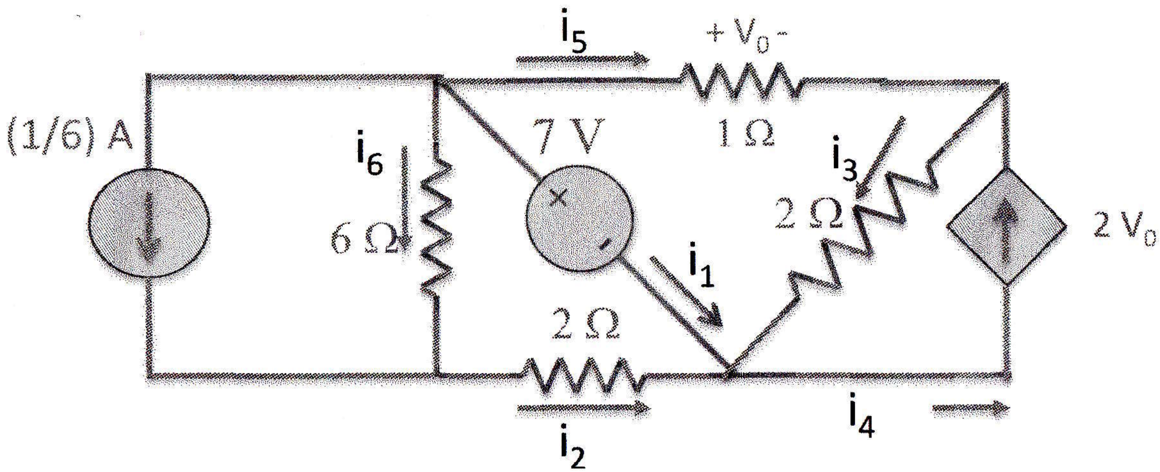
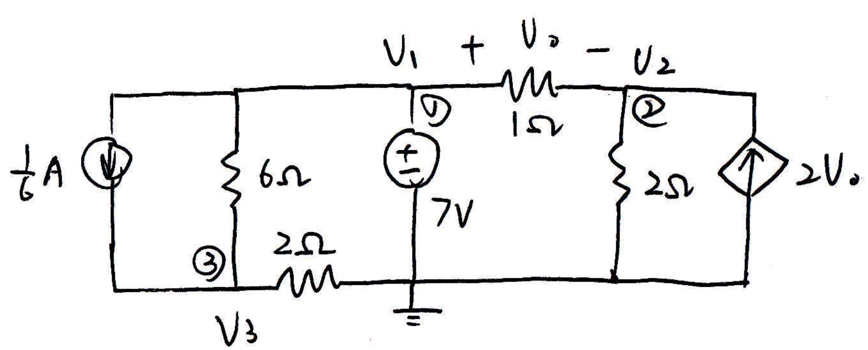
Problem 1:

-Use nodal analysis to find all node voltages.

-Find all the currents.



Solution:



let's set the cathod of the voltage source as the reference ground.

@ node 1 : $V_1 = 7V$

KCL @ node 2 : $\begin{cases} \frac{V_2 - V_1}{1\Omega} + \frac{V_2}{2\Omega} - 2V_0 = 0 \\ V_0 = V_1 - V_2 \end{cases}$

KCL @ node 3 : $\frac{V_1 - V_3}{6} + \frac{1}{6} - \frac{V_3}{2} = 0$

Solve for node 2 : $V_2 = 6V$.

