1) For a 2 dimensional electron gas in GaAs with a density of 10^{11} cm^{-2}, calculate the Fermi energy and the Fermi wavelength. (This will be important when we discuss quantum dots and quantum point contacts.)

The answers are: 3.5 meV and 80 nm.

2) What is the Fermi wavelength of electrons in aluminum? Is it possible to fabricate 1d Al wires using photolithography? Is it possible to fabricate Al wires using electron beam lithography?

3.6 angstroms.

No.

3) Find the resistivity of pure copper at room temperature. Now, find the density of electrons in copper, assuming one free electron per atom. Now, calculate the scattering time and the mean free path of the electrons from the Drude model. Is it possible to fabricate copper wires in the ballistic limit using photolithography? Is it possible to fabricate copper wires in the ballistic limit using electron beam lithography?

10 microohm-cm $\qquad$ $8.45\ 10^{22}\ cm^{-3}$ $\qquad$ $\frac{1}{\rho} = \sigma = \frac{ne^2\tau}{m}$

$\tau = 4\ 10^{-15}$ s $\quad l = v_{Fermi}\tau$

$v_{Fermi} = 1.6 \times 10^8$ cm/s so $l = 6.4$ nm

No

No

4) For a 2DEG in GaAs with $n=10^{11}$ cm^{-2} and a mobility of 8,000 cm^2/V-s (typical of room temperature HEMT operation), calculate the scattering time and the mean free path from the Drude model. Is it possible to fabricate devices using lithography that are smaller than the mean free path? Remember you must use the effective mass of electrons for the Fermi energy, etc.

$\mu = \frac{e\tau}{m^*}$ $\quad \tau = 0.3$ ps $l = v_{Fermi}\tau$ so $l = 1.3\ 10^5$ m/s x 0.3 ps = 40 nm

Yes

Note : This HW is a "toy model" of graphene.
Does not get at 1) Valley degeneracy 2) 2D–1D nanoribbon effects

1) In graphene, we have a linear relationship between energy and momentum:

$$E = \hbar v_F k = \hbar v_F \sqrt{(k_x)^2 + (k_y)^2} = \hbar v_F \sqrt{\left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_x}\right)^2}$$

↑ typo

$\Rightarrow k = \frac{1}{v_F} E$

$\Rightarrow \frac{dk}{dE} = \frac{1}{v_F}$

Derive the density of states vs. energy in graphene.

2) Now imagine you have a graphene nanoribbon. L_y is small. Calculate the density of states vs. energy of a 1d graphene nanoribbon.

$E = \hbar v_F \sqrt{k_x^2 + k_{y0}^2} \qquad k_{y0} = \frac{\pi}{L_y}$

1) $D(E)\, dE = D(k)\, dk$

$D(k)\,dk = \left[\frac{1 \text{ state}}{(\pi/L)^2} \times 2 \text{ (spin)}\right] \times$ area of disk of radius k in k-space $k_x > 0$ $k_y > 0$

$= \left[\frac{1}{(\pi/L)^2} \times 2\right] \cancel{2}\pi k\, dk \frac{1}{\cancel{4}}$

$= L^2 \frac{1}{\pi} k\, dk = L^2 \frac{1}{\pi} \frac{E}{\hbar v_F} dk$

$\Rightarrow D(k) = L^2 \frac{1}{\pi} k = L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F}$

$D(E) = D(k) \frac{dk}{dE} = \boxed{L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F} \frac{1}{\hbar v_F} = D(E)}$

$$\boxed{\rho(E) = \frac{E}{\hbar^2 \pi v_F^2}}$$

2) $E = \hbar v_F k$

$D(E)\, dE = D(k)\, dk$

$D(k)\, dk = \left[\frac{1}{\pi/L} \times 2 \text{ (spin)}\right] \times$ distance in k-space between $k, k+dk$

$= \frac{2}{\pi/L} \times dk \Rightarrow D(k) = L \frac{2}{\pi}$

$D(E) = D(k) \frac{dk}{dE} = L \frac{2}{\pi} \frac{1}{v_F} \Rightarrow \boxed{\rho(E) = \frac{2}{\pi} \frac{1}{v_F}}$