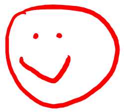
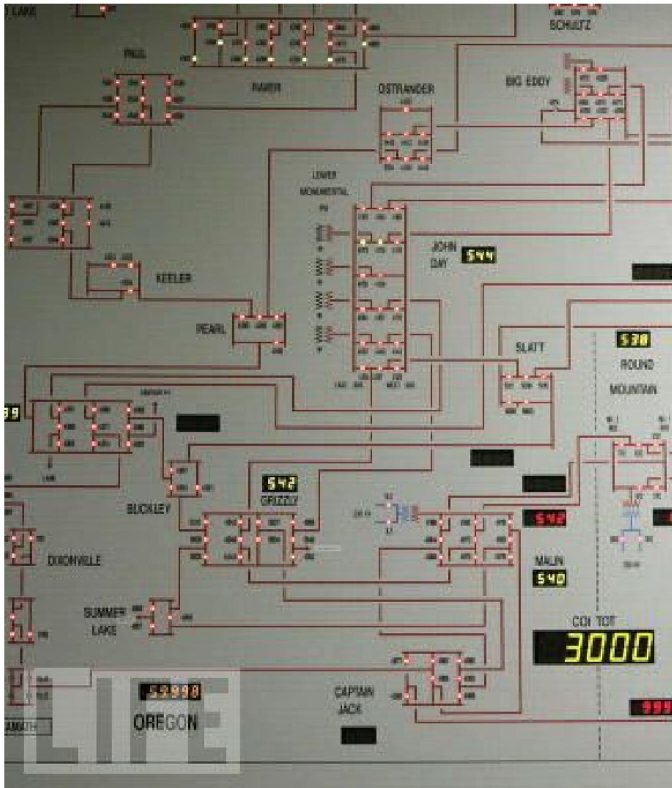
Announcements:

1. Announcement #1

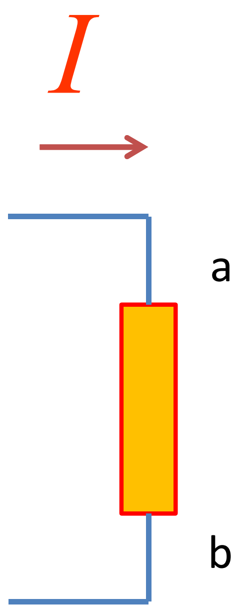
EECS 70A: Network Analysis

Lecture 11




SCHULTZ
VANTAGE
HANFORD
OSTRANDER
BIG EDDY
ASHE
JOHN DAY
KEELER
PEARL
McNARY
SLATT
ROUND MOUNTAIN
152
UF 66
TRACY
BUCKLEY
GRIZZLY
DIXONVILLE
SUMMER LAKE
MALIN
COI TOT
3000
CAPTAIN JACK
OREGON
LIFE

Power



$$I \, x \, V_{ab} = power$$

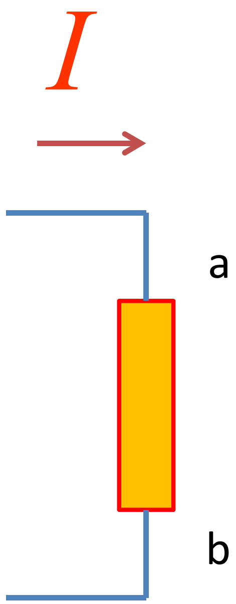
Watts [W] = Volt Amp [V-A]

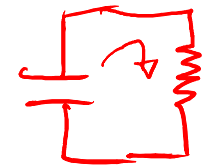
Note: MKSA unit system:
Meters Kilogram Second Amp

Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

Energy stored





$$I\,x\,V_{ab} = power$$

$$Power = \frac{Energy}{time}$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

$Q = CV$ CAP.

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}$$

Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

Inductor stored energy:

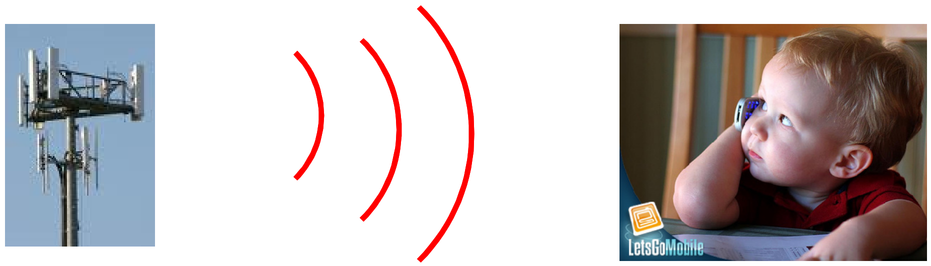
$V = L\frac{dI}{dt}$ INDUCTOR

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

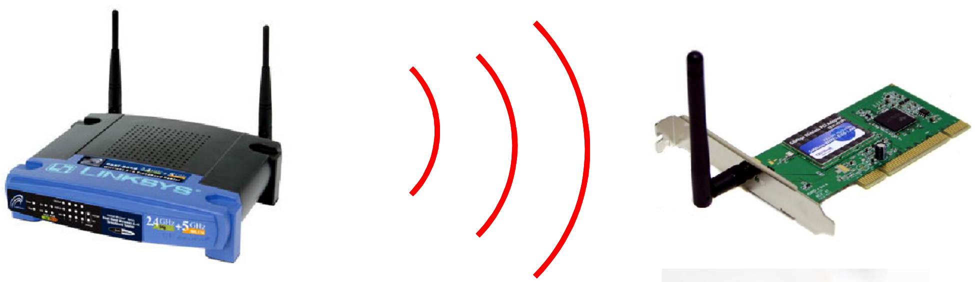
Wireless Communications

Broadcast Radio:

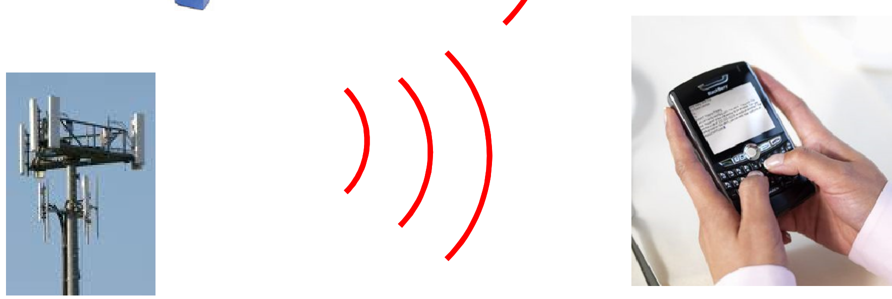
Telecom:



Internet:

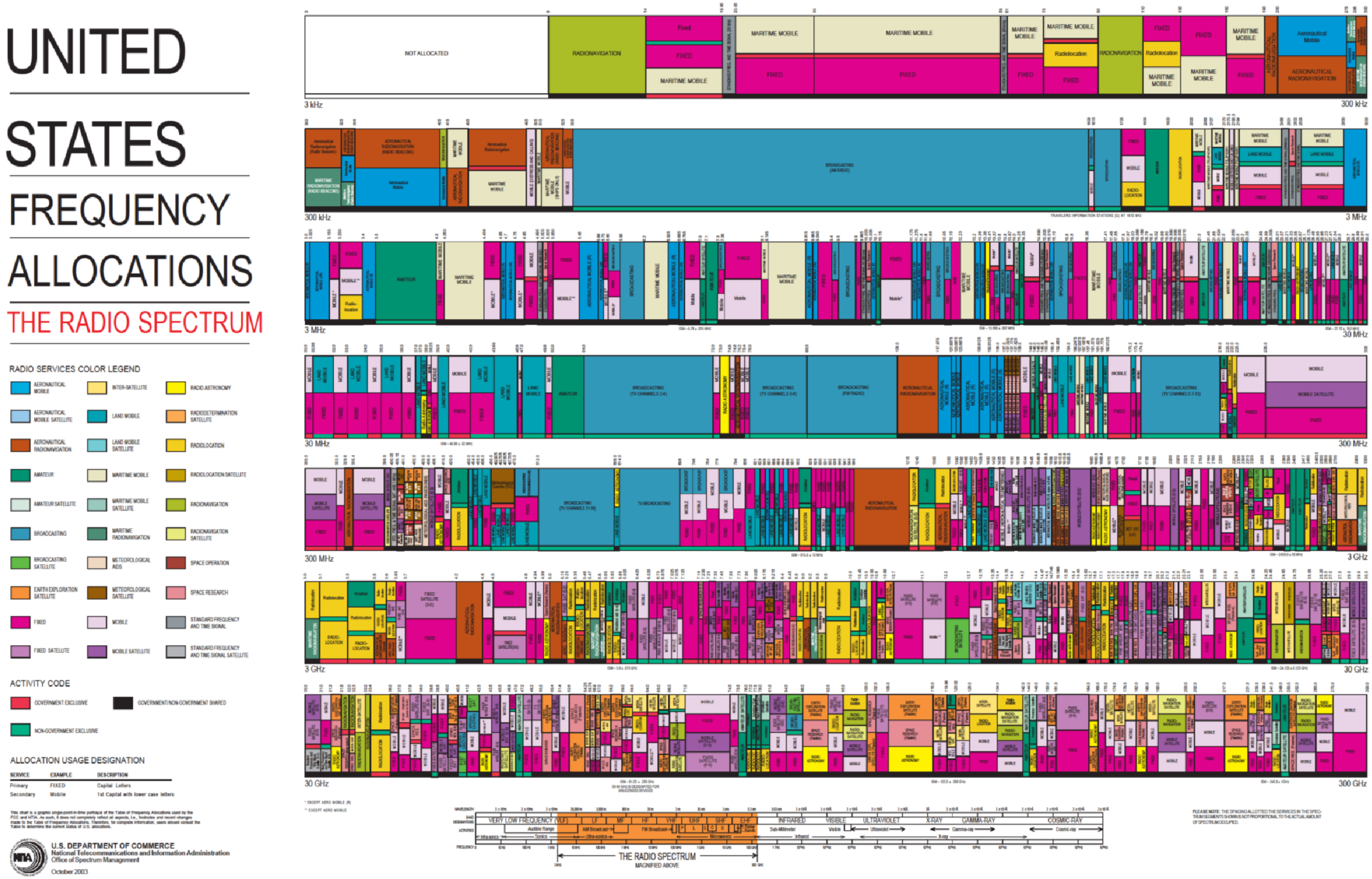


3G data:



All use sine waves (phasors) as way to describe signals and circuits.

