Announcements:

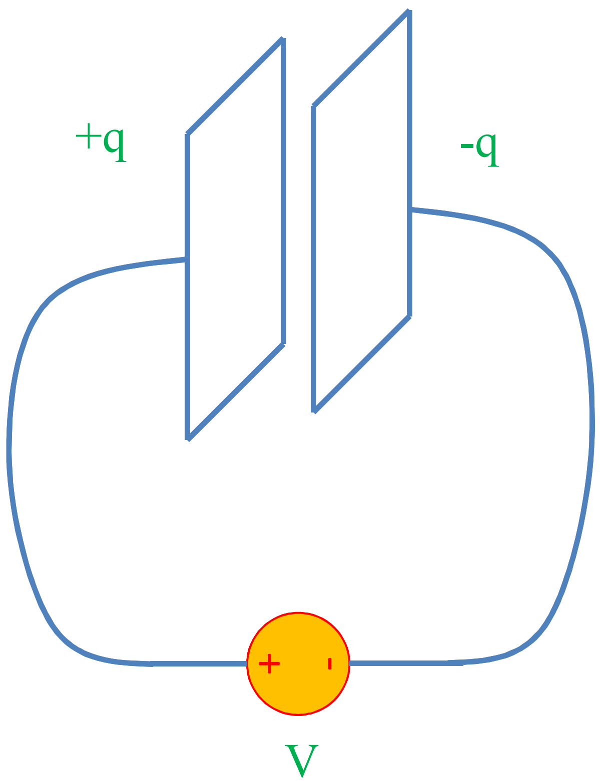
1. Etc etc etc

Note: Video did not get recorded on replay because of technical error.

EECS 70A: Network Analysis

Lecture 10

Capacitors



$$q = CV$$

$$C = \frac{\varepsilon A}{d}$$

A=area
d=plate separation

Farads[F] = Coulombs/Volt [C]/[V]

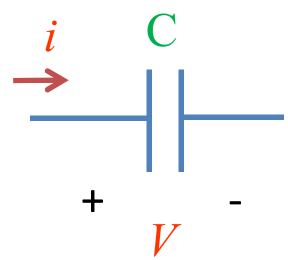
$$\varepsilon_0 = 8.85 \times 10^{-12} F/m$$

$$\varepsilon = \kappa \varepsilon_0$$

Dielectric constant:

$\kappa = 3.9$ SiO_2

$\kappa = 25$ HfO_2



Time dependence

$$q = CV \qquad i = \frac{dq}{dt} = C\frac{dV}{dt}$$

q, V, i can depend on time !

Implicit:

$$q(t) = CV(t) \qquad i(t) = \frac{dq(t)}{dt} = C\frac{dV(t)}{dt}$$

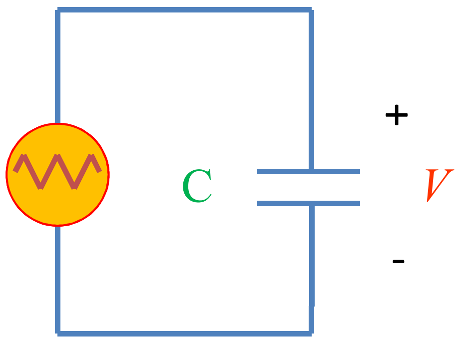
Will not always write (t), but it is assumed from now on.

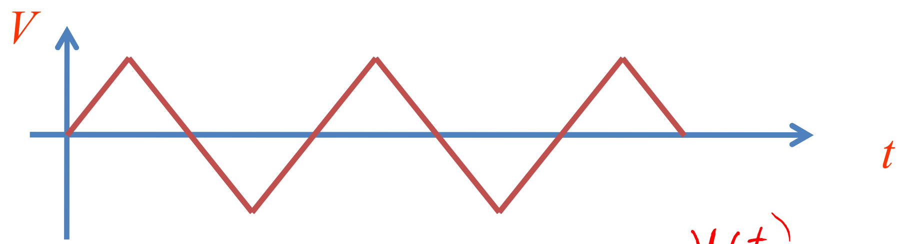
$$i(t) = C\frac{dV(t)}{dt} \Rightarrow V(t) = \frac{1}{C}\int i(t)dt$$

$$\Rightarrow q(t) = \int i(t)dt$$

Example Problem #2

(Students): Find i(t), q(t)



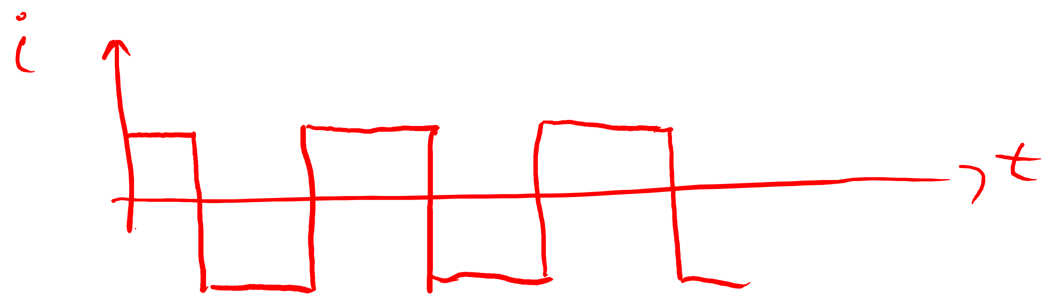


same waveform as V(t)

