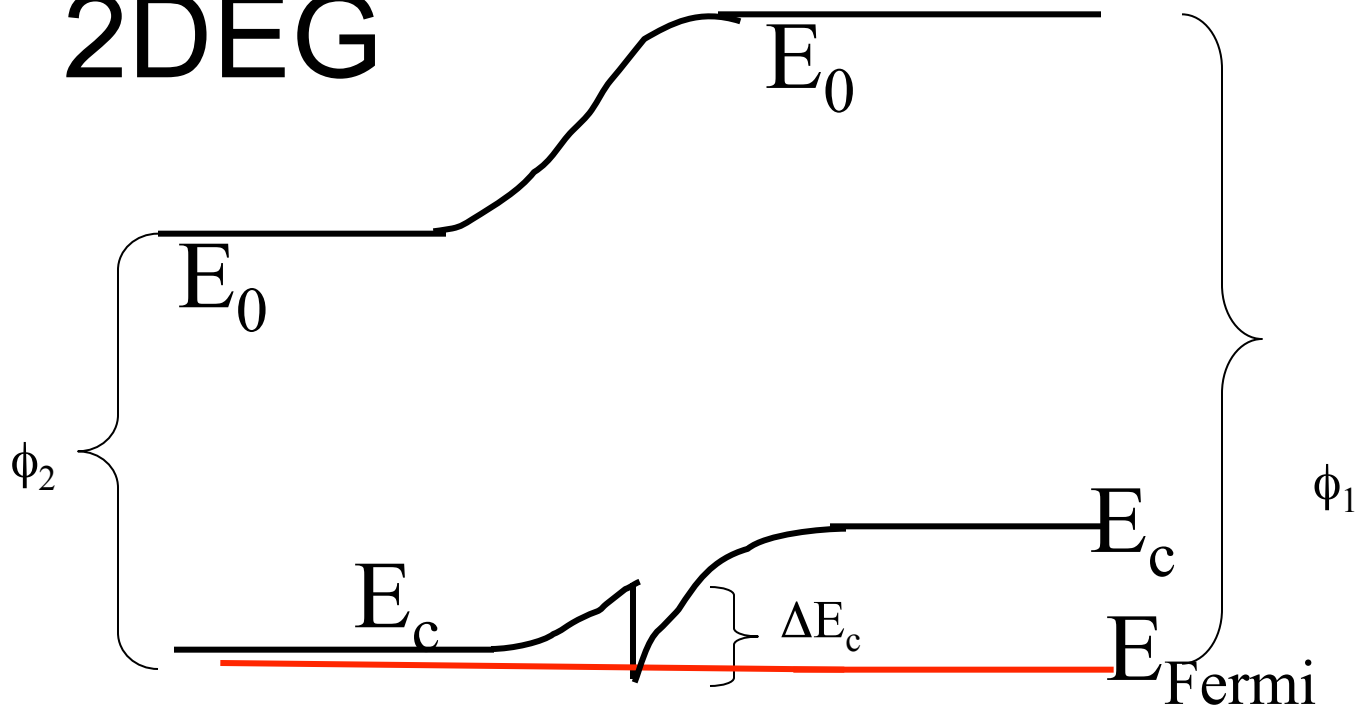


# Lecture 9: High electron mobility transistor (HEMT)

# 2DEG



$E_{\text{Fermi}}$

GaAs

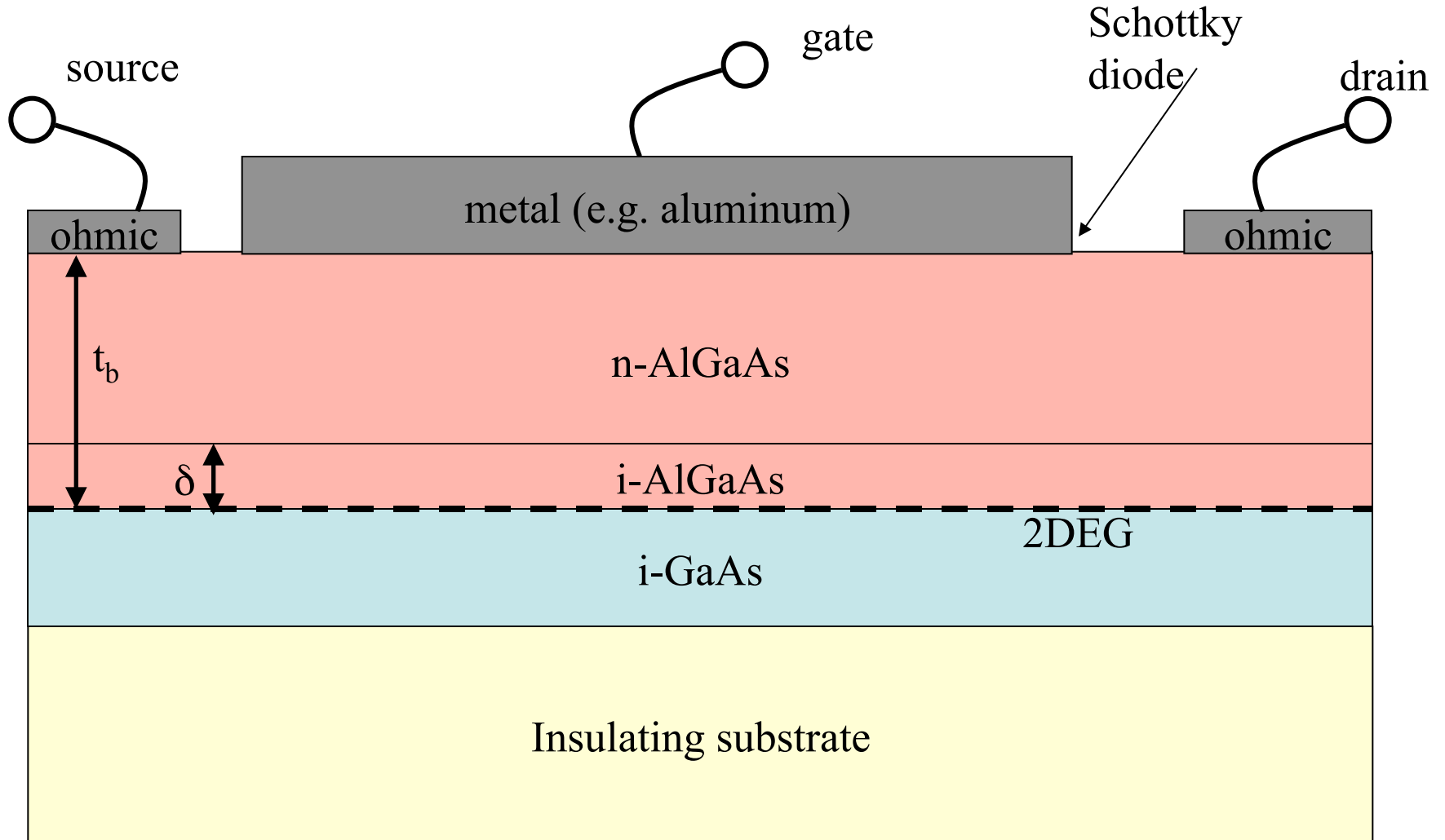
