# EECS 70A: Network Analysis

June 5, 2014

Comprehensive review

# Topics covered

- KCL, KVL
- Nodal analysis
- Mesh analysis
- Thevenin/Norton theorem
- RL, RC circuits (time dependence)
- R,L,C circuits
  - Phasors
  - Impedances
  - Transfer function/Bode
  - Power

# Nodal Analysis(Review)

Based on KCL, use node voltages as circuits variables.

1. Define a reference node.
2. Label remaining nodes. (n-1 nodes)
3. Apply KCL + ohm to all nodes and supernodes (e.g. $V_1$, $V_2$, $V_3$, …)
   Express all i's in terms of v's
4. Apply KVL to the voltage source
   If one end of voltage source connected to ground, don't need to
5. Solve the n-1 simultaneous equations, to find V's
6. Use Ohm's law to find the currents.

# Mesh analysis summary

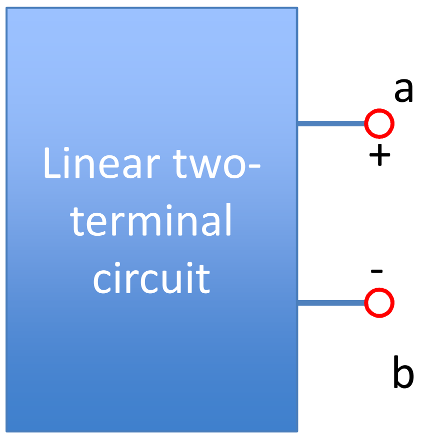
Based on KVL, use mesh currents as circuit variables.

1. Assign mesh currents $i_1$, $i_2$, … $i_n$
2. Apply KVL + Ohm's law to each mesh
3. Supermesh (if there is a current source present):

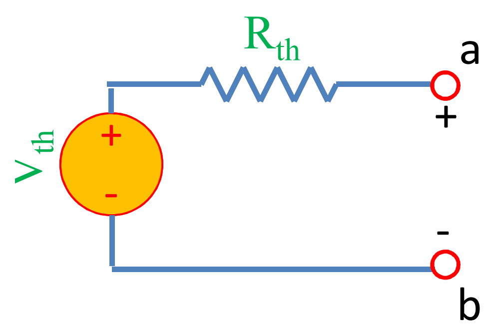
- CASE 1: current source only in one mesh.
  - Already have the current for that mesh => no need to write KVL for that mesh
- CASE 2: current source exits between two meshes. => create a supermesh
  - Apply KVL to the supermesh
  - Apply KCL to a node in the branch where two meshes intersect

1. Solve the equations for $i_1$ ,$i_2$ ,…$i_n$
   (e.g. using Kramer's rule)
2. Then solve for voltages using Ohm's law

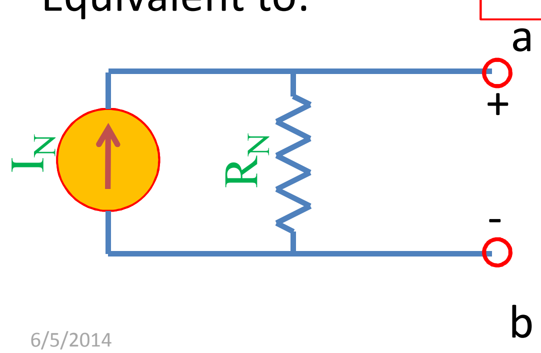
# Thevenin, Norton Theorems:




Equivalent to:




Equivalent to:




Thevenin:

1. **Calculating $V_{th}$:**
   Connect nothing to a-b. Calculate voltage. This is $V_{th}$.
2. **Calculating $R_{th}$:**
   Method 1:
   Connect terminal a to b (short).
   Calculate the current from a to b. This is call $I_{short\ circuit}$.
   $R_{th}=V_{th}/I_{short\ circuit}$.
   Method 2:
   Find the input resistance looking into terminals a-b after all the independent sources have been turned off. (Voltage sources become shorts, current sources become opens.)
   ***Trick (if dependent sources present):***
   Apply a 1 A current source to terminals a-b, find $V_{ab}$
   $R_{th} = V_{ab}/1A$.

