

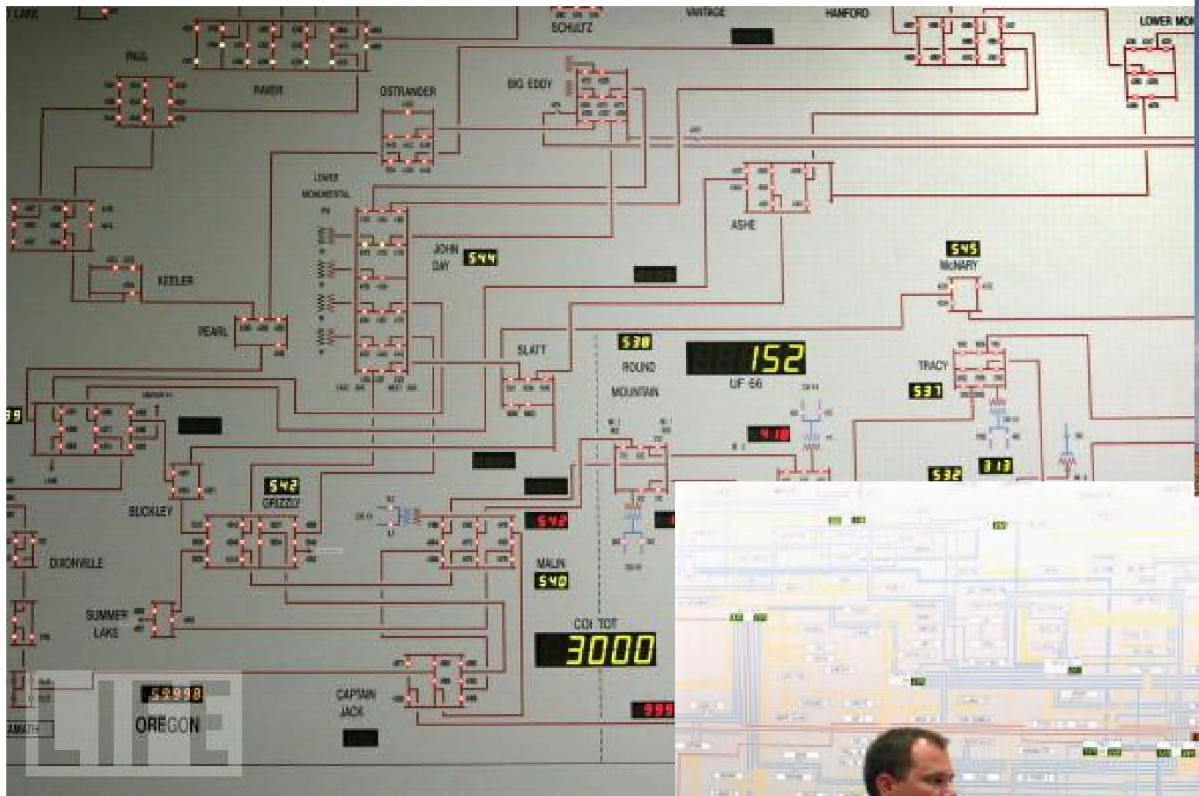
Announcements:

1. Announcement #1

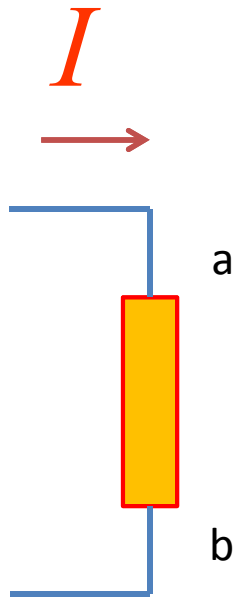
EECS 70A: Network Analysis

Lecture 11





Power



$$I \times V_{ab} = \text{power}$$

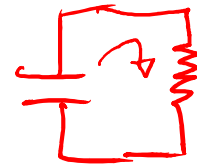
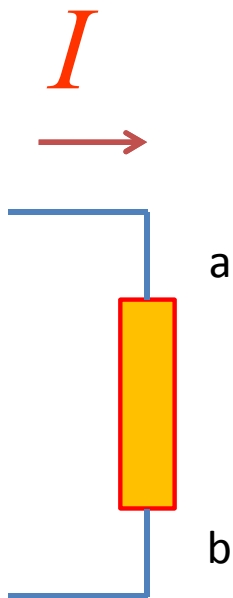
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
Meters Kilogram Second Amp

Resistor:
Energy lost to heat...

Inductor or capacitor:
Energy **STORED** and can be recovered...