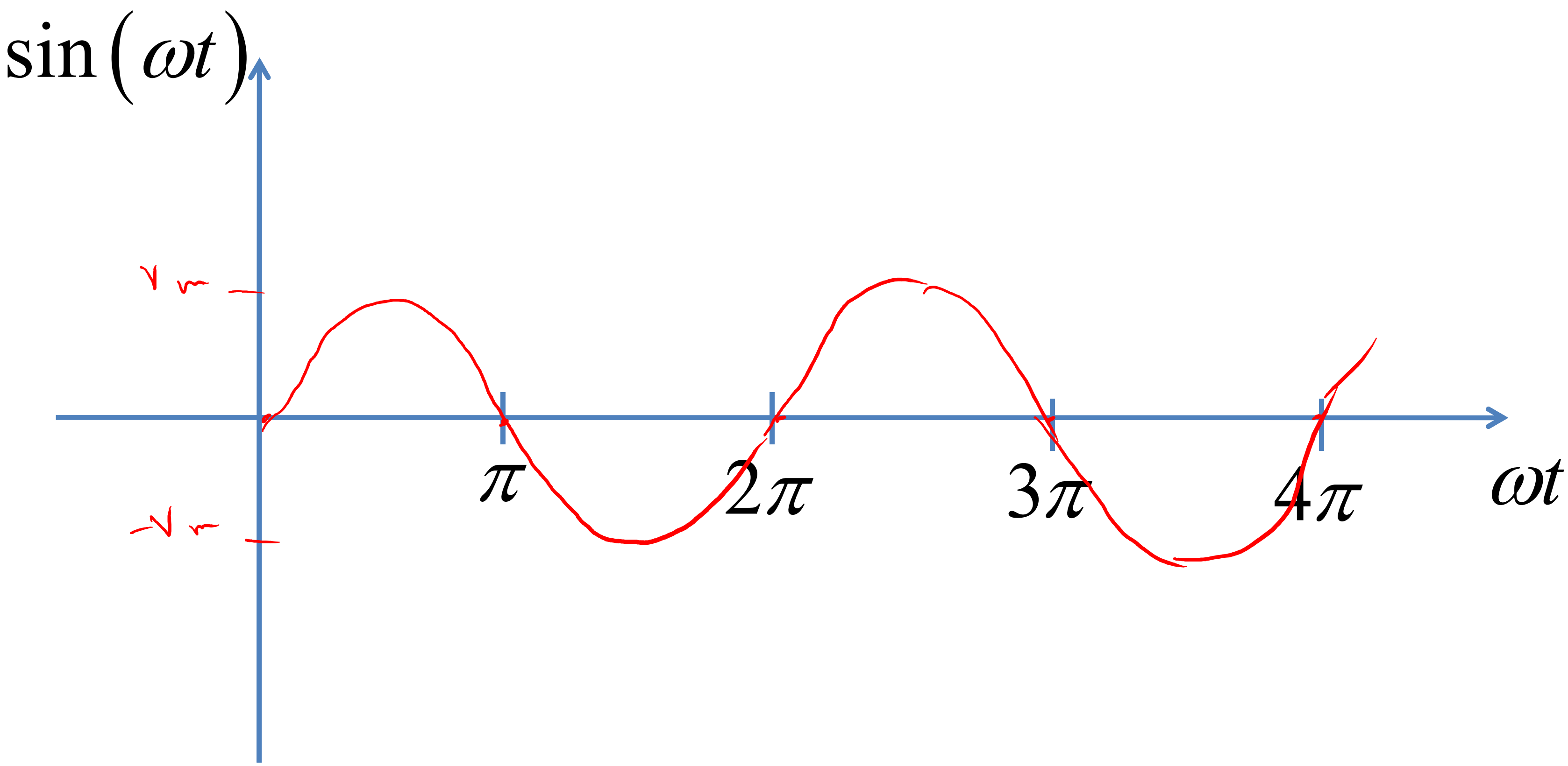
Frequency Allocations



http://www.ntia.doc.gov/osmhome/allochrt.PDF

Sine waves

$$V(t) = V_m \sin(\omega t)$$



Phase

$$V(t) = V_m \sin(\omega t + \phi)$$

$\sin(\phi_1 + \phi_2) = \ldots$

$\sin(\phi_1 - \phi_2) = \ldots$

$\cos(\phi_1 + \phi_2) = \ldots$

$\cos(\phi_1 + \phi_2) = \ldots$

