Problem 1:

Solution.

A): $v(t) = 8\cos(wt + \frac{\pi}{4})$ $\rightarrow$ $\underline{V} = 8e^{j\frac{\pi}{4}}$

$$\underline{V} = 8\cdot\cos\frac{\pi}{4} + j\,8\sin\frac{\pi}{4} = 4\sqrt{2} + j\,4\sqrt{2}$$

B): $i(t) = 4\sin(wt + \frac{\pi}{4})$

$$= 4\sin(wt + \frac{\pi}{2} - \frac{\pi}{4}) = 4\cos(wt - \frac{\pi}{4})$$

$$\underline{I} = 4e^{-j\frac{\pi}{4}}$$

$$\underline{I} = 4\cos(-\frac{\pi}{4}) - j\,4\sin(\frac{\pi}{4}) = 2\sqrt{2} - j\,2\sqrt{2}$$

c): $\underline{V} = 8 + j6$ $\quad |\underline{V}| = \sqrt{8^2+6^2} = 10$ $\quad \phi = \operatorname{atan}(\frac{6}{8}) \approx 37°$

$$= 0.646 \text{ rad}$$

$\underline{V} = 10e^{j0.646}$ $\quad v(t) = Re[\underline{V}e^{jwt}] = Re[10e^{j0.646}e^{jwt}]$

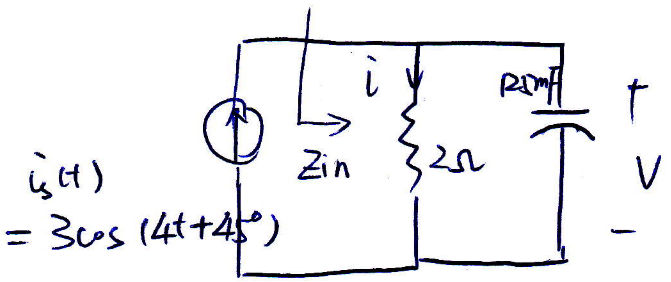
$$= 10\cos(wt + 0.646)$$

$$= 10\cos(wt + 37°)$$

D): $\underline{I} = 2 + 2j$ $\quad |\underline{I}| = \sqrt{2^2+2^2} = 2\sqrt{2}$. $\quad \phi = \operatorname{atan}(\frac{2}{2}) = 45° = \frac{\pi}{4}$

$\underline{I} = 2\sqrt{2}e^{j\frac{\pi}{4}}$ $\quad i(t) = Re[\underline{I}e^{jwt}] = 2\sqrt{2}\cos(wt + \frac{\pi}{4})$

Problem 2:



$\omega = 4$ rad/s.

$\underline{Z_C} = \dfrac{1}{j\omega C} = \dfrac{1}{j \times 4 \times 125 \times 10^{-3}} = -2j\ \Omega$

$\underline{Z_{in}} = R \,//\, \underline{Z_C} = 2 // -2j = 1-j = \sqrt{2}\angle -45^\circ\ \Omega$

$\underline{I_s} = 3e^{i\frac{\pi}{4}} = 3\angle 45^\circ\ A$

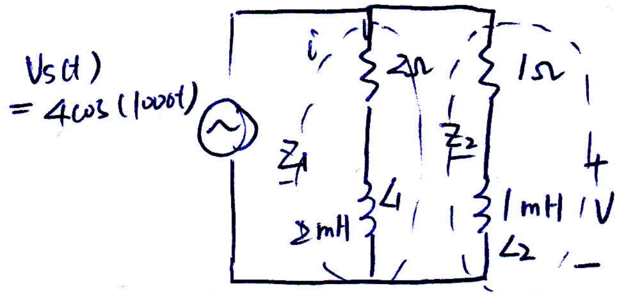
$\underline{V} = \underline{I_s} \cdot \underline{Z_{in}} = 3\angle 45^\circ \times \sqrt{2}\angle -45^\circ = 3\sqrt{2}\ V$

$\underline{I} = \dfrac{\underline{V}}{R} = \dfrac{3\sqrt{2}}{2}\ A$

$V(t) = Re[\underline{V} e^{j\omega t}] = 3\sqrt{2}\cos 4t\ \ V$

$i(t) = Re[\underline{I} e^{j\omega t}] = \dfrac{3\sqrt{2}}{2}\cos 4t\ \ A$

Problem 3:



$$Z_1 = 2\Omega + j\omega L_1 = 2\Omega + j \cdot \times 1000 \times 2 \times 10^{-3} = 2+2j\ \Omega$$

$$Z_2 = 1\Omega + j\omega L_2 = 1 + j \times 1000 \times 1 \times 10^{-3} = 1+j\ \Omega$$