AlGaAs

$E_v$

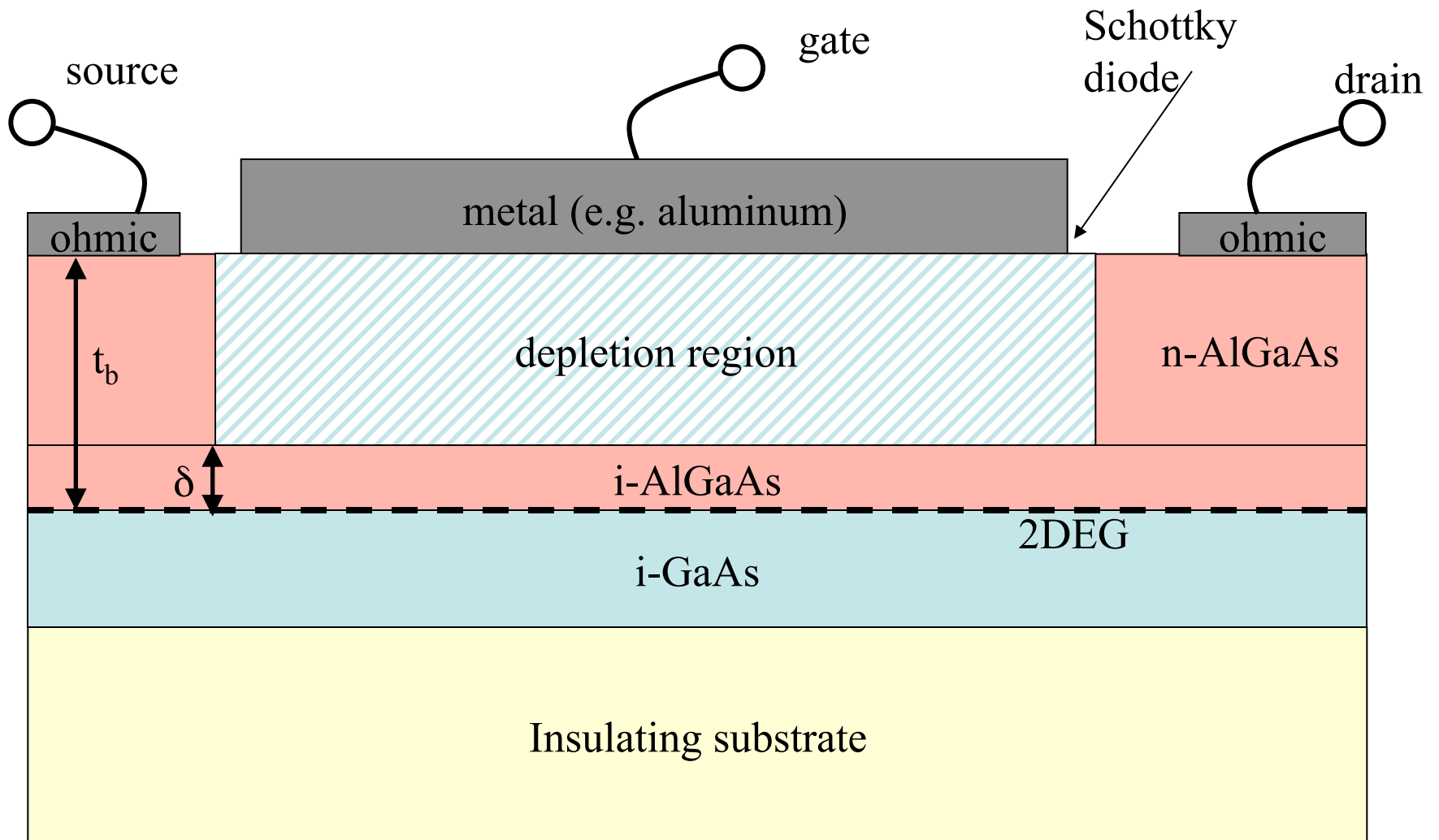
$E_v$

$\Delta E_v$

# HEMT:



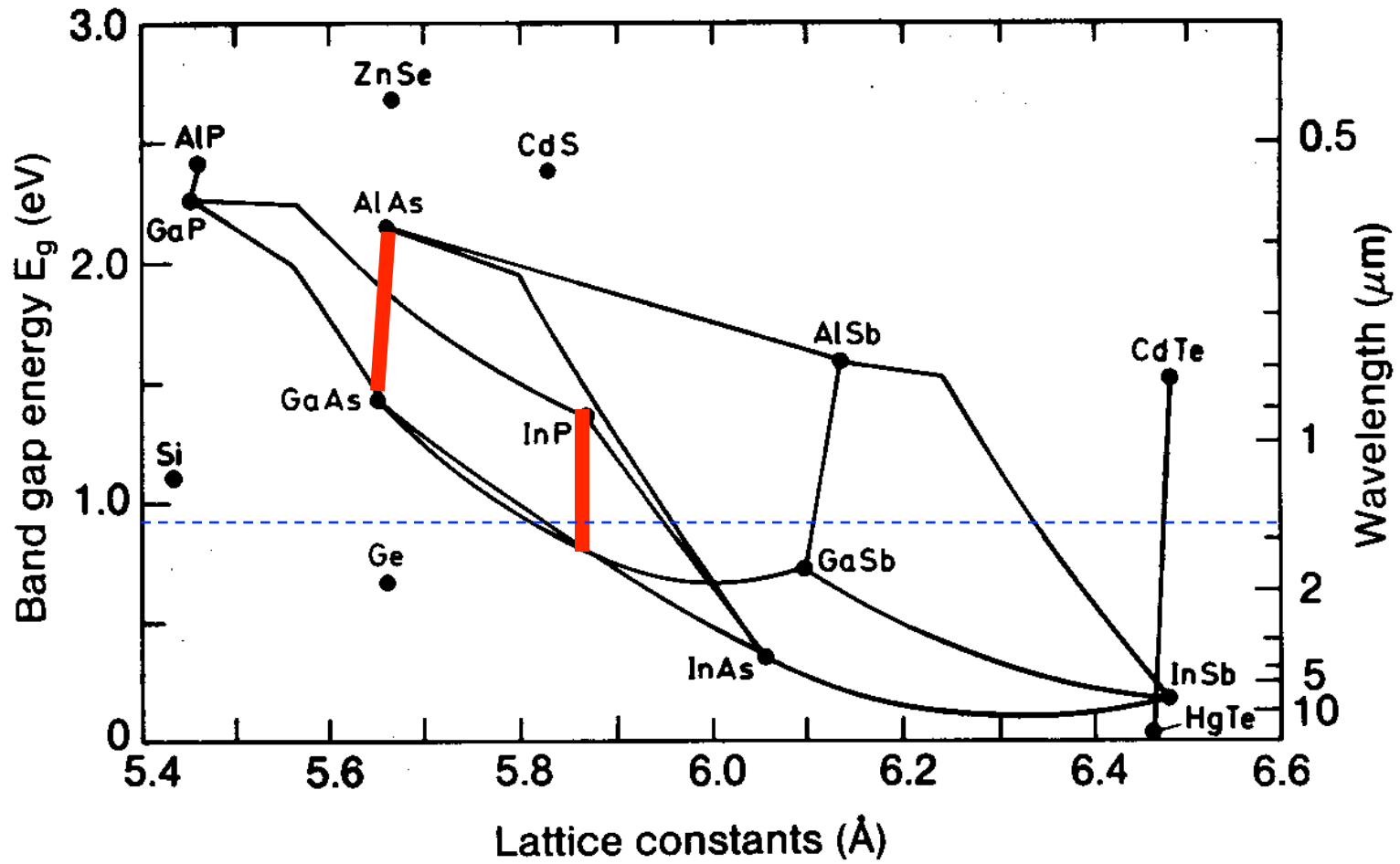
# HEMT:



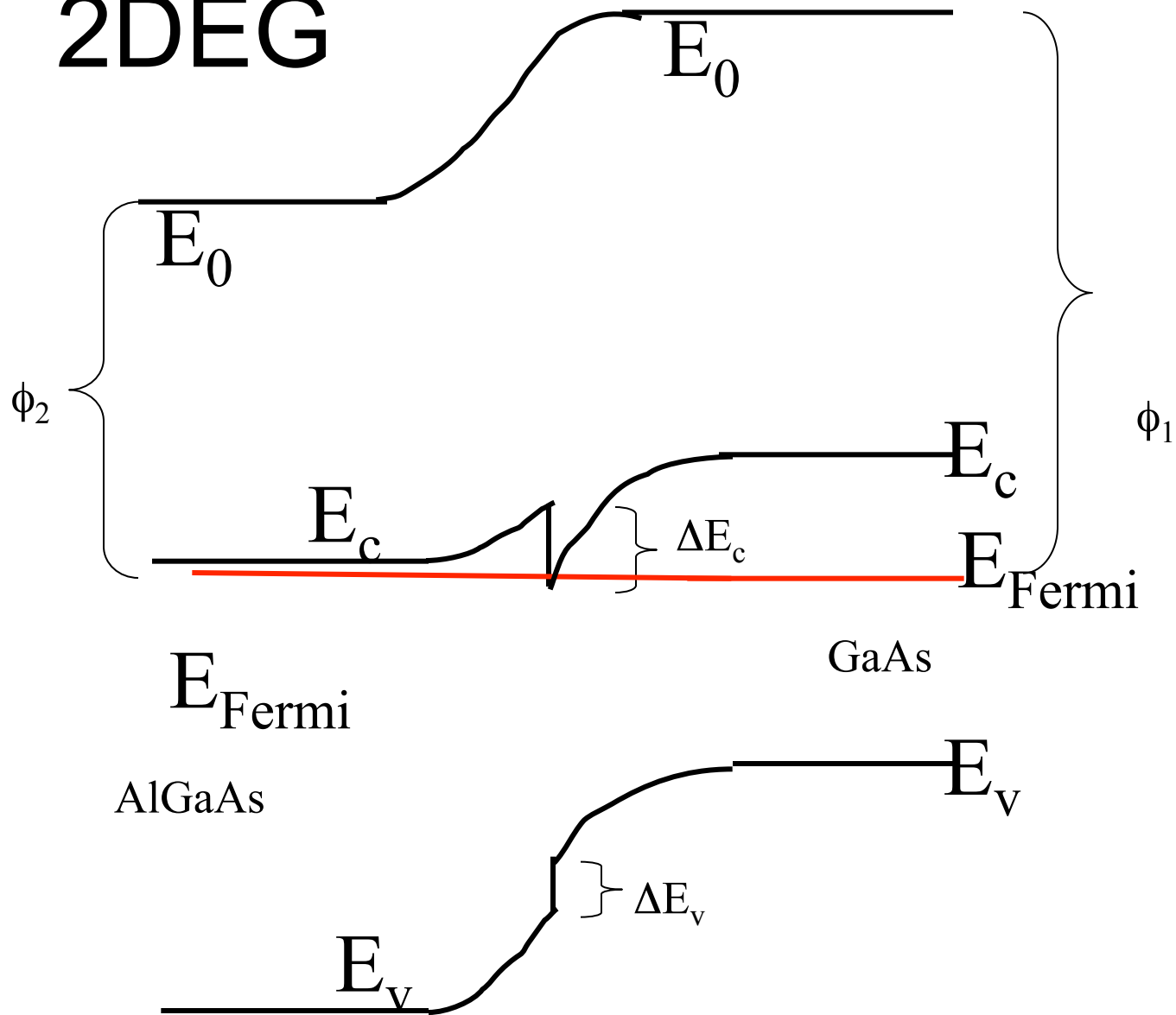
# Many possible variations

- “Quantum well” is also popular.
- Highly doped material under ohmics for low contact resistance
- InP based materials
- GaN based materials
- (pHEMT: strained materials)

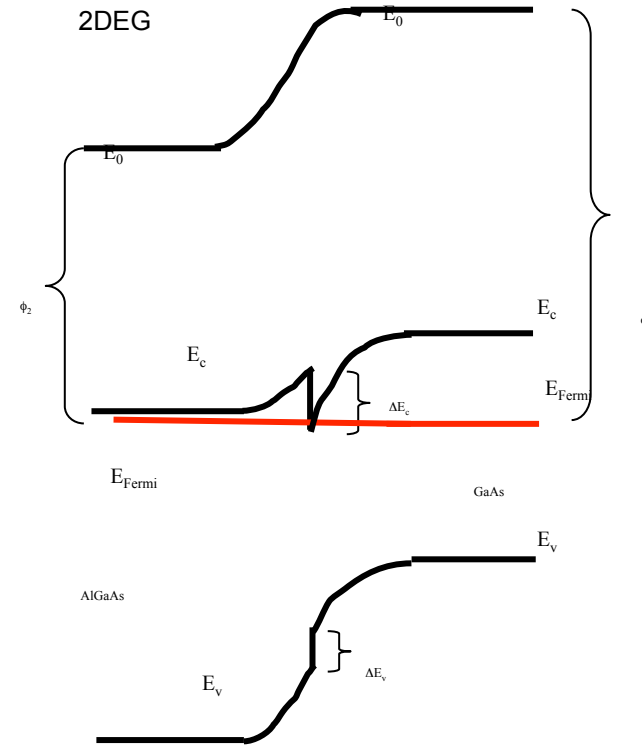
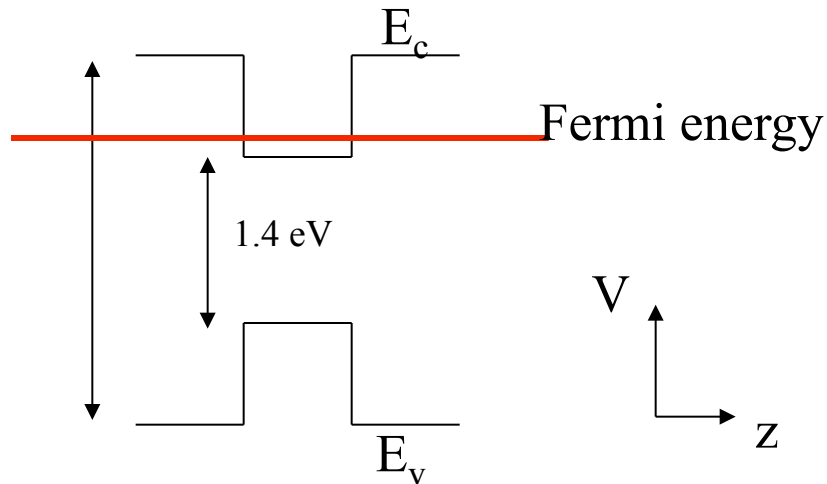
# Band gaps



# 2DEG



# Triangle vs. square well:



(Draw both bound states on board.  
In particular discuss figure 5.21 from Liu.)  
Also discuss shallow vs. wide wells.  
(Typically 100 angstroms works.)

