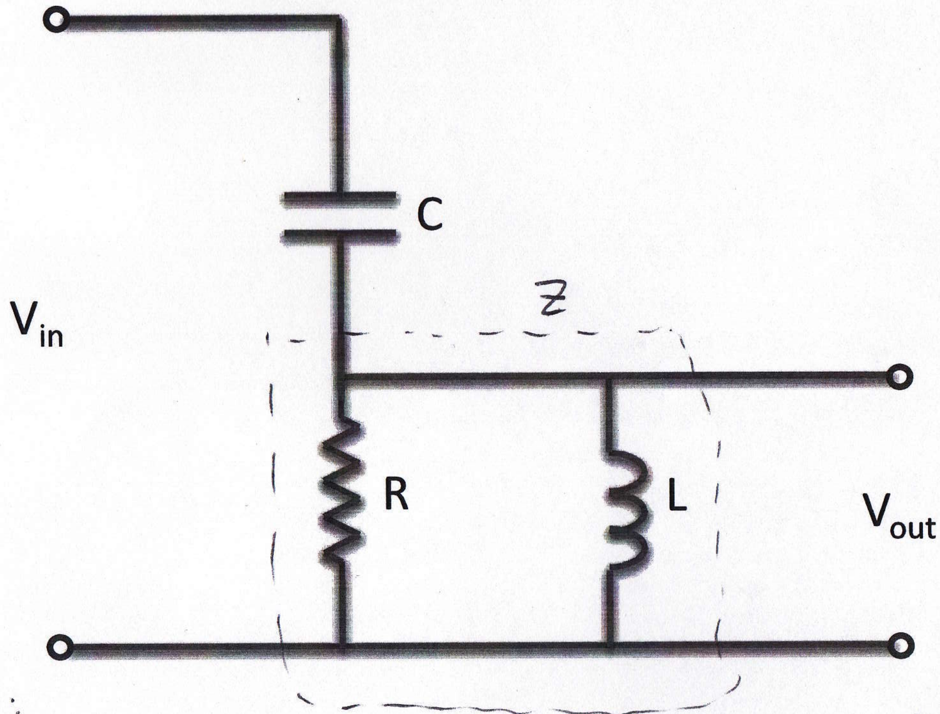
Problem 1: Find the transfer function of the circuit.



Solution:

$$\underline{H}(\omega) = \frac{\underline{V_{out}}}{\underline{V_{in}}} = \frac{\underline{Z}}{\underline{Z} + \frac{1}{j\omega C}}$$

$$\underline{Z} = R \parallel j\omega L = \frac{R \cdot j\omega L}{R + j\omega L}$$

$$\therefore \underline{H}(\omega) = \frac{\frac{R \cdot j\omega L}{R + j\omega L}}{\frac{R \cdot j\omega L}{R + j\omega L} + \frac{1}{j\omega C}}$$

$$H(\omega) = \frac{R \cdot j\omega L}{R \cdot j\omega L + \frac{1}{j\omega C}(R + j\omega L)}$$

$$= \frac{R \cdot j\omega L \cdot j\omega C}{R \cdot j\omega L \cdot j\omega C + R + j\omega L}$$

$$= \frac{-\omega^2 RLC}{-\omega^2 RLC + R + j\omega L}$$

Problem 2: Given $H(\omega) = \frac{1}{1+j\omega\tau_1} \cdot \frac{j\omega\tau_2}{1+j\omega\tau_2}$, where $\tau_2 > \tau_1$, please draw the Bode plot (only magnitude) of the transfer function.

Extra credit: Try to design a circuit having such a transfer function.

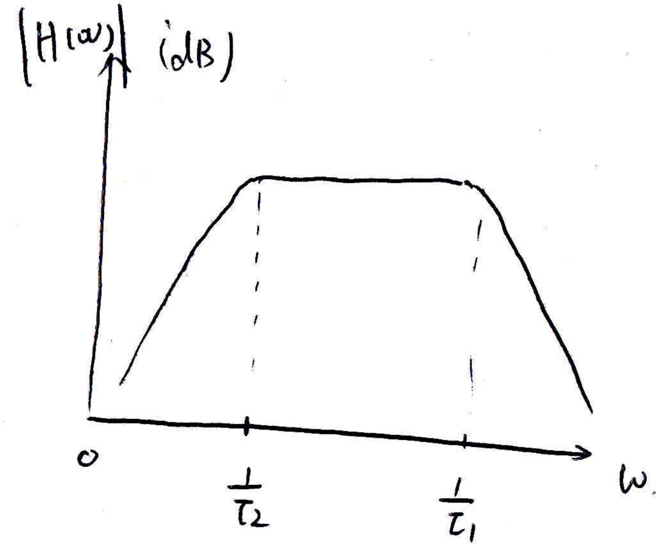
Solution:

$$H(w) = \frac{1}{1+jw\tau_1} \cdot \frac{jw\tau_2}{1+jw\tau_2}$$

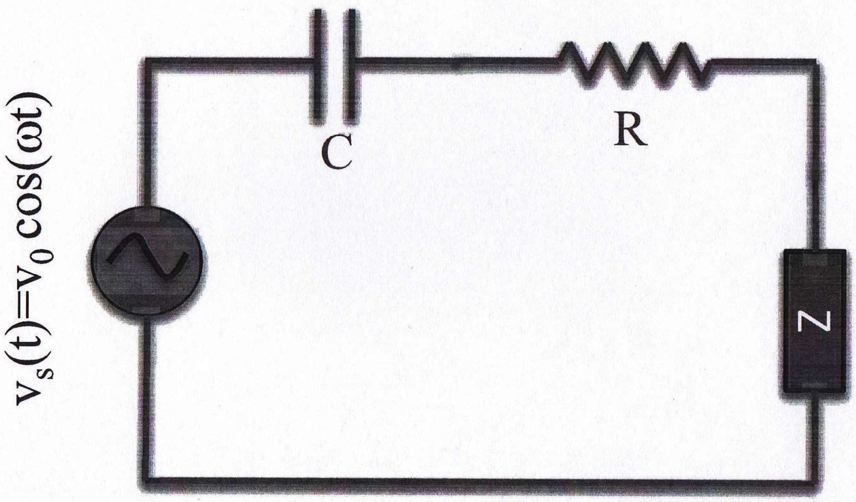
poles at $\begin{cases} w_{p1}\tau_1 = 1 \Rightarrow w_{p1} = \frac{1}{\tau_1} \\ w_{p2}\tau_2 = 1 \Rightarrow w_{p2} = \frac{1}{\tau_2} \end{cases}$

zero at $w_z \cdot \tau_2 = 0 \Rightarrow w_z = 0.$

The zero is located at the origin. and $|H(0)| = 0$, if the unit is dB, then $|H(0)|$ is negative infinity.



Problem 3: Given R = 5 Ω, C = 2 mF, ω = 100 rad/s. Find the impedance of *Z* so as to maximize the power transfer.



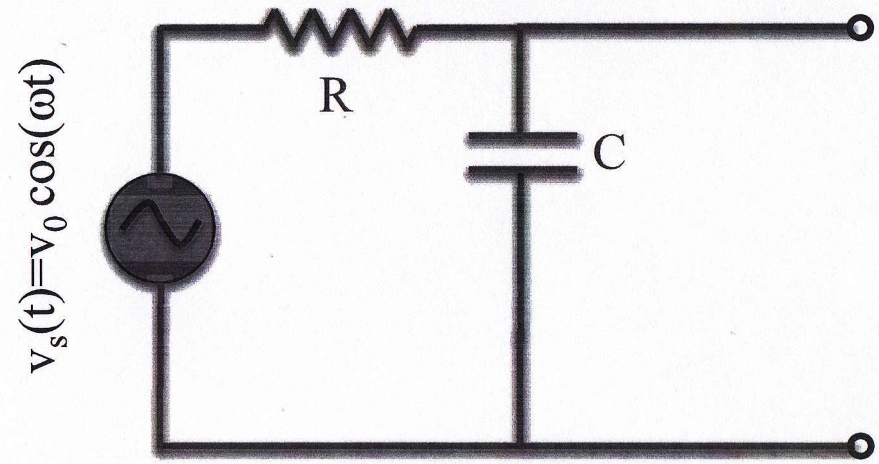
The Thevenin equivalent impedance is

$$Z_{th} = R + \frac{1}{j\omega C} = 5 + \frac{1}{j \times 2 \times 10^{-3} \times 100} = 5 - 5j$$

when the load matches the conjugate of Zth, the load get the maximum power.

$$Z = Z_{th}^{*} = 5 + 5j$$

Problem 4: Find V_{th} and R_{th}, and draw the Thevenin equivalent circuit for the following circuit.