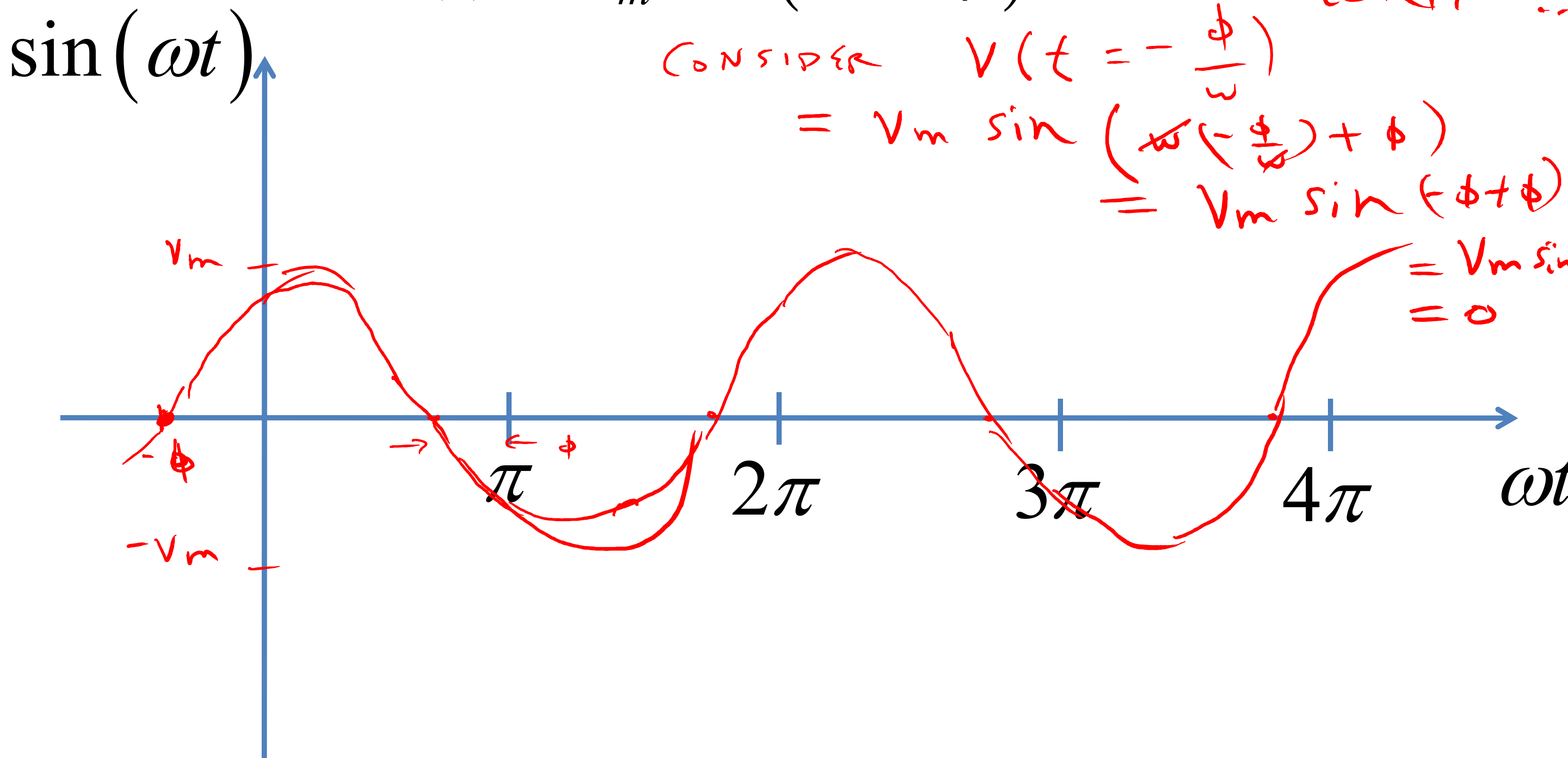
Consider $V\left(t = -\frac{\phi}{\omega}\right)$

$= V_m \sin\left(\omega\left(-\frac{\phi}{\omega}\right) + \phi\right)$

$= V_m \sin(-\phi + \phi)$

$= V_m \sin(0)$

$= 0$



$$A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t - \theta)$$

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) \qquad C = \sqrt{A^2 + B^2}$$