Norton:

1. **Calculating $R_N$:**
   $R_N=R_{th}$
2. **Calculating $I_N$:**
   $I_N=V_{th}/R_{th}$

# Conversion procedures

i(t) → $\mathbf{I}$

v(t) → $\mathbf{V}$

$\mathbf{I}$ → i(t)

$\mathbf{V}$ → v(t)

$$i(t) = I_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{I} = I_m e^{j\phi}$$

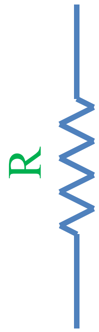
$$v(t) = V_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{V} = V_m e^{j\phi}$$

$$i(t) = \mathrm{Re}\left(\mathbf{I}\, e^{j\omega t}\right)$$

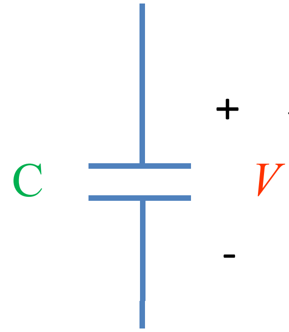
$$v(t) = \mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right)$$

For the exam, you should know how to carry out these procedures.

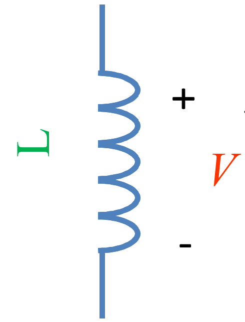
# Circuits




$\mathbf{V} = \mathbf{I}\, R$




$\mathbf{V}=\mathbf{I}/j\omega C$



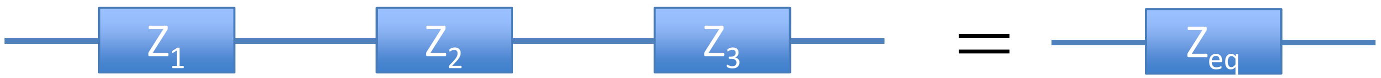

$\mathbf{V}=j\omega L\, \mathbf{I}$

"Impedance"

$Z = R$ $\qquad$ $Z=1/j\omega C$ $\qquad$ $Z = j\omega L$

KCL, KVL hold for relationship between **V**, **I**.

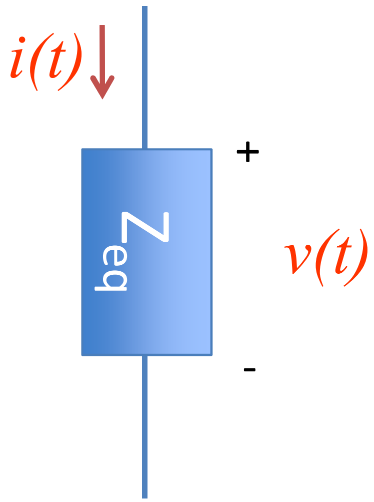
# Series/Parallel Impedances




$$Z_{eq} = Z_1 + Z_2 + Z_3$$


$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

# Conversion procedures




Given i(t) find v(t):

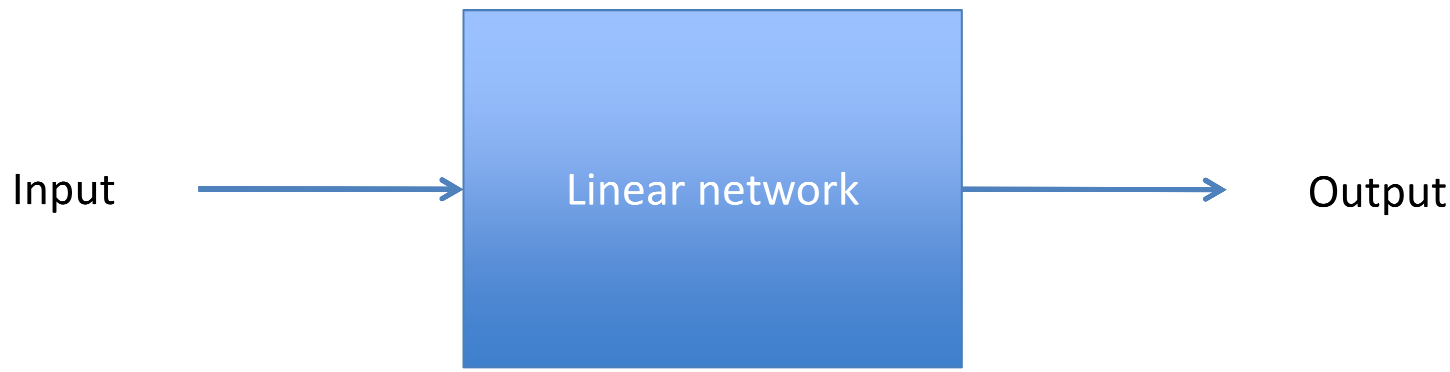
$$i(t) \rightarrow \mathbf{I} \rightarrow \mathbf{V} = \mathbf{I}\, Z_{eq} \rightarrow v(t)$$