$q = CV$

$i = \frac{dq}{dt} = C\frac{dV}{dt}$

$V = \frac{1}{C}\int i\,dt$



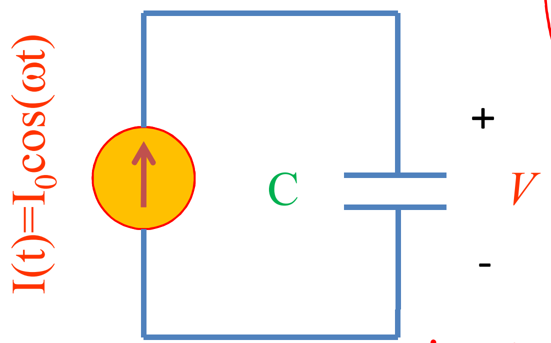
Example Capacitor Problem #2

$q = CV$

$i = C\frac{dV}{dt}$

$V =$

Find V(t), q(t)



$$V = \frac{1}{C}\int I(t)\,dt$$

$$= \frac{1}{C}\int I_0 \cos(\omega t) = \frac{I_0}{C}\int \cos(\omega t)\,dt$$

$u = \omega t \quad du = \omega\, dt$

$dt = \frac{1}{\omega}du$

$$= \frac{I_0}{C}\int \cos(u)\,du\,\frac{1}{\omega}$$

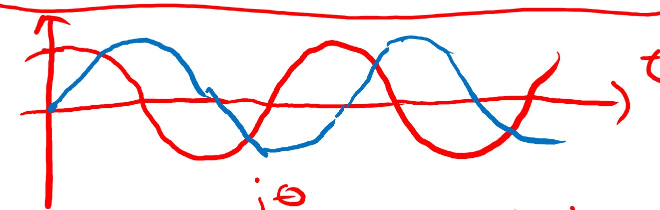
$$= \frac{I_0}{\omega C}\sin u = \boxed{\frac{I_0}{\omega C}\sin(\omega t) = V(t)}$$

blue V

red i

claim

$$I(t) = \mathrm{Re}\left[I_0 e^{j\omega t}\right]$$



why?

$z = x + jy \quad \mathrm{Re}[z] = x$

Euler $e^{j\theta} = \cos\theta + j\sin\theta$

"phasor"

$$\mathrm{Re}\left[I_0 e^{j\omega t}\right] = \mathrm{Re}\left[I_0\left(\cos(\omega t) + j\sin(\omega t)\right)\right]$$

$$= \mathrm{Re}\left[I_0\cos(\omega t) + jI_0\sin(\omega t)\right] = I_0\cos(\omega t)$$

$$V(t) = \frac{1}{C}\int I(t)\,dt = \frac{1}{C}\int \mathrm{Re}\left[I_0 e^{j\omega t}\right]dt = \frac{1}{C}I_0\int \mathrm{Re}\left[e^{j\omega t}\right]dt =$$

$$\frac{1}{C}I_0\,\mathrm{Re}\left[\int e^{j\omega t}\,dt\right] = \frac{1}{C}I_0\,\mathrm{Re}\left[\frac{1}{j\omega}e^{j\omega t}\right] = \boxed{\mathrm{Re}\left[\frac{I_0}{j\omega C}e^{j\omega t}\right] = V(t)}$$

OVER

$$\boxed{\mathrm{Re}\left[I_0 e^{j\omega t}\right] = I(t)}$$

CAP.

$$\int e^{j\omega t}\,dt = \int e^{u}\frac{du}{j\omega} = \frac{1}{j\omega}\int e^{u}\,du$$

$u = j\omega t$

$du = j\omega\,dt$

$dt = \frac{1}{j\omega}\,du$

$$= \frac{1}{j\omega}e^{u} = \frac{1}{j\omega}e^{j\omega t}$$

back

Phasors

$$I(t) = \mathrm{Re}\left[\underbrace{I_0}_{\text{CURRENT PHASOR}} e^{j\omega t}\right]$$