Complex numbers

* memorize

$e^{j\theta} = \cos\phi + j\sin\phi$

$$j \equiv \sqrt{-1} \qquad \frac{1}{j} = -j$$

$r = \sqrt{x^2+y^2}$

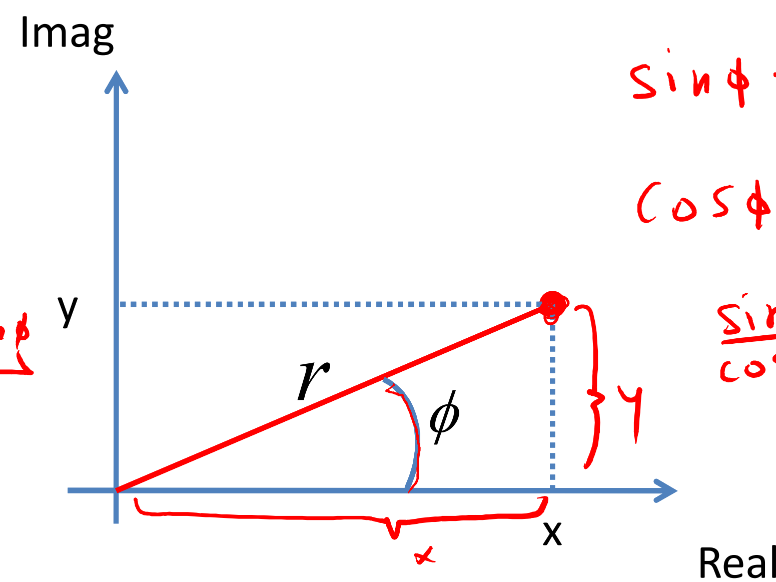
$$z = x + jy$$

$$z = re^{j\phi}$$

$= r(\cos\phi + j\sin\phi)$

$= \underbrace{r\cos\phi}_{x} + j\underbrace{r\sin\phi}_{y}$

$$z = r\angle\phi$$



$\sin\phi = \frac{y}{r}$

$\cos\phi = \frac{x}{r}$

$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{y}{x}$

$\phi = \tan^{-1}\left[\frac{y}{x}\right]$

$Re(z) = Real(z) = x$

$Im(z) = Imag(z) = y$

$Re(z) = r\cos\phi \qquad Im(z) = r\sin\phi$

Complex algebra

$$z_1 = x_1 + jy_1 = r_1 e^{j\phi_1} \qquad z_2 = x_2 + jy_2 = r_2 e^{j\phi_2}$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$e^x e^y = e^{x+y}$$

Multiplication:

$$z_1 z_2 = \left(r_1 e^{j\phi_1}\right)\left(r_2 e^{j\phi_2}\right) = r_1 r_2 e^{j\phi_1} e^{j\phi_2} = r_1 r_2 e^{j(\phi_1 + \phi_2)} = r_1 r_2 \angle(\phi_1 + \phi_2)$$

Division:

$$z_1 / z_2 = \frac{x_1 + jy_1}{x_2 + jy_2} = \frac{r_1 e^{j\phi_1}}{r_2 e^{j\phi_2}} = \frac{r_1}{r_2} \frac{e^{j\phi_1}}{e^{j\phi_2}} = \frac{r_1}{r_2} e^{j\phi_1} e^{-j\phi_2} = \frac{r_1}{r_2} e^{j(\phi_1 - \phi_2)}$$

Inversion:

$$1/z_1 = \frac{1}{r_1 e^{j\phi_1}} = \frac{1}{r_1} e^{-j\phi_1}$$

Square root:

$$\sqrt{z_1} = \left(r_1 e^{j\phi_1}\right)^{1/2} = r_1^{1/2}\left(e^{j\phi_1}\right)^{1/2} = r_1^{1/2} e^{j\phi_1/2}$$

Complex conjugate:

$$Z^* = x - jy = re^{-j\phi}$$

Euler relationship

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$\Rightarrow \cos\phi = \text{Re}(e^{j\phi})$$

$$e^{x}e^{y} = e^{x+y}$$

$$e^{z_1}e^{z_2} = e^{z_1+z_2}$$

Phasors:

$$V(t) = V_m\cos(\omega t + \phi) = \text{Re}\left(V_m e^{j(\omega t+\phi)}\right)$$

$$= V_m \,\text{Re}\left(e^{j(\omega t+\phi)}\right)$$

$$= \text{Re}\left(V_m e^{j\phi} e^{j\omega t}\right)$$

"Phasor" **V**

(Complex #)

Phasors

$$V(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}\left(V_m e^{j(\omega t+\phi)}\right)$$

$$= \mathrm{Re}\left(V_m e^{j\phi} e^{j\omega t}\right)$$

"Voltage Phasor" $\mathbf{V}$

(Complex #)

$$i(t) = I_m \cos(\omega t + \phi) = \mathrm{Re}\left(I_m e^{j(\omega t+\phi)}\right)$$

$i(t) = I_0 \cos(\omega t)$

$I_m = I_0$ $\mathbf{I} = I_0$

$\phi = 0$

$$= \mathrm{Re}\left(I_m e^{j\phi} e^{j\omega t}\right)$$

"Current Phasor" $\mathbf{I}$

$$\frac{dF(t)}{dt} = aF(t)$$

general solution $F(t) = Ce^{at}$

$$\frac{d}{dx}\left(Ce^{at}\right) = C\frac{d}{dx}\left(e^{at}\right) = Cae^{at}$$

$$= a\left(Ce^{a}\right)$$

$$= aF(\ )$$

$$\frac{d^2F(t)}{dt^2} = bF(t)$$

general solution $F(t) = C_1\sin(\sqrt{b}t) + C_2\cos(\sqrt{b}t)$

$$a\frac{d^2F(t)}{dt^2} + b\frac{dF(t)}{dt} + cF(t) = g(t)$$

general solution = . . .

Circuits

$v(t) = R\, i(t)$

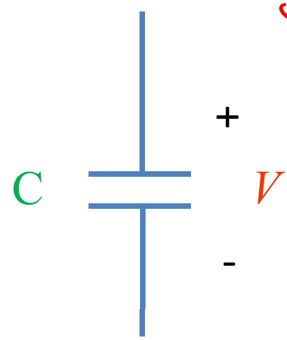
$q(t) = C\, v(t)$

$i(t) = C \frac{dv(t)}{dt}$

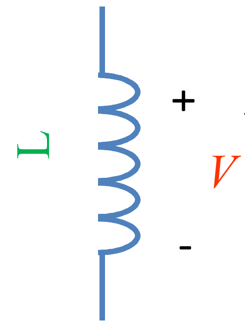
$v(t) = L \frac{di(t)}{dt}$

$\mathbf{V} = \mathbf{I}\, R$

$V = I\, R$



$\mathbf{V} = \mathbf{I}/j\omega C$



$\mathbf{V} = j\omega L\, \mathbf{I}$

"Impedance"

$Z = R$ $\qquad Z = 1/j\omega C$ $\qquad Z = j\omega L$

KCL, KVL hold for relationship between **V**, **I**.