Given v(t) find i(t):

$$v(t) \rightarrow \mathbf{V} \rightarrow \mathbf{I} = \mathbf{V} / Z_{eq} \rightarrow i(t)$$

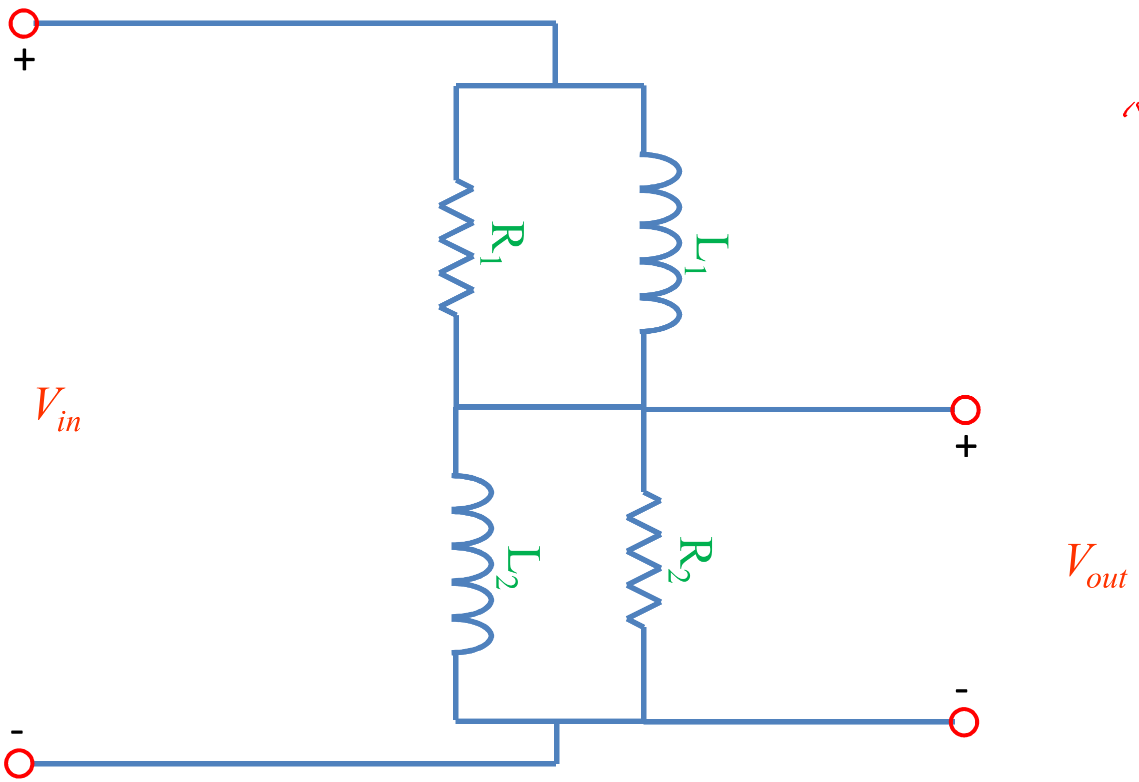
For the exam, you should know how to carry out these procedures.

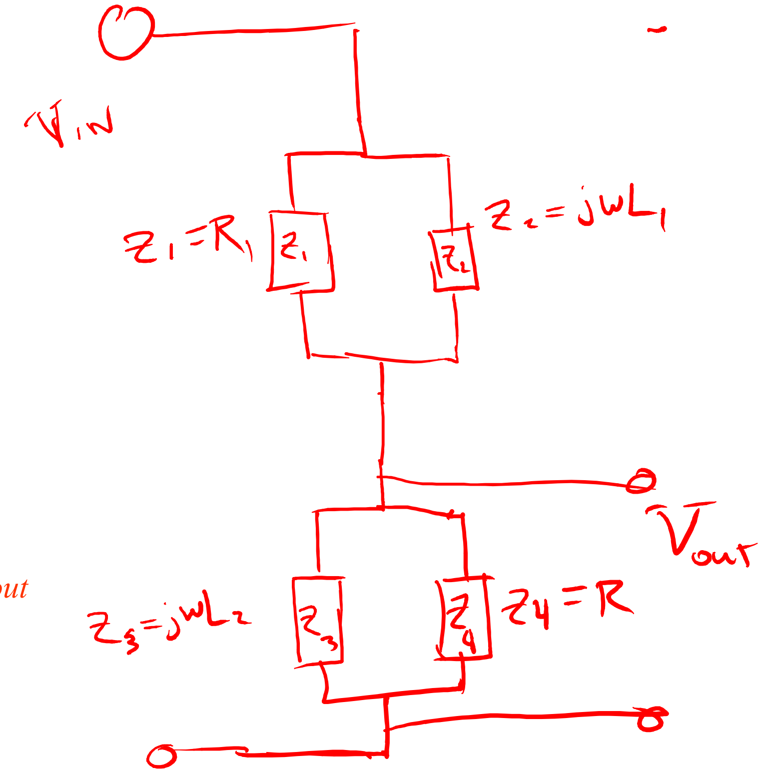
# “Transfer Function”