Energy stored



$$I \times V_{ab} = \text{power}$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

$$Q = CV \quad \text{CAP.}$$

$$I = \frac{dQ}{dt} = C \frac{dV}{dt}$$

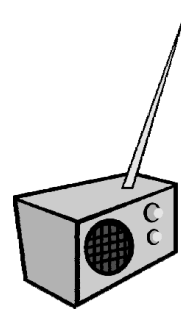
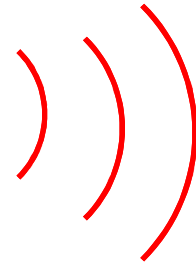
Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} LI^2$$

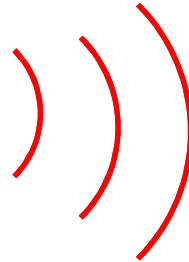
$$V = L \frac{dI}{dt} \quad \text{INDUCTOR}$$

Wireless Communications

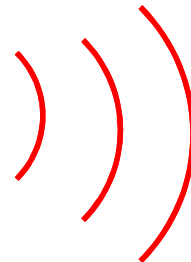
Broadcast Radio:



Telecom:



Internet:



3G data:



*All use sine waves
(phasors) as way to
describe signals and
circuits.*

Frequency Allocations

UNITED STATES FREQUENCY ALLOCATIONS THE RADIO SPECTRUM

RADIO SERVICES COLOR LEGEND

- AERONAUTICAL MOBILE
- INTER-SATELLITE
- RADIO ASTRONOMY
- AERONAUTICAL MOBILE SATELLITE
- LAND MOBILE
- BACKSCATTERING SATELLITE
- AERONAUTICAL RADIONAVIGATION
- LAND MOBILE SATELLITE
- RADIOLOGICAL
- AMATEUR
- MARITIME MOBILE
- RADIONAVIGATION SATELLITE
- AMATEUR SATELLITE
- MARITIME MOBILE SATELLITE
- RADIONAVIGATION
- BROADCASTING
- MARITIME RADIONAVIGATION
- RADIONAVIGATION SATELLITE
- BROADCASTING SATELLITE
- METEOROLOGICAL AIDS
- SPACE OPERATION
- EARTH EXPLORATION SATELLITE
- METEOROLOGICAL SATELLITE
- SPACE RESEARCH
- FIXED
- MOBILE
- STANDARD FREQUENCY AND TIME SIGNAL
- FIXED SATELLITE
- MOBILE SATELLITE
- STANDARD FREQUENCY AND TIME SIGNAL SATELLITE

