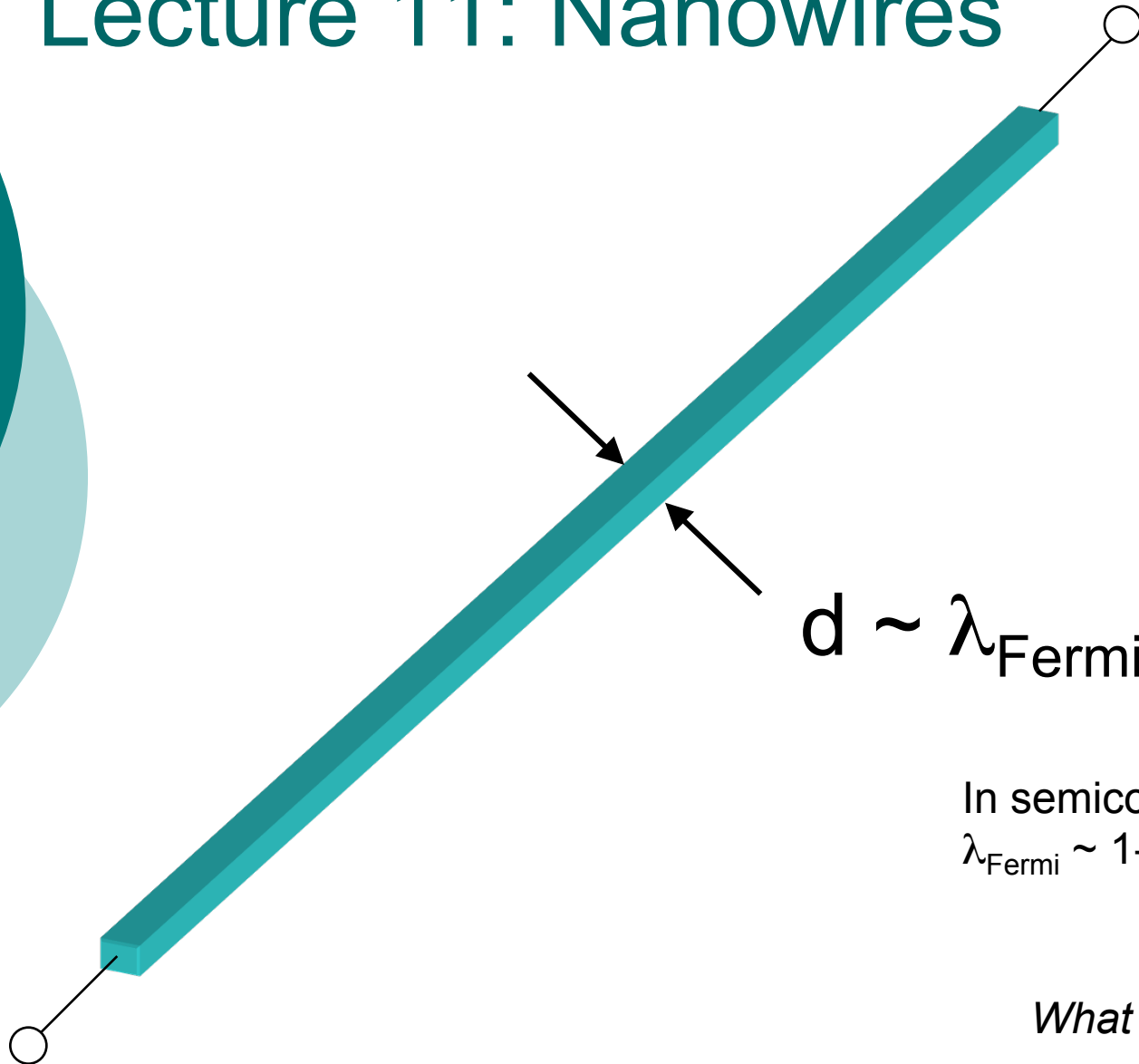


# Lecture 11: Nanowires



In semiconductors,  
 $\lambda_{\text{Fermi}} \sim 1\text{-}10 \text{ nm}$

*What is the resistance?*

# Readings this lecture covers

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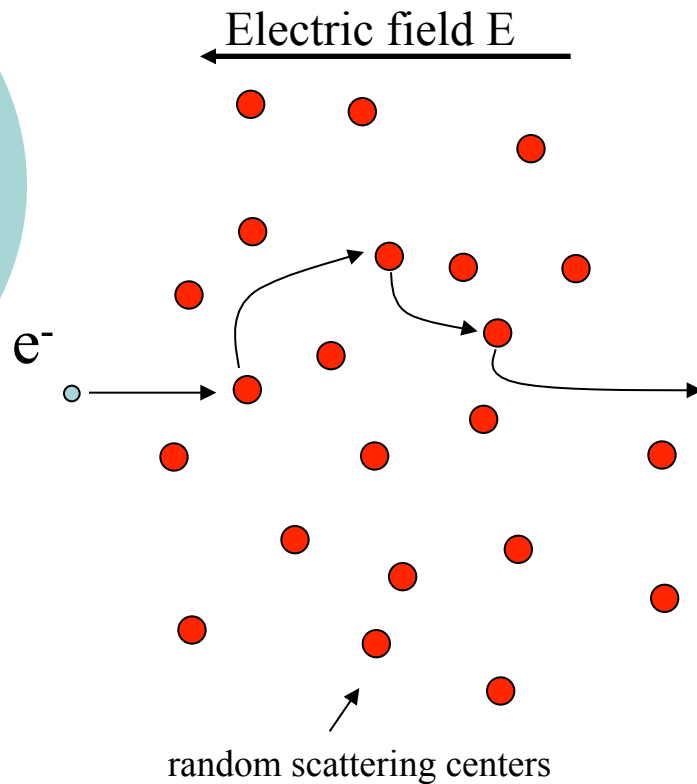
- Ferry pp. 39-46
- Hanson, pp. 124-125, 317-344

# Drift current

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- Caused by electric field
- Electron density constant
- Analogy: swarm of mosquitoes in the wind