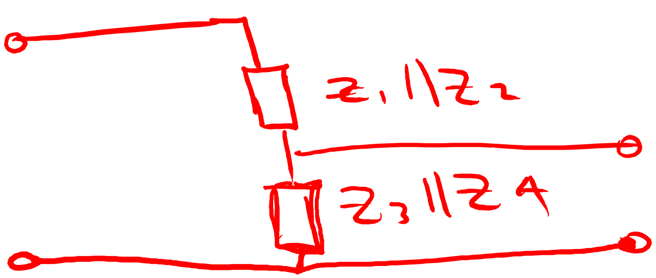

$$H(\omega) = \mathbf{V_{out}/V_{in}}$$

# Example Transfer function



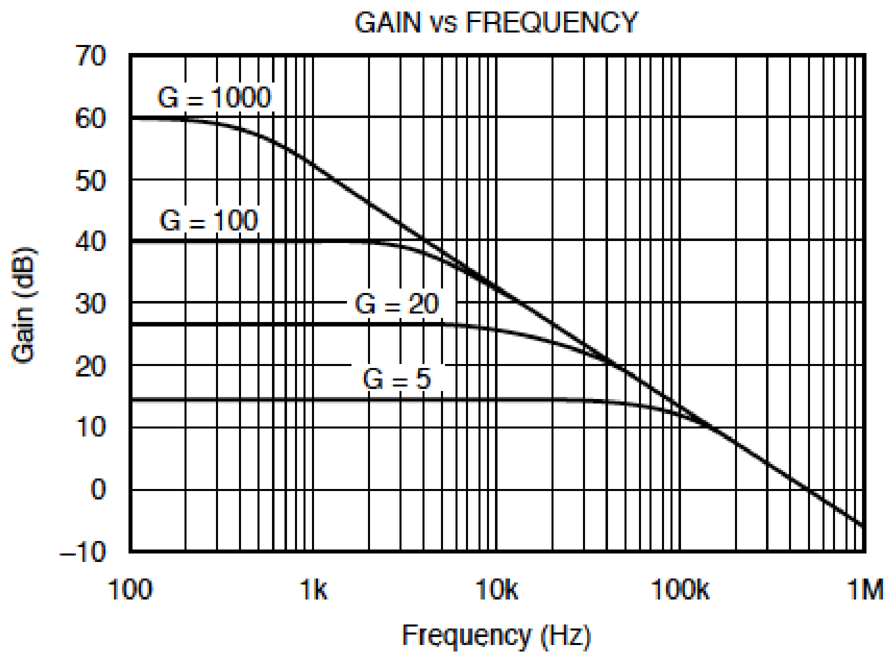




Calculate H(ω) for this circuit. Sketch the magnitude of H(ω) vs. ω.



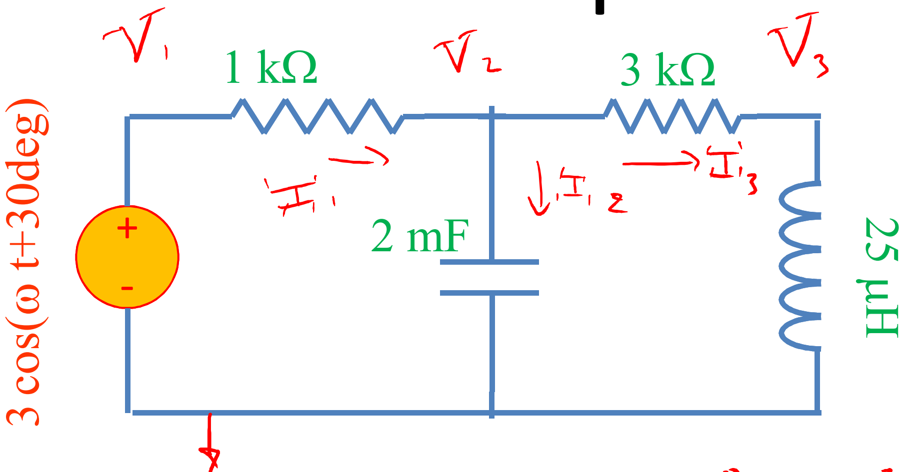

$$\frac{V_{out}}{V_{in}} = \frac{(Z_3 \| Z_4)}{(Z_3 \| Z_4) + (Z_1 \| Z_2)} = H(\omega)$$

