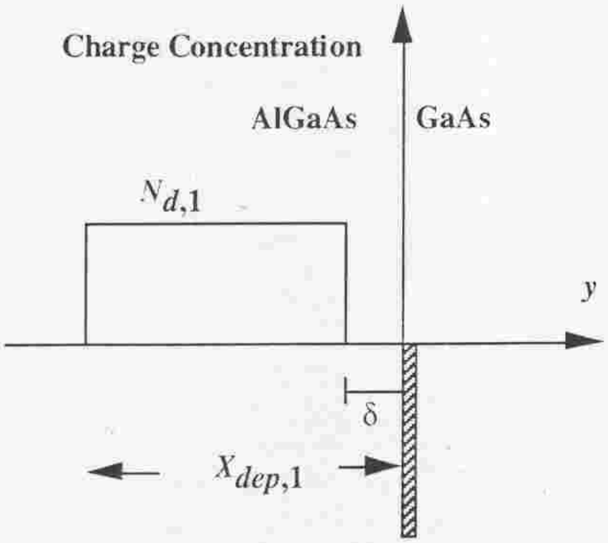
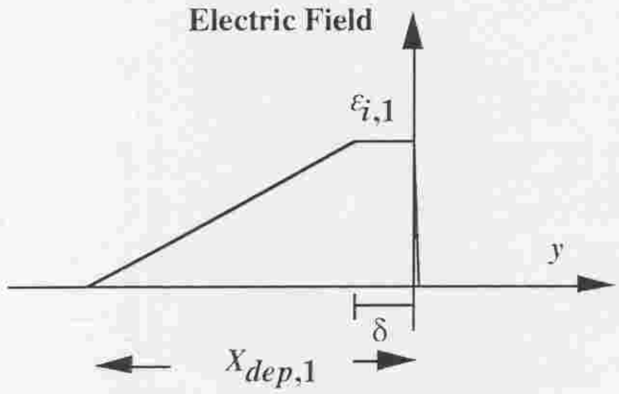
8





Charge Profile **Electric Field Profile**

(b) Because we have equated $\epsilon_{s,1} = \epsilon_{s,1}$ to ϵ_s, the electric fields at the two sides of the interface are equal. The slopes of the electric field are 0 and $qN_{d,1}/\epsilon_s$. in the undoped and doped portions of region 1, respectively. From an inspection of Fig. (old)5-24b, we can say,

$$\varepsilon_{i,1} = \frac{q}{\epsilon_s}\left(X_{dep,1} - \delta \right) N_{d,1} \tag{1}$$

The total amount of area enclosed by the electric field is the potential drop across the two ends. Hence, we can write,

$$\varepsilon_{i,1} \cdot \delta + \frac{\left(X_{dep,1} - \delta \right)}{2} \varepsilon_{i,1} = \phi_{bi,1} , \tag{2}$$

where $\phi_{bi,1}$ and $X_{dep,1}$ are the built-in voltage and the depletion thickness in region 1, respectively. Substituting Eq. (1) into Eq. (2) eliminates the variable $X_{dep,1}$. The resultant equation is quadratic in $\varepsilon_{i,1}$, which can be solved as,

$$\epsilon_s \varepsilon_{i,1} = \sqrt{\left(qN_{d,1}\delta \right)^2 + 2qN_{d,1}\epsilon_s \phi_{bi,1}} - qN_{d,1}\delta \tag{3}$$

(c) Since $\varepsilon_{i,1} = \varepsilon_{i,2}$, n_s and $\varepsilon_{i,1}$ are related through Eq. (5-105). We then arrive at the final set of equations which allow the simultaneous determination of the two unknowns: n_s and E_f.

$$n_s = \left[\sqrt{\left(N_{d,1}\delta \right)^2 + \frac{2N_{d,1}\epsilon_s}{q} \phi_{bi,1}(E_f)} - N_{d,1}\delta \right] \tag{4}$$

where n_s is in cm^{-2}; and E_f is in eV. At the room temperature, $kT = 0.0258$ eV. For clarity, we write $\phi_{bi,1}$ explicitly as a function of E_f. The dependence is clear from Fig. 5-21:

$$\phi_{bi,1} = \frac{\Delta E_c}{q} - \Phi_{N,1} - \frac{E_f}{q} \tag{5}$$

where $\Phi_{N,1}$ is $E_c - E_f$ at $x = -\infty$. The determination of $\Phi_{N,1}$ can be found in Examples 1-3 and 1-4.