Solve for node 3 : $V_3 = 2V$

Therefore $V_1 = 7V$, $V_2 = 6V$, $V_3 = 2V$

For the currents

$i_2 = \frac{V_3}{2\Omega} = 1A$

$i_3 = \frac{V_1 - V_2}{1\Omega} + 2(V_1 - V_2) = 3A$

$i_4 = 2(V_1 - V_2) = 2A$

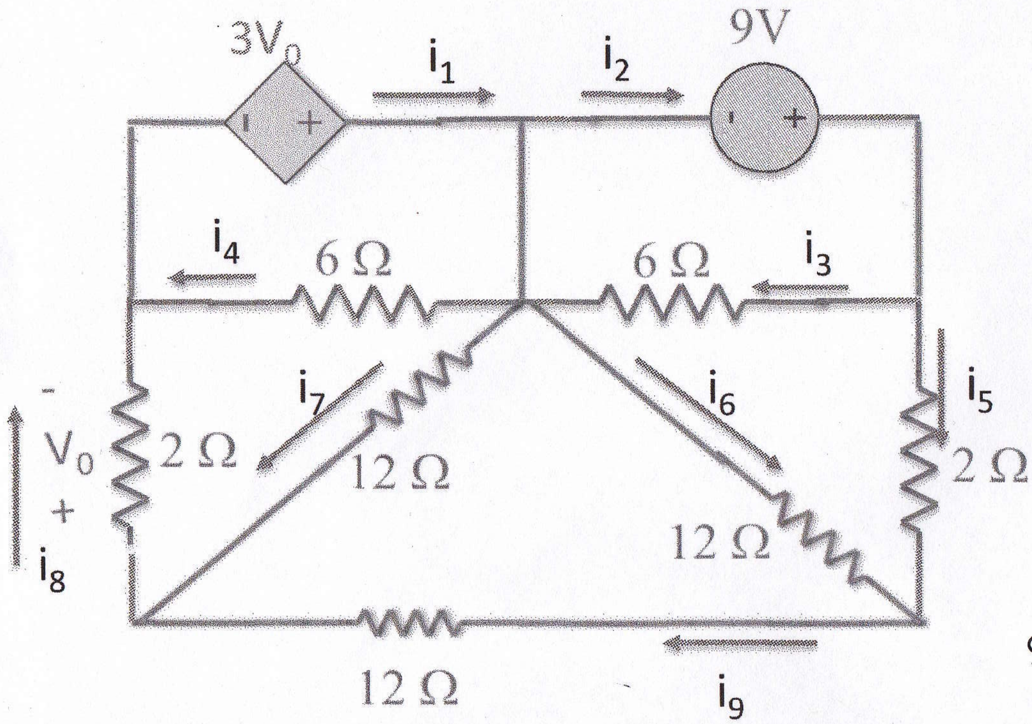
$i_5 = \frac{V_1 - V_3}{1\Omega} = 1A$

$i_6 = \frac{V_1 - V_3}{6\Omega} = \frac{5}{6}A$

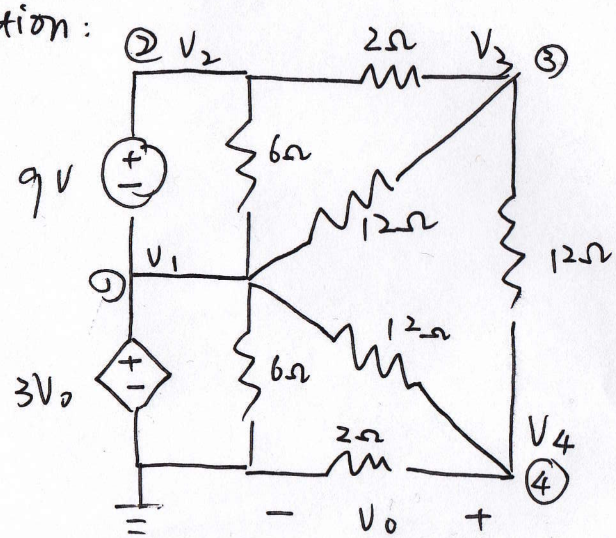
$i_1 = -(i_5 + i_6) = -\frac{11}{6}A$ not right

i1= -(i5+i6+1/6)= -2A

Problem 2: Use nodal analysis to find all the node voltages and currents



Solution:



KCL @ node 3:

$$\frac{V_3-V_2}{2\Omega}+\frac{V_3-V_1}{12\Omega}+\frac{V_3-V_4}{12\Omega}=0 \qquad (1)$$

KCL @ node 4:

$$\frac{V_4}{2\Omega}+\frac{V_4-V_3}{12\Omega}+\frac{V_4-V_1}{12\Omega}=0 \qquad (2)$$

$$\begin{cases} V_2-V_1=9\,V \\ V_1=3V_0 \\ \cancel{V_0=-3V_4} \end{cases} \Rightarrow \begin{cases} V_1=3V_4 \\ V_2=3V_4+9 \end{cases}$$

$V_0 = V_4$

Substitute V_1, V_2 into Eq.(1)

$$\frac{V_3-(3V_4+9)}{2}+\frac{(V_3-3V_4)}{12}+\frac{V_3-V_4}{12}=0$$

$$\Rightarrow \quad 8V_3-22V_4-54=0 \qquad (3)$$

Substitute V_1, V_2 into Eq (2)

$$\frac{V_4}{2}+\frac{V_4-V_3}{12}+\frac{V_4-3V_4}{12}=0$$

$$\Rightarrow \quad 5V_4-V_3=0 \qquad (4)$$

substitute Eq (4) into Eq (3)

$$\Rightarrow \begin{cases} V_4=3\,V \\ V_3=15\,V \end{cases}$$

So $V_1 = 9V$

$V_2 = 18V$

$V_3 = 15V$

$V_4 = 3V$

For the currents:

$$i_3 = \frac{V_2 - V_1}{6\Omega} = \frac{18-9}{6} = \frac{3}{2} A$$

$$i_4 = \frac{V_1}{6\Omega} = \frac{3}{2} A$$

$$i_5 = \frac{V_2 - V_3}{2\Omega} = \frac{18-15}{2} = \frac{3}{2} A$$

$$i_6 = \frac{V_1 - V_3}{12\Omega} = \frac{9-15}{12} = -\frac{1}{2} A$$

$$i_7 = \frac{V_1 - V_4}{12\Omega} = \frac{9-3}{12} = \frac{1}{2} A$$

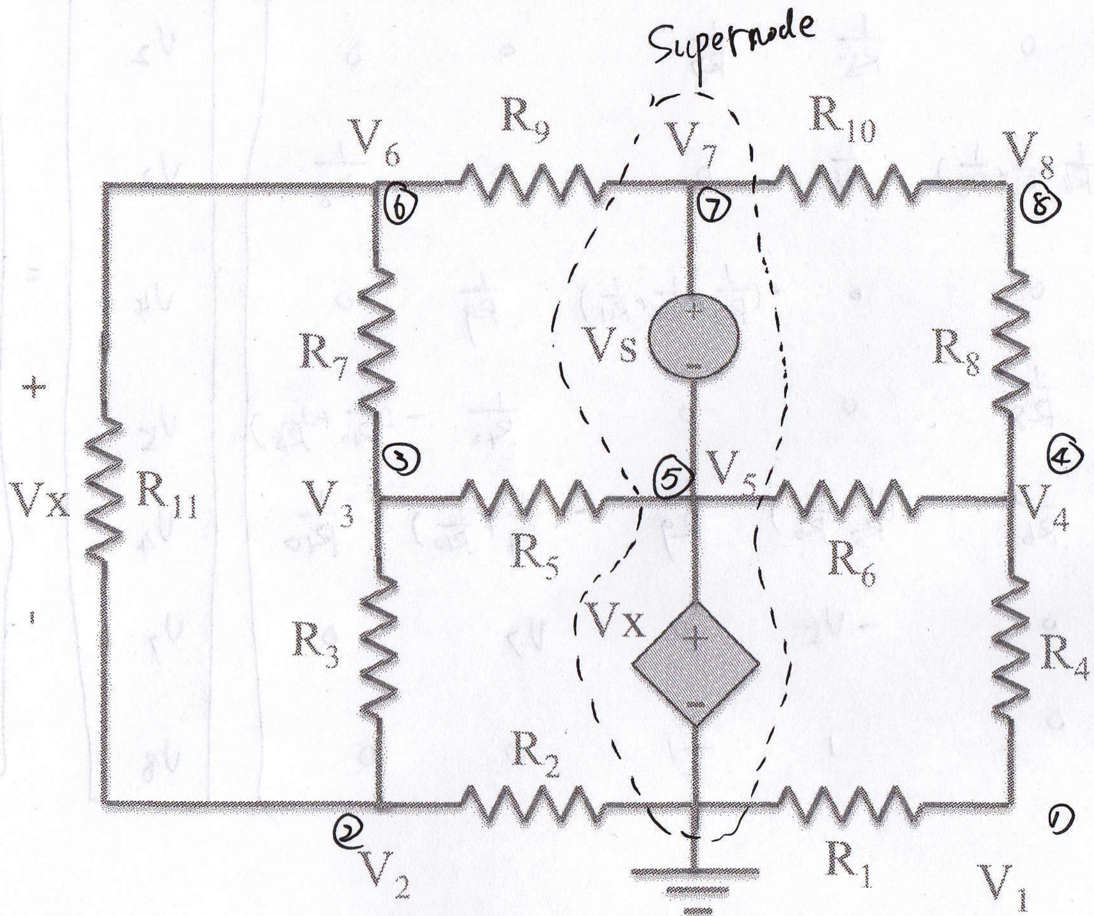
$$i_8 = \frac{V_4}{2\Omega} = \frac{3}{2} = 1.5 A$$

$$i_9 = \frac{V_3 - V_4}{12\Omega} = \frac{15-3}{12} = 1 A$$

$$i_1 = i_4 + i_8 = 3 A$$

$$i_2 = i_3 + i_5 = 3 A$$

Problem 3: Write all the node voltage equations and put them in the matrix form. You don't need to solve



@ node 6 : $\frac{V_7-V_6}{R_9} - \frac{V_6-V_3}{R_7} - \frac{V_6-V_2}{R_{11}} = 0$

$\Rightarrow \frac{V_2}{R_{11}} - \left(\frac{1}{R_7}+\frac{1}{R_9}+\frac{1}{R_{11}}\right)V_6 + \frac{1}{R_9}V_7 = 0 \quad (4)$

@ node 8: $\frac{V_7-V_8}{R_{10}} = \frac{V_8-V_4}{R_8}$

$\Rightarrow \frac{V_4}{R_8} + \frac{V_7}{R_{10}} - \left(\frac{1}{R_{10}}+\frac{1}{R_8}\right)V_8 = 0 \quad (5)$

@ supernode

$\frac{V_6-V_7}{R_9} + \frac{V_8-V_7}{R_{10}} + \frac{V_3-V_5}{R_5} + \frac{V_4-V_5}{R_6} + \frac{V_2}{R_2} + \frac{V_1}{R_1} = 0$

$\Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_5} + \frac{V_4}{R_6} - \left(\frac{1}{R_5}+\frac{1}{R_6}\right)V_5 + \frac{V_6}{R_9} - \left(\frac{1}{R_9}+\frac{1}{R_{10}}\right)V_7 + \frac{V_8}{R_{10}} = 0 \quad (6)$

In addition:

$V_7 - V_5 = V_s \quad (7)$

$V_5 = V_x = V_6 - V_2$

$V_2 + V_5 - V_6 = 0 \quad (8)$

@ node 2:

$\frac{V_3-V_2}{R_3} + \frac{V_6-V_2}{R_{11}} - \frac{V_2}{R_2} = 0 \Rightarrow -\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_{11}}\right)V_2 + \frac{1}{R_3}V_3 + \frac{1}{R_{11}}V_6 = 0 \quad (1)$

We have eight equations for eight unknown numbers.

@ node 3 :

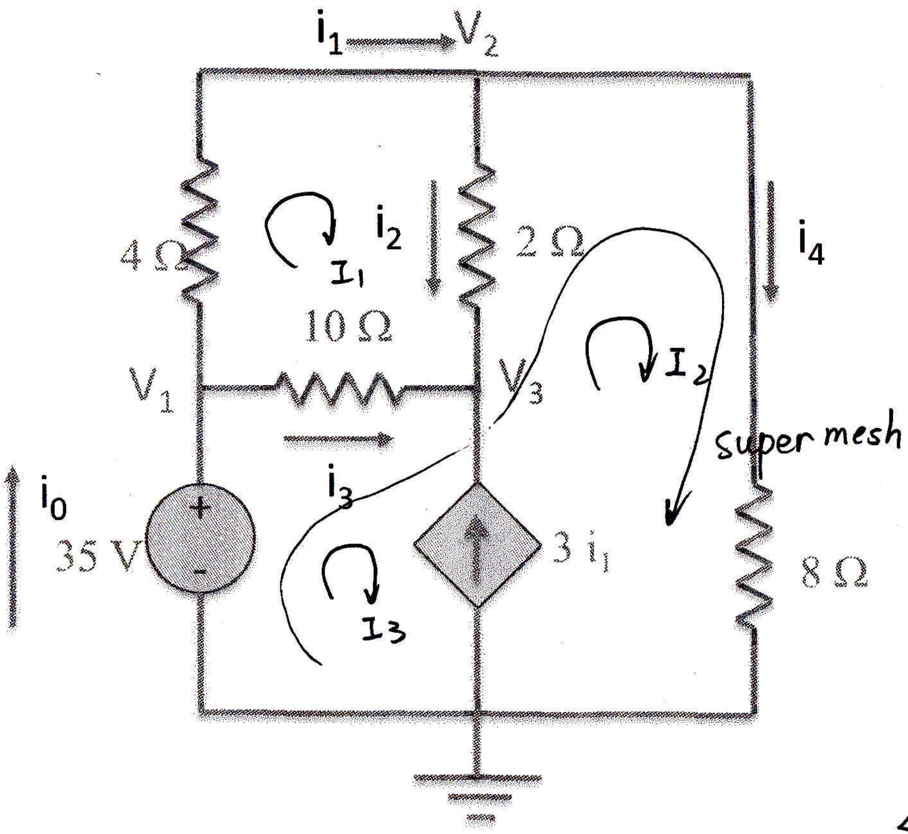
$\frac{V_6-V_3}{R_7} + \frac{V_5-V_3}{R_5} - \frac{V_3-V_2}{R_3} = 0 \Rightarrow \frac{1}{R_3}V_2 - \left(\frac{1}{R_3}+\frac{1}{R_7}+\frac{1}{R_5}\right)V_3 + \frac{1}{R_5}V_5 + \frac{1}{R_7}V_6 = 0 \quad (2)$