# Bode example




Find Vout(t) given:
Vin(t)= 1 mV cos ( 2*pi*1kHz*t)

# Comprehensive Example




Solve all currents, voltages using Nodal. Do again using mesh.

$i_1(t) = \mathrm{Re}\left[I_1 e^{j\omega t}\right]$

$V$

$$I_1 = \frac{V_1 - V_2}{1k\Omega}$$

$$I_2 = \frac{V_2}{\frac{1}{j\omega 2mF}}$$

$$I_3 = \frac{V_3}{j\omega 25\mu H}$$

N1 $\quad V_1 = 3e^{j30^\circ} = 3e^{j\pi/6}$

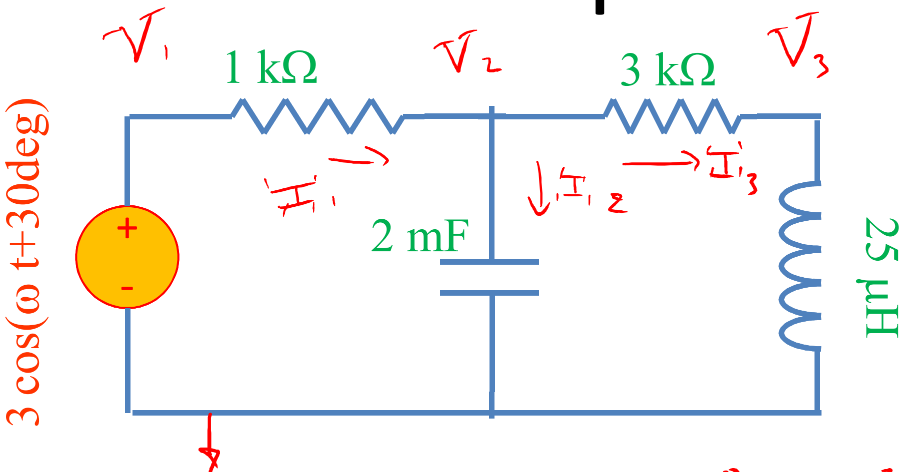
N2 $\quad I_1 = I_2 + I_3$

$$\frac{V_1 - V_2}{1k\Omega} = \frac{V_2 - 0}{\frac{1}{j\omega 2mF}} + \frac{V_2 - V_3}{3k\Omega}$$

N3 $\quad I_3 = I_3 \Rightarrow \frac{V_2 - V_3}{3k\Omega} = \frac{V_3 - 0}{j\omega 25\mu H}$

2 eq. 2 unknowns Find $V_2$, $V_3$.

# Comprehensive Example




Solve all currents, voltages using Nodal. Do again using mesh.

$i_1(t) = \mathrm{Re}\left[\bar{I}_1 e^{j\omega t}\right]$

$\bar{V}$

$\bar{I}_1 = \dfrac{\bar{V}_1 - \bar{V}_2}{1\,k\Omega}$

$\bar{I}_2 = \dfrac{\bar{V}_2}{\frac{1}{j\omega\, 2mF}}$

$\bar{I}_3 = \dfrac{\bar{V}_3}{j\omega\, 25\mu H}$

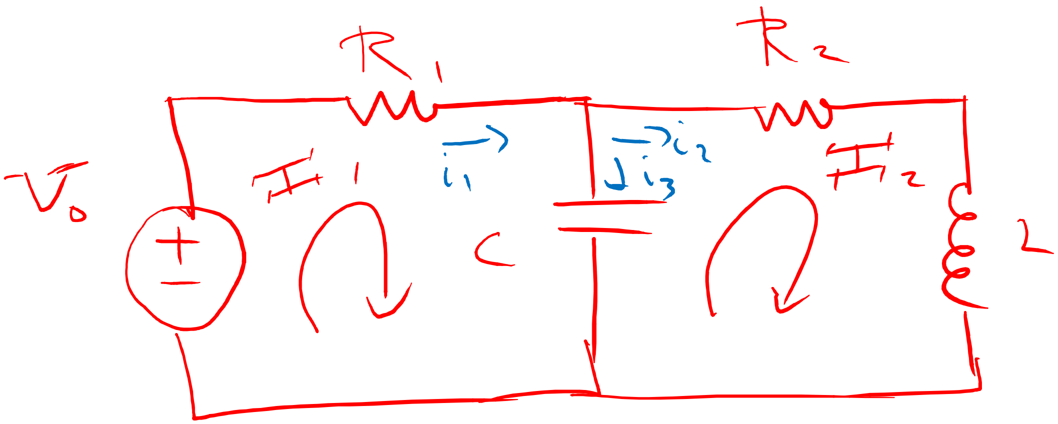
(N1) $\bar{V}_1 = 3e^{j30^\circ} = 3e^{j\pi/6}$