(d) The other equation, based on Eq. (5-98), is,

$$\ln\left[1 + \exp\left(\frac{E_f - 1.11 \times 10^{-9} (n_s)^{2/3}}{kT} \right) \right] + \ln\left[1 + \exp\left(\frac{E_f - 1.95 \times 10^{-9} (n_s)^{2/3}}{kT} \right) \right]$$

$$= \frac{n_s}{D\,kT} \qquad (6)$$

As mentioned, the zero energy level is at the bottom tip of the triangular well. The procedure to numerically solve for n_s and E_f is to first guess a negative E_f, such that the guessed E_f is below the tip of the triangular well. From this initial guess, $\phi_{bi,1}$ is evaluated in accordance with Eq. (5), and n_s is determined from Eq. (4). These two values of E_f and n_s are substituted into Eq. (6) to check for equality. If the two sides of Eq. (6) are different, another value of E_f is guessed. The iteration continues until a certain convergence criterion is met.

9. (a) $E_n = 0.056\, n^2$ eV. So, $E_1 = 0.056$ eV and $E_2 = 0.224$ eV.

(b) $n_s = D\,kT \ln\left[\,1 + \exp\left(\frac{E_f - E_1}{kT}\right)\right]$

(c)
$$\frac{dn_s}{dE_f} = D\,kT \frac{1}{1 + \exp\left(\frac{E_f - E_1}{kT}\right)} \frac{1}{kT} \exp\left(\frac{E_f - E_1}{kT}\right)$$

$$\frac{dn_s}{dE_f} = \frac{D}{1 + \exp\left(\frac{E_1 - E_f}{kT}\right)}$$

so,
$$\Delta t_b = \frac{\epsilon_s}{q^2} \frac{1 + \exp\left(\frac{E_1 - E_f}{kT}\right)}{D} = \frac{\epsilon_s}{q^2 D} \frac{\exp\left(\frac{n_s}{DkT}\right)}{\exp\left(\frac{n_s}{DkT}\right) - 1}$$

(d) when n_s is large,

$$\Delta t_b \approx \frac{\epsilon_s}{q^2 D} = \frac{1.159 \times 10^{-12}}{\left(1.6 \times 10^{-19}\right)^2 \cdot 1.743 \times 10^{32}} = 2.6 \times 10^{-7} \text{ cm}$$

10. (a) It is obtained by taking the derivative of Eq. (5-151) with respect to x, evaluating the derivative at $x = L$, and seek the condition such that the derivative is equal to $-\infty$ (rather than ε_{sat} as in § 5-7).

$$\frac{(\alpha - 1)^2}{2\alpha} = \frac{\varepsilon_{sat} L}{U_{CH}(0)}$$

(b) $\alpha = 2 - \sqrt{3}$, or $U_{CH}(L) = 0.268$ V. When $U_{CH}(L) = 0.3$ V, it is larger than 0.268 V. Hence, we expect Eq. (5-151) to hold. When $U_{CH}(L) = 0.25$ V is smaller than 0.268 V, the saturation region has been formed and Eq. (5-151) fails.

(c) 0.642 V.

11. (a)

$$\int_0^L I_D\left(1 - \frac{1}{\varepsilon_{sat}} \frac{d}{dx} U_{CH}\right) dx = -\,W\mu_0\, C'_{ox} \int_{U_{CH}(0)}^{U_{CH}(L)} U_{CH}(x)\, dU_{CH}$$

$$I_D = \frac{W\mu_0 C'_{ox}}{2} \times \frac{\left[\,U_{CH}^2(0) - U_{CH}^2(L)\right]}{L + (U_{CH}(0) - U_{CH}(L)) / \varepsilon_{sat}}$$

With $U_{CH}(0) = V_{GS} - V_T$ and $U_{CH}(L) = V_{GS} - V_T - V_{DS}$, we get

$$I_D = \frac{W\mu_0 C'_{ox}}{L + V_{DS} / \varepsilon_{sat}} \times \left[\,(V_{GS} - V_T)\, V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$\Delta I_D^2 + B\,\Delta I_D - \frac{2I_{DS,\text{sat}}\,\epsilon_s W h \mu_0 \varepsilon_{sat}\,(V_{DS} - V_{DS,\text{sat}})}{\left(L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}\right)^2} = 0$$

where B is,

$$B = \frac{2\epsilon_s W h \mu_0 \varepsilon_{sat}^2}{L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}} - \frac{2\,\epsilon_s W h \mu_0 \varepsilon_{sat}\,(V_{DS} - V_{DS,\text{sat}})}{\left(L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}\right)^2}$$

$$= \frac{2\epsilon_s W h \mu_0 \varepsilon_{sat}}{L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}}\left(\varepsilon_{sat} - \frac{(V_{DS} - V_{DS,\text{sat}})}{L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}}\right)$$

Hence, the drain current as a function of V_{DS} is given by,

$$I_D = I_{DS,\text{sat}} - \frac{B}{2} + \left[\frac{B^2}{4} + \frac{2I_{DS,\text{sat}}\,\epsilon_s W h \mu_0 \varepsilon_{sat}\,(V_{DS} - V_{DS,\text{sat}})}{\left(L + \dfrac{V_{DS,\text{sat}}}{\varepsilon_{sat}}\right)^2}\right]^{1/2}$$