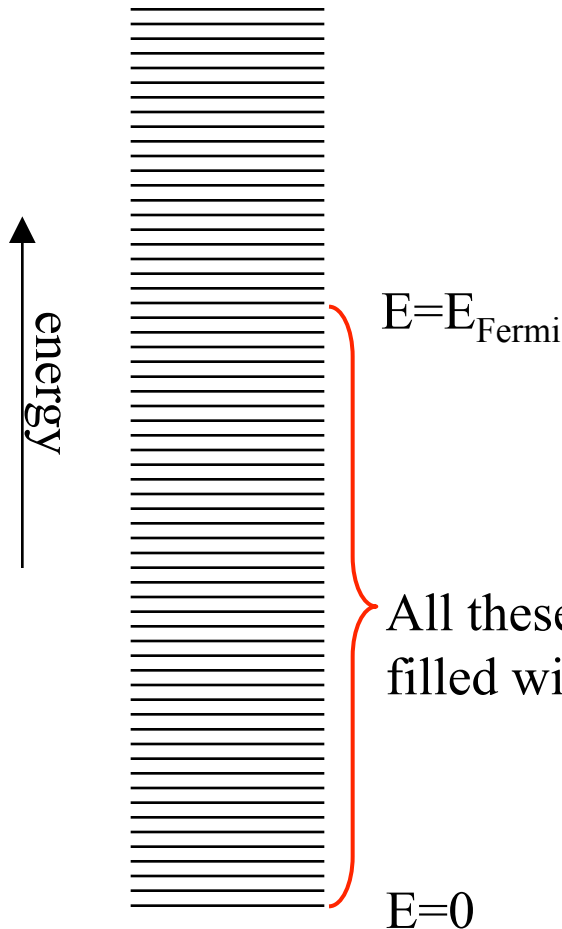


# Fermi energy in 2 dimensions

$$\# \text{ electrons} = \int_0^{E_f} N_E dE = \int_0^{E_f} L^2 \frac{m}{\pi \hbar} dE$$

$$\# \text{ electrons} = L^2 \frac{m}{\pi \hbar} E_f$$

$$\Rightarrow E_f = \frac{\hbar \pi}{m} \left( \frac{\# \text{ electrons}}{L^2} \right) = \frac{\hbar \pi}{m} n$$



All these states are filled with electrons.

In GaAs,  $10^{11} \text{cm}^{-2}$  gives

$$E_f \sim 1-10 \text{ meV}$$

But  $10^{12} \text{cm}^{-2}$  gives more than first subband.

Discuss “subband”, how above integral gets modified.

# Problem

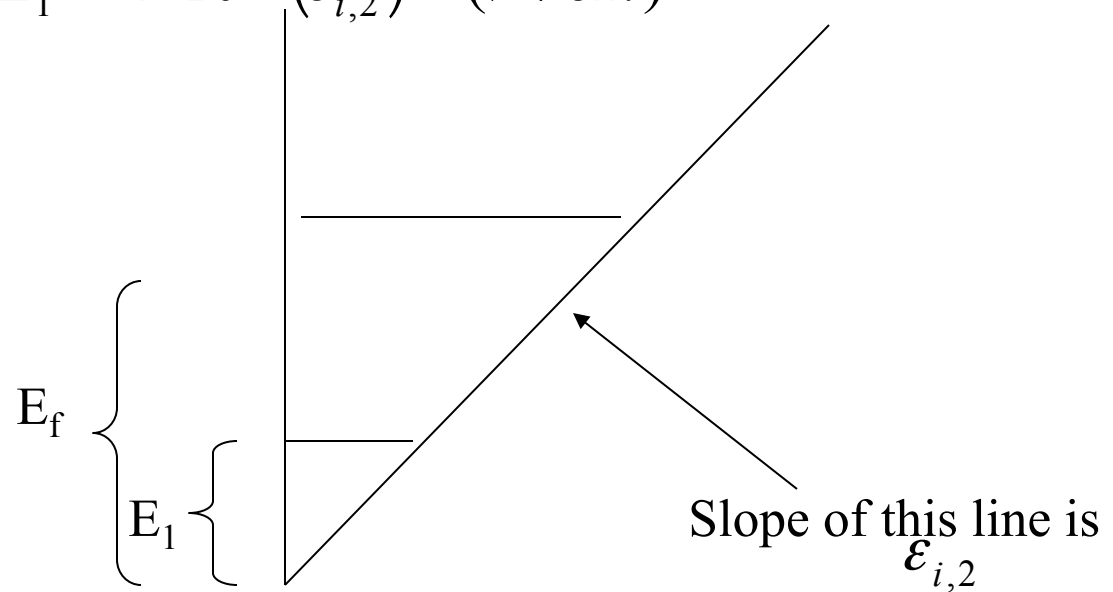
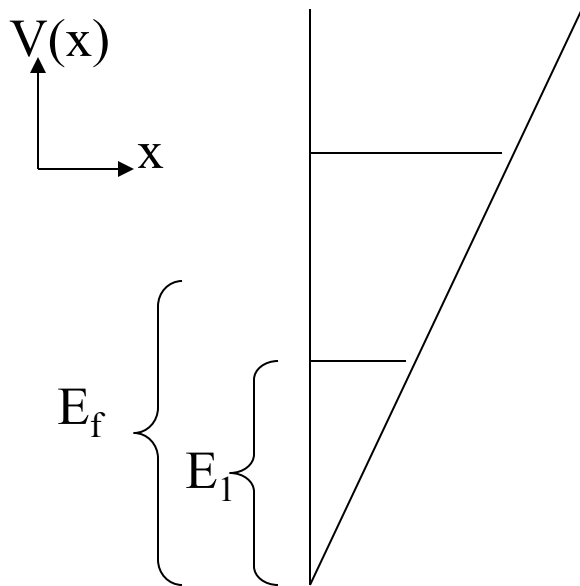
- Presence of electrons changes shape of potential well.
- We need a way to account for this.
- Will do NOW.
- Why? We want to know how many electrons there are!
- Later, we want to know how gate voltage changes that.

# Triangular wells

From quantum mechanics:

$$E_n \approx \left( \frac{\hbar^2}{8\pi^2 m^2} \right)^{1/3} \left( \frac{3\pi}{2} e \epsilon_{i,2} \right)^{2/3} \left( n - \frac{1}{4} \right)^{2/3}$$

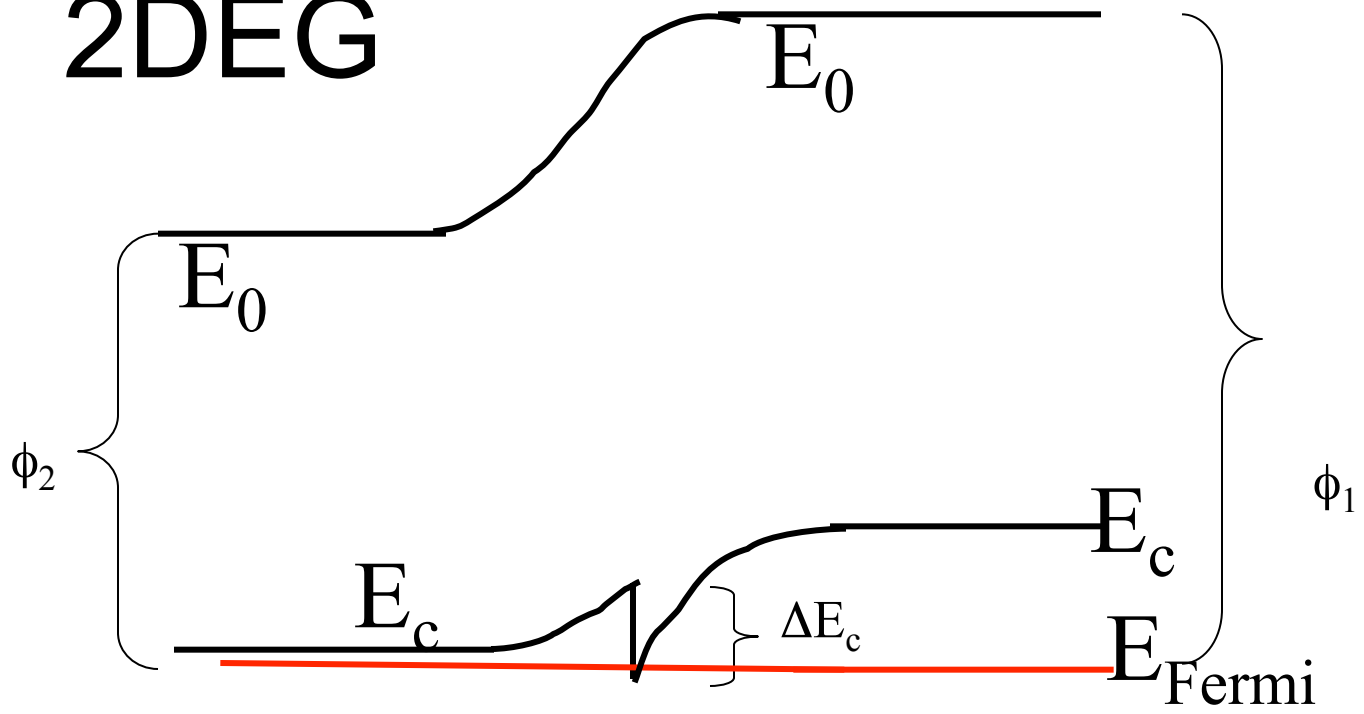
$$E_1 \approx 4 \cdot 10^{-5} (\epsilon_{i,2})^{2/3} (V / cm)$$



$$E_f = \frac{\hbar^2 \pi}{m} n_s \rightarrow (E_f - E_1) = \frac{\hbar \pi}{m} n_s$$

Note: Lecture 10 had error of hbar.