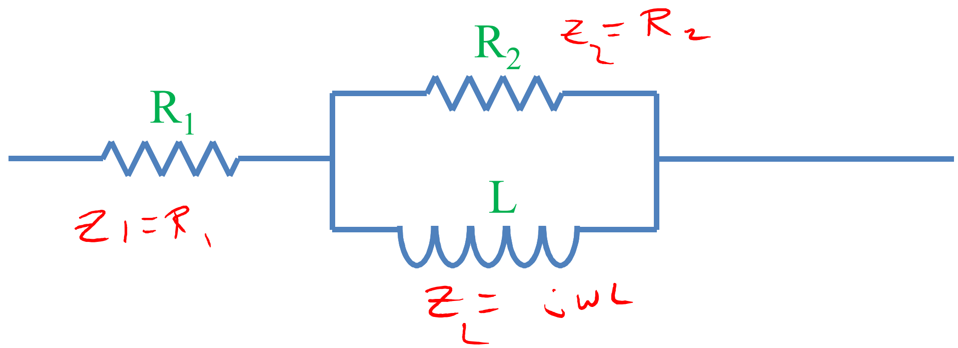
Series/Parallel Impedances

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

Example problem #1

Find Z_{eq} for this circuit: (instructor)

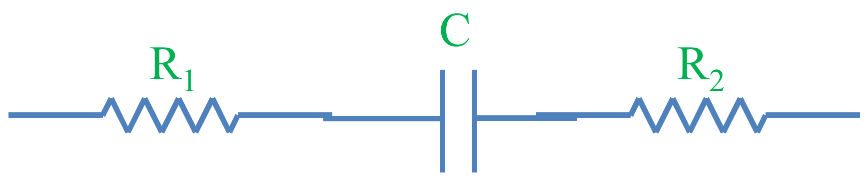


$$\frac{j\omega L\, R_2}{j\omega L + R_2}$$

$$R_1 + \frac{(j\omega L\, R_2)\frac{1}{R_2}}{(R_2 + j\omega L)\frac{1}{R_2}} = \boxed{R_1 + \frac{j\omega L}{1 + j\omega (L/R_2)}}$$

Example problem #2

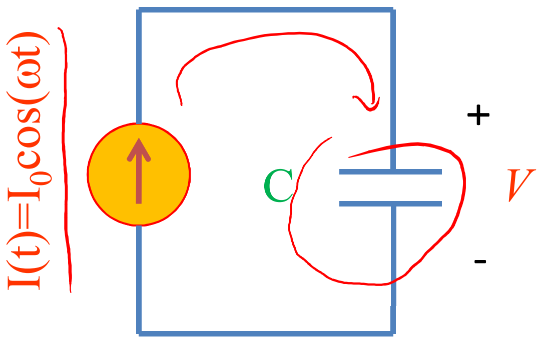
Find Z_{eq} for this circuit: (students)



$R_1 + \frac{1}{j\omega C} + R_2$

Phasor to voltage conversion

Find V(t), q(t)



$$I(t) = \mathrm{Re}\left(\tilde{I} e^{j\omega t}\right) = I_0 \cos(\omega t)$$

$$\Rightarrow \tilde{I} = I_0$$

$$Z = \frac{1}{j\omega C} \qquad \frac{\tilde{V}}{\tilde{I}} = \frac{1}{j\omega C}$$

$$\tilde{V} = \frac{1}{j\omega C}\tilde{I} = \frac{I_0}{j\omega C}$$

$$q = CV$$

$$i = C\frac{dV}{dt}$$

$$V = \frac{1}{C}\int i\,dt = \frac{1}{C}\int I_0 \cos(\omega t)\,dt = \frac{I_0}{C\omega}\sin(\omega t)$$

$$q = \frac{I_0}{\omega}\cos(\omega t)$$

q should be sin not cos

$$\frac{1}{j} = -j$$

$$V(t) = \mathrm{Re}\left(\tilde{V} e^{j\omega t}\right) = \mathrm{Re}\left[\frac{I_0}{j\omega C} e^{j\omega t}\right] =$$