# Drift: Drude model



$$F = ma$$

$$eE = m \frac{\partial v}{\partial t}$$

$$v_{avg} = \underbrace{\frac{e\tau}{m}}_{\mu} E$$

$$j = ne v_{avg} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} E$$

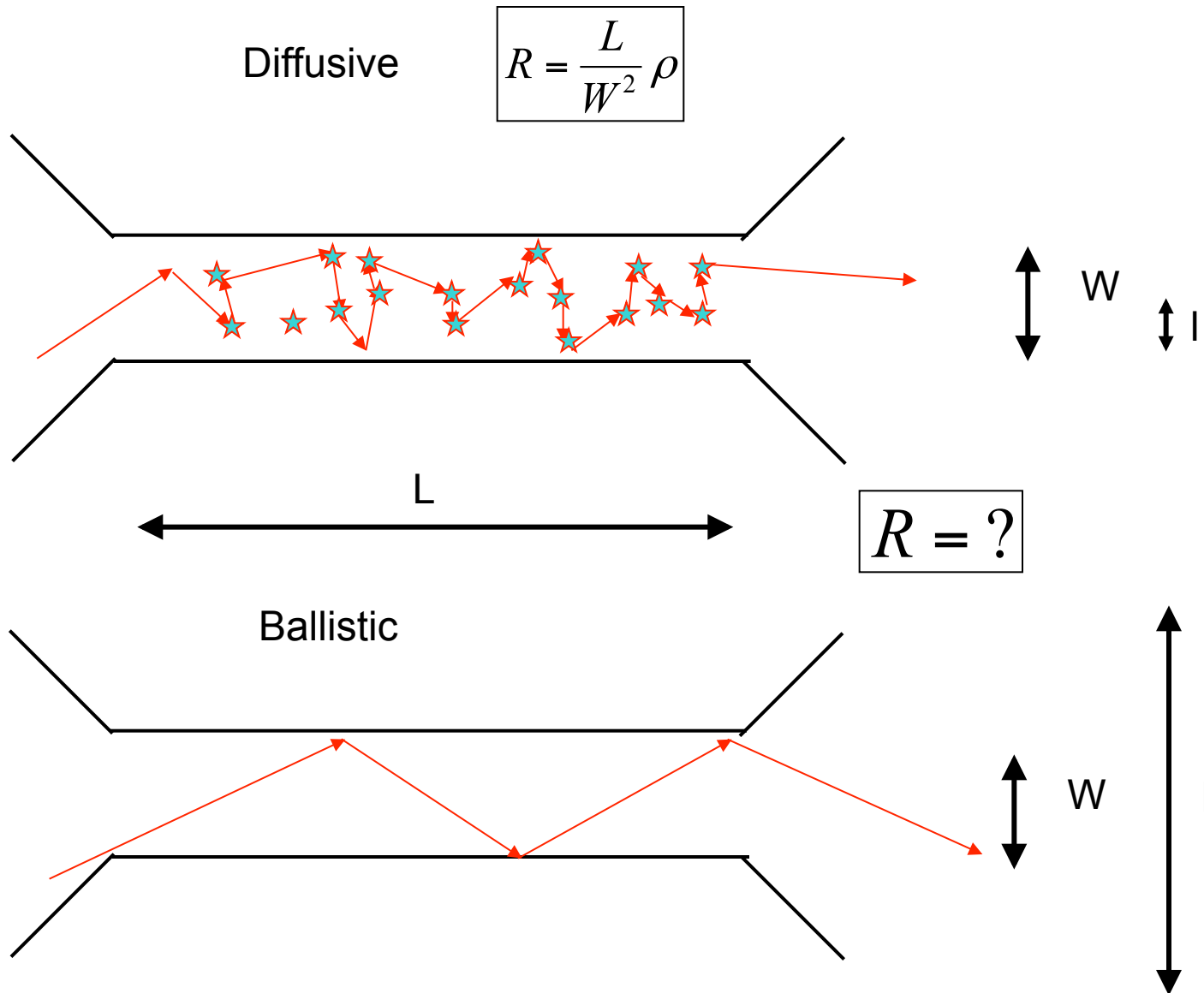
# Types of scattering

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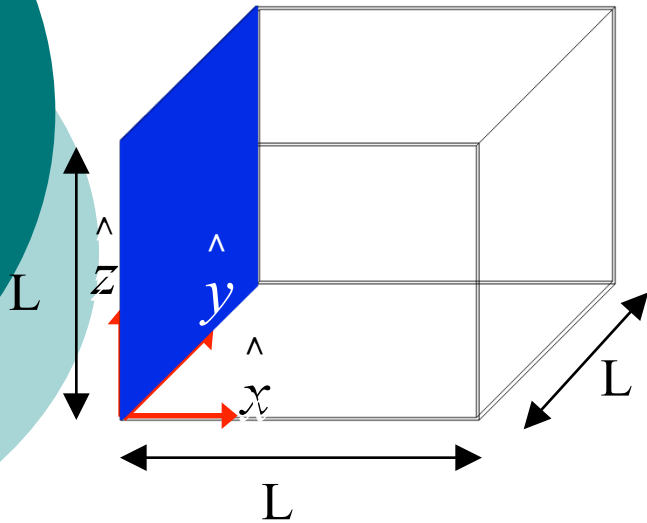
- Electron-phonon:
  - Very temperature dependent
  - Phonons are lattice vibrations