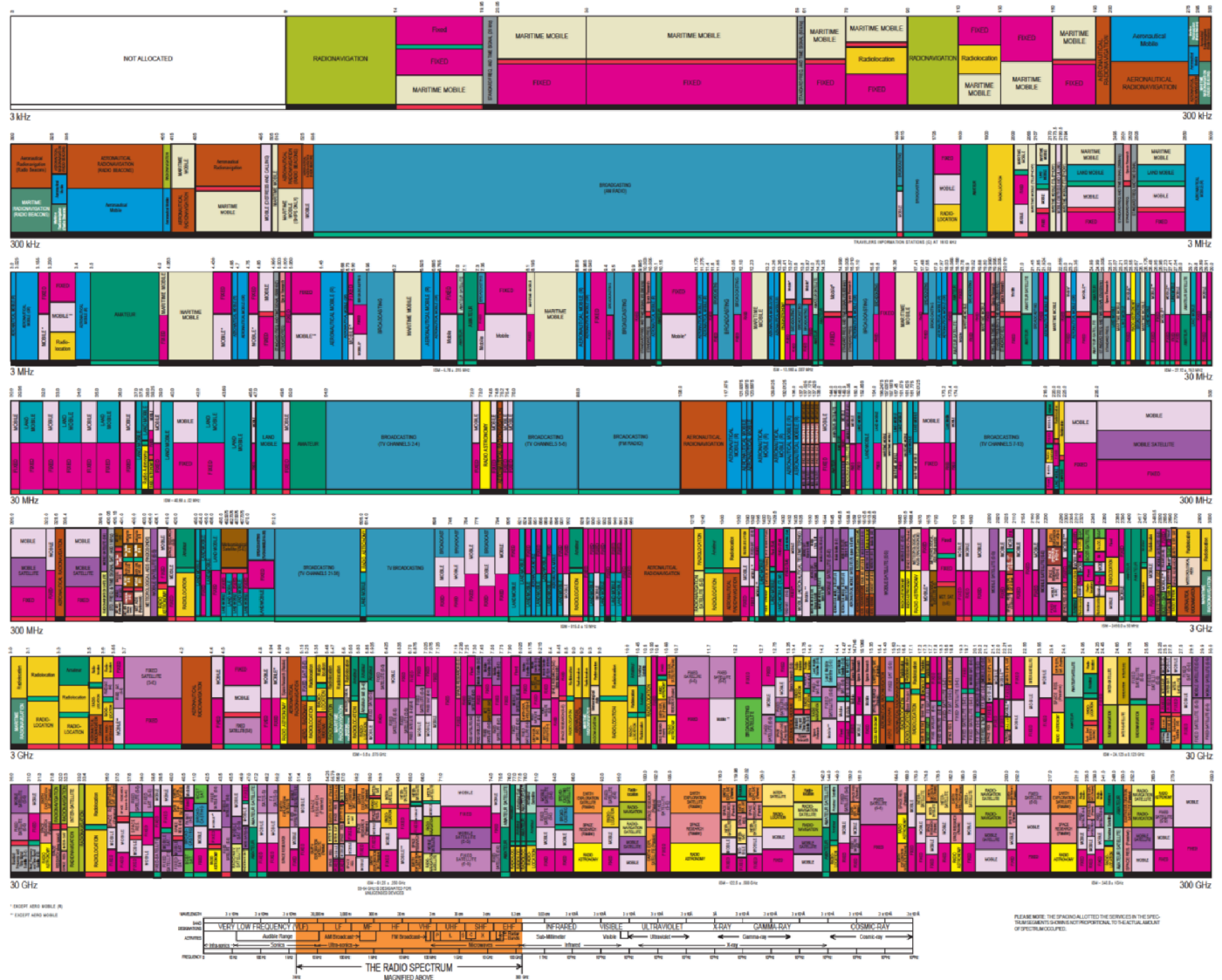
ACTIVITY CODE

- GOVERNMENT EXCLUSIVE
- GOVERNMENT/NON-GOVERNMENT SHARED
- NON-GOVERNMENT EXCLUSIVE

ALLOCATION USAGE DESIGNATION

SERVICE	EXAMPLE	DESCRIPTION
Primary	FIXED	Capital Letters
Secondary	Mobile	1st Capital with lower case letters

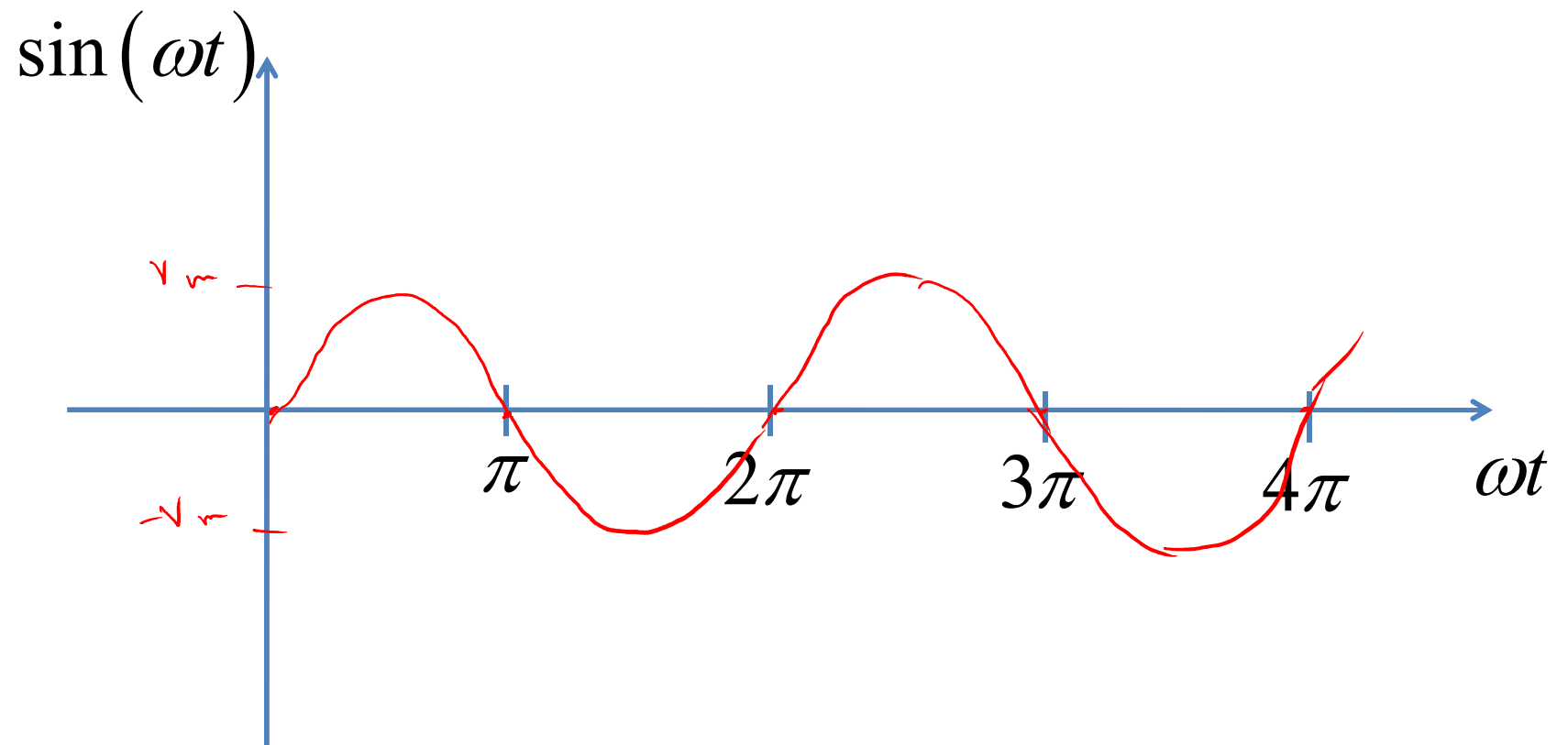
This chart is a graphic representation in part of the Table of Frequency Allocations used by the FCC and the ITU. It does not constitute a license or permit, i.e., facilities and services are subject to the Table of Frequency Allocations. It is intended for reference purposes only. Wherever possible, the Table is identified by the service code of the ITU. Revisions, by frequency allocation, are shown within the Table.