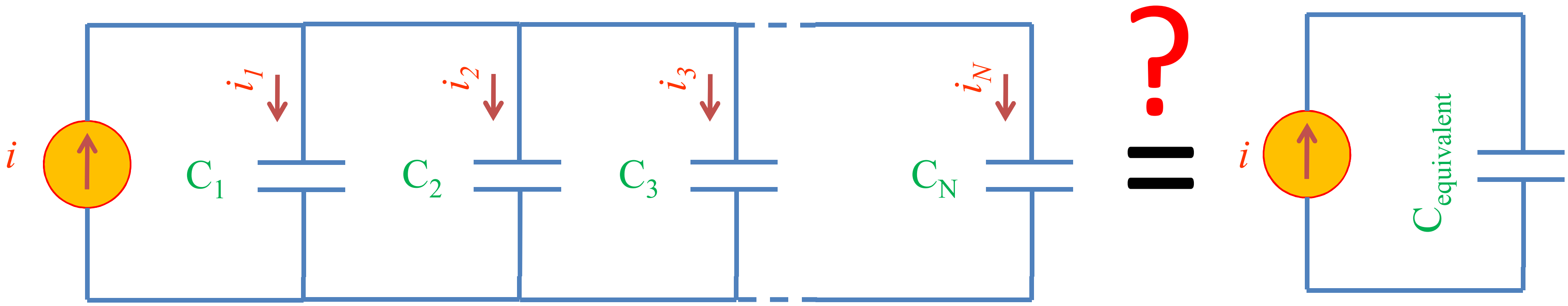
$$V(t) = \mathrm{Re}\left[\underbrace{\frac{1}{j\omega C} I_0}_{\text{VOLTAGE PHASOR}} e^{j\omega t}\right]$$

Voltage phasor $\mathbb{V}$

Current phasor $\mathbb{I}$

For a capacitor we have $\mathbb{V} = \mathbb{I} \underbrace{\frac{1}{j\omega C}}_{\text{"impedance"}}$

Parallel Capacitors



KCL $\quad i = i_1 + i_2 + i_3 + \cdots + i_N$

$$i_1 = C_1 \frac{dV}{dt} \qquad i_2 = C_2 \frac{dV}{dt} \quad \cdots \quad i_N = C_N \frac{dV}{dt}$$

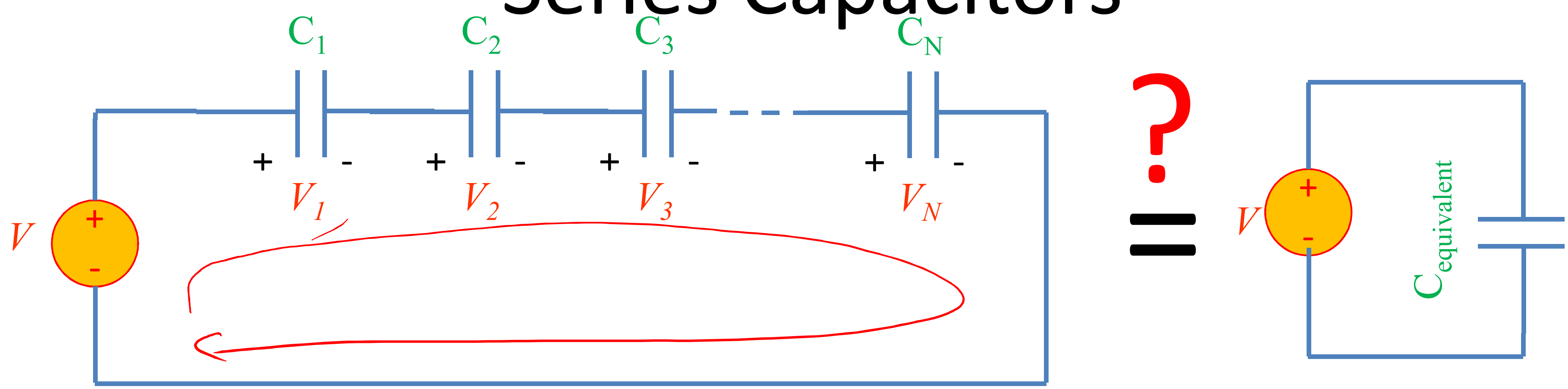
$$i = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + \cdots C_N \frac{dV}{dt}$$

$$= \frac{dV}{dt}(C_1 + C_2 + \cdots C_N) = \frac{dV}{dt} \boxed{\sum_{k=1}^{N} C_k}$$