  - At low temperatures, lattice is “perfectly still”
- Impurity scattering
  - Temperature independent
  - Depends on impurity concentration

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_{electron-phonon}} + \frac{1}{\tau_{impurity}}$$

# Ballistic vs. diffusive transport



# Particle in a box



$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

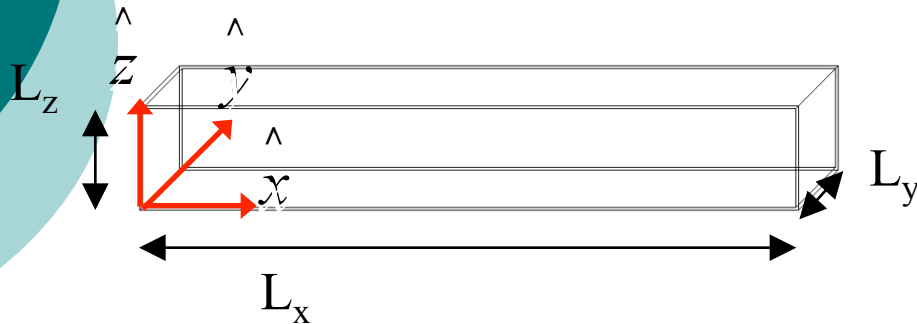
$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x^2 + n_y^2 + n_z^2)$$

