

HW1 EECS 277B Fall 2013

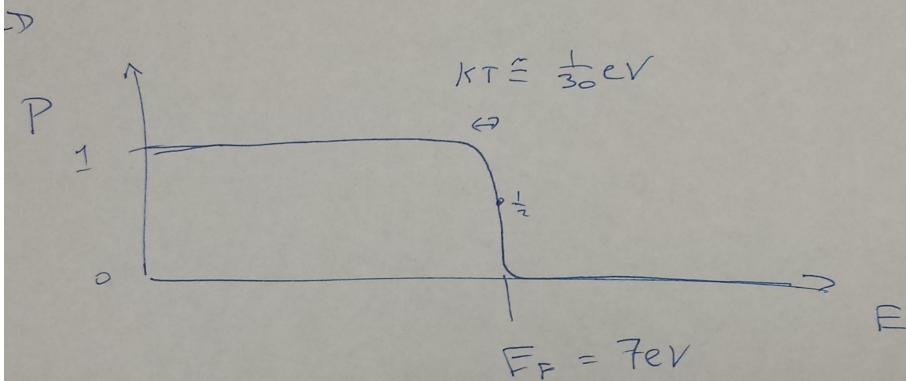
Liu 1.1

Liu 1.6

Consider a particle in a box. What is the size the box must be for the energy levels to be larger than  $kT$ ?

Derive the density of states in a 2D world.

Sketch the probability that each energy level is occupied for a piece of copper as a function of energy at room temperature. Hint: You need to look up the Fermi energy of copper.



$$E = \frac{\hbar^2}{2m} \left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

$$\Rightarrow \Delta E \sim \frac{\hbar^2}{2m} \left(\frac{\pi}{L}\right)^2$$

$$\Delta E > kT \Rightarrow \frac{\hbar^2}{2m} \left(\frac{\pi}{L}\right)^2 > kT$$

$$\Rightarrow L < \sqrt{\frac{\hbar^2 \pi^2}{2m kT}}$$

$$\hbar = 10^{-34} \frac{\text{m}^2}{\text{s}} \text{ kg}$$

$$kT = 4 \times 10^{-21} \text{ J}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\Rightarrow L < \sqrt{\frac{(10^{-34})^2 \pi^2}{2 \cdot 9.1 \cdot 10^{-31} \cdot 4 \cdot 10^{-21}}}$$
$$\sim \sqrt{\frac{10^{-34-34+1-1+31+21-1}}{10^{-17}}} \text{ m}$$
$$\sim \sqrt{10^{-17}} \text{ m} \approx 3 \text{ nm}$$

2 dimensions

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

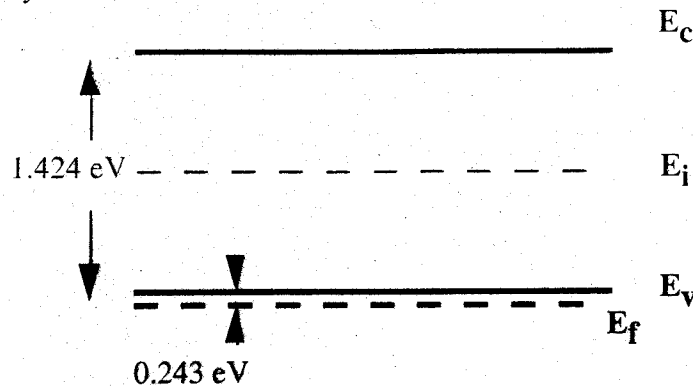
$$\rho_k dk = \frac{k dk}{\pi}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