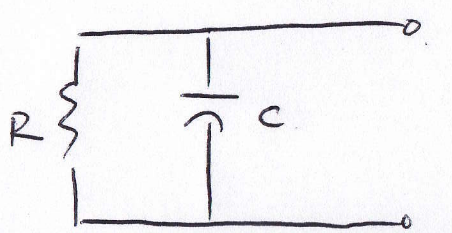
$$V_{th} = V_{oc} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot V_s$$

$$= \frac{1}{1+j\omega RC} \cdot V_s$$

$$= \frac{V_0}{1+j\omega RC}$$

Solution:

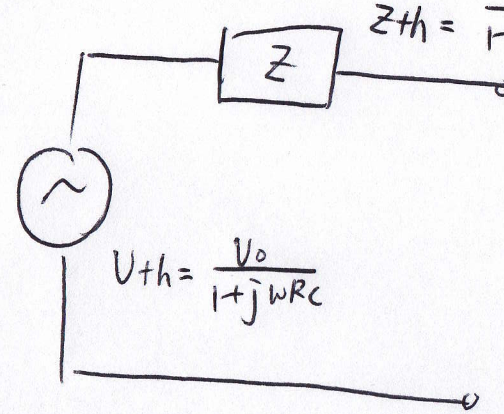
Z_{th}:



$$Z_{th} = R // \frac{1}{j\omega C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{R}{1+j\omega RC}$$

Thevenin equivalent circuit



$$Z_{th} = \frac{R}{1+j\omega RC}$$

$$V_{th} = \frac{V_0}{1+j\omega RC}$$

Problem 5: Given the transfer function *H(ω)* of a system,
if the input is $V_{in}(t) = \mathrm{Re}(\sum_n a_n e^{j\omega_n t})$

then the output can be expressed as $V_{out}(t) = \mathrm{Re}\left[\sum_n a_n H(\omega_n) e^{j\omega_n t}\right]$

where $H(\omega) = |H(\omega)| e^{j\phi(\omega)}$

Now, for a system, the Bode plot of its transfer function is illustrated in the next page, and $\omega_0 \tau = 1$
If the input voltage is $V_{in}(t) = \mathrm{Re}\left(1V \cdot e^{j\omega_1 t} + 1V \cdot e^{j\omega_2 t}\right)$
Find the output voltage for the following cases:

a) $\omega_1 = 0.1\tau^{-1}$ $\omega_2 = 10\tau^{-1}$
b) $\omega_1 = 0.01\tau^{-1}$ $\omega_2 = 100\tau^{-1}$