These are the allowed energy levels, or “quantum states”

# Particle in a nanowire

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$



$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)}{2m} = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 n_y^2 + \left( \frac{\pi}{L_z} \right)^2 n_z^2 \right]$$

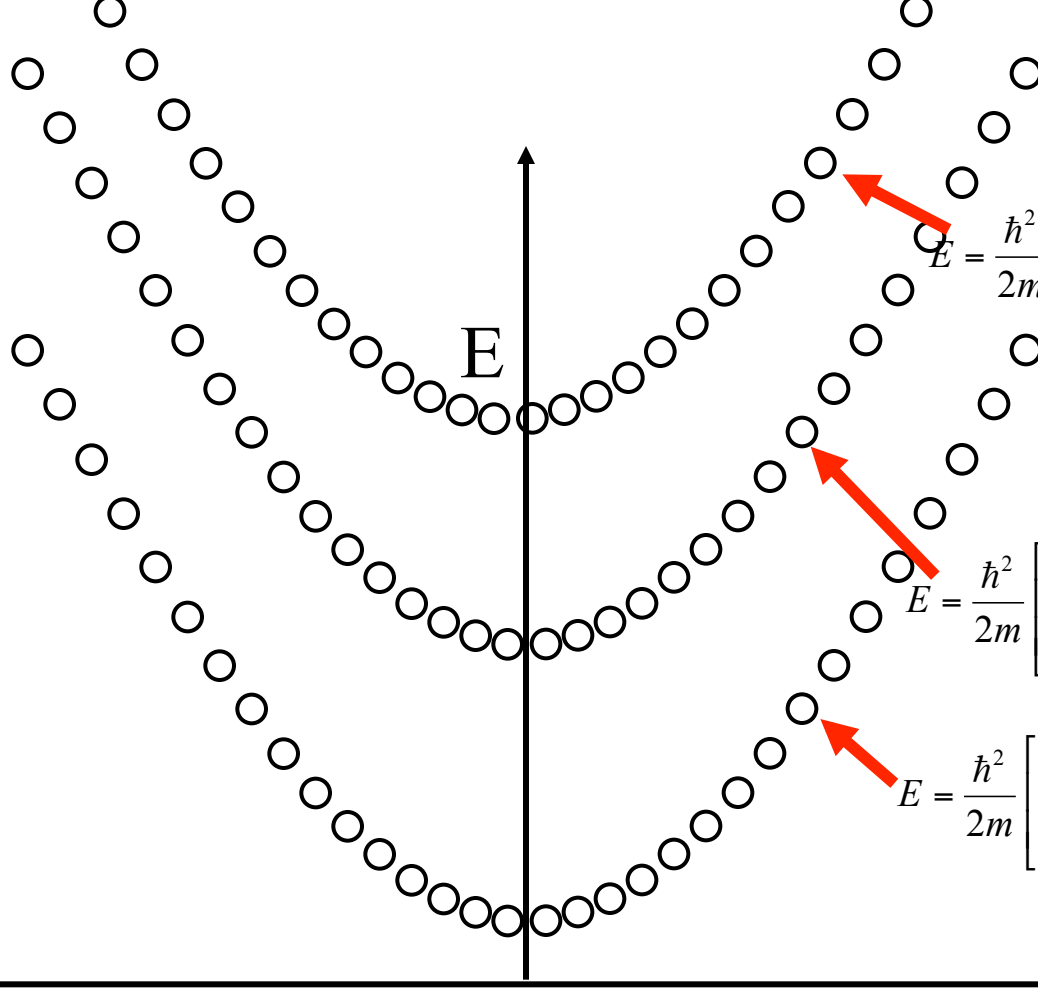
These are the allowed energy levels, or “quantum states”



# Limits:

$$E = \frac{\hbar^2(k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)}{2m} = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 n_y^2 + \left( \frac{\pi}{L_z} \right)^2 n_z^2 \right]$$

$$L_x \rightarrow \infty \quad L_y \rightarrow 0 \quad L_z \rightarrow 0$$



$$E = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 3^2 + \left( \frac{\pi}{L_z} \right)^2 \right]$$

$$E = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 2^2 + \left( \frac{\pi}{L_z} \right)^2 \right]$$