(N2) $\bar{I}_1 = \bar{I}_2 + \bar{I}_3$

$\dfrac{\bar{V}_1 - \bar{V}_2}{1\,k\Omega} = \dfrac{\bar{V}_2 - 0}{\frac{1}{j\omega 2mF}} + \dfrac{\bar{V}_2 - \bar{V}_3}{3\,k\Omega}$

(N3) $\bar{I}_3 = \bar{I}_3 \Rightarrow \dfrac{\bar{V}_2 - \bar{V}_3}{3\,k\Omega} = \dfrac{\bar{V}_3 - 0}{j\omega\, 25\mu H}$

2 eq. 2 unknowns. Find $\bar{V}_2$, $\bar{V}_3$.




$$i_1(t) = \mathrm{Re}\left[ I_1 \, e^{j\omega t} \right]$$

$$i_3(t) = \mathrm{Re}\left[ (I_1 - I_2) e^{j\omega t} \right]$$

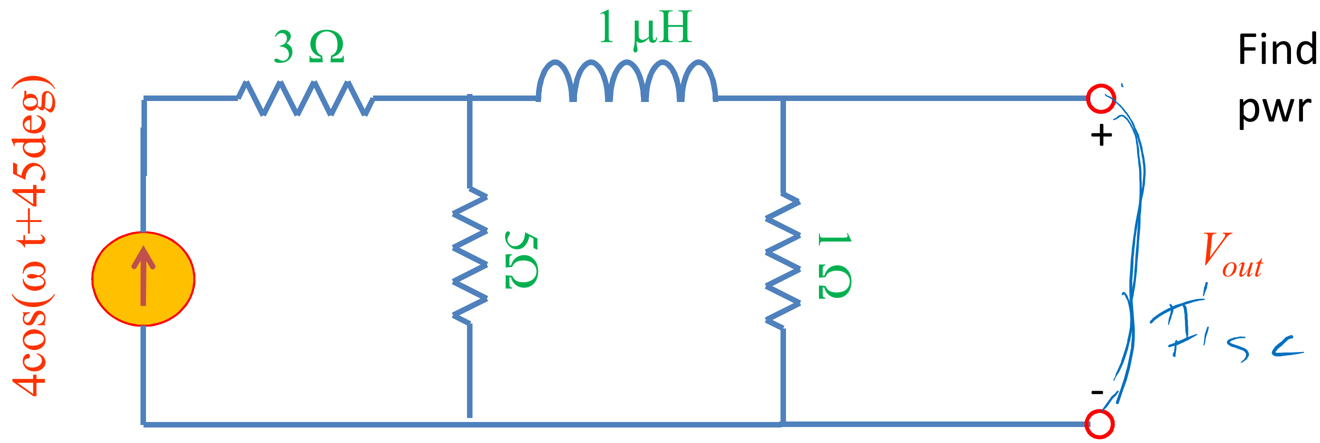
$$i_2(t) = \mathrm{Re}\left[ I_2 \, e^{j\omega t} \right]$$