@ node 4:

$\frac{V_8-V_4}{R_8} + \frac{V_5-V_4}{R_6} - \frac{V_4-V_1}{R_4} = 0 \Rightarrow \frac{1}{R_4}V_1 - \left(\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_8}\right)V_4 + \frac{1}{R_6}V_5 + \frac{1}{R_8}V_8 = 0 \quad (3)$

$$
\begin{bmatrix}
0 & -\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) & \frac{1}{R_3} & 0 & 0 & \frac{1}{R_{11}} & 0 & 0 \\
0 & \frac{1}{R_3} & -\left(\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_7}\right) & 0 & \frac{1}{R_5} & \frac{1}{R_7} & 0 & 0 \\
\frac{1}{R_4} & 0 & 0 & -\left(\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_8}\right) & \frac{1}{R_6} & 0 & 0 & \frac{1}{R_8} \\
0 & \frac{1}{R_{11}} & 0 & 0 & 0 & -\left(\frac{1}{R_7}+\frac{1}{R_9}+\frac{1}{R_{11}}\right) & \frac{1}{R_9} & 0 \\
0 & 0 & 0 & \frac{1}{R_8} & 0 & 0 & \frac{1}{R_{10}} & -\left(\frac{1}{R_{10}}+\frac{1}{R_8}\right) \\
\frac{1}{R_1} & \frac{1}{R_2} & \frac{1}{R_5} & \frac{1}{R_6} & -\left(\frac{1}{R_5}+\frac{1}{R_6}\right) & \frac{1}{R_9} & -\left(\frac{1}{R_9}+\frac{1}{R_{10}}\right) & \frac{1}{R_{10}} \\
0 & 0 & 0 & 0 & -V_5 & 0 & V_7 & 0 \\
0 & 1 & 0 & 0 & 1 & -1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_4 \\ V_7 \\ V_8
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0
\end{bmatrix}
$$

Problem 4: use mesh analysis to find all the currents and node voltages.



Solution:

KVL @ mesh I_1

$4I_1 + 2(I_1 - I_2) + 10(I_1 - I_3) = 0$

$\Rightarrow \quad 8I_1 - I_2 - 5I_3 = 0 \quad (1)$

KVL @ mesh I_2 & mesh I_3 $\rightarrow$ super mesh

$-35 + (I_3 - I_1)\times 10 + (I_2 - I_1)\times 2 + I_2 \times 8 = 0$

$\Rightarrow \quad -12I_1 + 10I_2 + 10I_3 = 35 \quad (2)$

$I_2 - I_3 = 3i_1 = 3I_1 \Rightarrow 3I_1 - I_2 + I_3 = 0 \quad (3)$

matrix form:

$$\begin{bmatrix} 8 & -1 & -5 \\ -12 & 10 & 10 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 35 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -1 & -5 \\ -12 & 10 & 10 \\ 3 & -1 & 1 \end{vmatrix} = 208$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & -5 \\ 35 & 10 & 10 \\ 0 & -1 & 1 \end{vmatrix} = 210$$

$$\Delta_2 = \begin{vmatrix} 8 & 0 & -5 \\ -12 & 35 & 10 \\ 3 & 0 & 1 \end{vmatrix} = 805$$

$$\Delta_3 = \begin{vmatrix} 8 & -1 & 0 \\ -12 & 10 & 35 \\ 3 & 1 & 0 \end{vmatrix} = 175$$

$I_1 = \frac{\Delta_1}{\Delta} = 1.0096\ A$

$I_2 = \frac{\Delta_2}{\Delta} = \frac{805}{208} = 3.8702A$

$I_3 = \frac{\Delta_3}{\Delta} = 0.8413\ A$

$i_1 = I_1 = 1.0096\ A$

$i_4 = I_2 = 3.8702A$

$i_2 = I_1 - I_2 = -2.8606A$

$i_3 = I_3 - I_1 = 0.8413 - 1.0096 = -0.1683A$

For the node voltages:

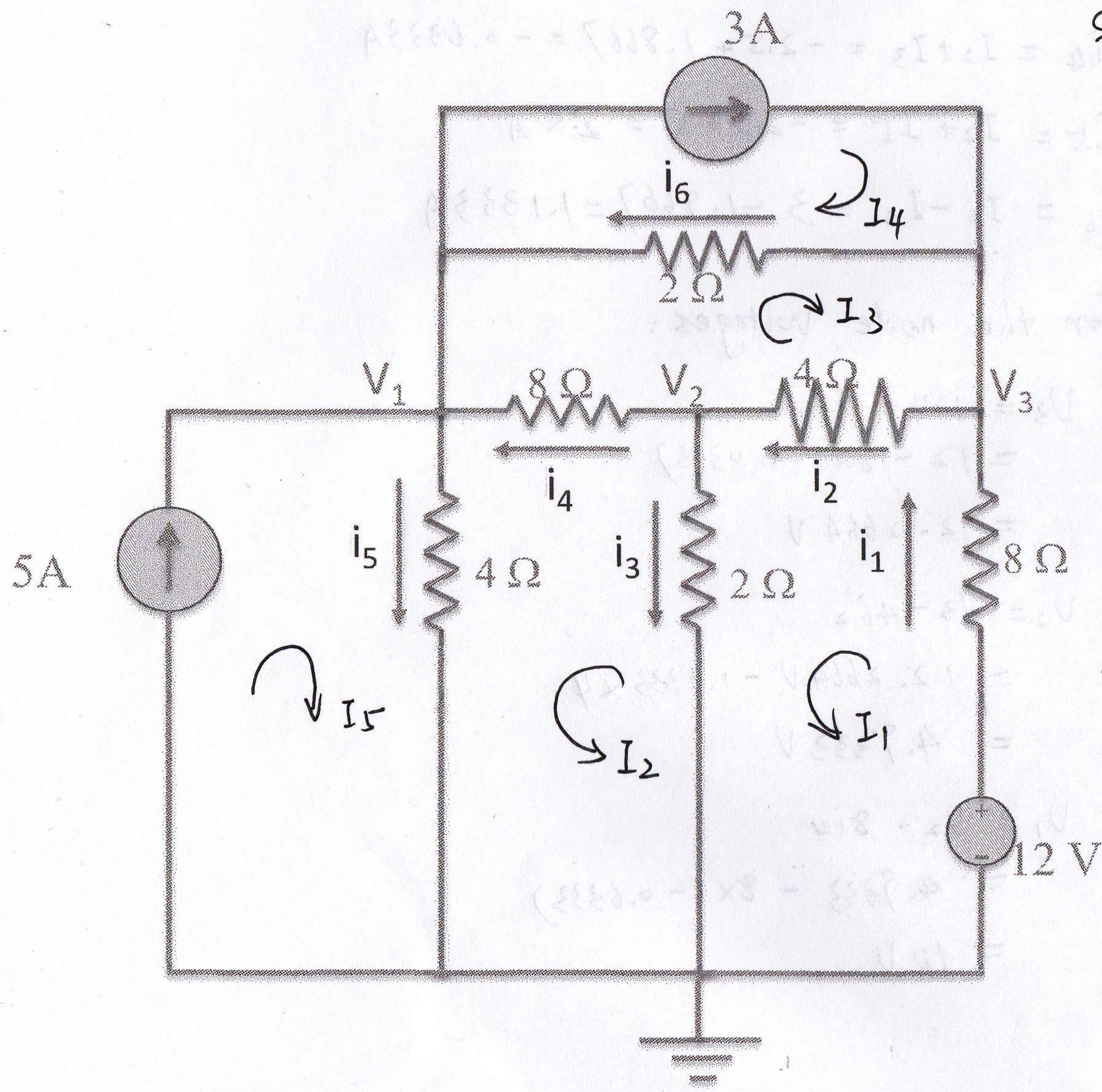
$V_1 = 35V$

$V_2 = V_1 - 4i_1 = 30.9616V$

$V_3 = V_1 - 10i_3 = 35 - 10\times(-0.1683) = 36.683\ V$

Problem 5:

-Use mesh analysis to find resistor currents and the all node voltages



Solution:

KVL @ mesh I_1

$$-12 + 8I_1 + 4(I_1 + I_3) + 2(I_1 - I_2) = 0$$

$$\Rightarrow 7I_1 - I_2 + 2I_3 = 6 \quad (1)$$

KVL @ mesh I_2

$$\begin{cases} 2(I_2 - I_1) + 8(I_2 + I_3) + 4(I_2 + I_5) = 0 \\ I_5 = 5A \end{cases}$$

$$\Rightarrow I_1 - 7I_2 - 4I_3 = 10 \quad (2)$$

KVL @ mesh I_3

$$\begin{cases} 4(I_1 + I_3) + 8(I_2 + I_3) + 2(I_3 - I_4) = 0 \\ I_4 = 3A \end{cases}$$

$$\Rightarrow 2I_1 + 4I_2 + 7I_3 = 3 \quad (3)$$

matrix form

$$\begin{bmatrix} 7 & -1 & 2 \\ 1 & -7 & -4 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -1 & 2 \\ 1 & -7 & -4 \\ 2 & 4 & 7 \end{vmatrix} = -180$$

$$\Delta_1 = \begin{vmatrix} 6 & -1 & 2 \\ 10 & -7 & -4 \\ 3 & 4 & 7 \end{vmatrix} = 6$$

$$\Delta 2 = \begin{vmatrix} 7 & 6 & 2 \\ 1 & 10 & -4 \\ 2 & 3 & 7 \end{vmatrix} = 450$$

$$\Delta 3 = \begin{vmatrix} 7 & -1 & 6 \\ 1 & -7 & 10 \\ 2 & 4 & 3 \end{vmatrix} = -336$$

$$I_1 = \frac{\Delta 1}{\Delta} = -0.0333\ A$$

$$I_2 = \frac{\Delta 2}{\Delta} = -2.5\ A$$

$$I_3 = \frac{\Delta 3}{\Delta} = 1.8667\ A$$

$i_1 = I_1 = -0.0333 A$

$i_2 = I_1 + I_3 = -0.0333 + 1.8667 = 1.8334 A$

$i_3 = I_1 - I_2 = -0.0333 + 2.5 = 2.4667 A$

$i_4 = I_2 + I_3 = -2.5 + 1.8667 = -0.6333 A$

$i_5 = I_2 + I_5 = -2.5 + 5 = 2.5 A$

$i_6 = I_4 - I_3 = 3 - 1.8667 = 1.1333 A$

For the node voltages:

$V_3 = 12V - 8i_1$

$= 12 - 8 \times (-0.0333)$

$= 12.2664 V$

$V_2 = V_3 - 4i_2$

$= 12.2664 V - 1.8333 \times 4$

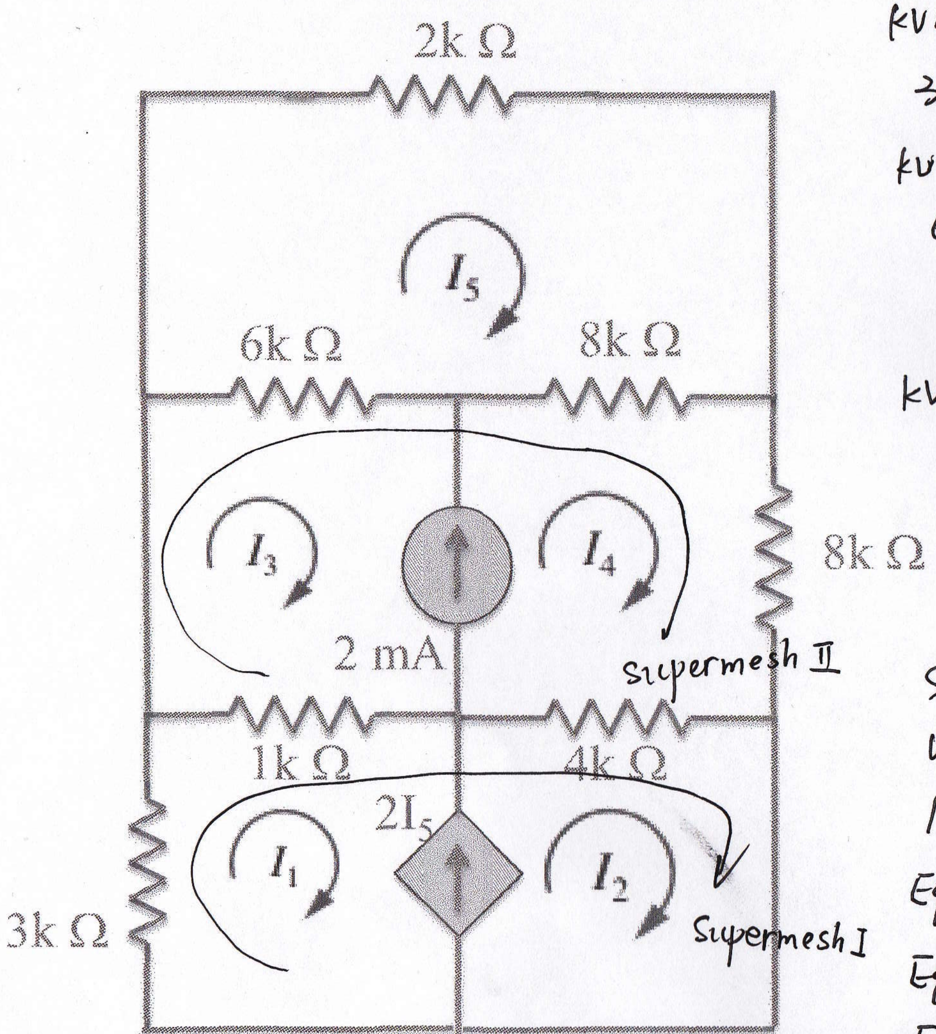
$= 4.9333 V$

$V_1 = V_2 - 8i_4$

$= 4.9333 - 8 \times (-0.6333)$

$= 10 V$

Problem 6: Write all the mesh current equations and put them in the matrix form. You don't need to solve



Solution:

KVL @ supermesh I:

$3k\Omega\, I_1 + (I_1 - I_3)\times 1k\Omega + (I_2 - I_4)\, 4k\Omega \Rightarrow$ (1)

KVL @ supermesh II:

$6k\Omega\,(I_3 - I_5) + 8k\Omega\,(I_4 - I_5) + 8k\Omega\, I_4 + 4k\Omega\,(I_4 - I_2)$
$+ 1k\Omega\,(I_3 - I_1) = 0$ (2)

KVL @ mesh 5:

$2k\Omega\, I_5 + 8k\Omega\,(I_5 - I_4) + 6k\Omega\,(I_5 - I_3) = 0$ (3)

$-I_3 + I_4 = 2mA$ (4)

$I_1 - I_2 + 2I_5 = 0$ (5)

Suppose all the current are in the order of mA range. We can remove kΩ to simplify the equations, because $1k\Omega \times 1mA = 1\Omega \times 1A = 1V$.