$$i = [C_{eq}] \frac{dV}{dt}$$

$$C_{eq} = \sum_{k=1}^{N} C_k$$

Series Capacitors



$$V = V_1 + V_2 + \cdots V_N$$

$$V_1 = \frac{1}{C_1}\int i\,dt \qquad V_2 = \frac{1}{C_2}\int i\,dt$$

$$i_1 = i_2 = i_3 = i \ldots$$

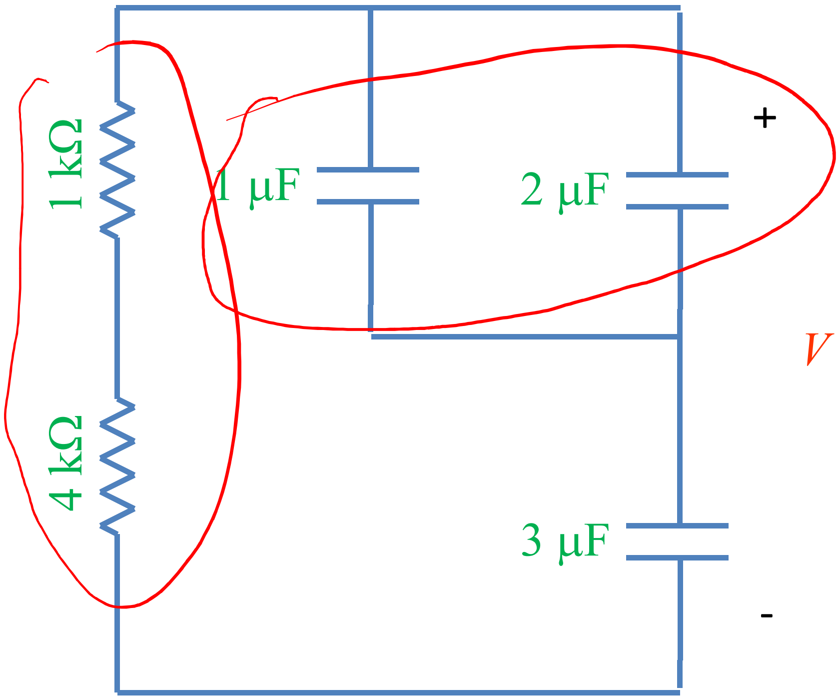
$$= \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \cdots \frac{1}{C_N}\int i\,dt$$

$$= \int i\,dt \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}\right)}_{C_{eq}^{-1}}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}}$$

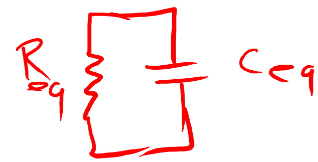
Example problem #4

(Students) Find V(t), given that V(t=0) = 5 Volts



$K = 10^3$

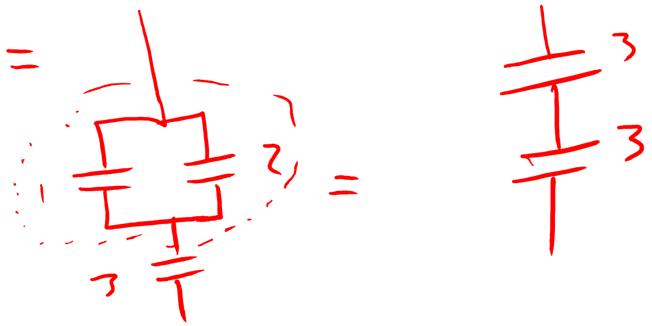
$\mu = 10^{-6}$



$R_{eq} = 5\,k\Omega$

$V(t) = V(t=0)\,e^{-t/RC}$

$C_{eq} =$



$=$

$\dfrac{1}{\frac{1}{3}+\frac{1}{3}} = \dfrac{3}{2}\,\mu F$

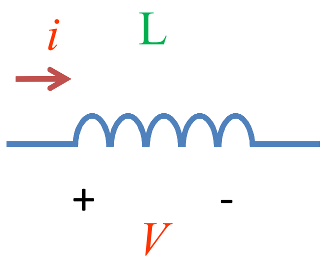
$V(t) = 5\,V\,e^{-t/\tau}$

$\tau = R_{eq} C_{eq} = 5\,k\Omega\,\frac{3}{2}\,\mu F = 7.5\;10^{3}\,\Omega\;10^{-6}\,F$

$= 7.5\;10^{-3}\,s = \underline{\underline{7.5\,ms}}$

Inductors

$I(t) = I_0 \cos(\omega t)$



$$L = \frac{N^2 \mu A}{l}$$

A=area
l=wire length
N = # of turns
$\mu = 4\,\pi\,10^{-6}$ H/m

Henry[H]

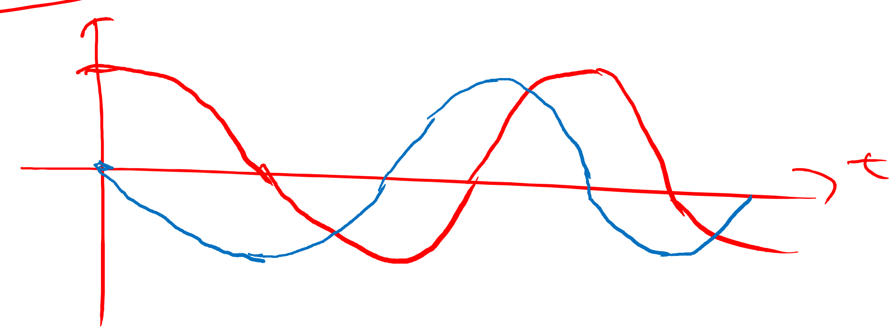
$$V = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int V\,dt$$

compare cap.