<http://www.ntia.doc.gov/osmhome/allochrt.PDF>

Sine waves

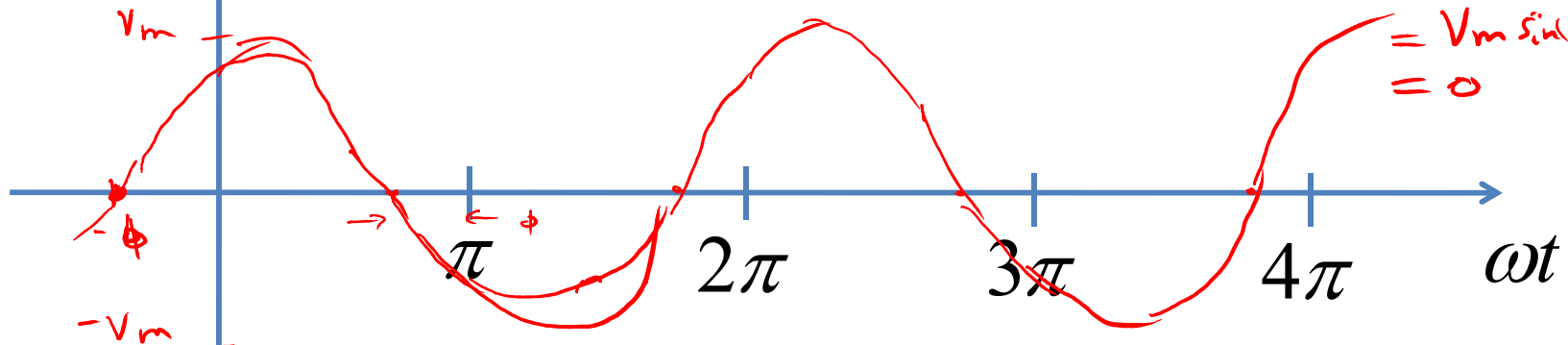
$$V(t) = V_m \sin(\omega t)$$



Phase

$$V(t) = V_m \sin(\omega t + \phi)$$

$\sin(\omega t)$



$$\begin{aligned} \sin(\phi_1 + \phi_2) &= \dots \\ \sin(\phi_1 - \phi_2) &= \dots \\ \cos(\phi_1 + \phi_2) &= \dots \\ \cos(\phi_1 - \phi_2) &= \dots \end{aligned}$$

CONSIDER $V(t = -\frac{\phi}{\omega})$

$$\begin{aligned} &= V_m \sin\left(\omega\left(-\frac{\phi}{\omega}\right) + \phi\right) \\ &= V_m \sin(-\phi + \phi) \\ &= V_m \sin(0) \\ &= 0 \end{aligned}$$

$$A \cos(\omega t) + B \sin(\omega t) = C \cos(\omega t - \theta)$$

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) \quad C = \sqrt{A^2 + B^2}$$

* memorize

Complex numbers

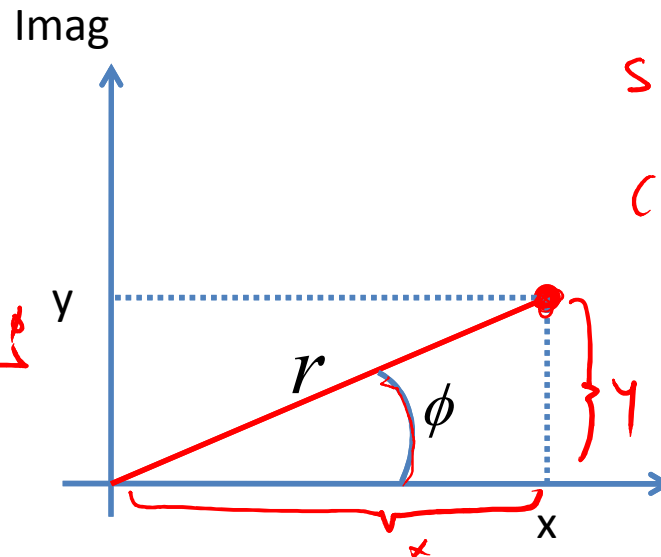
$$e^{j\theta} = \cos \phi + j \sin \phi$$

$$j \equiv \sqrt{-1} \quad \frac{1}{j} = -j$$

$$r = \sqrt{x^2 + y^2}$$

$$z = x + jy$$

$$z = r e^{j\phi} \\ = r (\cos \phi + j \sin \phi) \\ = r \cos \phi + j r \sin \phi \\ z = r \angle \phi$$



