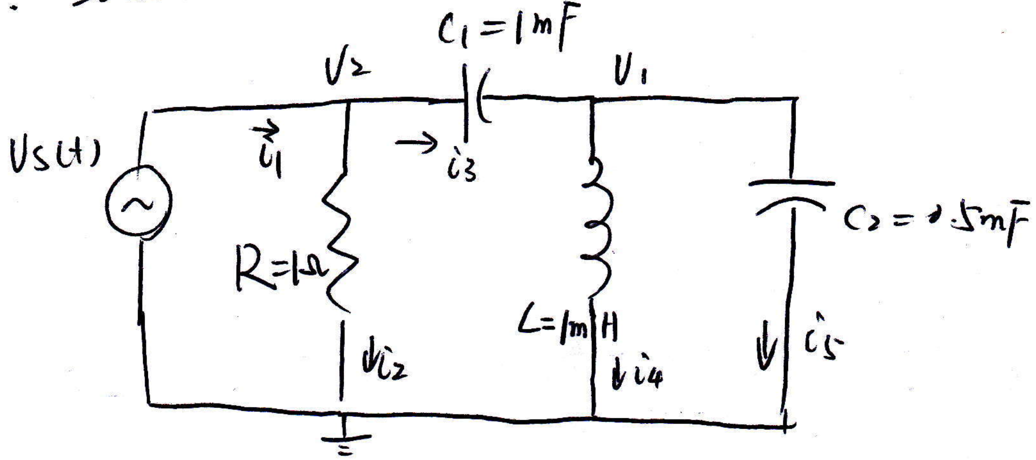
1. Solution:




use nodal analysis to find the node voltages and currents in phasor

$\underline{V_s} = 1\ V \qquad \omega = 1000\ rad/s$

$\frac{1}{j\omega C_1} = -j\ \Omega \quad , \quad \frac{1}{j\omega C_2} = -2j\ \Omega \quad , \quad j\omega L = j\ \Omega$

$$\begin{cases} \underline{V_2} = 1 \\ \dfrac{\underline{V_2} - \underline{V_1}}{\frac{1}{j\omega C_1}} = \dfrac{\underline{V_1}}{j\omega L} + \dfrac{\underline{V_1}}{\frac{1}{j\omega C_2}} \end{cases} \Rightarrow \frac{1 - \underline{V_1}}{-j} = \frac{\underline{V_1}}{j} + \frac{\underline{V_1}}{-2j}$$

$$\underline{V_1} - 1 = \underline{V_1} - \frac{\underline{V_1}}{2} \Rightarrow \underline{V_1} = 2$$

$$\underline{i_2} = \frac{\underline{V_2}}{R} = \frac{1}{1} = 1\ A$$

$$\underline{i_3} = \frac{\underline{V_2} - \underline{V_1}}{-j} = \frac{1-2}{-j} = -j\ A$$

$$\underline{i_4} = \frac{\underline{V_1}}{j\omega L} = \frac{2}{j} = -2j\ A$$

$$\underline{i_5} = \frac{\underline{V_1}}{\frac{1}{j\omega C_2}} = \frac{2}{-2j} = j\ A$$

$$\underline{i_1} = \underline{i_2} + \underline{i_3} = 1 - j\ A$$

in time domain

$$V_2(t) = Re[\underline{V_2}\, e^{j\omega t}]$$

$$= \cos(\omega t) = \cos(1000t) \ V.$$

$$V_1(t) = Re[\underline{V_1}\, e^{j\omega t}] = 2\cos(1000t) \ V.$$

$$i_1(t) = Re[\underline{i_1}\, e^{j\omega t}] = Re[\sqrt{2}\, e^{-j\frac{\pi}{4}} e^{j\omega t}] = \sqrt{2}\cos(\omega t - \tfrac{\pi}{4}) \ A$$