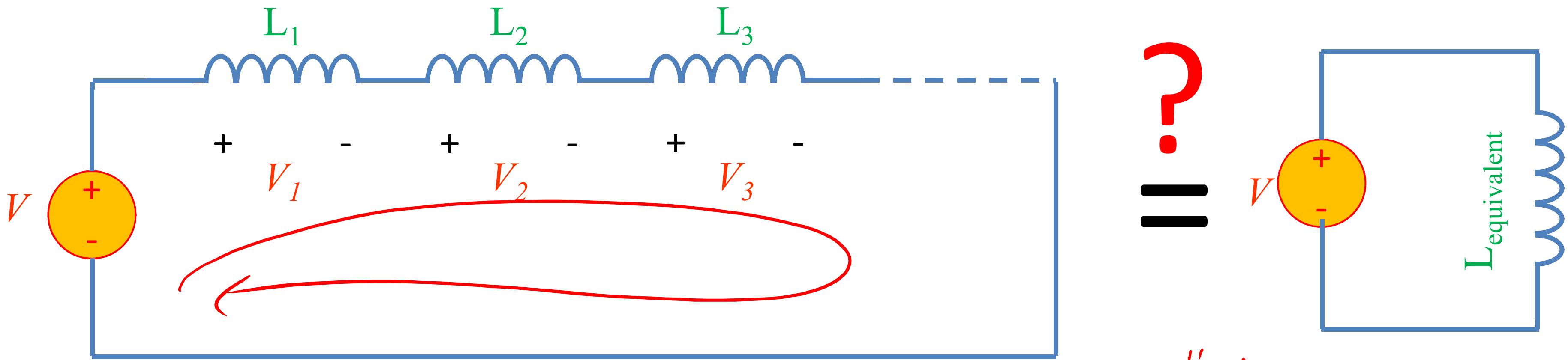
$$i = C\frac{dV}{dt}$$

Blue V

Red i



Series Inductors



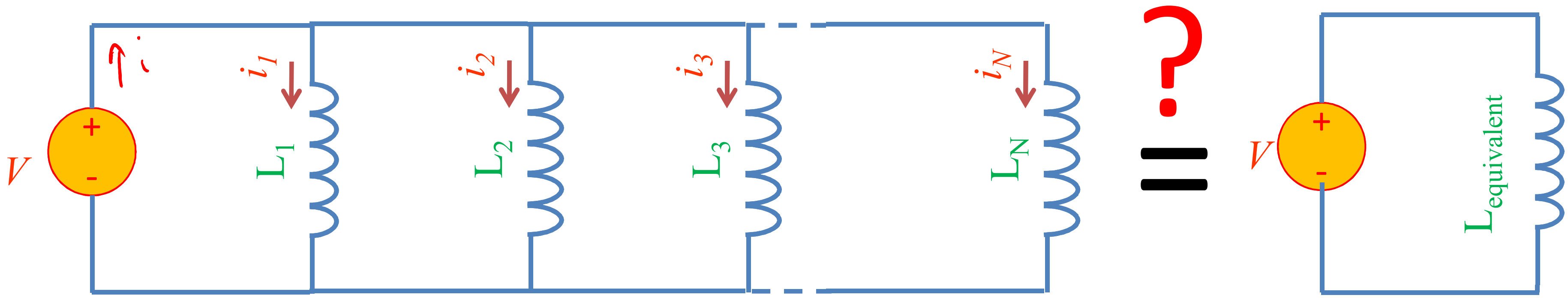
KVL $\quad V = V_1 + V_2 \cdots V_N \qquad i_1 = i_2 \cdots i_N = \text{"}i\text{"}$

$$V_1 = L_1 \frac{di}{dt} \quad V_2 = L_2 \frac{di}{dt} \cdots V_N L_N \frac{di}{dt}$$

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots L_N \frac{di}{dt} = \frac{di}{dt}(L_1 + L_2 \cdots L_N)$$

$$L_{eq} = L_1 + L_2 + \cdots L_N = \sum_{i=1}^{k} L_N$$

Parallel Inductors



KCL $i = i_1 + i_2 + \cdots i_N$ $\qquad v_1 = v_2 = \cdots v_N = \text{"}v\text{"}$

$i_1 = \frac{1}{L_1}\int v\,dt \quad i_2 = \frac{1}{L_2}\int v\,dt \quad \cdots$

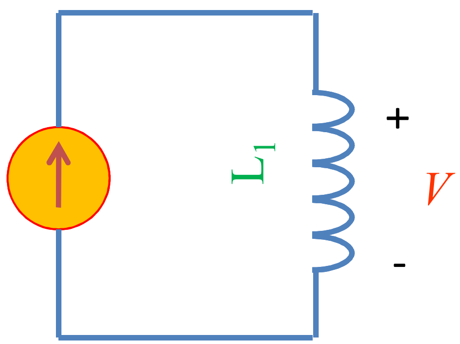
$i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \cdots \frac{1}{L_N}\int v\,dt$