Eq.(1): $4I_1 + 4I_2 - I_3 - 4I_4 = 0$

Eq(2): $-I_1 - 4I_2 + 7I_3 + 20I_4 - 14I_5 = 0$

Eq(3): $-6I_3 - 8I_4 + 16I_5 = 0$

Eq(4): $-I_3 + I_4 = 2$

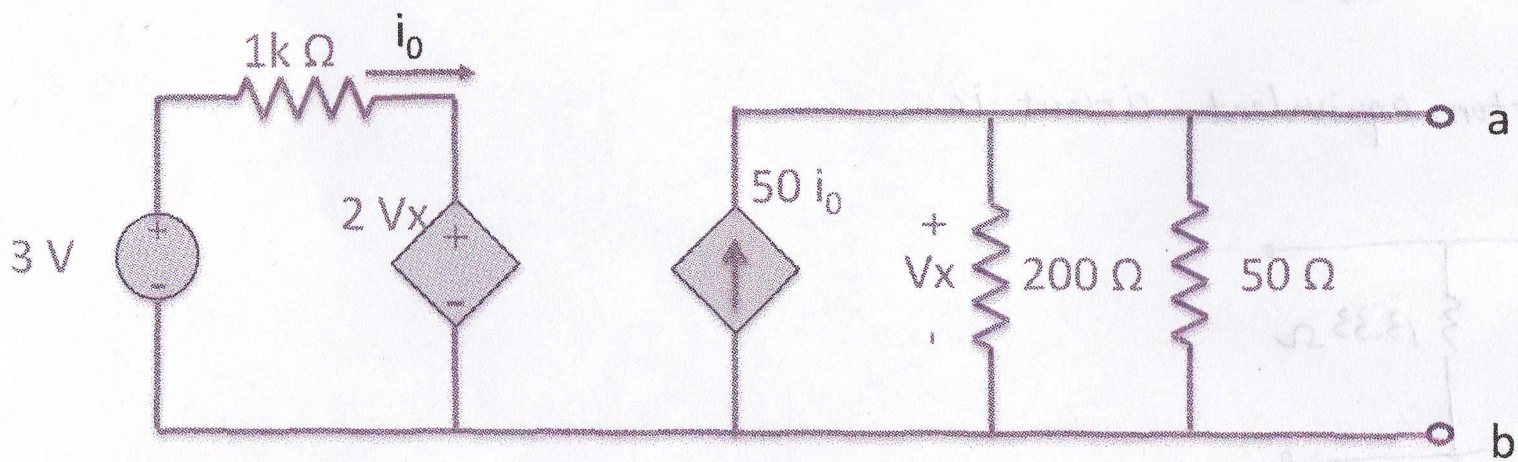
Eq(5): $I_1 - I_2 + 2I_5 = 0$

matrix form:

$$\begin{bmatrix} 4 & 4 & -1 & 4 & 0 \\ -1 & -4 & 7 & 20 & -14 \\ 0 & 0 & -6 & -8 & 16 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$I_1, I_2 \ldots I_6$ should have units of mA.

Problem 7: Obtain the Thevenin and Norton equivalent as seen from a-b terminals:



Solution :