$$\sin \phi = \frac{y}{r}$$

$$\cos \phi = \frac{x}{r}$$

$$\frac{\sin \phi}{\cos \phi} = \tan \phi \\ = \frac{y}{x}$$

$$\text{Re}(z) = \text{Real}(z) = x$$

$$\text{Im}(z) = \text{Imag}(z) = y$$

$$\text{Re}(z) = r \cos \phi \quad \text{Im}(z) = r \sin \phi$$

$$\phi = \tan^{-1} \left[\frac{y}{x} \right]$$

Complex algebra

$$z_1 = x_1 + jy_1 = r_1 e^{j\phi_1} \quad z_2 = x_2 + jy_2 = r_2 e^{j\phi_2}$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$e^x e^y = e^{x+y}$$

Multiplication:

$$z_1 z_2 = (r_1 e^{j\phi_1}) (r_2 e^{j\phi_2}) = r_1 r_2 e^{j\phi_1} e^{j\phi_2} = \underline{r_1 r_2} e^{j(\phi_1 + \phi_2)} = r_1 r_2 \angle (\phi_1 + \phi_2)$$

Division:

$$z_1 / z_2 = \frac{x_1 + jy_1}{x_2 + jy_2} = \frac{r_1 e^{j\phi_1}}{r_2 e^{j\phi_2}} = \frac{r_1}{r_2} \frac{e^{j\phi_1}}{e^{j\phi_2}} = \frac{r_1}{r_2} e^{j\phi_1} e^{-j\phi_2} = \frac{r_1}{r_2} e^{j(\phi_1 - \phi_2)}$$

Inversion:

$$1/z_1 = \frac{1}{r_1 e^{j\phi_1}} = \frac{1}{r_1} e^{-j\phi_1}$$

Square root:

$$\sqrt{z_1} = (r_1 e^{j\phi_1})^{1/2} = r_1^{1/2} (e^{j\phi_1})^{1/2} = r_1^{1/2} e^{j\phi_1/2}$$

Complex conjugate:

$$Z^* = x - jy = r e^{-j\phi}$$

Euler relationship

$$e^{j\phi} = \cos \phi + j \sin \phi$$

$$\Rightarrow \cos \phi = \operatorname{Re}(e^{j\phi})$$

$$e^x e^y = e^{x+y}$$
$$e^{z_1} e^{z_2} = e^{z_1+z_2}$$

Phasors:

$$V(t) = V_m \cos(\omega t + \phi) = \operatorname{Re}(V_m e^{j(\omega t + \phi)})$$
$$= V_m \operatorname{Re}(e^{j(\omega t + \phi)}) = \operatorname{Re}(V_m e^{j\phi} e^{j\omega t})$$

“Phasor” \mathbf{V}

(Complex #)

Phasors

$$V(t) = V_m \cos(\omega t + \phi) = \operatorname{Re}\left(V_m e^{j(\omega t + \phi)}\right)$$

$$= \operatorname{Re}\left(\underbrace{V_m e^{j\phi}} e^{j\omega t}\right)$$

“Voltage Phasor” **V**

(Complex #)

$$i(t) = I_m \cos(\omega t + \phi) = \operatorname{Re}\left(I_m e^{j(\omega t + \phi)}\right)$$

$$= \operatorname{Re}\left(\underbrace{I_m e^{j\phi}} e^{j\omega t}\right)$$

“Current Phasor” **I**

$$i(t) = I_0 \cos(\omega t)$$

$$I_m = I_0 \quad \vec{I}_1 = I_0 \\ \phi = 0$$

$$\frac{dF(t)}{dt} = a F(t)$$

general solution $F(t) = C e^{at}$

$$\begin{aligned} \frac{d}{dx} (e^{at}) &= c \frac{d}{dx} (e^{at}) = c a e^{at} \\ &= a (c e^a) \\ &= a F(t) \end{aligned}$$

$$\frac{d^2 F(t)}{dt^2} = b F(t)$$

general solution $F(t) = C_1 \sin(\sqrt{b}t) + C_2 \cos(\sqrt{b}t)$

$$a \frac{d^2 F(t)}{dt^2} + b \frac{dF(t)}{dt} + c F(t) = g(t)$$

general solution = ...

Circuits

$$v(t) = R i(t)$$

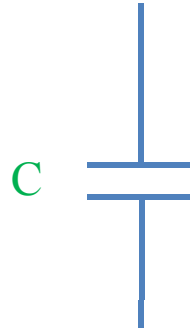


$$\mathbf{V} = \mathbf{I} R$$

$$V = I R$$

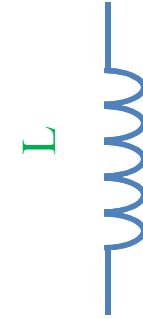
$$q(t) = C v(t)$$

$$i(t) = C \frac{dv(t)}{dt}$$



$$\mathbf{V} = \mathbf{I} / j\omega C$$

$$v(t) = L \frac{di(t)}{dt}$$



$$\mathbf{V} = j\omega L \mathbf{I}$$

“Impedance”

$$Z = R$$

$$Z = 1 / j\omega C$$

$$Z = j\omega L$$

KCL, KVL hold for relationship between \mathbf{V} , \mathbf{I} .