# 2DEG



$E_{\text{Fermi}}$

GaAs

AlGaAs

$E_v$

$E_v$

$\Delta E_v$

# Poisson equation

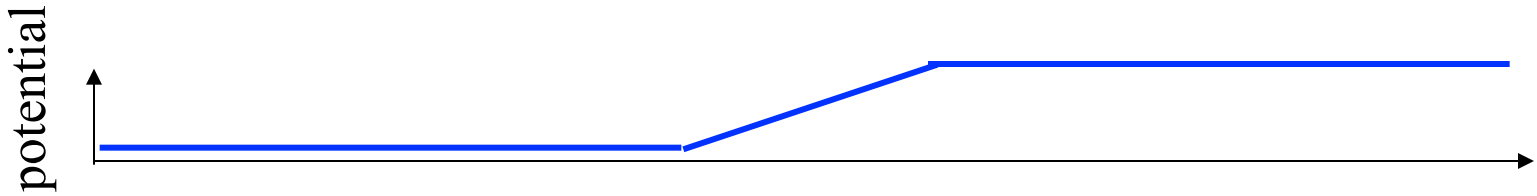
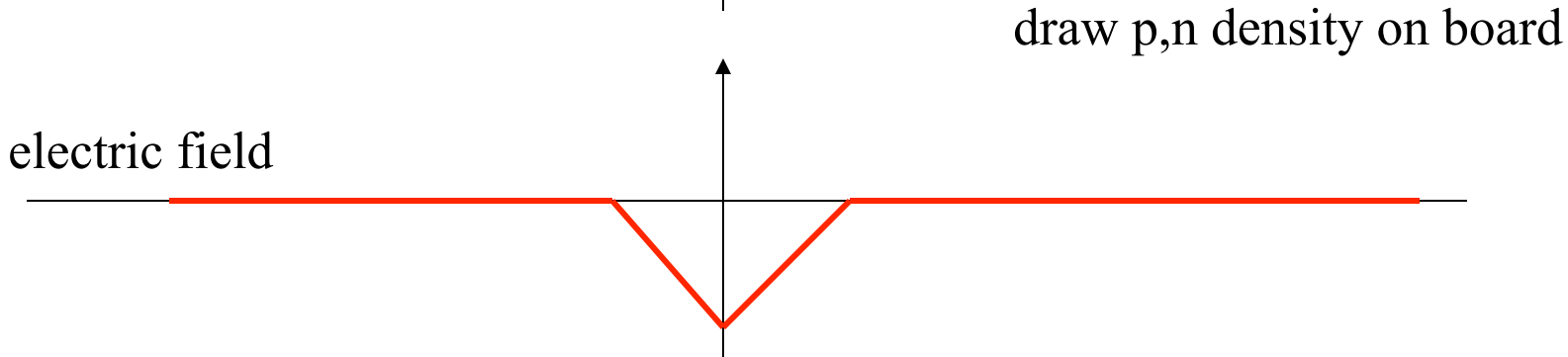
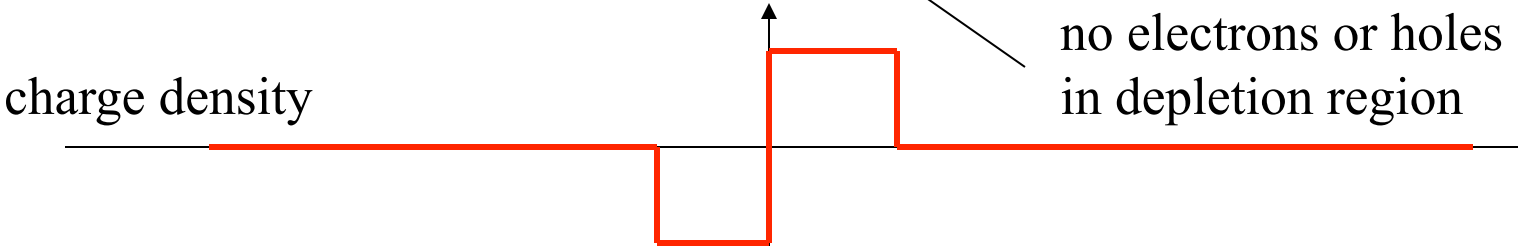
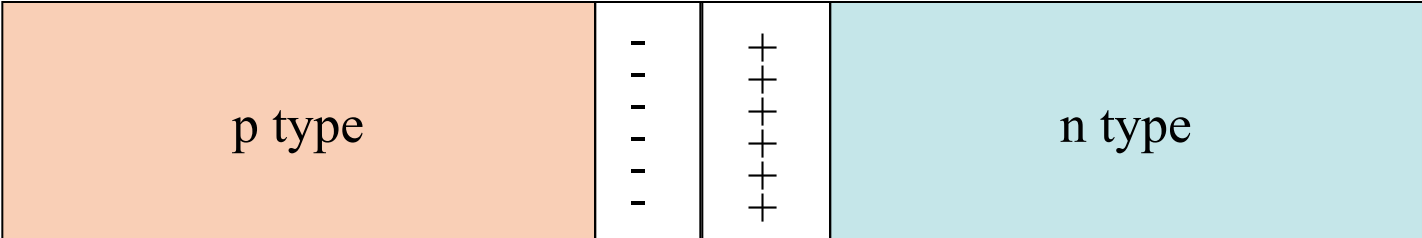
Actually first of Maxwell's four equations:

$$\vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\epsilon}$$

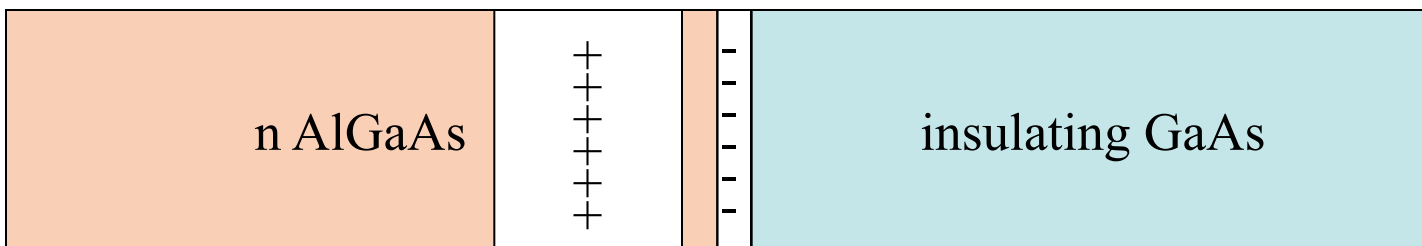
In the x-direction only:

$$\frac{dE}{dx} = \frac{q}{\epsilon_p} \left( p - n + N_d^+ - N_a^- \right)$$

# p n diode



# Hint for HW #5.8

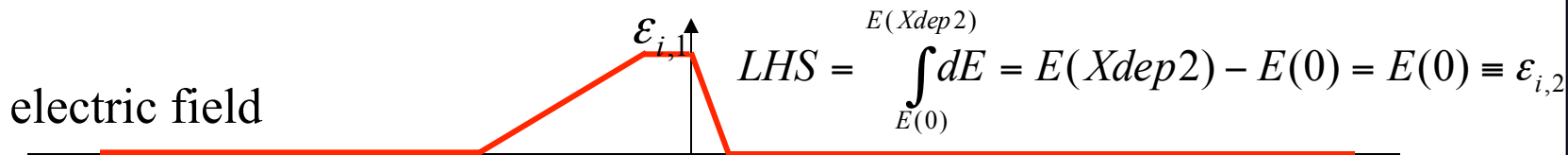
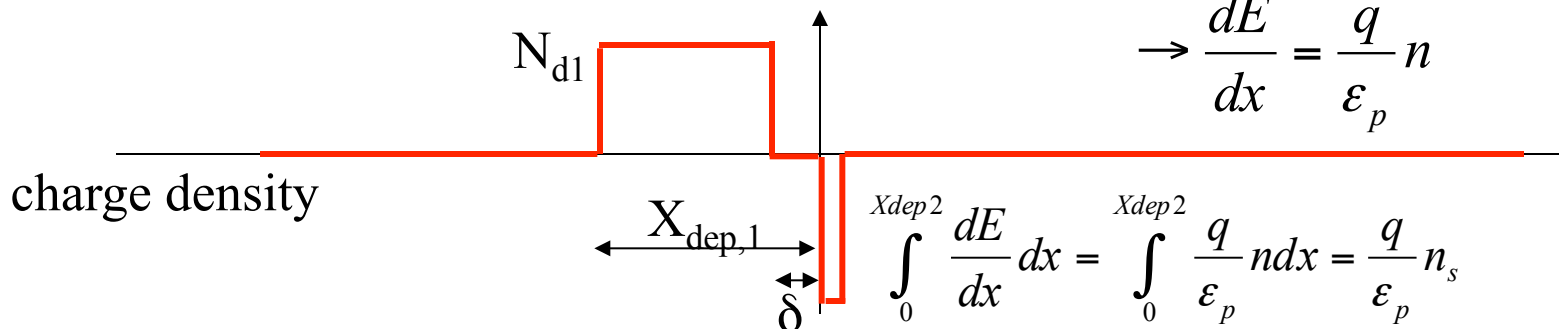


no electrons or holes  
in depletion region

undoped AlGaAs 2DEG

$$\frac{dE}{dx} = \frac{q}{\epsilon_p} (p - n + N_d^+ - N_a^-)$$

$$\rightarrow \frac{dE}{dx} = \frac{q}{\epsilon_p} n$$



Part b, calculate  $\epsilon_{i,1}$

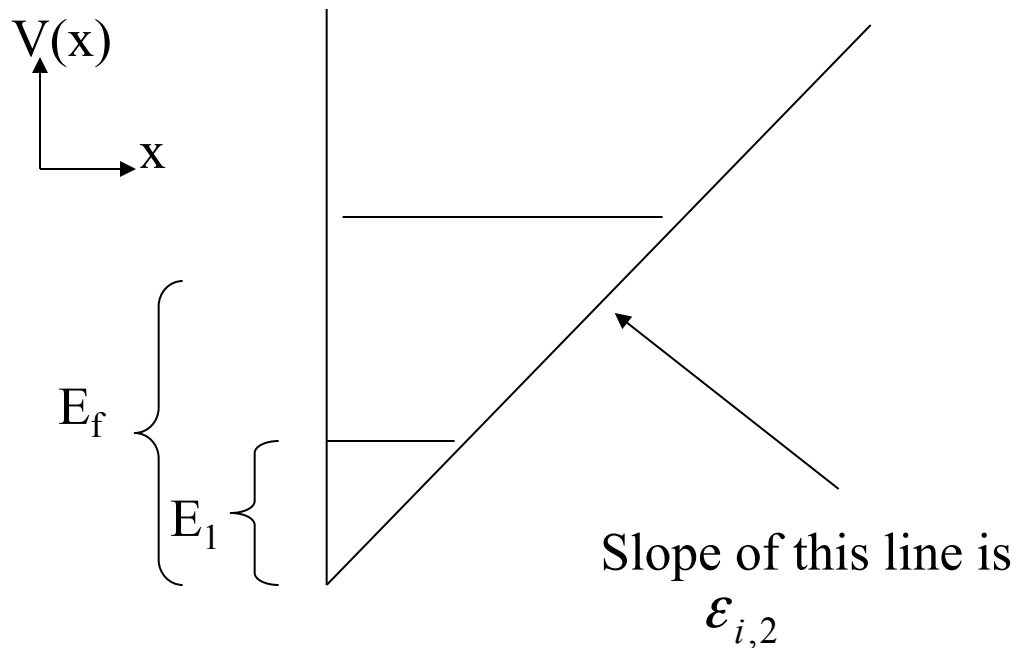
Then calculate  $\varphi_{bi,1} \equiv \int_{-\infty}^0 E(x) dx$

Then eliminate  $X_{dep,1}$

$$\epsilon_{i,1} = \epsilon_{i,2}$$

$$\Rightarrow \epsilon_{i,2} = \frac{q}{\epsilon_p} n_s$$

# Triangular wells



$$E_1 \approx 4 \cdot 10^{-5} (\epsilon_{i,2})^{2/3} (V / cm)$$

$$(E_f - E_1) = \frac{\hbar\pi}{m} n_s$$

$$\epsilon_{i,2} = \frac{q}{\epsilon_p} n_s$$

$$E_1 \approx 4 \cdot 10^{-5} \left( \frac{q}{\epsilon_p} n_s \right)^{2/3} (V / cm)$$

Discuss intuitively: adding electrons changes slope which changes Fermi energy.

That gives one relationship between  $E_f$  and  $n_s$ .

To solve for both of them, you need another relationship: HW 5.8



# From HW 5.8

You will find:

$$n_s \propto \sqrt{\Delta E_c} \sqrt{N_d}$$

Want to engineer material so that  $\Delta E_c$  large.

For GaAs, there is a limit.