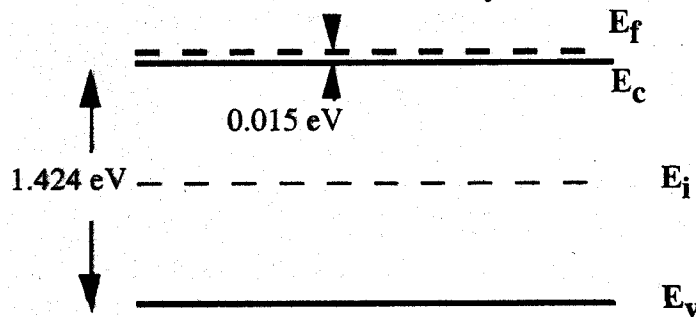
$$\rho(E)dE = \frac{m}{\pi\hbar^2} dE$$

**Solutions to Chapter 1 Problems**  
**Fundamentals of III-V Devices, by W. Liu**

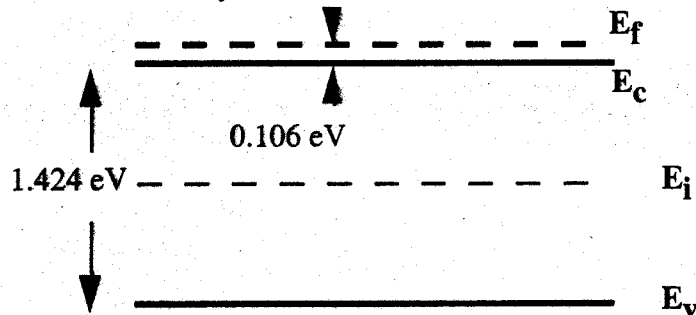
1. (a) For a *p*-type GaAs layer,  $N_V = 4.7 \times 10^{18} \text{ cm}^{-3}$ .  $p/N_V = 1 \times 10^{20} / 4.7 \times 10^{18} = 21.3$ . From Fig. 1-21, we locate 21.3 from the *y*-axis. The corresponding *x*-axis value is 9.4. Therefore,  $E_V - E_f = 9.4 \times 0.0258 = 0.243 \text{ eV}$ . The Fermi-level is below the valence band by 0.243 eV.



(b) According to Example 1-3,  $N_c$  for  $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$  is  $3.72 \times 10^{17} \text{ cm}^{-3}$ .  $n/N_c = 5 \times 10^{17} / 3.72 \times 10^{17} = 1.34$ . From Fig. 1-21, we locate 1.34 from the *y*-axis. The corresponding *x*-axis value is 0.6. Therefore,  $E_f - E_c = 0.6 \times 0.0258 = 0.015 \text{ eV}$ . The Fermi-level is above the conduction band by 0.015 eV.



(c) For GaAs,  $n_i = 1.79 \times 10^6 \text{ cm}^{-3}$  and  $N_c = 4.7 \times 10^{17} \text{ cm}^{-3}$ . The electron concentration is  $n_i^2/p = (1.79 \times 10^6)^2 / 1 \times 10^{-6} = 3.2 \times 10^{18} \text{ cm}^{-3}$ .  $n/N_c = 3.2 \times 10^{18} / 4.7 \times 10^{17} = 6.8$ . From Fig. 1-21, we locate 6.8 from the *y*-axis. The corresponding *x*-axis value is 4.1. Therefore,  $E_f - E_c = 4.1 \times 0.0258 = 0.106 \text{ eV}$ . The Fermi-level is above the conduction band by 0.106 eV.



2. Let us suppose that the material is *p*-type. If region A is hotter, then the holes tend to

$$\delta p = \delta p(0) \exp\left(-\frac{t}{\tau_p}\right)$$

The time constant is  $\tau_p$ .

5. (a) At 10 V, the electric field is 10 V/cm. For the  $5 \times 10^{18} \text{ cm}^{-3}$  sample, the mobility is given by Eq. (1-73), is  $7200/(1 + 5.51 \times 10^{-17} \cdot 5 \times 10^{18})^{0.233} = 1943 \text{ cm}^2/\text{V}\cdot\text{s}$ . We use Eq. (1-35) to solve for the current density. Since there is no concentration gradient, the diffusion component is zero.  $J_n = 1.6 \times 10^{-19} \cdot 1943 \cdot 5 \times 10^{18} \cdot 10 = 1.55 \times 10^4 \text{ A/cm}^2$ . The current is  $1.55 \times 10^4 \text{ A/cm}^2 \cdot 100 \mu\text{m}^2 = 0.0155 \text{ A}$ .