M1: $V_0 + I_1 R + (I_1 - I_2) \frac{1}{j\omega C} = 0$

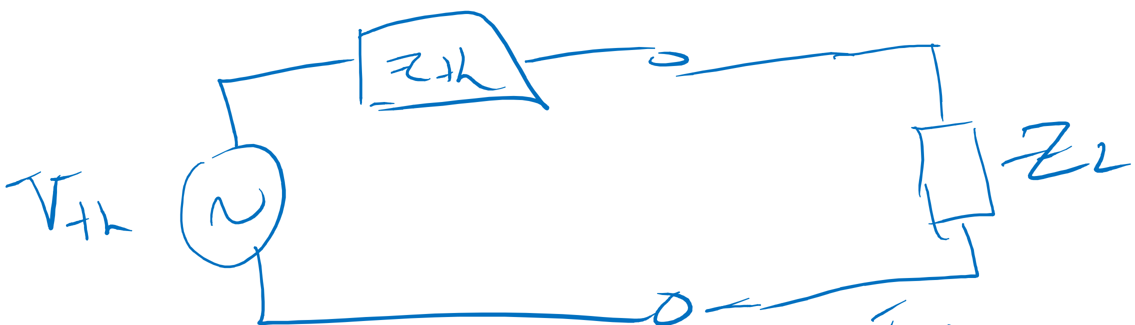
M2: $(I_2 - I_1) \frac{1}{j\omega C} + I_2 R_2 + I_2 \, j\omega L = 0$

Solve $I_1$, $I_2$

# Phasor Example 2




Find Thev Eq Circ + max pwr txfr load impedance.