12. From Eqs. (5-99), (5-106) and (5-107),

$$n_s = D\left[1.95 \times 10^{-9}\,(n_s)^{2/3} - 1.11 \times 10^{-9}\,(n_s)^{2/3}\right] + 2D\left[E_f - 1.95 \times 10^{-9}\,(n_s)^{2/3}\right]$$

$$\text{or,}\quad E_f = \frac{n_s}{2D} + \frac{3.06 \times 10^{-9}}{2}\,n_s^{2/3}$$

when $n_s = 2 \times 10^{12}$ cm^{-2}, with $D = 2.79 \times 10^{13}$ cm$^{-2}\cdot$eV^{-1}, $E_f = 0.279$ eV.

13. (a) The various parameter values given in the description are: $t_b = 280$ Å; $\delta = 30$ Å; $N_{d,1} = 1 \times 10^{18}$ cm^{-3}; and $\phi_B = 1$ eV. According to the description about Eq. (5-115), E_{f0} = 0.0518 eV. From Eq. (1-90), ΔE_c of an $Al_{0.35}Ga_{0.65}As/GaAs$ heterojunction is 0.244 eV. According to Eq. (5-113):

$$\phi_{00}' = \frac{1.6 \times 10^{-19} \cdot 1 \times 10^{18}}{2 \cdot 1.159 \times 10^{-12}}\left(280 \times 10^{-8} - 30 \times 10^{-8}\right)^2 = 0.431 \text{ V}$$

The threshold voltage is found from Eq. (5-119) as:

$$V_T = \phi_B + \frac{E_{f0}}{q} - \phi_{00}' - \frac{\Delta E_c}{q}$$
$$= 1.0 + 0.0518 - 0.431 - 0.244 = 0.377 \text{ V}$$

(b) $V_{DS,\text{sat}} = V_{GS} - V_T = 0.5 - 0.377 = 0.123$ V. Since $V_{DS} > V_{DS,\text{sat}}$, the transistor is in saturation and the saturation index $\alpha = 0$. The gate capacitance per area is found from Eq. (5-121):

$$C_{ox}' = \frac{1.159 \times 10^{-12}}{280 \times 10^{-8} + 68 \times 10^{-8}} = 3.33 \times 10^{-7}\,\frac{\text{F}}{\text{cm}^2}$$

The current is given by Eq. (5-133):

$$I_D = \frac{W C_{ox}' \mu_n}{L}\,\frac{(V_{GS} - V_T)^2}{2}$$
$$= \frac{500 \times 10^{-4} \cdot 3.33 \times 10^{-7} \cdot 6500}{0.25 \times 10^{-4}}\,\frac{(0.5 - 0.377)^2}{2} = 0.033 \text{ A}$$

14. (a) False. The statement is true only under the d.c. condition. During transient, there is also gate current.

```
      z12= -ygd/dely
      z21= -ydg/dely
      z22= ygg/dely
      z11prime= z11 + RG + RS
      z12prime= z12 + RS
      z21prime= z21 + RS
      z22prime= z22 + RG + RS
      h21= cabs(- z21prime/z22prime)
      U= cabs(z21prime - z12prime)* cabs(z21prime- z12prime)
     + /(Real(z11prime)*Real(z22prime)-Real(z12prime)*Real(z21prime))
     + /4.
      write (1,*) h21, U
      end
```

11. The factor $(1 - \alpha)$ in Eq. (6-170) can be found from Eq. (5-132) as V_{DS}/V_{Dsat}. Therefore, according to Eq. (6-170)

$$R_{ch} \to \frac{1}{g_m}\frac{V_{DS}}{V_{Dsat}}\left[\frac{3\alpha^3 + 15\alpha^2 + 10\alpha + 2}{10(1+\alpha)(1+2\alpha)^2}\right|_{\alpha=1} = \frac{1}{6\,g_m}\frac{V_{DS}}{V_{Dsat}}$$

12. (a) $R_G = \frac{1}{3} R_{SHG}\frac{W}{L} = \frac{1}{3}\cdot 0.03 \cdot \frac{1000}{0.5} = 20\ \Omega$

(b) $R_G = \frac{1}{N}\times\frac{1}{3} R_{SHG}\frac{W}{L} = \frac{1}{10}\times\frac{1}{3}\cdot 0.03\cdot\frac{100}{0.5} = 0.2\ \Omega$

(c) $R_G = \frac{1}{N}\times\frac{1}{12} R_{SHG}\frac{W}{L} = \frac{1}{5}\times\frac{1}{12}\cdot 0.03\cdot\frac{200}{0.5} = 0.2\ \Omega$