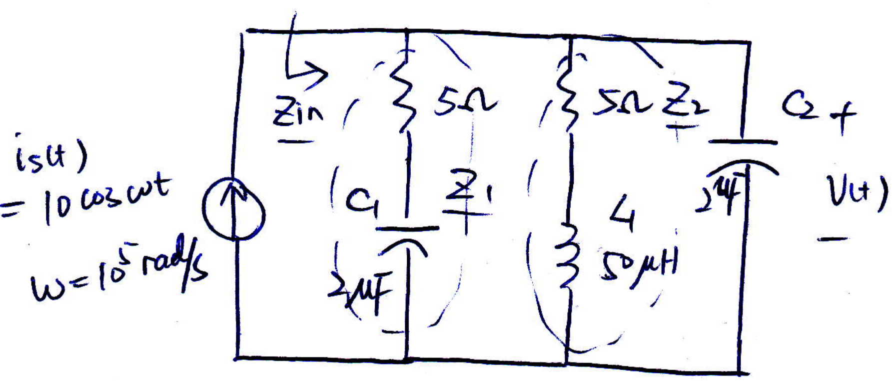
$$\underline{I} = \frac{V_s}{Z_1} = \frac{4}{2+2j} = 1-j\ A = \sqrt{2}\, e^{-j\frac{\pi}{4}}\ A$$

$$\underline{V} = \frac{j\omega L_2}{1+j\omega L_2} \cdot \underline{V_s} = \frac{j}{1+j} \times 4 = 2+2j\ V = 2\sqrt{2}\, e^{j\frac{\pi}{4}}\ V$$

$$i(t) = Re[\underline{I}\, e^{j\omega t}] = \sqrt{2}\cos(\omega t - \tfrac{\pi}{4})\ A$$

$$v(t) = Re[\underline{V}\, e^{j\omega t}] = 2\sqrt{2} \cdot \cos(\omega t + \tfrac{\pi}{4})\ V.$$

Problem 4 :



$$\underline{Z_{C1}} = \frac{1}{j\omega C_1} = \frac{1}{j\times 10^5 \times 2\times 10^{-6}} = -5j \ \Omega$$

$$\underline{Z_{C2}} = \frac{1}{j\omega C_2} = -5j \ \Omega$$

$$\underline{Z_L} = j\omega L = j\times 50\times 10^{-6}\times 10^5 = 5j \ \Omega$$

$$\underline{Z_{in}} = \underline{Z_1} \,//\, \underline{Z_2} \,//\, \underline{Z_{C2}}$$

$$= (5-5j)\,//\,(5+5j)\,//\,(-5j)$$

$$= \frac{(5-5j)(5+5j)}{5-5j+5+5j} \,//\, -5j$$

$$= 5 \,//\, -5j$$

$$= \frac{5}{2}(1-j) \ \Omega$$

$$\underline{V} = \underline{I_s}\cdot\underline{Z_{in}} = 10\cdot\frac{5}{2}(1-j) = 25(1-j) = 25\cdot\sqrt{2}\, e^{-j\frac{\pi}{4}}$$

$$v(t) = Re[\underline{V} e^{j\omega t}] = 25\sqrt{2}\cos(\omega t - \frac{\pi}{4})$$

Problem 5

using KVL: $V_s = V_1 + V_2$

$V_2 = iR$

$i = C\dfrac{dV_1}{dt}$

therefore $V_s = RC \cdot \dfrac{dV_1}{dt} + V_1 \Rightarrow \dfrac{dV_1}{dt} + \dfrac{V_1}{RC} = \dfrac{V_0 \cos wt}{RC}$ (*)

The solution should obey the form of $A\cos wt + B\sin wt$

Assume $V_1 = A\cos wt + B\sin wt$

Then $\dfrac{dV_1}{dt} = -Aw\cos wt + Bw\sin wt$, substitute into (*)

$$-Aw\sin wt + Bw\cos wt + \frac{A}{RC}\cos wt + \frac{B}{RC}\sin wt = \frac{V_0}{RC}\cos wt$$

$$\left(Bw + \frac{A}{RC}\right)\cos wt + \left(\frac{B}{RC} - Aw\right)\sin wt = \frac{V_0}{RC}\cos wt$$

Compare the coefficients.

$$\begin{cases} Bw + \dfrac{A}{RC} = \dfrac{V_0}{RC} \\ \dfrac{B}{RC} - Aw = 0 \end{cases} \Rightarrow B = AwRC$$

$$AwRC \cdot w + \frac{A}{RC} = \frac{V_0}{RC} \Rightarrow A = \frac{V_0}{RC\left(w^2RC + \frac{1}{RC}\right)} = \frac{V_0}{w^2R^2C^2+1}$$

$$B = AwRC = \frac{wRC}{w^2R^2C^2+1} \cdot V_0$$

$$V_1 = A\cos wt + B\sin wt$$

$$= \frac{V_0}{1+w^2R^2C^2}\cos wt + \frac{wRC}{w^2R^2C^2+1}V_0\sin wt$$

$$= \sqrt{\left(\frac{V_0}{1+w^2R^2C^2}\right)^2 + \left(\frac{V_0 wRC}{1+w^2R^2C^2}\right)^2}\cos(wt+\varphi)$$

where $\varphi = \operatorname{atan}\left(\dfrac{-\dfrac{wRCV_0}{w^2R^2C^2+1}}{\dfrac{V_0}{w^2R^2C^2+1}}\right) = \operatorname{atan}(-wRC)$

$$\therefore V_1 = \frac{V_0}{\sqrt{1+w^2R^2C^2}}\cos(wt+\varphi), \quad \varphi = \operatorname{atan}(-wRC)$$

$$V_2 = V_S - V_1$$

$$= V_0 \cos \omega t - \frac{V_0}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\omega t + \varphi)$$

where $\varphi = \mathrm{atan}(-\omega RC)$