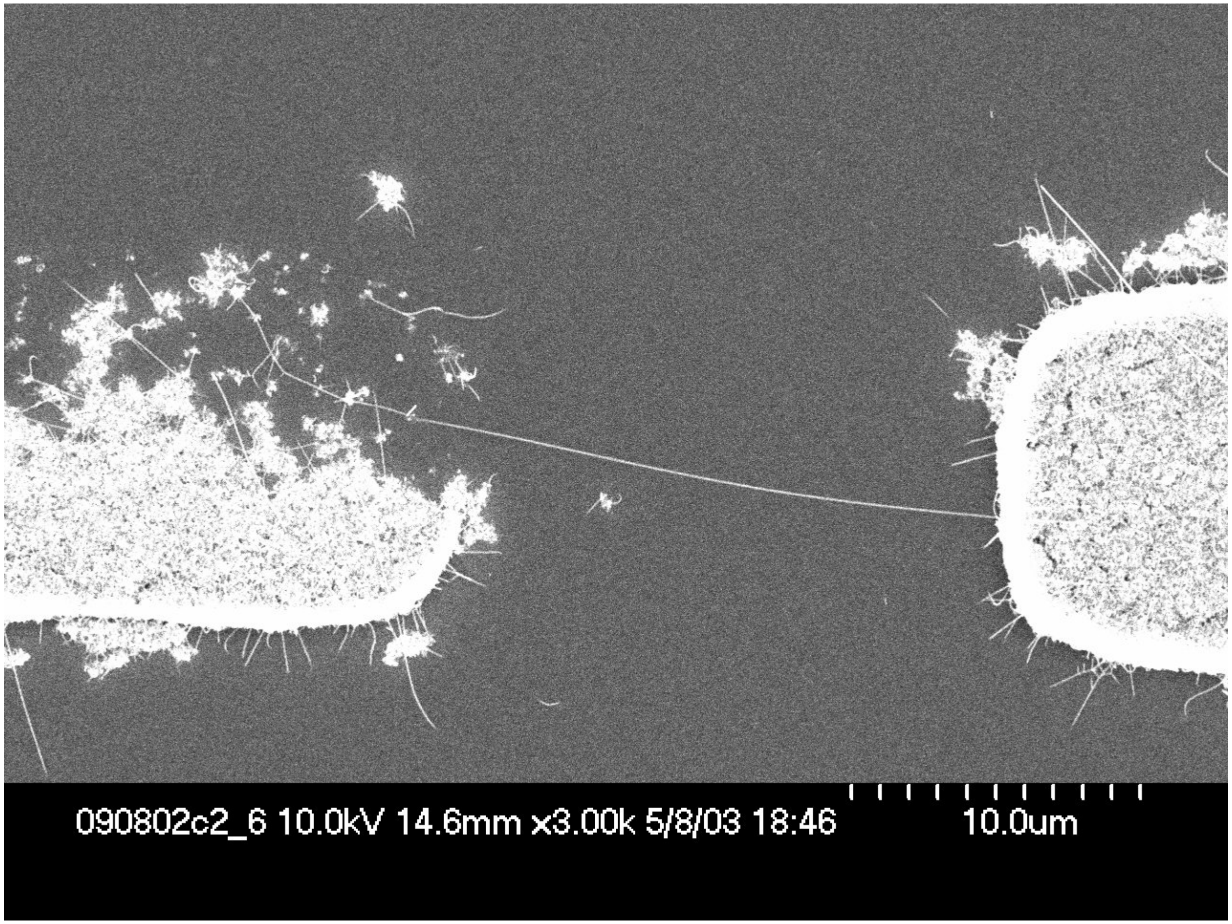
CVD





Adapted from Dai in *Carbon Nanotubes*, by Dresselhaus and Dresselhaus

Lithographically defined catalysts



Shengdong Li, unpublished

Electrical contact?

- $4e^2/h$ can be to achieved
  - Achieved in J Kong, et al, Phys. Rev. Lett. **87**, 106801 (2001)
  - Pd as contact metal is supposed to be better

Circuits

- p-dope, n-dope
- Nano p-n junctions demonstrated
- Complementary logic demonstrated
  - Inverters
  - Logic

Other applications

- Nanomaterials:
  - Nanotubes are strongest materials known to man
- STM:
  - Nanotube tips give very high horizontal resolution
- Nano-bio sensors
- RF MEMS resonators