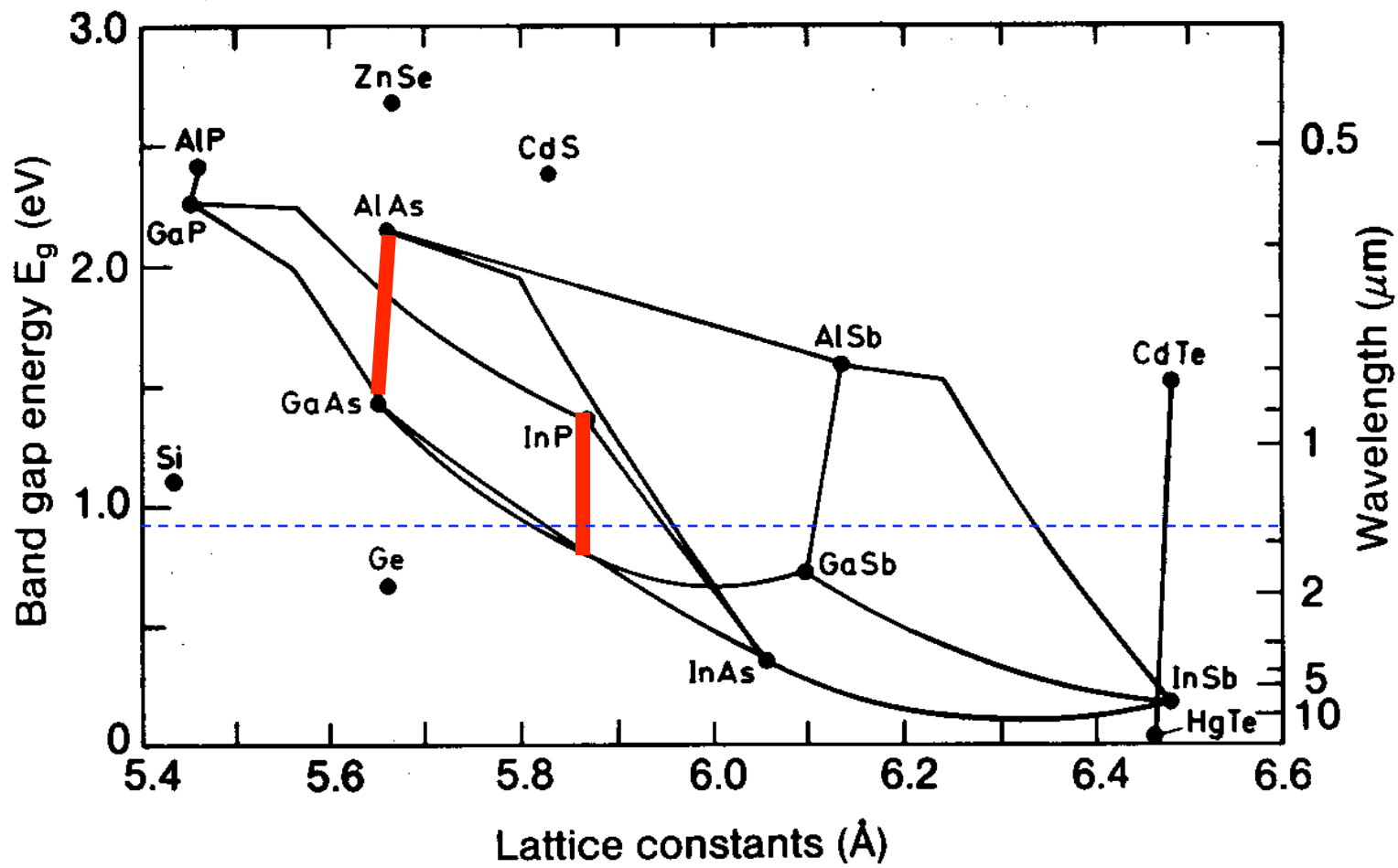
For  $\text{In}_x\text{Ga}_{1-x}\text{As}/\text{In}_x\text{Al}_{1-x}\text{As}$ ,

use strained layers to get larger  $\Delta E_c$  (discuss).

(InP has higher mobility, peak velocity than GaAs.)

Called pseudomorphic HEMT: pHEMT.

# Band gaps



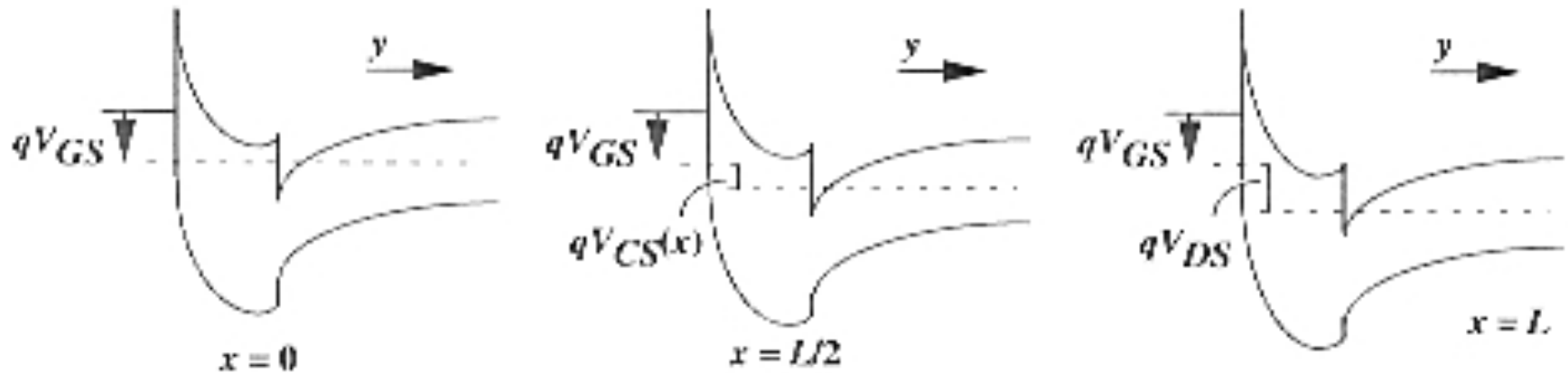
# $n_s$ vs $E_f$

After all that mumbo-jumbo, we know it is complicated.  
We approximate it many times as:

$$E_f(n_s) = E_{f,0} + a \cdot n_s$$



# Vary gate voltage



Changes Fermi energy which changes density.  
(Draw better pictures on board.)

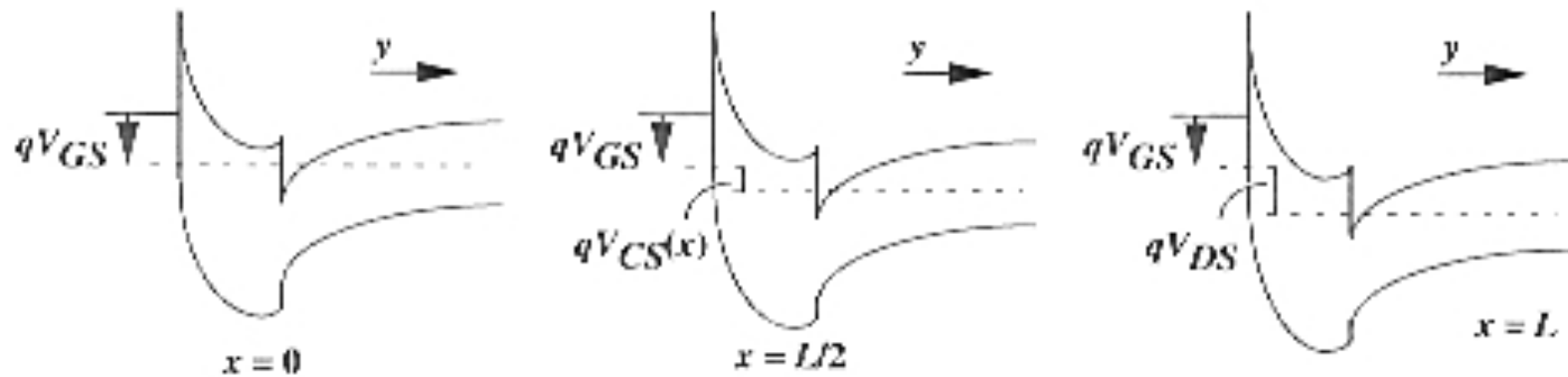
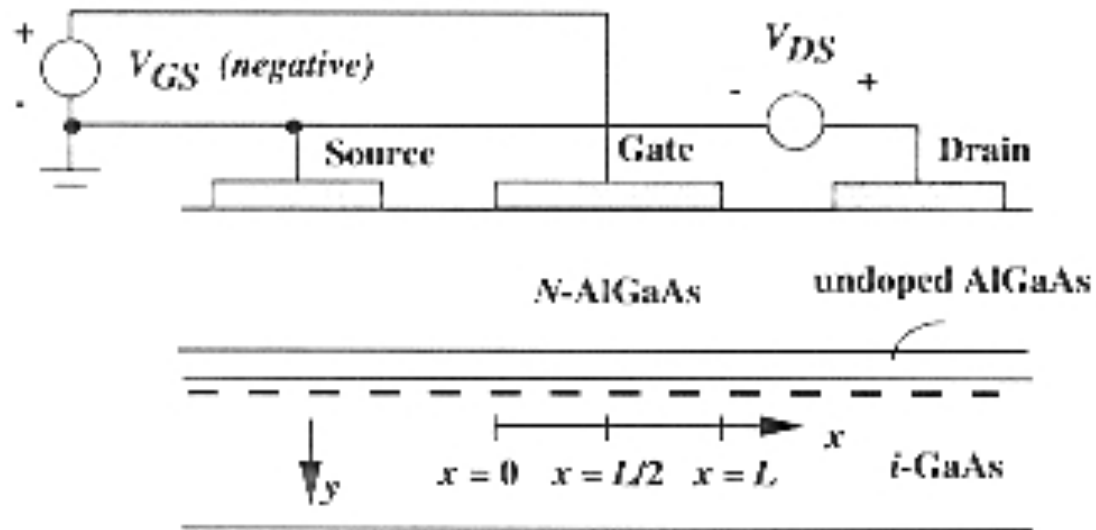
From Liu.