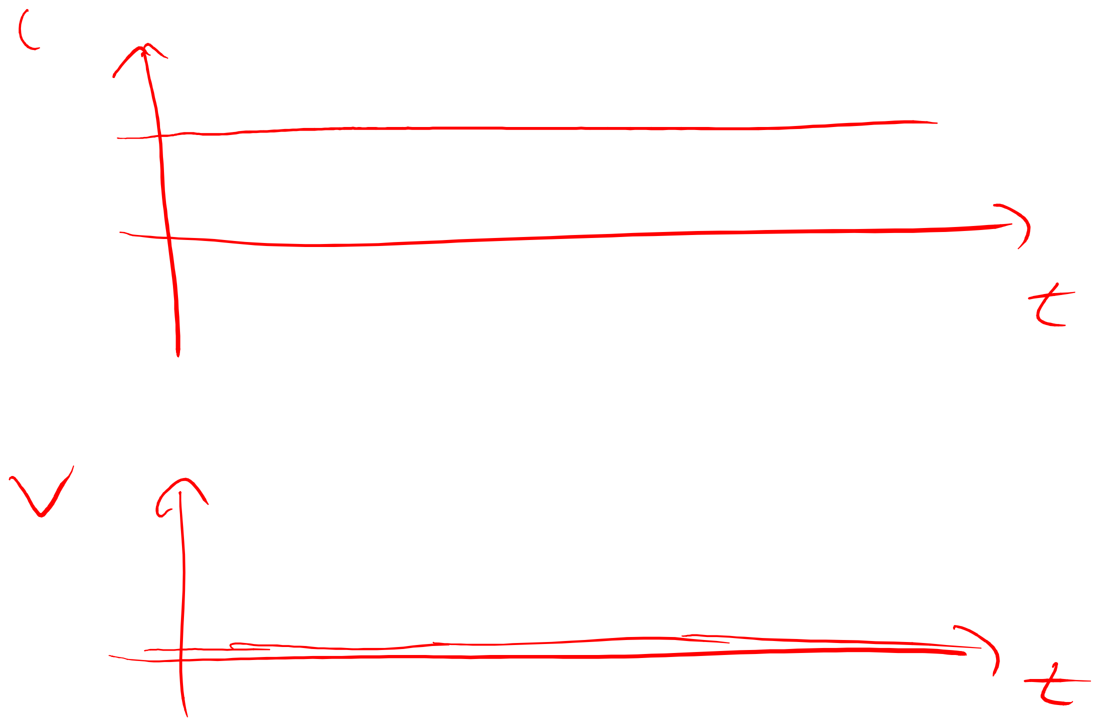
$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}\right)\int v\,dt$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}$$

Example Inductor Problem

(Students): Find V(t).



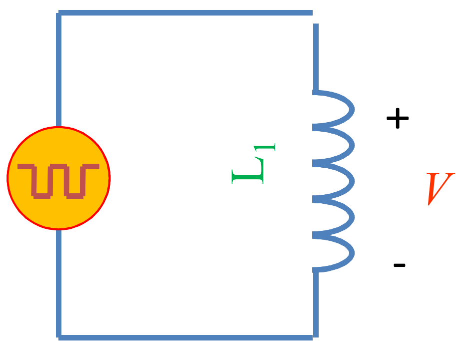


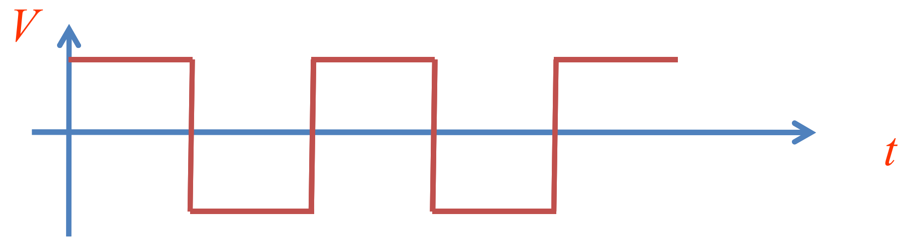
$V = L \frac{di}{dt}$

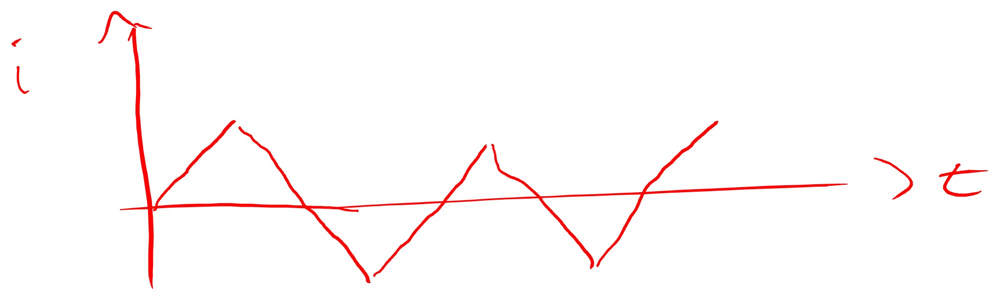
DC: cap looks like "open"
ind looks like "short"