$i_0 = \dfrac{3V - 2V_x}{1k\Omega}$

$V_x = (200\Omega // 50\Omega) \times 50\, i_0$

$= 40 \times 50\, i_0$

$i_0 = \dfrac{3 - \cancel{2000 i_0}\ 4000*i0}{1000}$

$3000\, i_0 = 3$

$i_0 = \cancel{10^{-3} A}$ i0 = 6e-4 A

$V_x = 50\, i_0 \times (200\Omega // 50\Omega)$

$= 50\, i_0 \times 40$

$= 2000 \times 10^{-3}$

$= \cancel{2V}$ Vx = 1.2 V

$\therefore V_{ab} = \cancel{2V}$ Vab = 1.2 V

now let's short a.b ports with a wire, then $V_x = 0$

So $i_0 = \dfrac{3V}{1k\Omega} = 3\, mA$

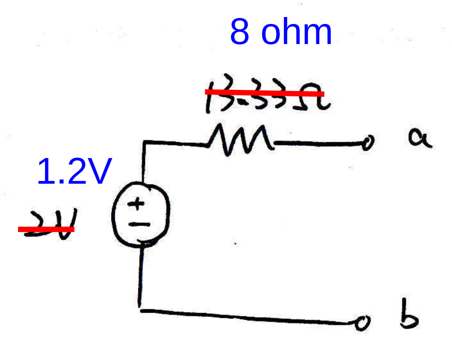
$I_{ab} = 50\, i_0 = 150\, mA$

$R_{th} = \dfrac{V_{ab}}{I_{ab}} = \dfrac{2V}{150mA} = \cancel{13.33\Omega}$

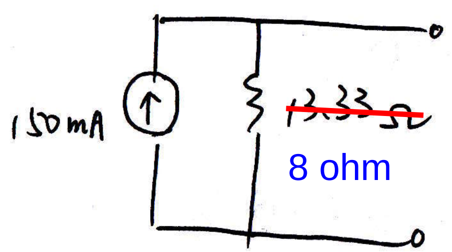
Rth = 1.2/0.15 = 8 ohm

$I_N = \dfrac{V_{ab}}{R_{th}} = 150\, mA$

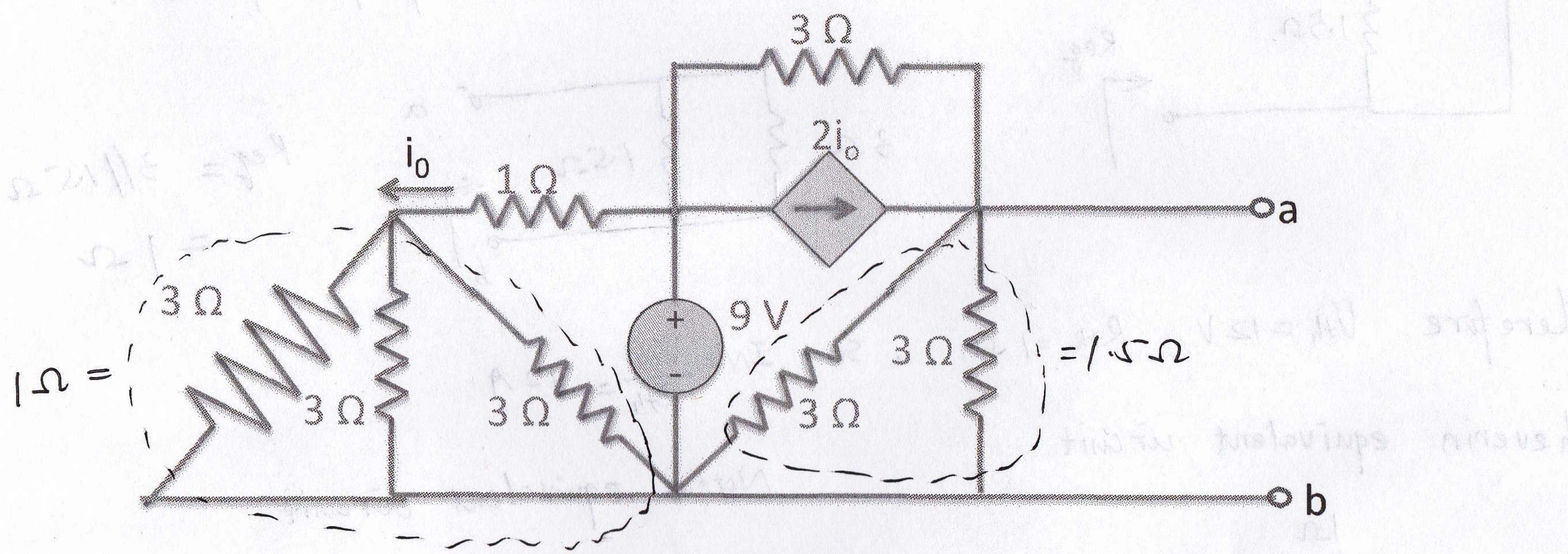
Therefore, the Thevenin equivalent circuit is



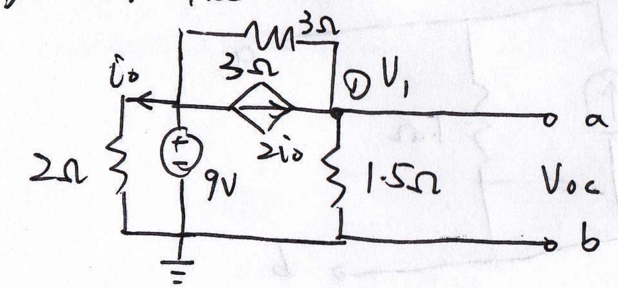
The Norton equivalent circuit is



Problem 8: Obtain the Thevenin and Norton equivalent as seen from a-b terminals:



Solution: The above circuit can be simplied as the following equivalent circuit



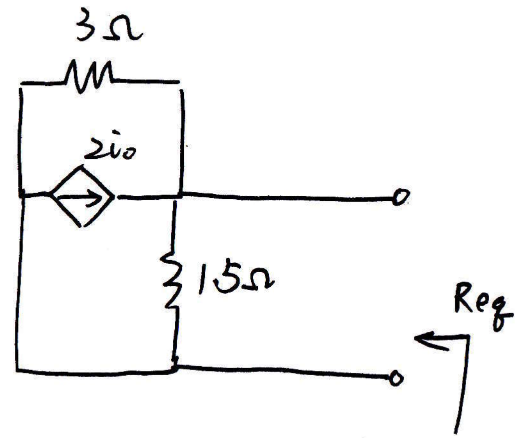
$i_o = \frac{9V}{2\Omega} = 4.5A$

KCL @ node 1 : $2i_o + \frac{9 - V_1}{3\Omega} - \frac{V_1}{1.5\Omega} = 0$

$\Rightarrow V_1 = 12V$

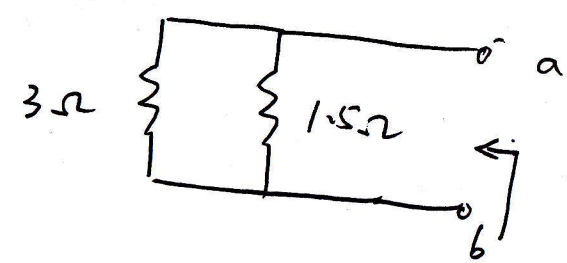
$\Rightarrow V_{oc} = 12V$

to obtain Thevenin equivalent resistance, we short circuit the independent voltage source, the equivalent circuit is as follows:



Since $i_o = 0$, therefore the CCCS can be viewed as open circuit

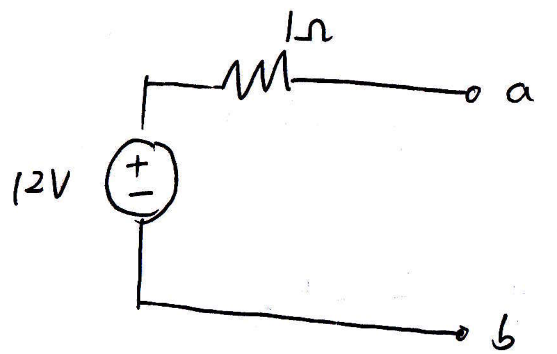
the circuit can be simplified further:



$R_{eq} = 3 // 1.5\ \Omega$
$= 1\ \Omega$

Therefore $V_{th} = 12V$, $R_{th} = 1\ \Omega$, so $I_N = \frac{V_{th}}{R_{th}} = 12A$

Thevenin equivalent circuit



Norton equivalent circuit