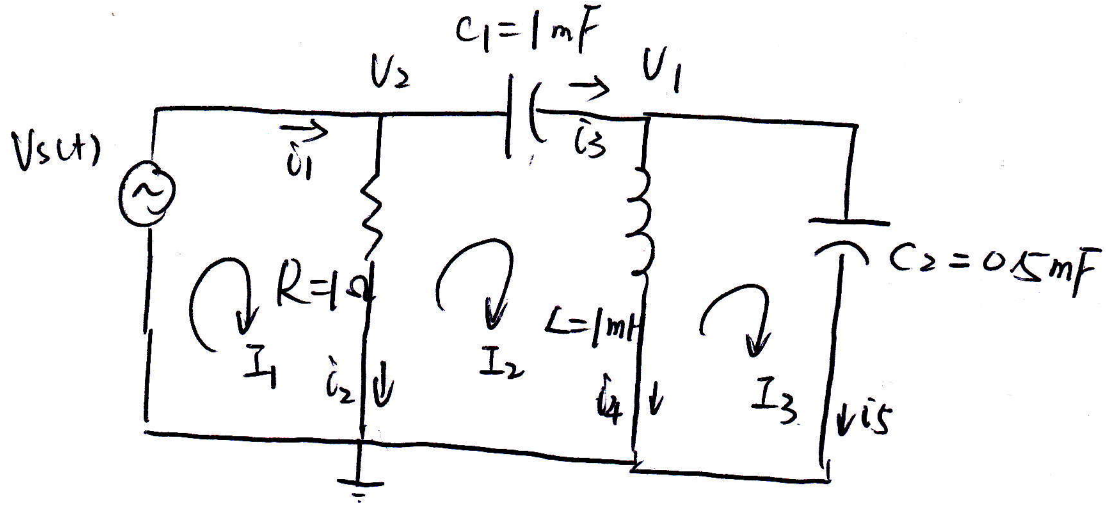
$$i_2(t) = Re[\underline{i_2}\, e^{j\omega t}] = \cos(1000t) \ A$$

$$i_3(t) = Re[\underline{i_3}\, e^{j\omega t}] = Re[e^{-j\frac{\pi}{2}} e^{j\omega t}] = \cos(\omega t - \tfrac{\pi}{2}) \ A$$

$$i_4(t) = Re[\underline{i_4}\, e^{j\omega t}] = Re[2e^{-j\frac{\pi}{2}} e^{j\omega t}] = 2\cos(\omega t - \tfrac{\pi}{2}) \ A$$

$$i_5(t) = Re[\underline{i_5}\, e^{j\omega t}] = Re[e^{j\frac{\pi}{2}} e^{j\omega t}] = \cos(\omega t + \tfrac{\pi}{2}) \ A$$

2. Solution:




Use mesh analysis to find the nodel voltages and currents in phasor, and then change them into time domain.

Loop I:

$$1 = 1\Omega(\underline{I}_1 - \underline{I}_2) \qquad (1)$$

Loop II:

$$1\Omega(\underline{I}_2 - \underline{I}_1) + \underline{I}_2(-j) + (\underline{I}_2 - \underline{I}_3) \cdot j = 0 \quad (2)$$

Loop III:

$$(\underline{I}_3 - \underline{I}_2) \cdot j + \underline{I}_3(-2j) = 0 \qquad (3).$$

Substitute (1) into (2)

$$-j\underline{I}_2 + j \cdot \underline{I}_2 - \underline{I}_3 \cdot j - 1 = 0$$

$$\underline{I}_3 = \frac{-1}{j} = j \quad A \qquad (4)$$

substitute (4) into (3)

$$\underline{I}_3 \cdot j - 2j\underline{I}_3 - j\underline{I}_2 = 0$$

$$-j\underline{I}_2 = j\underline{I}_3$$

$\underline{I_2} = -\underline{I_3} = -j \quad A \qquad (5)$

Substitute (5) into (1)

$\underline{I_1} = 1 + \underline{I_2} = 1 - j \quad A$

$\underline{i_1} = \underline{I_1} = 1 - j \quad A$

$\underline{i_2} = I_1 - I_2 = 1 \ A$

$\underline{i_3} = I_2 = -j \ A$

$\underline{i_4} = I_2 - I_3 = -2j \ A$

$\underline{i_5} = \underline{I_3} = j$

$\underline{V_2} = \underline{V_S} = 1 V$

$\underline{V_1} = \underline{i_4} \cdot j\omega L = -2j \cdot j = 2 V$

in time domain:

$v_1(t) = 2\cos 1000t \ V$

$v_2(t) = \cos 1000t \ V$

$i_1(t) = \sqrt{2}\cos(1000t - \frac{\pi}{4}) \ A$

$i_2(t) = \cos(1000t) \ A$

$i_3(t) = \cos(1000t - \frac{\pi}{2}) \ A$

$i_4(t) = 2\cos(1000t - \frac{\pi}{2}) \ A$

$i_5(t) = \cos(1000t + \frac{\pi}{2}) \ A$

3. Solution :

A). $|H(f_1)| = 0\ dB \Rightarrow |H(f_1)| = 1$

$V_{out}(t) = 4\cos(2\pi f_1 t + \varphi_1)$ . $\varphi_1$ is unknown.

B) $|H(f_2)| = 0\,dB \Rightarrow |H(f_2)| = 1$

$V_{out}(t) = 3\cos(2\pi f_2 t + \varphi_2)$ , $\varphi_2$ is unknown.

c) $|H(f_3)| = -10\ dB \Rightarrow |H(f_3)| = 10^{-\frac{10}{20}} = 10^{-0.5} = \frac{1}{\sqrt{10}}$

$V_{out}(t) = \frac{5}{\sqrt{10}}\cos(2\pi f_3 t + \varphi_3)$ , $\varphi_3$ is unknown.

D) $|H(f_4)| = -20\,dB \Rightarrow |H(f_4)| = 10^{-1} = 0.1$

$V_{out}(t) = \frac{1}{10}\cdot 10\cdot \cos(2\pi f_4 t + \varphi_4)$

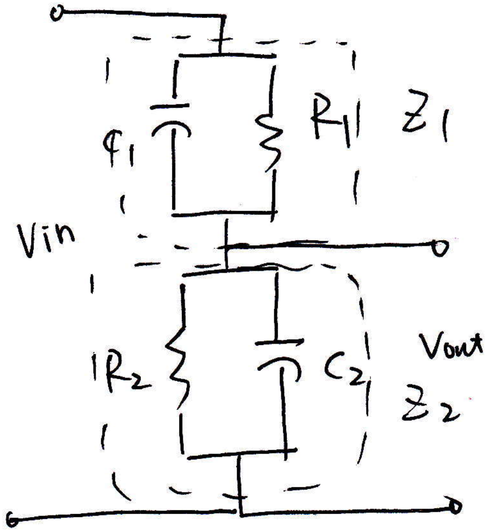
$= \cos(2\pi f_4 t + \varphi_4)$ , $\varphi_4$ is unknown

E) $|H(f_5)| = -40\ dB \Rightarrow |H(f_5)| = 10^{-2} = 0.01$

$V_{out}(t) = 0.01 \times 10\cos(2\pi f_5 t + \varphi_5)$

$= 0.1\cos(2\pi f_5 t + \varphi_5)$ , $\varphi_5$ is unknown.

4. solution.




$$Z_1 = R_1 // \frac{1}{j\omega C_1} = \frac{R_1 \cdot \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$Z_2 = R_2 // \frac{1}{j\omega C_2} = \frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$H(\omega) = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{\frac{R_2}{1 + j\omega R_2 C_2}}{\frac{R_2}{1 + j\omega R_2 C_2} + \frac{R_1}{1 + j\omega R_1 C_1}}$$

$$H(\omega) = \frac{R_2}{R_2 + \frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1} \cdot R_1}$$

$$= \frac{1}{1 + \frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1} \cdot \frac{R_1}{R_2}}$$