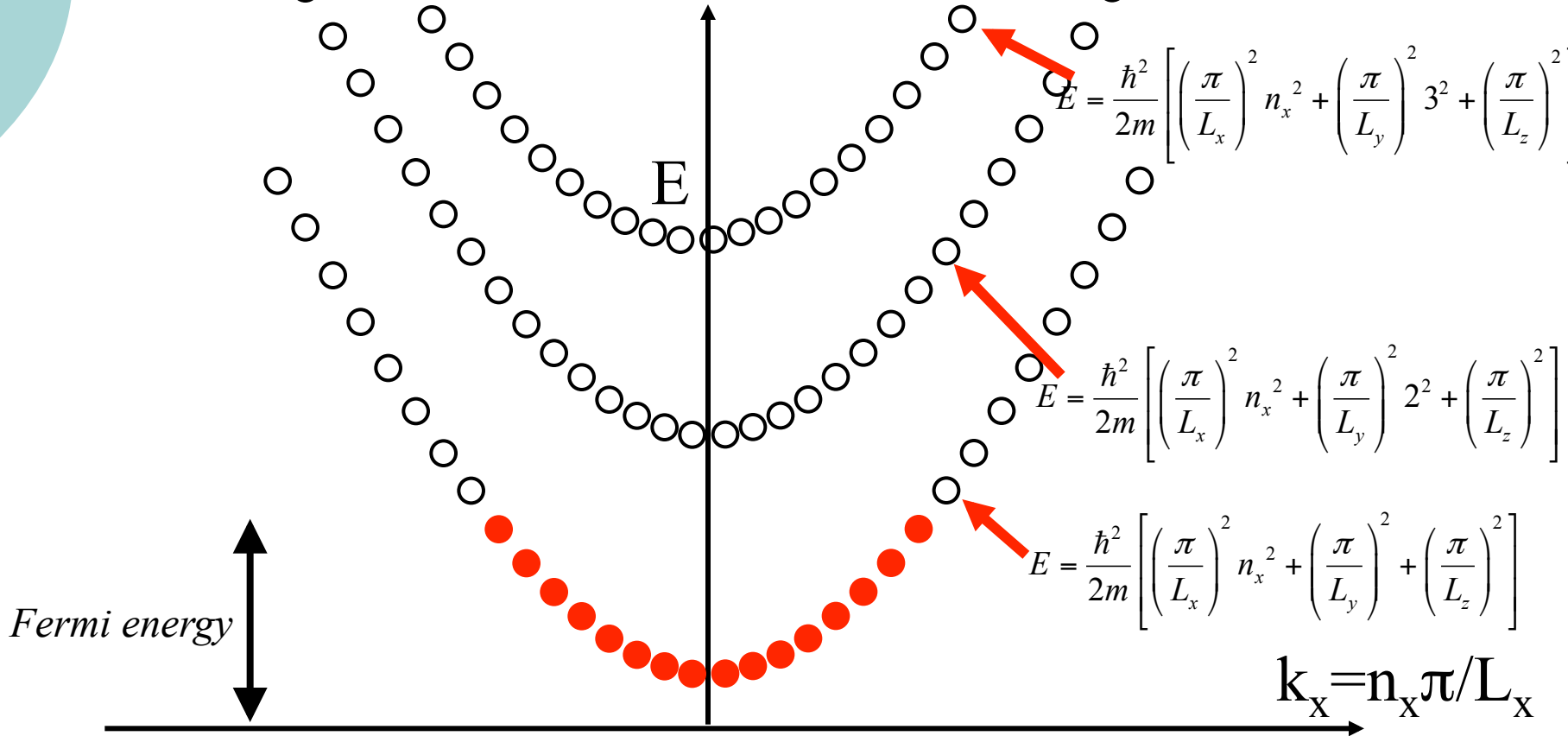
$$E = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 + \left( \frac{\pi}{L_z} \right)^2 \right]$$

$$k_x = n_x \pi / L_x$$

# 1d system:

$$E = \frac{\hbar^2(k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)}{2m} = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 n_y^2 + \left( \frac{\pi}{L_z} \right)^2 n_z^2 \right]$$