Example Inductor Problem #2

(Students): Find i(t)





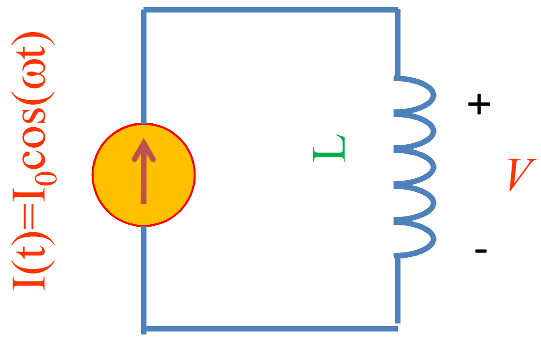


$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt$$

Example Inductor Problem #3

Find V(t)



$$V(t) = L\frac{d}{dt}\left[I_o \cos(\omega t)\right]$$

$$= -I_o L \omega \left[\sin(\omega t)\right]$$ ✓

current phasor

$$I(t) = Re\left[I_o e^{j\omega t}\right] = Re\left[\tilde{I} e^{j\omega t}\right]$$

$$V(t) = L\frac{d}{dt}\left[Re\, I_o e^{j\omega t}\right] = Re\left[L I_o \frac{d}{dt} e^{j\omega t}\right]$$

$$= Re\left[L I_o j\omega e^{j\omega t}\right] = Re\left[\underbrace{I_o j\omega L}_{\text{voltage phasor } \tilde{V}} e^{j\omega t}\right]$$

Inductor $\tilde{V} = \underbrace{j\omega L}_{\text{"impedance"}} \tilde{I}$

Impedance $= Z$

R

$\frac{1}{j\omega C}$

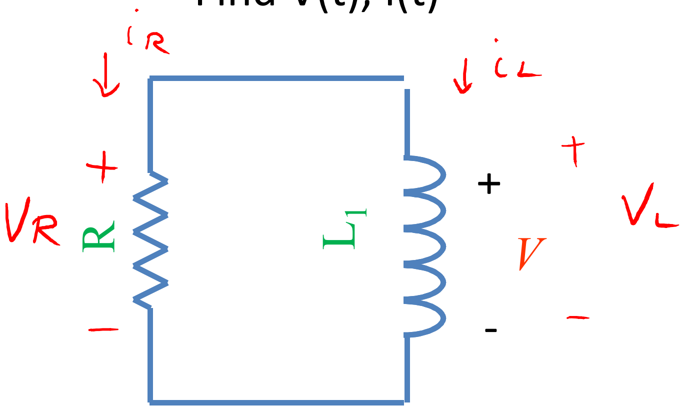
$j\omega L$

$$\bar{V} = Z \, \underline{\bar{I}}$$

KVL, KCL, Node/Mesh, Thev. Norton

LR circuit

Find V(t), i(t)



$$\boxed{\frac{df(x)}{dx} = a f(x)}$$

$$f(x) = c e^{ax}$$

$$c \frac{d}{dx} e^{ax} = a c e^{ax} = a f(x)$$

KVL $\Rightarrow$ $V_L = V_R$

KCL $\Rightarrow$ $i_R = -i_L$

$$L \frac{di_L}{dt} = V_L = V_R = +i_R R = -i_L R$$

$$\boxed{-i_L R = L \frac{di_L}{dt}}$$

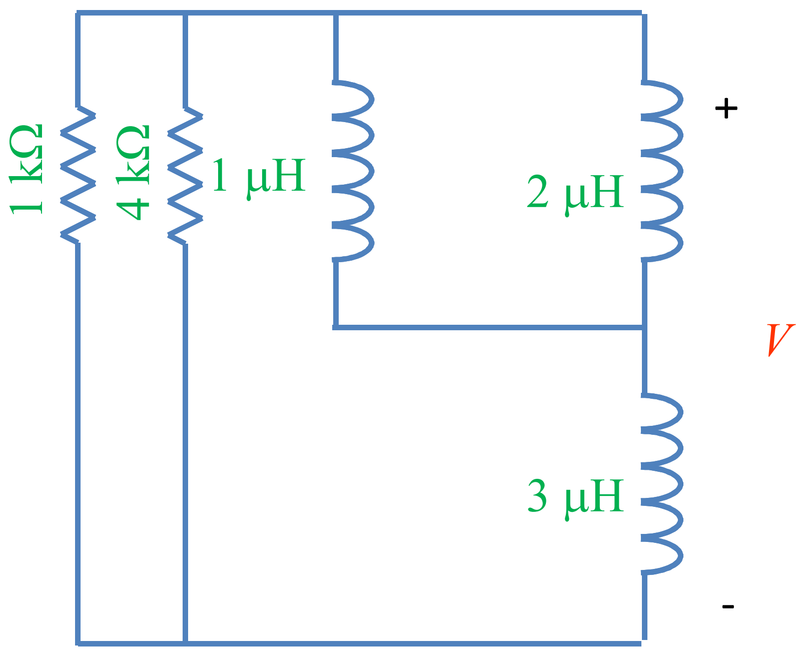
$$\boxed{\frac{di_L}{dt} = -\frac{1}{L/R} i_L}$$