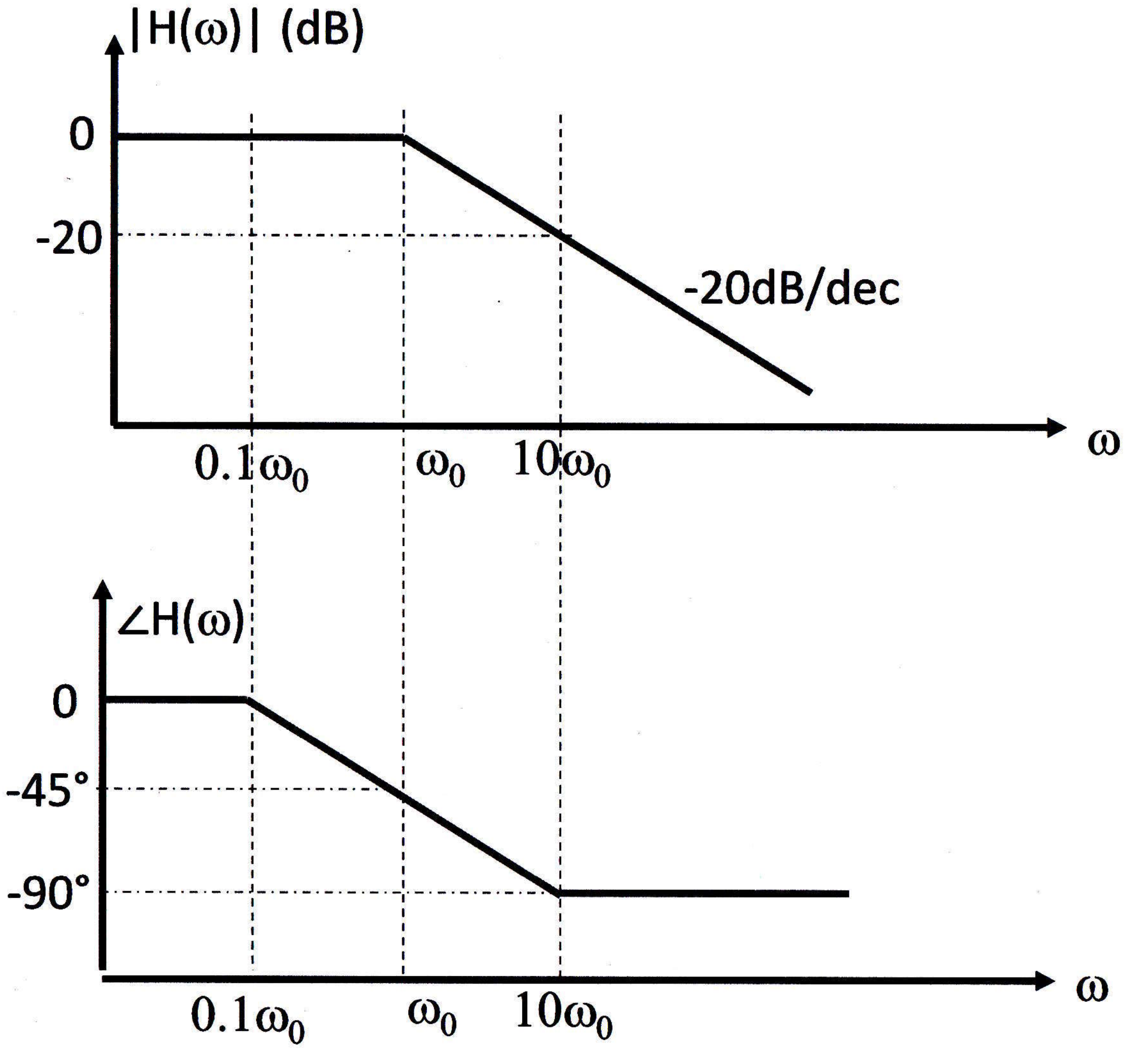
Solution $\omega_0 \tau = 1. \Rightarrow \omega_0 = \frac{1}{\tau}$

a) $\omega_1 = 0.1\tau^{-1} = 0.1\omega_0$
$\omega_2 = 10\tau^{-1} = 10\omega_0$
from Bode plot, $|H(\omega_1)| = 0\,dB$.
$|H(\omega_2)| = -20\,dB$

$\therefore |H(\omega_1)| = 1$
$|H(\omega_2)| = \frac{1}{10}$

$\angle H(\omega_1) = 0.$
$\angle H(\omega_2) = -90° = -\frac{\pi}{2}$

$V_{out}(t) = \mathrm{Re}[\,1V \cdot H(\omega_1) \cdot e^{j\omega_1 t} + 1V \cdot H(\omega_2) \cdot e^{j\omega_2 t}\,]$
$= \mathrm{Re}[\,1 \cdot e^{j\omega_1 t} + \frac{1}{10} e^{-j\frac{\pi}{2}} \cdot e^{j\omega_2 t}\,]$
$= \cos\omega_1 t + \frac{1}{10}\cos(\omega_2 t - \frac{\pi}{2})$



b). $\omega_1 = 0.01\,\tau^{-1} = 0.01\,\omega_0$

$\omega_2 = 100\,\tau^{-1} = 100\,\omega_0$

from Bode plot

$|H(\omega_1)| = 1 \qquad \angle H(\omega_1) = 0$

$|H(\omega_2)| = -40dB = \frac{1}{100}, \quad \angle H(\omega_2) = -90^\circ$

$$V_{out}(t) = Re[1\cdot H(\omega_1)e^{j\omega_1 t} + 1\cdot H(\omega_2)\cdot e^{j\omega_2 t}]$$

$$= Re[e^{j\omega_1 t} + \frac{1}{100}e^{-j\frac{\pi}{2}}e^{j\omega_2 t}]$$

$$= \cos\omega_1 t + \frac{1}{100}\cos(\omega_2 t - \frac{\pi}{2})$$