Series/Parallel Impedances



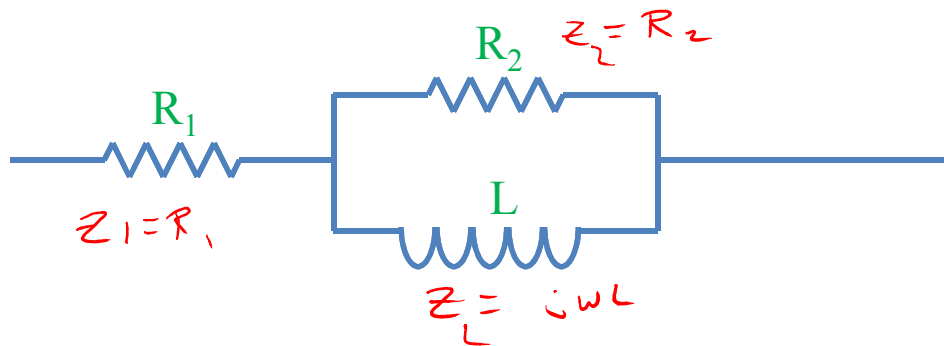
$$Z_{eq} = Z_1 + Z_2 + Z_3$$



$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

Example problem #1

Find Z_{eq} for this circuit: (instructor)

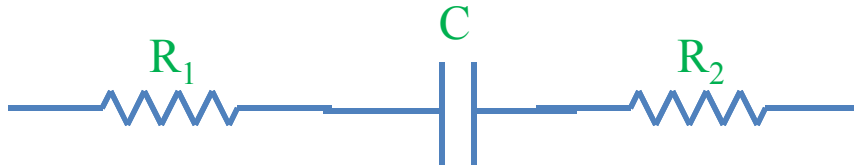


$$\frac{j\omega L R_2}{j\omega L + R_2}$$

$$R_1 + \frac{(j\omega L R_2) \frac{1}{R_2}}{(R_2 + j\omega L) \frac{1}{R_2}} = R_1 + \frac{j\omega L}{1 + j\omega(L/R_2)}$$

Example problem #2

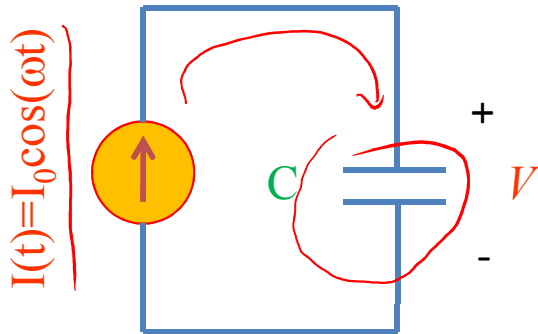
Find Z_{eq} for this circuit: (students)



$$R_1 + \frac{1}{j\omega C} + R_2$$

Phasor to voltage conversion

Find $V(t)$, $q(t)$



$$I(t) = \text{Re} (I e^{j\omega t}) = I_0 \cos(\omega t)$$

$$\Rightarrow \underline{I} = I_0$$

$$Z = \frac{1}{j\omega C} \quad \frac{V}{I} = \frac{1}{j\omega C}$$

$$V = \frac{1}{j\omega C} I = \frac{I_0}{j\omega C}$$

$$q = cv$$

$$i = c \frac{dv}{dt}$$

$$v = \frac{1}{c} \int i dt$$

$$= \frac{1}{c} \int I_0 \cos(\omega t) dt$$

$$= \frac{I_0}{\omega C} \sin(\omega t)$$

$$q = \frac{I_0}{\omega C} \cos(\omega t)$$

q should be sin not cos

$$\frac{1}{j} = -j$$

$$V(t) = \text{Re} (v e^{j\omega t}) = \text{Re} \left[\frac{I_0}{j\omega C} e^{j\omega t} \right] =$$

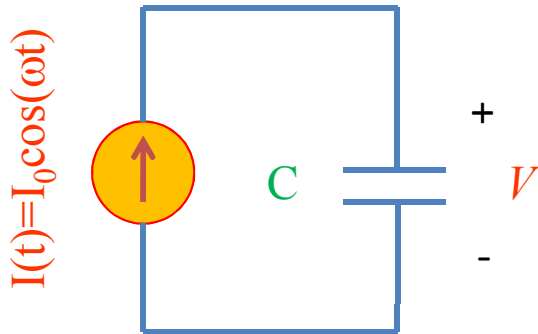
$$= \text{Re} \left[\frac{I_0}{\omega C} (-j) e^{j\omega t} \right] = -\frac{I_0}{\omega C} \text{Re} [j e^{j\omega t}]$$

$$= -\frac{I_0}{\omega C} \text{Re} [j (\cos(\omega t) + j \sin(\omega t))] = -\frac{I_0}{\omega C} \text{Re} [j \cos(\omega t) + j^2 \sin(\omega t)]$$

$$= -\frac{I_0}{\omega C} \text{Re} [-\sin(\omega t) + j \cos(\omega t)] = \frac{I_0}{\omega C} \sin(\omega t)$$

