Solutions

HW1

EECS 277B

1) Calculate the density of states vs. E in a one dimensional world
2) Find the spacing between the lowest two energy levels in a 3d box of size 1 m.
3) Consider a single electron in a box. Calculate the size of the box at which only the ground state is occupied at room temperature.

2) $$\Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (1^2+1^2+2^2) - (1^2+1^2+1^2)$$

$$= \frac{3\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$$

$$m = 9.1 \times 10^{-31}\ \text{Kg}$$

$$\hbar = 1.05 \times 10^{-34}\ \text{J-s}$$

$$\Delta E = 1.8 \times 10^{-37}\ \text{J}$$

$$= 10^{-18}\ \text{eV}$$

3) $$\Delta E > KT \iff \frac{3\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 > KT$$

$$\iff L^2 < \frac{3\hbar^2}{2m}\frac{\pi^2}{KT}$$

From above $\Delta E = 10^{-18}\ \text{eV}$ @ $L = 1\ \text{m}$

So @ $L = 1\text{m}$ $$\frac{\Delta E}{KT} = \frac{10^{-18}\ \text{eV}}{\frac{1}{30}\ \text{eV}} = 3.3\ 10^{-17}$$

$$\Rightarrow L = \left[3.3\ 10^{-17}\right]^{\frac{1}{2}}\ \text{m} = 5.7\ \text{nm}$$

$\Rightarrow$ Need $\underline{L < 5.7\ \text{nm}}$

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1 dimension

$$N_k dk = ?$$

"Volume" of line segment
=dk

Number of states in length=
length x States/length

States/length = 1 / (π/L) :

$$N_k dk = (dk) \cdot \left(\frac{1}{(\pi / L)} \right) \cdot 2 = \frac{2}{\pi} L dk$$

$$\rho_k dk \equiv \frac{N_k dk}{\text{length}} = \frac{2}{\pi} dk$$

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1 dimension

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{2}{\pi} dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\rho(E)dE = \frac{2}{\pi}\sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}} = \frac{1}{\hbar\pi}\sqrt{\frac{2m}{E}} dE$$