13. From Eq. (5-117), Δt_b = 68 Å. According to Eq. (5-121), the total gate oxide capacitance is:

$$C_{ox} = \frac{13.1\times 8.85\times 10^{-14}}{(300+68)\times 10^{-8}} 0.29\times 32\times 10^{-8} = 2.92\times 10^{-14}\ \mathrm{F}$$

The overlap gate-to-drain capacitance (which is identical to the gate-to-source capacitance) is:

$$C_{gd,\mathrm{p}} = 2.92\times 10^{-14}\times 0.1 = 2.9\ \times 10^{-15}\ \mathrm{F}$$

According to Eqs. (6-16) and (6-17), the intrinsic C_{gg} and C_{gd} at $\alpha = 0$ are:

$$C_{gg} = 2.92\times 10^{-14}\times\left[\frac{2}{3}\right] = 1.95\times 10^{-14}\ \mathrm{F}$$

$$C_{gd} = 4.6\times 10^{-14}\times[\,0\,] = 0$$

The total $C_{gg,\mathrm{t}}$ is the intrinsic component plus two times the overlap capacitance. It is two times because the overlap exists at both the drain and the source sides:

$$C_{gg,\mathrm{t}} = 1.95\times 10^{-14} + 2\times 2.9\ \times 10^{-15} = 2.53\times 10^{-14}\ \mathrm{F}\ .$$

The total $C_{gd,\mathrm{t}}$ is equal to the intrinsic component plus the overlap component:

$$C_{gd,\mathrm{t}} = 0 + 2.9\ \times 10^{-15} = 2.9\ \times 10^{-15}\ F$$

The gate resistance is calculated from Eq. (6-193):

$$R_G = \frac{1}{3}\,5\,\frac{32}{0.29}\times\frac{1}{8} = 23\ \Omega$$

According to Eq. (6-235), we have,

$$\frac{1}{2\pi f_T} = \frac{2.53\times 10^{-14}}{0.07} + \frac{2.53\times 10^{-14}}{0.48}(1+3)\cdot 0.00573 + (1+3)\cdot 2.9\times 10^{-15}$$

$$= 3.74\times 10^{-13}\ \mathrm{s}$$

Therefore, f_T = 425 GHz. To find the maximum oscillation frequency, we determine the parameter Ψ from Eq. (6-239):

$$\Psi = (3+1)\frac{\left(2.53\times10^{-14}\right)^2(0.00573)^2}{(0.07)^2} + (3+1)\frac{\left(2.53\times10^{-14}\right)\left(2.9\times10^{-15}\right)(0.00573)}{(0.07)}$$

$$+\frac{\left(2.53\times10^{-14}\right)^2(0.00573)}{(0.07)^2}$$

$$= 1.0\times10^{-27}\ \text{F·s}$$

From Eq. (6-238), we then have,

$$f_{max} = \sqrt{\frac{425\times10^9}{8\pi\cdot23\cdot2.9\times10^{-15}\left(1+\frac{2\pi\cdot425\times10^9}{2.9\times10^{-15}}1\times10^{-27}\right)}}$$

So, f_{max} is 363 GHz.

14. The change in geometry modifies the gate resistance. Because f_T does not depend on the gate resistance, the cutoff frequency is still 425 GHz. However, f_{max} is affected.

$$R_G = \frac{1}{3}5\frac{32\times8}{0.29} = 1471\ \Omega$$

From Eq. (6-238) and the Ψ from the solution of Problem 13:

$$f_{max} = \sqrt{\frac{425\times10^9}{8\pi\cdot1471\cdot2.9\times10^{-15}\left(1+\frac{2\pi\cdot425\times10^9}{2.9\times10^{-15}}1\times10^{-27}\right)}}$$

The maximum oscillation frequency is 45 GHz.

15. MSG is equal to the magnitude of y_{21}/y_{12}. From Eq. (2-226), we have, $y_{21} = g_m - j\omega C_{dg,t}$ and $y_{12} = -j\omega C_{gd,t}$. Therefore, MSG is,

$$MSG = \left|\frac{g_m - j\omega\,C_{dg,t}}{-j\omega\,C_{gd,t}}\right| = \frac{\sqrt{g_m^2+\omega^2C_{dg,t}^2}}{\omega^2C_{gd,t}^2}$$

When the parasitic capacitances are neglected, then,

$$MSG = \frac{\sqrt{g_m^2+\omega^2C_{dg}^2}}{\omega^2C_{gd}^2}$$

The second expression is problematic at times because C_{gd}, the intrinsic device capacitance, is zero when the device is in saturation. $C_{gd,t}$, in contrast, includes the parasitic capacitance between the gate and the drain and is never zero.