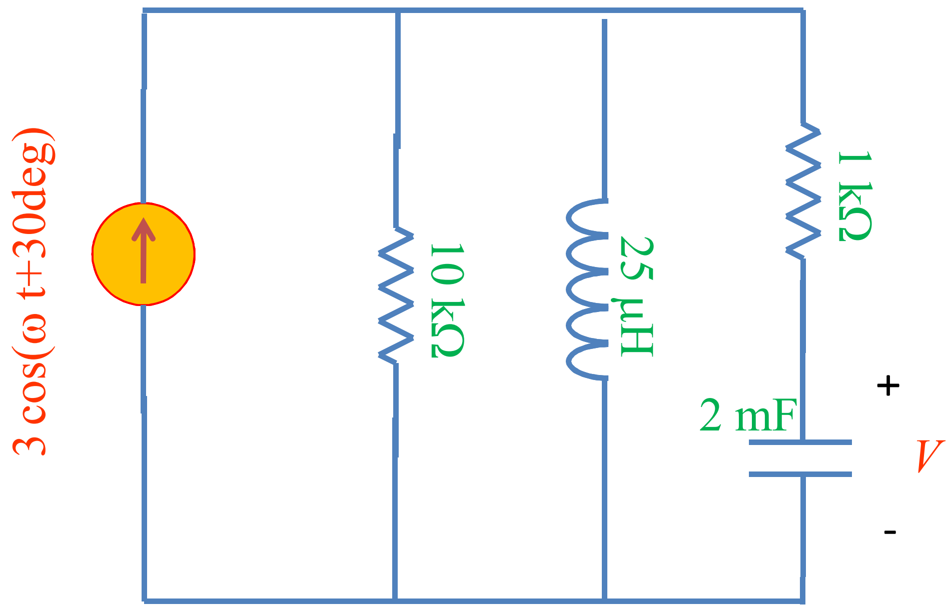



$$Z_L = Z_{th}^*$$

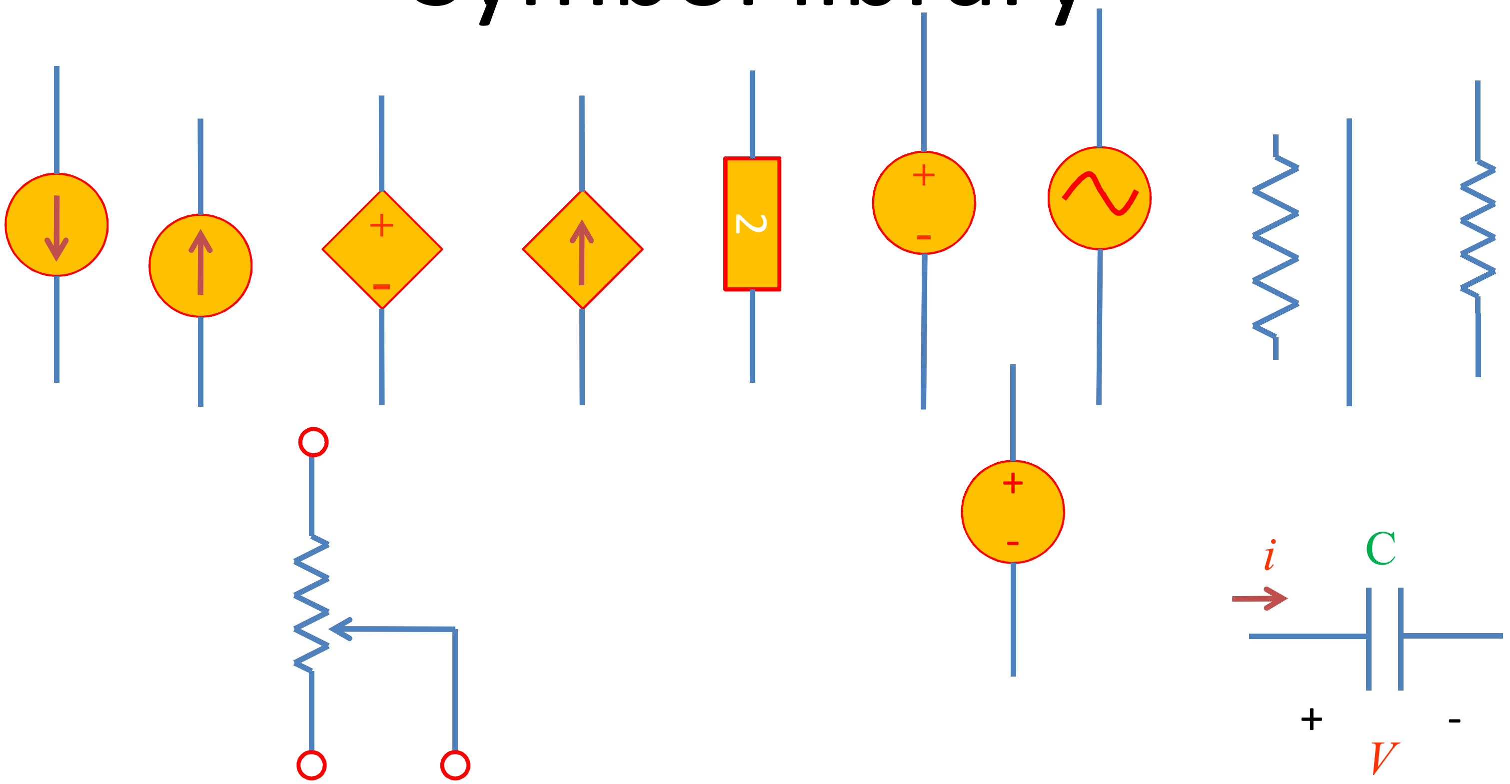
$$Z_{th} = \frac{V_{th}}{I_{sc}}$$

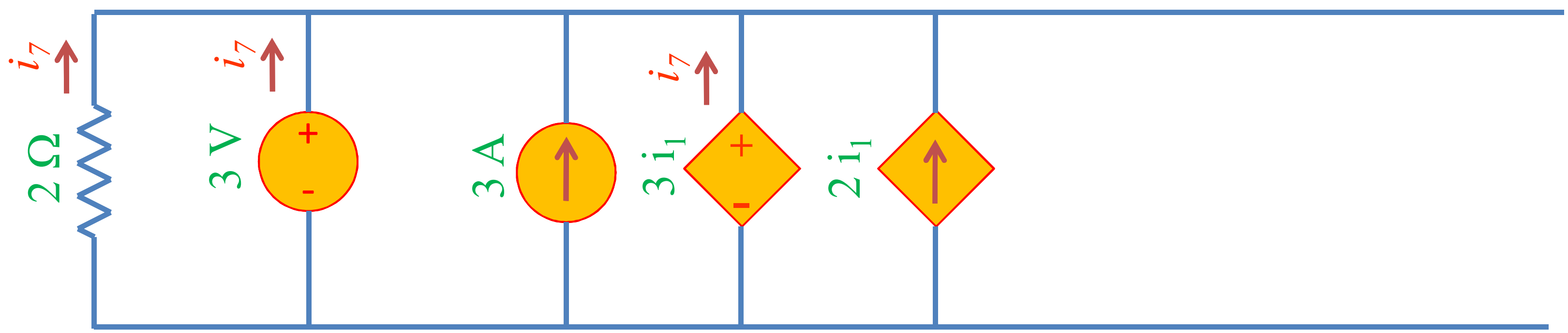
# Phasor Example 1

Find v(t).



# Symbol library



# Exam cheat sheet

This will be provided with the exam.

| radians : | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| sin | $\frac{\sqrt{0}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{4}}{2}$ | $0$ |
| cos | $\frac{\sqrt{4}}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{0}}{2}$ | $-1$ |
| tan | $\frac{\sqrt{0}}{\sqrt{4}}$ | $\frac{\sqrt{1}}{\sqrt{3}}$ | $\frac{\sqrt{2}}{\sqrt{2}}$ | $\frac{\sqrt{3}}{\sqrt{1}}$ | DNE | $0$ |

where $\sqrt{\cdot}$ always denotes the positive square root, and DNE means does not exist.