$$L_x \rightarrow \infty \quad L_y \rightarrow 0 \quad L_z \rightarrow 0$$



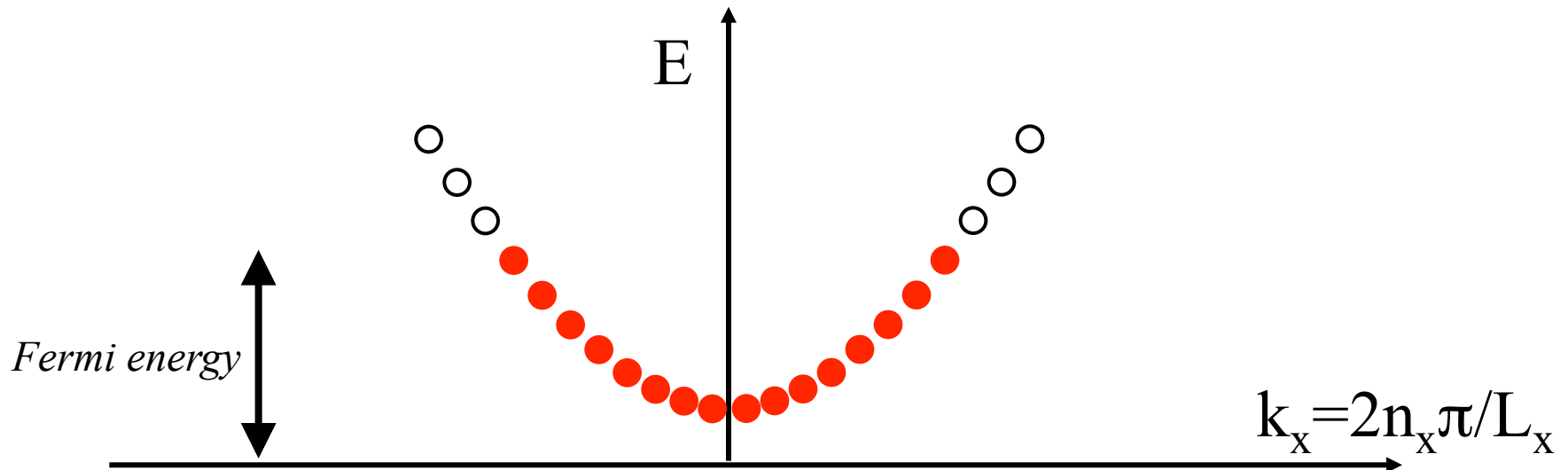
# Positive and negative k-vectors:

Particle in a box: (positive k-vectors only)

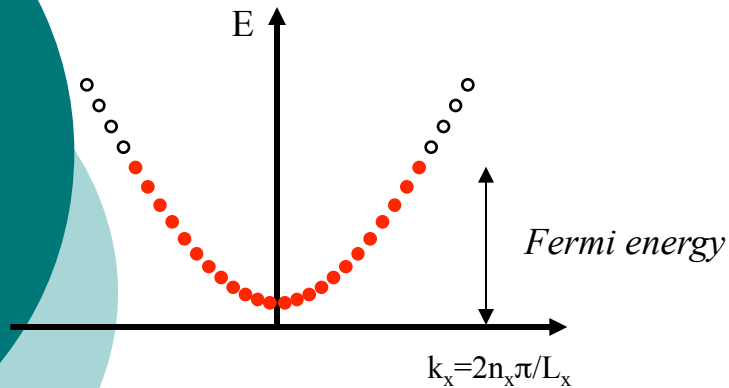
$$E = \frac{\hbar^2(k_{n_x}^2 + k_{n_y}^2 + k_{n_z}^2)}{2m} = \frac{\hbar^2}{2m} \left[ \left( \frac{\pi}{L_x} \right)^2 n_x^2 + \left( \frac{\pi}{L_y} \right)^2 n_y^2 + \left( \frac{\pi}{L_z} \right)^2 n_z^2 \right]$$

“Born-Von Karman” boundary conditions: (positive *and* negative k-vectors)

$$E = \frac{\hbar^2}{2m} \left[ \left( \frac{2\pi}{L_x} \right)^2 n_x^2 + \left( \frac{2\pi}{L_y} \right)^2 n_y^2 + \left( \frac{2\pi}{L_z} \right)^2 n_z^2 \right]$$



# Single sub-band:



$$I = \frac{\text{charge}}{\text{time}} = e \cdot \frac{\#\text{elec}}{\text{time}} = e \cdot v \frac{\#\text{elec}}{\text{length}}$$

Different electrons have different velocities.

