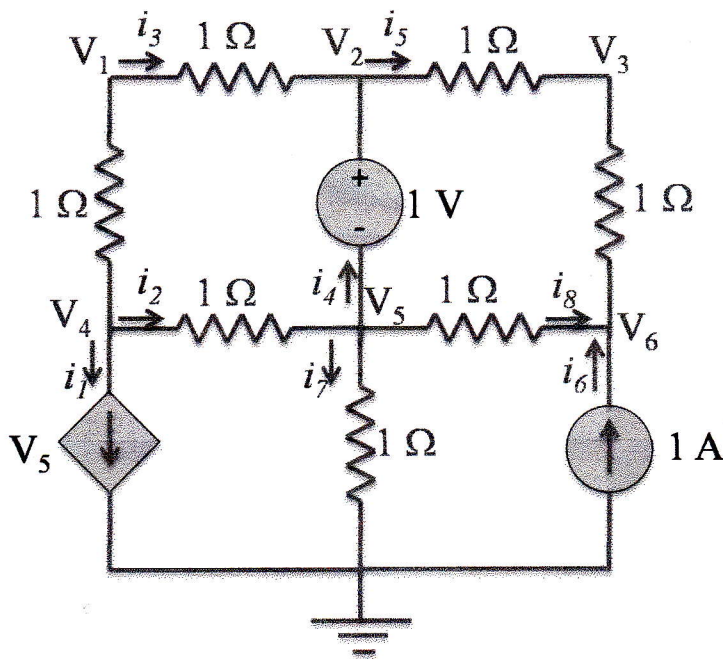


PROBLEM ONE: (48 points)

Use nodal analysis to find 6 equations for V_1 through V_6 . For your convenience, please fill in the blanks for the coefficients in the equations in the following page. You don't need to solve them. (Hint: you may want to use supernode to enclose the voltage source.) Please write all necessary steps to earn full credit. (42 + 6 = 48 points)



kCL @ supernode

$$\frac{V_1 - V_2}{1} + \frac{V_3 - V_2}{1} + \frac{V_4 - V_5}{1} + \frac{V_6 - V_5}{1} - \frac{V_5}{1} = 0$$

$$V_1 - 2V_2 + V_3 + V_4 - 3V_5 + V_6 = 0 \quad (1)$$

kCL @ node 1:

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{1} = 0$$

$$2V_1 - V_2 - V_4 = 0 \quad (2)$$

kCL @ node 3:

$$\frac{V_2 - V_3}{1} + \frac{V_6 - V_3}{1} = 0$$

$$V_2 - 2V_3 + V_6 = 0 \quad (3)$$

KCL @ node 4:

$$\begin{cases} \frac{V_1 - V_4}{1} + \frac{V_5 - V_4}{1} = \dot{v}_1 \\ \dot{v}_1 = V_5 \end{cases}$$

$$V_1 - 2V_4 = 0 \quad (4)$$

KCL @ node 6

$$\frac{V_3 - V_6}{1} + \frac{V_5 - V_6}{1} + 1 = 0$$

$$V_3 + V_5 - 2V_6 + 1 = 0$$

Voltage drop of the voltage source

$$V_2 - V_5 = 1V$$

Supernode	<u>1</u>	<u>V₁</u> +	<u>-2</u>	<u>V₂</u> +	<u>1</u>	<u>V₃</u> +	<u>1</u>	<u>V₄</u> +	<u>-3</u>	<u>V₅</u> +	<u>1</u>	<u>V₆</u> =	<u>0</u>
Node 1	<u>2</u>	<u>V₁</u> +	<u>-1</u>	<u>V₂</u> +	<u> </u>	<u>V₃</u> +	<u>-1</u>	<u>V₄</u> +	<u> </u>	<u>V₅</u> +	<u> </u>	<u>V₆</u> =	<u>0</u>
Node 3	<u> </u>	<u>V₁</u> +	<u>1</u>	<u>V₂</u> +	<u>-2</u>	<u>V₃</u> +	<u> </u>	<u>V₄</u> +	<u> </u>	<u>V₅</u> +	<u>1</u>	<u>V₆</u> =	<u>0</u>
Node 4	<u>1</u>	<u>V₁</u> +	<u> </u>	<u>V₂</u> +	<u> </u>	<u>V₃</u> +	<u>-2</u>	<u>V₄</u> +	<u> </u>	<u>V₅</u> +	<u> </u>	<u>V₆</u> =	<u>0</u>
Node 6	<u> </u>	<u>V₁</u> +	<u> </u>	<u>V₂</u> +	<u>1</u>	<u>V₃</u> +	<u> </u>	<u>V₄</u> +	<u>1</u>	<u>V₅</u> +	<u>-2</u>	<u>V₆</u> =	<u>-1</u>
Voltage source	<u> </u>	<u>V₁</u> +	<u>1</u>	<u>V₂</u> +	<u> </u>	<u>V₃</u> +	<u> </u>	<u>V₄</u> +	<u>-1</u>	<u>V₅</u> +	<u> </u>	<u>V₆</u> =	<u>1</u>