(b) Let us assume that the current flow under this scenario is identical to that obtained from part (a). This is our initial guess, and we shall obtain the solution by iteration. The temperature rise is  $200 \text{ K/W} \cdot 10 \text{ V} \cdot 0.0155 \text{ A} = 31 \text{ }^\circ\text{C}$ . We shall approximate it as  $30 \text{ }^\circ\text{C}$ ; therefore, the mobility of the transistor is  $1943 \cdot 70\% = 1360 \text{ cm}^2/\text{V}\cdot\text{s}$ . With this new mobility value, the current density is  $1.085 \times 10^4 \text{ A/cm}^2$  and the current is  $0.01085 \text{ A}$ . If this were indeed the current, then the temperature rise would be  $21.7 \text{ }^\circ\text{C}$  and the mobility would become  $1521 \text{ cm}^2/\text{V}\cdot\text{s}$ . With this new mobility value, the current density is  $1.21 \times 10^4 \text{ A/cm}^2$  and the current is  $0.0121 \text{ A}$ . With this current, the temperature rise is  $24.2 \text{ }^\circ\text{C}$  and the mobility becomes  $1472 \text{ cm}^2/\text{V}\cdot\text{s}$ . With this new mobility value, the current density is  $1.17 \times 10^4 \text{ A/cm}^2$  and the current is  $0.0117 \text{ A}$ . We can repeat the iteration more if a more accurate solution is needed. However, at this point we note that  $0.0117 \text{ A}$  is not too far from our previous guess of  $0.0121 \text{ A}$ . Hence, we approximate the current flow to be  $0.0117 \text{ A}$ .

6. The electric field is ~~200 V/cm~~  $2 \times 10^2 \text{ V/cm}$ . From the equation in Example 1-7,  $\mu_p = x/et = 100 \mu\text{m} / (5 \times 10^2 \text{ V/cm}) / (20 \text{ ns}) = 100 \text{ cm}^2/\text{V}\cdot\text{s}$ .

7. (a) At  $x \neq 0$ , the generation rate is zero. Further, since there is no external electric field and the problem is static,  $\epsilon = \partial(\delta p)/\partial t = 0$ . Therefore, Eq. (1-66) becomes:

$$D_p \frac{d^2(\delta p)}{dx^2} - \frac{\delta p}{\tau_p} = 0.$$

The above equation applies to both  $x > 0$  and  $x < 0$ . Let  $L_p$  be the square root of  $D_p \cdot \tau_p$ . Consider the case of  $x > 0$  first, with a boundary condition that  $\delta p(x = \infty) = 0$ . Because the above differential equation has the solution of  $A \cdot \exp(-x/L_p) + B \exp(+x/L_p)$ , the boundary condition implies that  $B = 0$ . Therefore, the solution at  $x > 0$  is  $A \cdot \exp(-x/L_p)$ . From symmetry, the solution at  $x < 0$  is  $A \cdot \exp(+x/L_p)$ .

(b) At  $x = 0$ , we still have  $\epsilon = \partial(\delta p)/\partial t = 0$ . However, the generation rate is no longer zero. Let the generation rate be  $g \text{ cm}^{-3}\cdot\text{s}^{-1}$ , then Eq. (1-66) at  $x = 0$  is:

$$D_p \frac{d^2(\delta p)}{dx^2} + g - \frac{\delta p}{\tau_p} = 0.$$

Let us integrate the above differential equation. We obtain the following identity:

$$D_p \int_{-\Delta}^{\Delta} \frac{d^2(\delta p)}{dx^2} dx + \int_{-\Delta}^{\Delta} g dx - \int_{-\Delta}^{\Delta} \frac{\delta p}{\tau_p} dx = 0 \quad (a)$$

From elementary calculus, we find that if  $f(x)$  is a continuous function, then