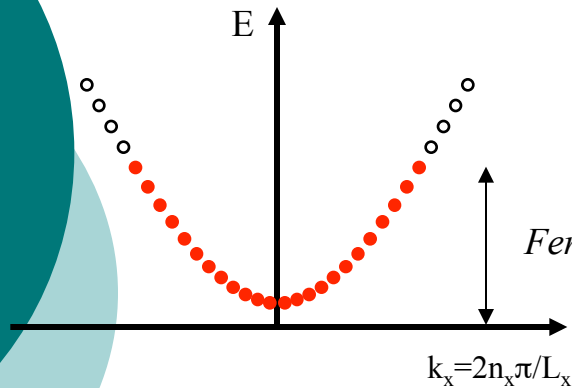
$$v = \frac{\text{momentum}}{\text{mass}} = \frac{p}{m} = \frac{\hbar k}{m}$$

$$I = I_{\text{right goers}} - I_{\text{left goers}}$$

$$I_{\text{right goers}} = \sum_{\text{right going electrons}} e v \frac{1}{\text{length}} = \sum_{\text{occupied states (right goers)}} e v \frac{1}{\text{length}}$$

$$I_{\text{right goers}} = \frac{e}{L_x} \sum_{k_x=0}^{k_F} \frac{\hbar k_x}{m} = \frac{e}{L_x} \sum_{n_x=0}^{n_F} \frac{\hbar (n_x 2\pi / L_x)}{m} = \frac{e 2\pi \hbar}{m L_x^2} \sum_{n_x=0}^{n_F} n_x$$

# Single sub-band:



$$I_{\text{right goes}} = \frac{e2\pi\hbar}{mL_x^2} \sum_{n_x=0}^{n_F} n_x \rightarrow \frac{e2\pi\hbar}{mL_x^2} \int_0^{n_F} n_x dn_x$$

Change of variables:

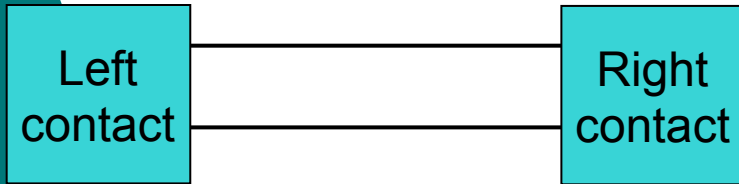
$$E = \frac{\hbar^2 k_x^2}{2m} = \frac{\hbar^2 (2n_x \pi / L_x)^2}{2m} \Rightarrow dE = \frac{4\hbar^2 (\pi / L_x)^2}{m} n_x dn_x$$

$$\Rightarrow n_x dn_x = \frac{m}{4\hbar^2 (\pi / L_x)^2} dE$$

$$I_{\text{right goes}} = \frac{e\pi\hbar}{2mL_x^2} \int_0^{n_F} n_x dn_x \rightarrow \frac{e\pi\hbar}{2mL_x^2} \frac{m}{\hbar^2 (\pi / L_x)^2} \int dE = \frac{e}{h} \int dE$$

# Resistance quantum

Ballistic conductor



$$I_{\text{right goes}} = \frac{e}{h} \int dE \quad I_{\text{right goes}} = \frac{e}{h} \int dE$$

$$I = \frac{e}{h} \left[ \int dE_{\text{right goes}} - \int dE_{\text{left goes}} \right]$$

$$I = \frac{e}{h} \left[ (E_F + eV) - E_F \right] = \frac{e^2}{h} V$$

$$V = I \frac{h}{e^2} = IR_{\text{quantum}}$$

$$R_{\text{quantum}} = \frac{h}{e^2} = 25 \text{ k}\Omega$$

With spin:

$$R_{\text{quantum}} = \frac{h}{2e^2} = 12.5 \text{ k}\Omega$$