# Density

$$en_s = \frac{\epsilon}{t_b + \epsilon a / e^2} (V_{GB} - V_T)$$

$$V_T \equiv \phi_B + \frac{E_{f,0}}{e} - \frac{eN_{d,1}}{2\epsilon} (t_b - \delta)^2 - \frac{\Delta E_c}{e}$$

# HEMT analysis



From Liu.

# Density

$$en_s = \frac{\epsilon}{t_b + \epsilon a / e^2} (V_{GB} - V_T)$$

$$\rightarrow en_s(x) = C_{ox} (V_{GS} - V_T - V_{CS}(x))$$

$$C_{ox} \equiv \frac{\epsilon}{t_b + \epsilon a / e^2}$$

$$V_{CS}(0) = 0$$

$$V_{CS}(L) = V_{SD}$$



# Current

$J$  is 2d,  $n_s$  is 2d. (Discuss).

$$J = e \cdot \mu \cdot n_s \cdot E$$

$$I_D = J \cdot (\text{width}) = e \cdot \mu \cdot n(x) \cdot E(x) \cdot W$$

$$I_D = \mu \cdot C_{ox} (V_{GS} - V_T - V_{CS}(x)) \cdot E(x) \cdot W$$

$$= \mu \cdot C_{ox} (V_{GS} - V_T - V_{CS}(x)) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$

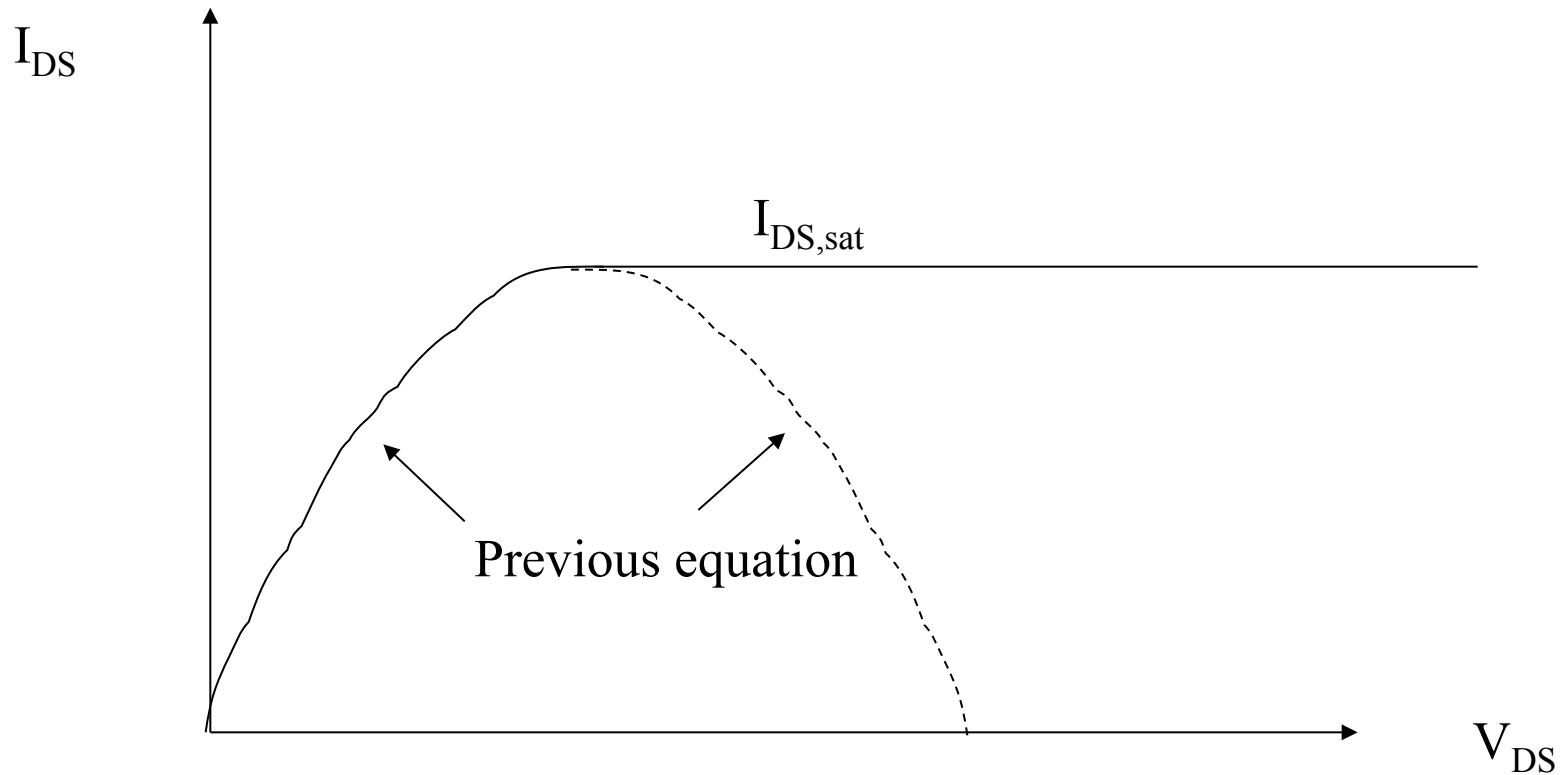
# Integrating:

$$I_D = \mu \cdot C_{ox} (V_{GS} - V_T - V_{CS}(x)) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$

$$\begin{aligned} \int_0^L I_D dx &= \int_0^L \mu \cdot C_{ox} (V_{GS} - V_T - V_{CS}(x)) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W dx \\ &= \int_{V_{CS}(0)}^{V_{CS}(L)} \mu \cdot C_{ox} (V_{GS} - V_T - V_{CS}(x)) \cdot \partial V_{CS}(x) \cdot W = doable \end{aligned}$$

$$I_D = \frac{W \cdot \mu \cdot C_{ox}}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

# Conclusion:



# Channel potential:

$$J = e \cdot \mu \cdot n_s \cdot E$$

$$I_D = J \cdot (\text{width}) = e \cdot \mu \cdot n(x) \cdot E(x) \cdot W$$

Since we are in 2d, no position dependent thickness  $b(x)$ . Life is easier.  
It can be shown that:

$$V_{CS}(x) = (V_{GS} - V_T) \left[ 1 - \sqrt{1 - \frac{x}{L} (1 - \alpha^2)} \right]$$

$$\alpha \equiv \begin{cases} 1 - V_{DS} / V_{DS,sat} & \text{for } V_{DS} < V_{DS,sat} \\ 0 & \text{for } V_{DS} > V_{DS,sat} \end{cases}$$

# Velocity saturation

- Just like MESFETs
- Important in short channel HEMTs
- Need to model channel as to regions: saturated and unsaturated
- Qualitative IVs are similar