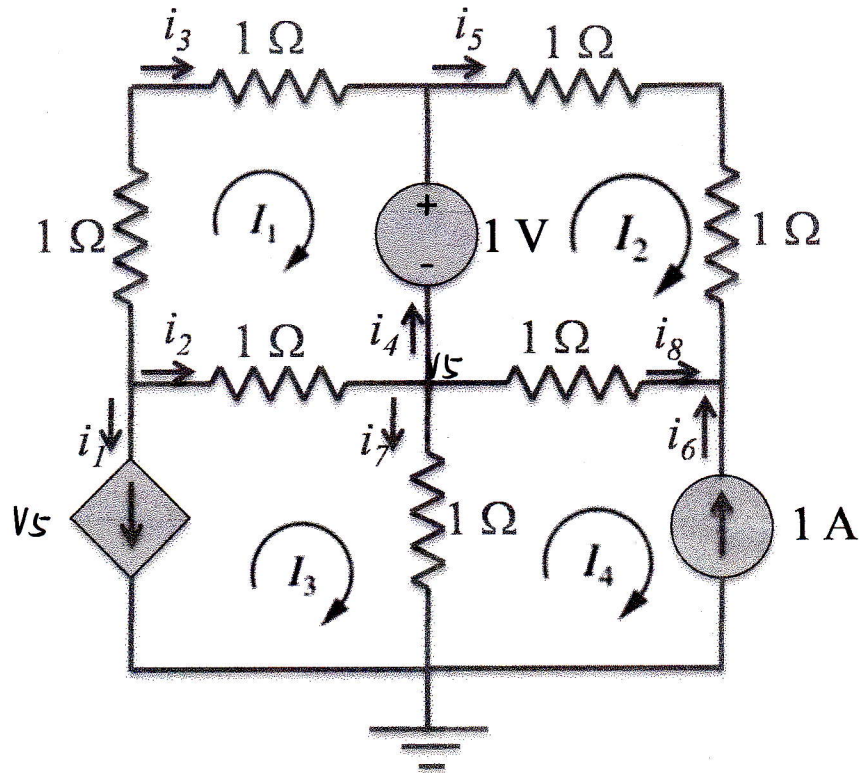
solutions are not unique. For example:

Node 4: $-V_1 + V_2 - V_4 = 0$

Node 6: $V_2 - V_3 + V_5 - V_6 = -1$

PROBLEM TWO: (24 points):

Use mesh analysis to find 4 equations for I_1 through I_4 . For your convenience, please fill in the blanks for the coefficients in the equations in the following page. You don't need to solve them. Please write all necessary steps to earn full credit. (20 + 4 = 24 points)



Solution:

KVL @ mesh I_1

$$1V + 1\Omega \times (I_1 - I_3) + I_1 \times 1\Omega + I_1 \times 1\Omega = 0$$

$$3I_1 - I_3 + 1 = 0 \quad (1)$$

KVL @ mesh I_2

$$-1V + 1\Omega \times I_2 + 1\Omega \times I_2 + 1\Omega \times (I_2 - I_4) = 0$$

$$3I_2 - I_4 = 1 \quad (2)$$

@ mesh I_3

$$\begin{cases} I_3 = -\hat{v}_1 = -V_5 \\ V_5 = 1\Omega \times (I_3 - I_4) \end{cases}$$