$$i_L(t) = i_L(t=0)\, e^{-t/\tau}$$

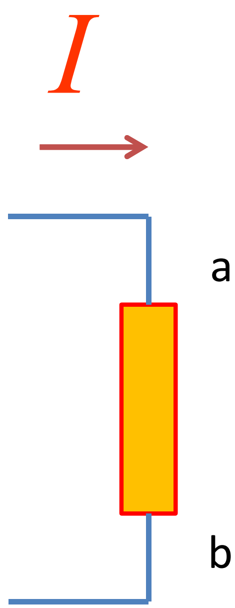
$$\tau = \frac{L}{R}$$

Example LR problem

(Students) Find V(t), given that V(t=0) = 5 Volts



Power



$$I \times V_{ab} = power$$

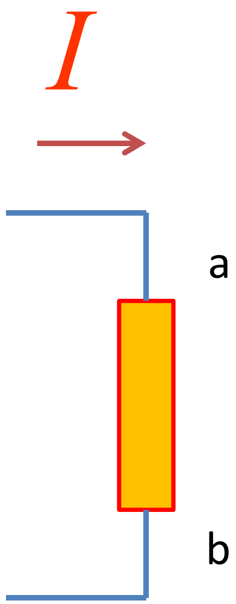
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
Meters Kilogram Second Amp

Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

Energy stored



$$I\ x\ V_{ab} = power$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

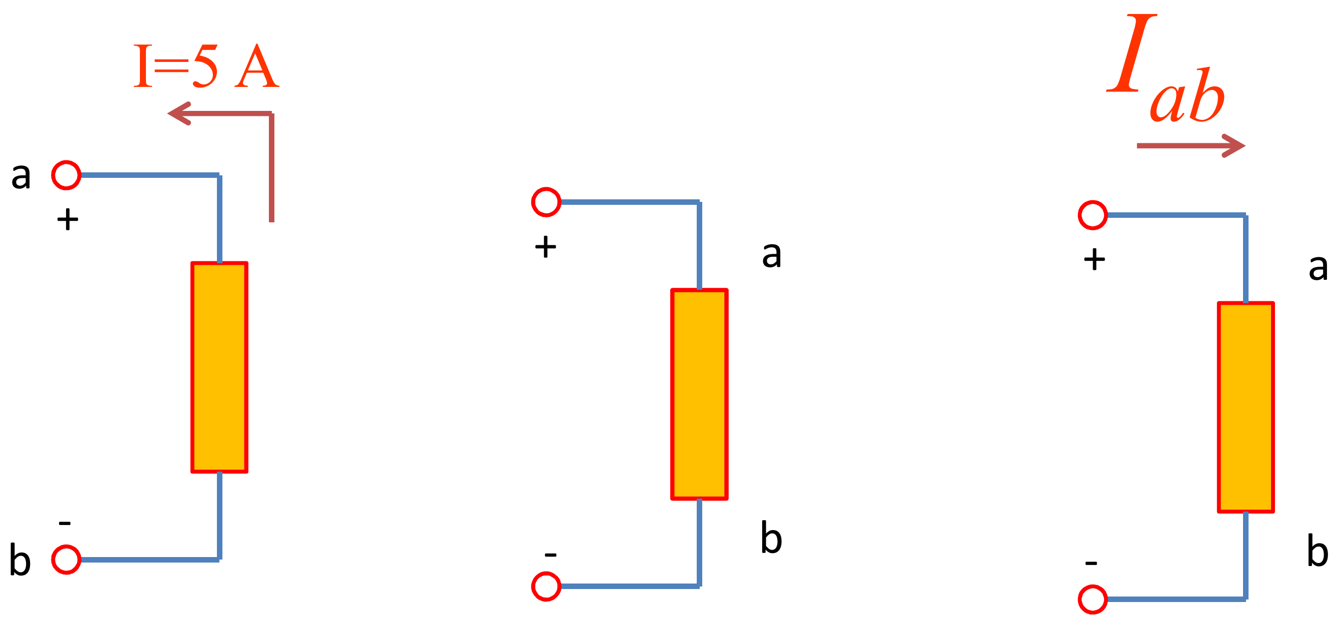
Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

Symbol library



Symbol library