Phasor to voltage conversion

Find $V(t)$, $q(t)$



Problem gives us:

$$I(t) = I_0 \cos(\omega t)$$

Compare to definition of current phasor:

$$i(t) = I_m \cos(\omega t + \phi) = \text{Re} \left(I_m e^{j\phi} e^{j\omega t} \right) = \text{Re} \left(\mathbf{I} e^{j\omega t} \right)$$

$$\Rightarrow \mathbf{I} = I_0$$

Find voltage phasor using generalized Ohm's law:

$$\mathbf{V} = \mathbf{I} / j\omega C$$

Find $v(t)$ from voltage phasor:

$$v(t) = \text{Re} \left[\mathbf{V} e^{j\omega t} \right] = \text{Re} \left[\left(\frac{\mathbf{I}}{j\omega C} \right) e^{j\omega t} \right] = \text{Re} \left[\left(\frac{I_0}{j\omega C} \right) e^{j\omega t} \right]$$

$$= \frac{I_0}{\omega C} \text{Re} \left[\left(\frac{1}{j} \right) e^{j\omega t} \right] = \frac{I_0}{\omega C} \text{Re} \left[-j e^{j\omega t} \right] = \frac{I_0}{\omega C} \text{Re} \left[-j (\cos(\omega t) + j \sin(\omega t)) \right]$$

$$= \frac{I_0}{\omega C} \text{Re} \left[-j \cos(\omega t) + (-j)j \sin(\omega t) \right] = \frac{I_0}{\omega C} \text{Re} \left[\sin(\omega t) - j \cos(\omega t) \right] = \frac{I_0}{\omega C} \sin(\omega t)$$

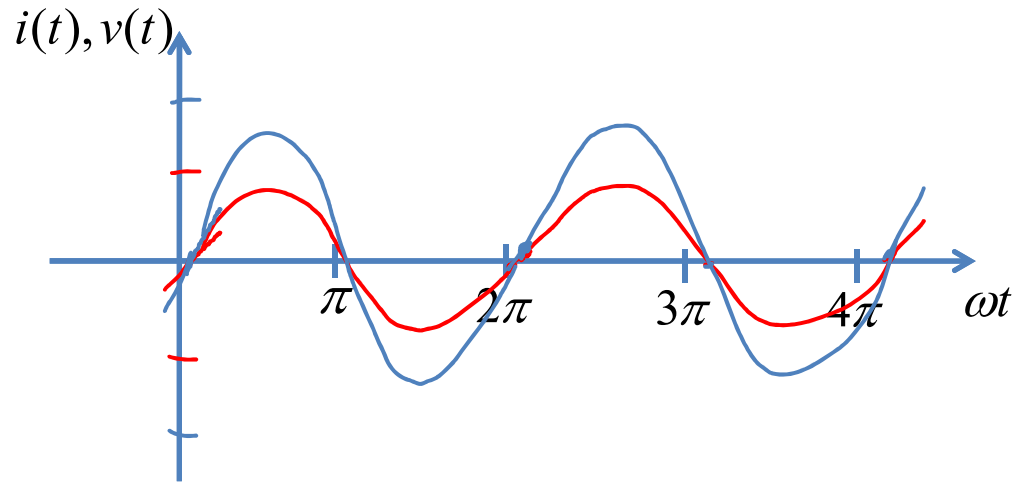
Phase vs. impedance (Z)

In general:

$$Z \text{ real i.e. } Z = x + jy$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \neq 0 & = 0 \end{array}$$

$\Rightarrow i(t), v(t)$ “in phase”

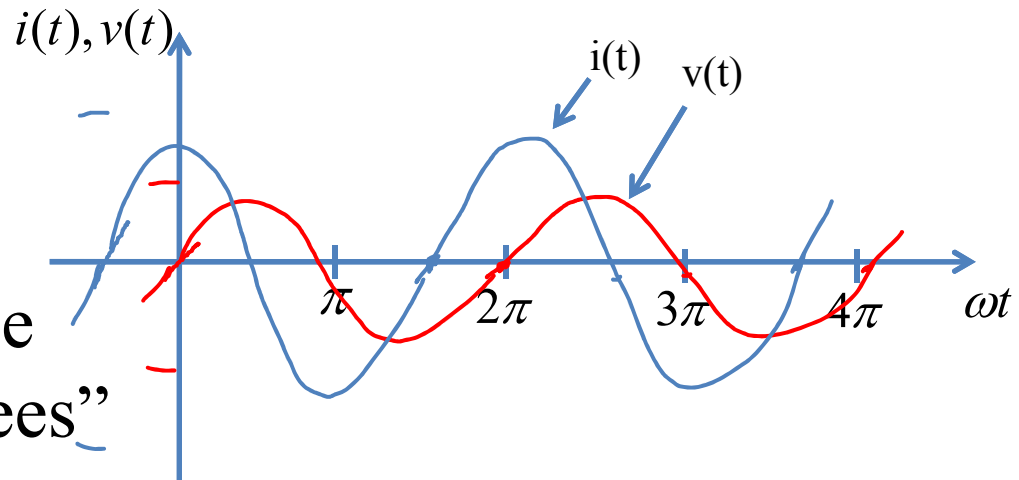


In general:

$$Z \text{ imag i.e. } Z = x + jy$$

$$\begin{array}{cc} \uparrow & \uparrow \\ = 0 & \neq 0 \end{array}$$

$\Rightarrow i(t), v(t)$ “out of phase by 90 degrees”

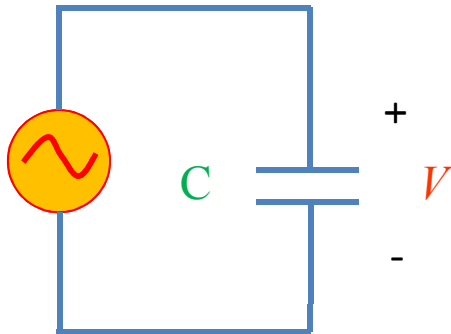


HW problem: Find relationship between phase shift and impedance (Z).

Example phasor problem

Find $i(t)$ (students)

$$v(t) = 3 \cos(10t + 30^\circ) \text{ Volts}$$



Problem gives us:

$$v(t) = 3 \cos(10t + 30^\circ) \text{ Volts}$$

Compare to definition of voltage phasor:

$$v(t) = V_m \cos(\omega t + \phi) = \text{Re}(V_m e^{j\phi} e^{j\omega t}) = \text{Re}[\mathbf{V} e^{j\omega t}]$$

$$\Rightarrow \mathbf{V} = ?$$

Find current phasor using generalized Ohm's law:

$$\mathbf{I} = j\omega C \mathbf{V} = \dots$$

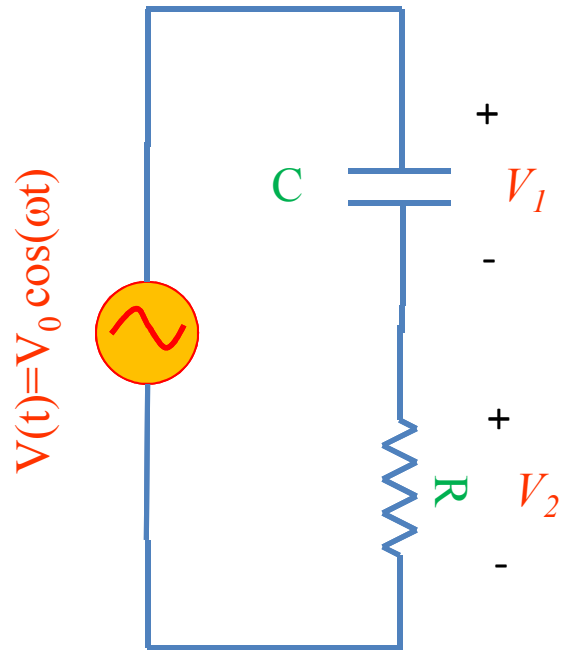
Find $i(t)$ from current phasor:

$$i(t) = \text{Re}(\mathbf{I} e^{j\omega t}) = \dots$$

Trick #1

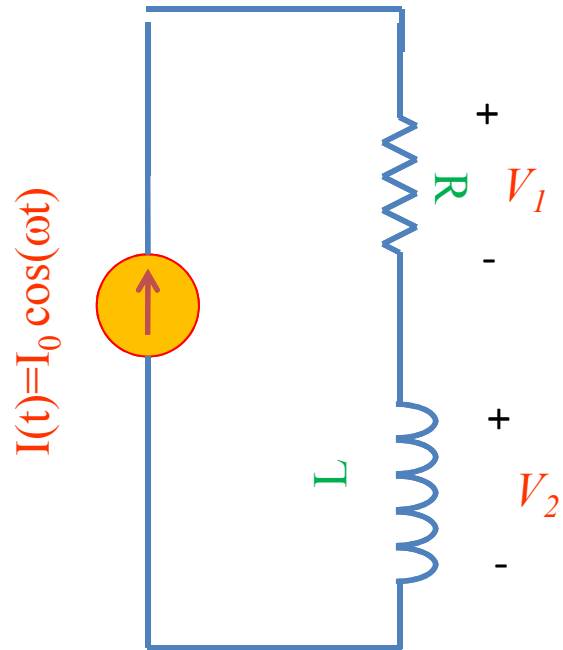
Example problem #3

Find $i(t)$, $V_1(t)$, $V_2(t)$ for this circuit: (instructor)



Example problem #4

Find $i(t)$, $V_1(t)$, $V_2(t)$ for this circuit: (students)



Low pass filter