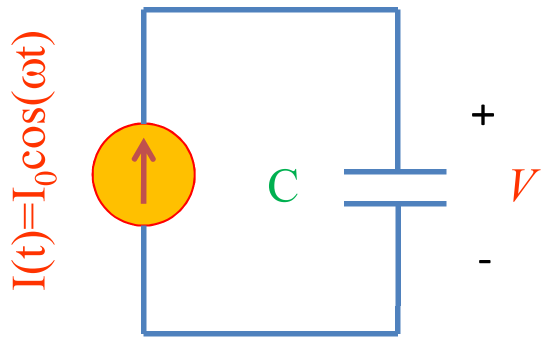
$$= \mathrm{Re}\left[\frac{I_0}{\omega C}(-j) e^{j\omega t}\right] = -\frac{I_0}{\omega C}\mathrm{Re}\left[j e^{j\omega t}\right]$$

$$= -\frac{I_0}{\omega C}\mathrm{Re}\left[j\left(\cos(\omega t) + j\sin(\omega t)\right)\right] = -\frac{I_0}{\omega C}\mathrm{Re}\left[j\cos(\omega t) + j^2\sin(\omega t)\right]$$

$$= -\frac{I_0}{\omega C}\mathrm{Re}\left[-\sin(\omega t) + j\cos(\omega t)\right] = -\frac{I_0}{\omega C}\left[-\sin(\omega t)\right] = \frac{I_0}{\omega C}\sin(\omega t)$$

Phasor to voltage conversion

Find V(t), q(t)



Problem gives us:

$I(t)=I_0\cos(\omega t)$

Compare to definition of current phasor:

$$i(t) = I_m \cos\left(\omega t + \phi\right) = \mathrm{Re}\left(I_m e^{j\phi} e^{j\omega t}\right) = \mathrm{Re}\left(\mathbf{I} e^{j\omega t}\right)$$

$$\Rightarrow \mathbf{I} = I_0$$

Find voltage phasor using generalized Ohm's law:

$$\mathbf{V}=\mathbf{I}/j\omega C$$

Find v(t) from voltage phasor:

$$v(t) = \mathrm{Re}\left[\mathbf{V} e^{j\omega t}\right] = \mathrm{Re}\left[\left(\frac{\mathbf{I}}{j\omega C}\right) e^{j\omega t}\right] = \mathrm{Re}\left[\left(\frac{I_0}{j\omega C}\right) e^{j\omega t}\right]$$

$$= \frac{I_0}{\omega C}\mathrm{Re}\left[\left(\frac{1}{j}\right) e^{j\omega t}\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[-j e^{j\omega t}\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[-j\left(\cos(\omega t) + j\sin(\omega t)\right)\right]$$

$$= \frac{I_0}{\omega C}\mathrm{Re}\left[-j\cos(\omega t) + (-j)\, j\sin(\omega t)\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[\sin(\omega t) - j\cos(\omega t)\right] = \frac{I_0}{\omega C}\sin(\omega t)$$

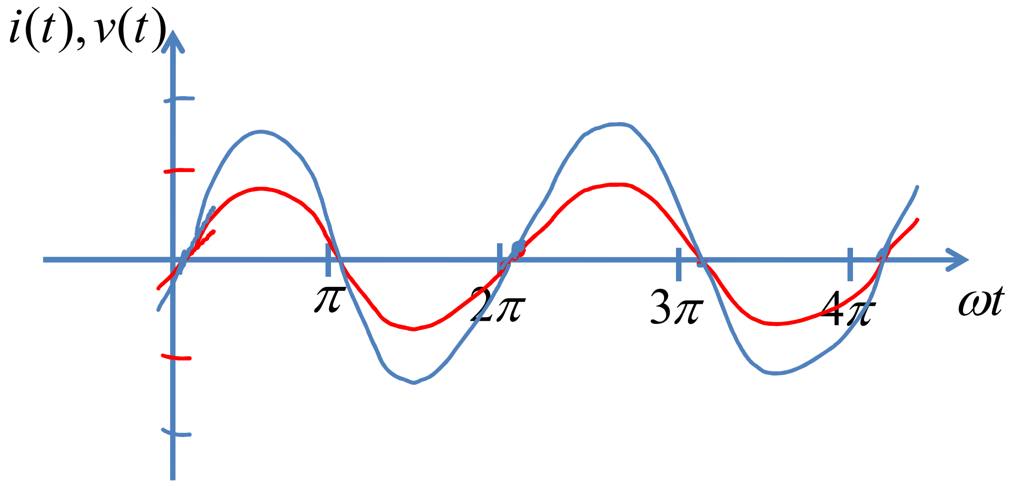
Phase vs. impedance (Z)

In general:

Z real i.e. $Z = x + jy$

$x \neq 0$, $y = 0$

=> i(t), v(t) "in phase"

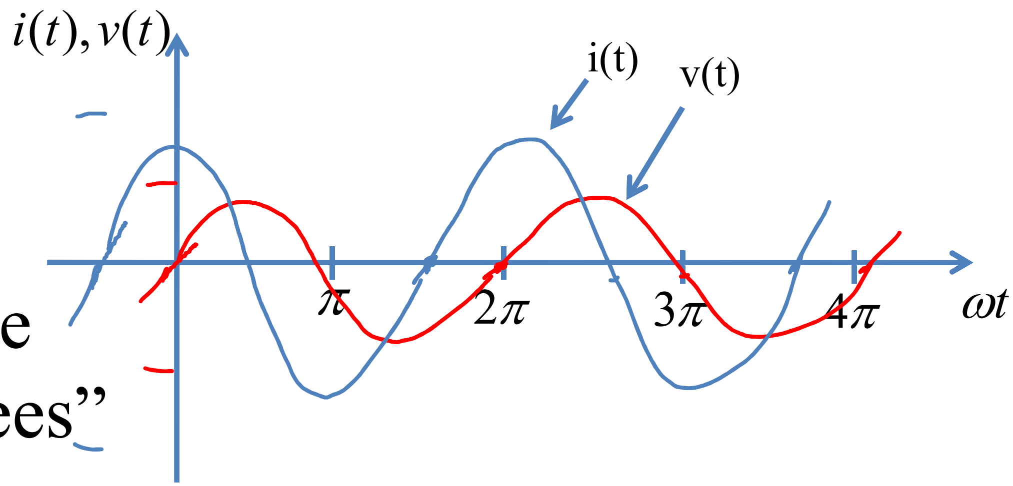


In general:

Z imag i.e. $Z = x + jy$

$x = 0$, $y \neq 0$

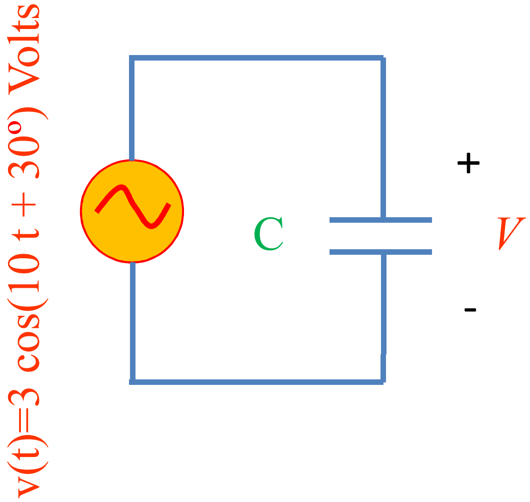
=> i(t), v(t) "out of phase
by 90 degrees"



HW problem: Find relationship between phase shift and impedance (Z).

Example phasor problem

Find i(t) (students)



Problem gives us:

v(t)=3 cos(10 t + 30º) Volts

Compare to definition of voltage phasor:

$$v(t) = V_m \cos\left(\omega t + \phi\right) = \mathrm{Re}\left(V_m e^{j\phi} e^{j\omega t}\right) = \mathrm{Re}\left[\mathbf{V} e^{j\omega t}\right]$$

$$\Rightarrow \mathbf{V} = ?$$

Find current phasor using generalized Ohm's law:

$$\mathbf{I} = j\omega C\ \mathbf{V} = \ldots$$

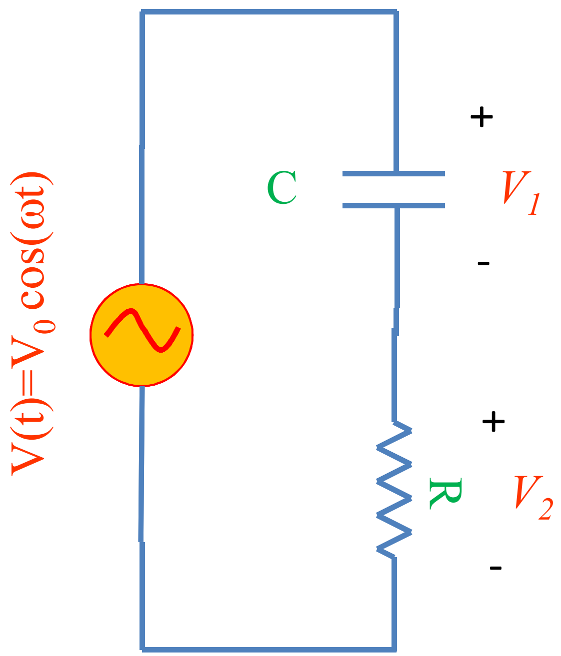
Find i(t) from current phasor:

$$i(t) = \mathrm{Re}\left(\mathbf{I} e^{j\omega t}\right) = \ldots$$

Trick #1

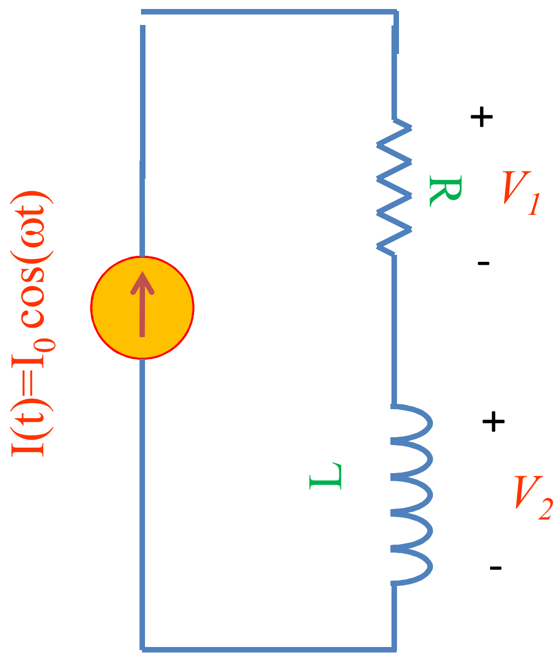
Example problem #3

Find i(t), V_1(t), V_2(t) for this circuit: (instructor)



Example problem #4

Find i(t), V_1(t), V_2(t) for this circuit: (students)



Low pass filter