$$2I_3 - I_4 = 0 \quad (3)$$

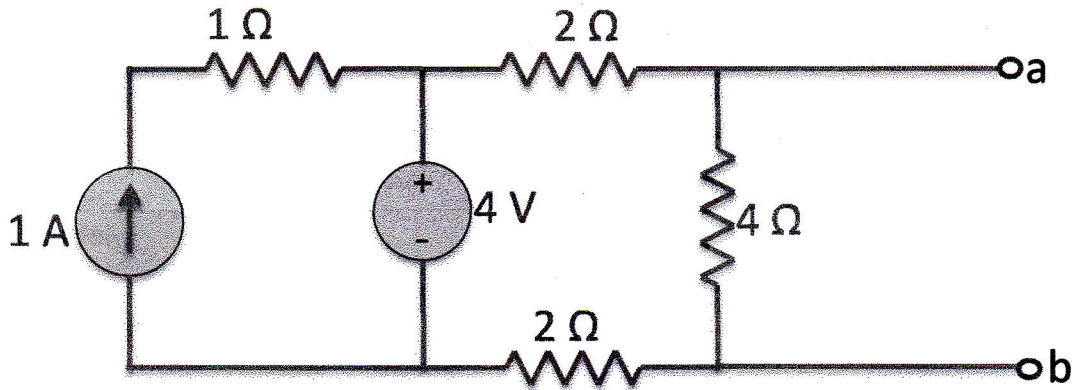
@ mesh I_4

$$I_4 = -1A$$

Mesh 1	<u>3</u>	$I_1 +$	<u> </u>	$I_2 +$	<u>-1</u>	$I_3 +$	<u> </u>	$I_4 =$	<u>-1</u>
Mesh 2	<u> </u>	$I_1 +$	<u>3</u>	$I_2 +$	<u> </u>	$I_3 +$	<u>-1</u>	$I_4 =$	<u>1</u>
Mesh 3	<u> </u>	$I_1 +$	<u> </u>	$I_2 +$	<u>2</u>	$I_3 +$	<u>-1</u>	$I_4 =$	<u>0</u>
Mesh 4	<u> </u>	$I_1 +$	<u> </u>	$I_2 +$	<u> </u>	$I_3 +$	<u>+1</u>	$I_4 =$	<u>-1</u>

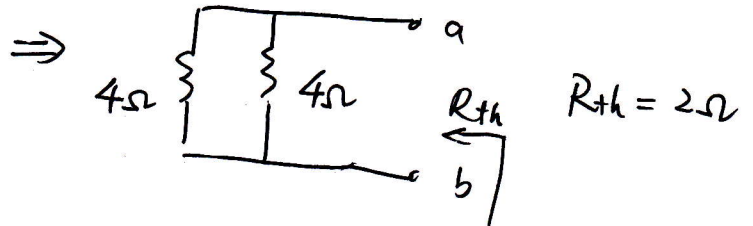
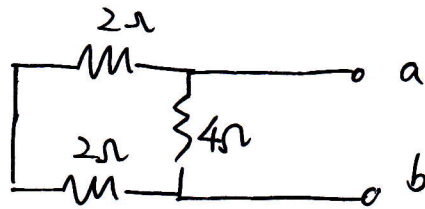
PROBLEM THREE: (28 points):

Find V_{th} , R_{th} , I_N and R_N for the following circuit, and draw the Thevenin and Norton equivalent circuits. Please write all necessary steps to earn full credit.



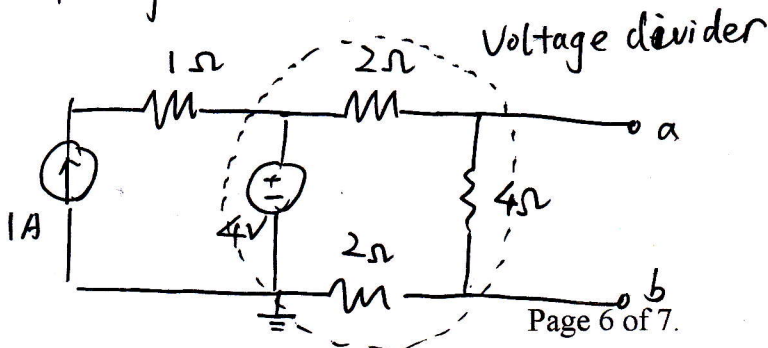
Solution:

To find R_{th} ,



$R_N = R_{th} = 2\Omega$

To find V_{th} ,



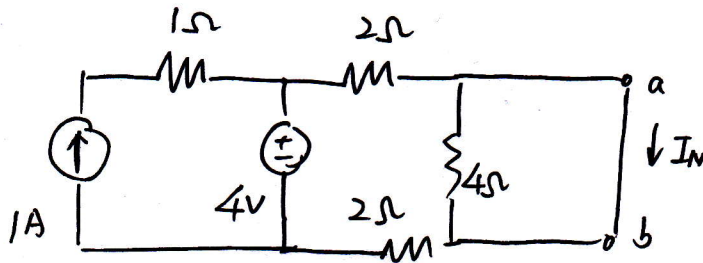
$$V_{ab} = 4 \times \frac{4\Omega}{2\Omega + 2\Omega + 4\Omega}$$

$$= 2V$$

$V_{th} = 2V$

$$I_N = \frac{V_{th}}{R_{th}} = \frac{2V}{2\Omega} = 1A$$

Another way to solve I_N

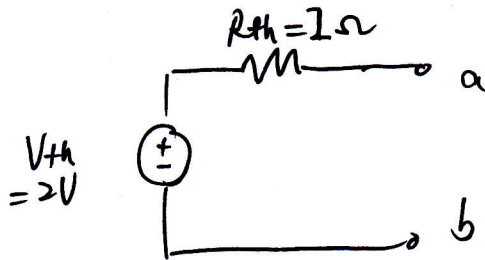


The 4Ω resistor is shorted

Thus,

$$I_N = \frac{4V}{2\Omega + 2\Omega} = 1A$$

Thevenin equivalent



Norton equivalent