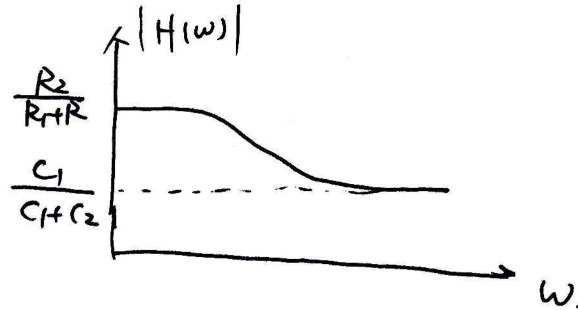
Two extreme cases:

$$|H(0)| = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

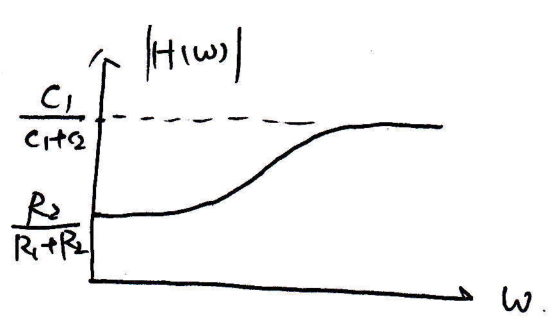
$$|H(\infty)| = \frac{1}{1 + \frac{\cancel{\infty}}{\cancel{\infty}} \cdot \frac{R_2 C_2}{R_1 C_1} \cdot \frac{R_1}{R_2}}$$

$$= \frac{1}{1 + \frac{C_2}{C_1}} = \frac{C_1}{C_1 + C_2}$$

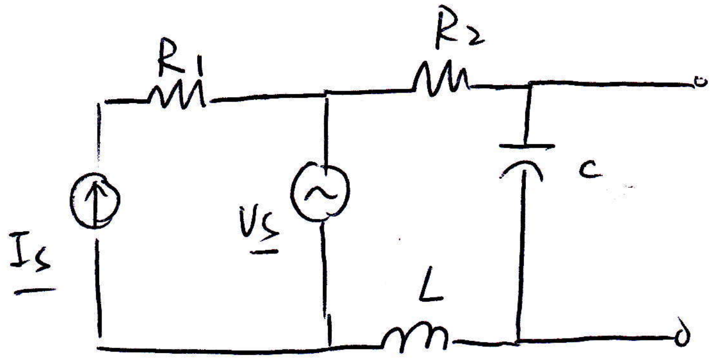
if $\frac{R_2}{R_1 + R_2} > \frac{C_1}{C_1 + C_2}$




if $\frac{R_2}{R_1 + R_2} < \frac{C_1}{C_1 + C_2}$

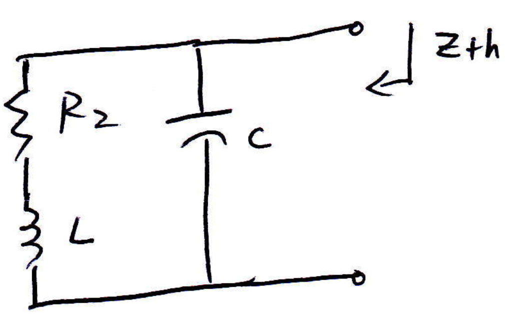


5. Solution:




$w = 1000$ rad/s

To find $Z_{th}$, leave the current source as open circuit, leave the voltage source as short circuit.




$$\underline{Z_{th}} = (R_2 + jwL) \parallel \frac{1}{jwC}$$

$$= (2 + 2j) \parallel -2j$$

$$= \frac{(2+2j)\cdot(-2j)}{2 + 2j - 2j}$$

$$= 2 - 2j \ \Omega$$

$V_{th} = V_{oc}$, the current source won't contribute anything to the open circuit voltage, because it is fixed by $V_s$.

$$\underline{V_{oc}} = \frac{\frac{1}{jwC}}{R_2 + \frac{1}{jwC} + jwL} \cdot \underline{V_s} = \frac{-2j \times 10}{2 - 2j + 2j} = -j10$$

$\therefore \underline{V_{th}} = -10j$ V

when $Z = Z_{th}^*$, the load gets maximum transfer power

$\therefore Z = Z_{th}^* = 2 + 2j$

and, $P_{max} = \dfrac{|V_{th}|^2}{8R_{th}} = \dfrac{|10|^2}{8 \times 2} = 6.25$ W, where $R_{th} = Re[\underline{Z_{th}}]$