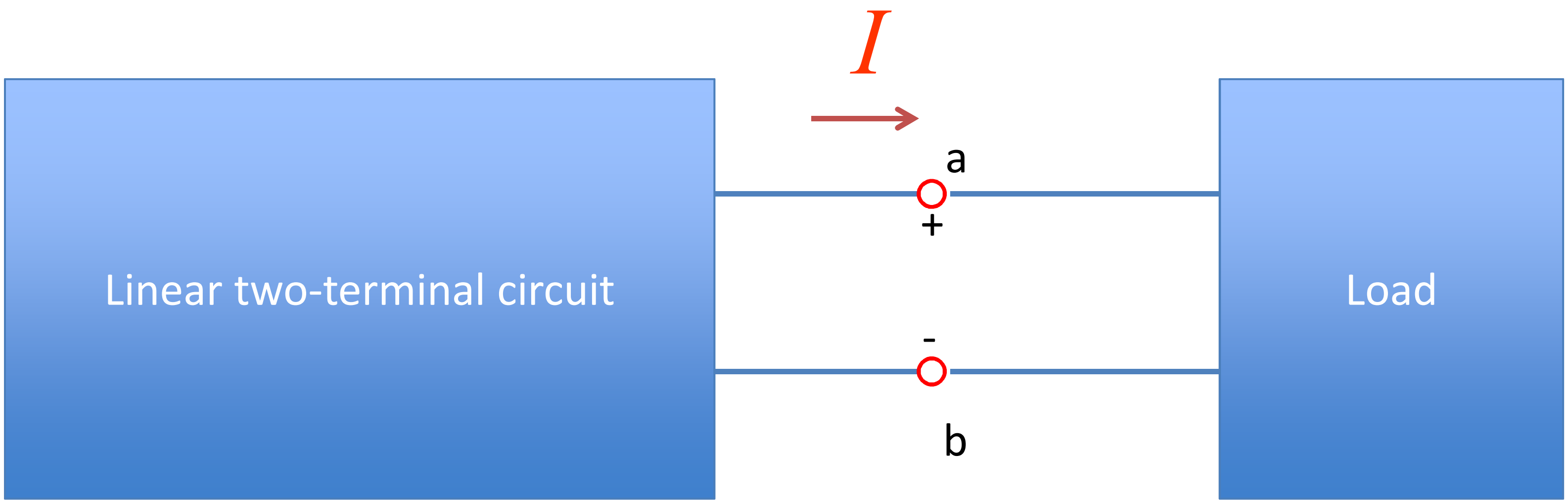
Announcements:

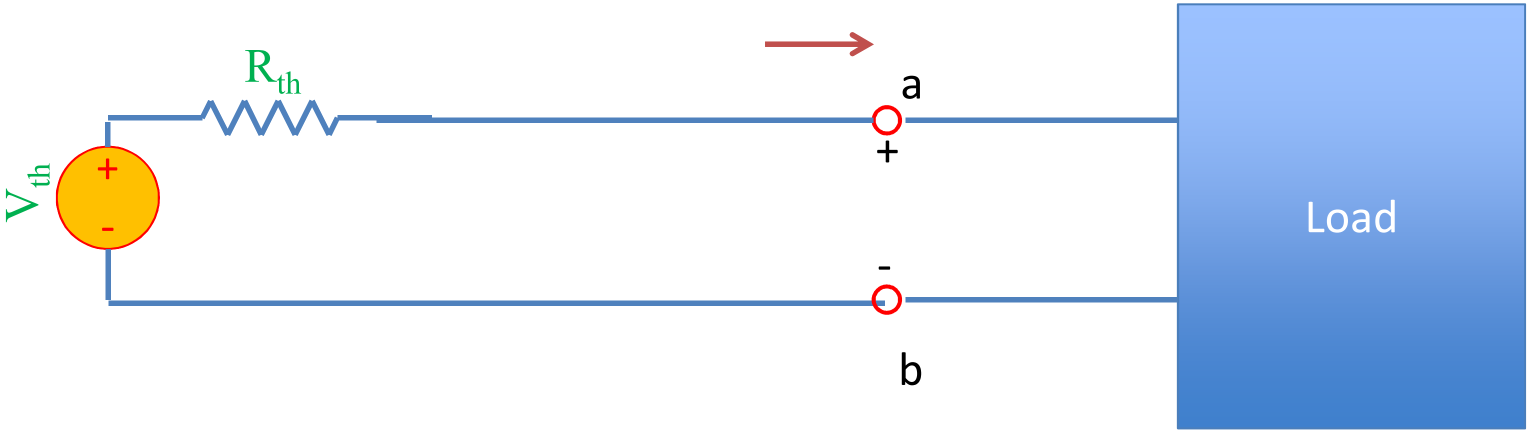
1. Announcements

EECS 70A: Network Analysis

Thevenin's Theorem



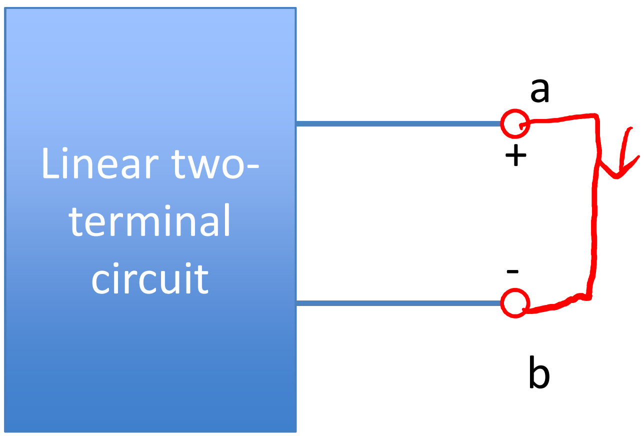
Equivalent to:



Finding V_{th}, R_{th}

Goal: Find V_{th}, R_{th}

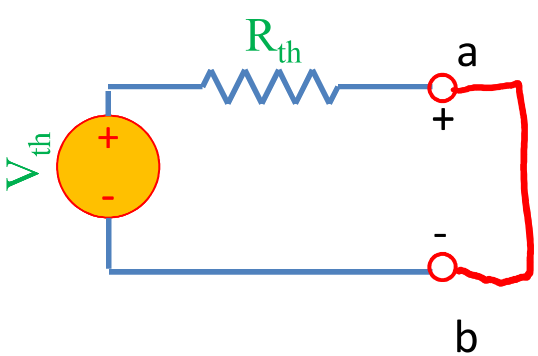
Given: Circuit



1) Find V_{th}

Calculate V_{ab} (when no external circuit connected)

Equivalent to:

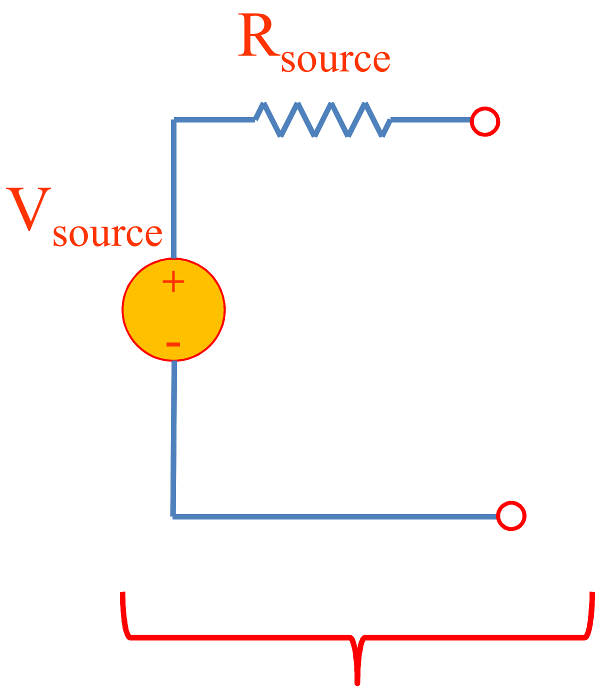


$I_{s.c.} = \frac{V_{th}}{R_{th}}$

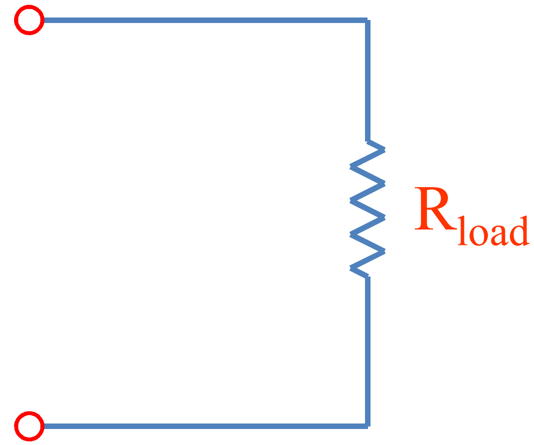
2) Find R_{th}

$$R_{th} = \frac{V_{th}}{I_{\text{short circuit}}}$$

Source/load



Thevenin Thm:
Any circuit can be represented by this equivalent circuit.



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Case 1:

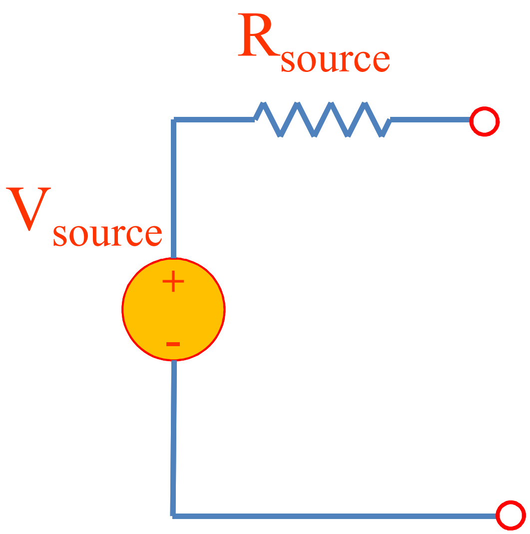
$$R_{load} >> R_{source}$$

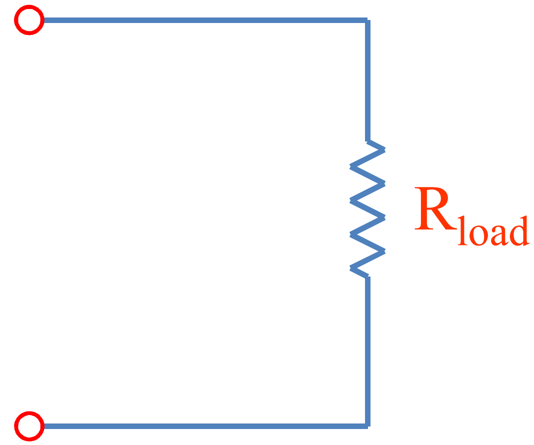
Case 2:

$$R_{source} >> R_{load}$$

We say R_{load} *"loads down"* the source.

Source/load





$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Thevenin Thm:
<u>Any</u> circuit can be represented by this equivalent circuit.

Case 1:

$$R_{load} >> R_{source}$$

Case 2:

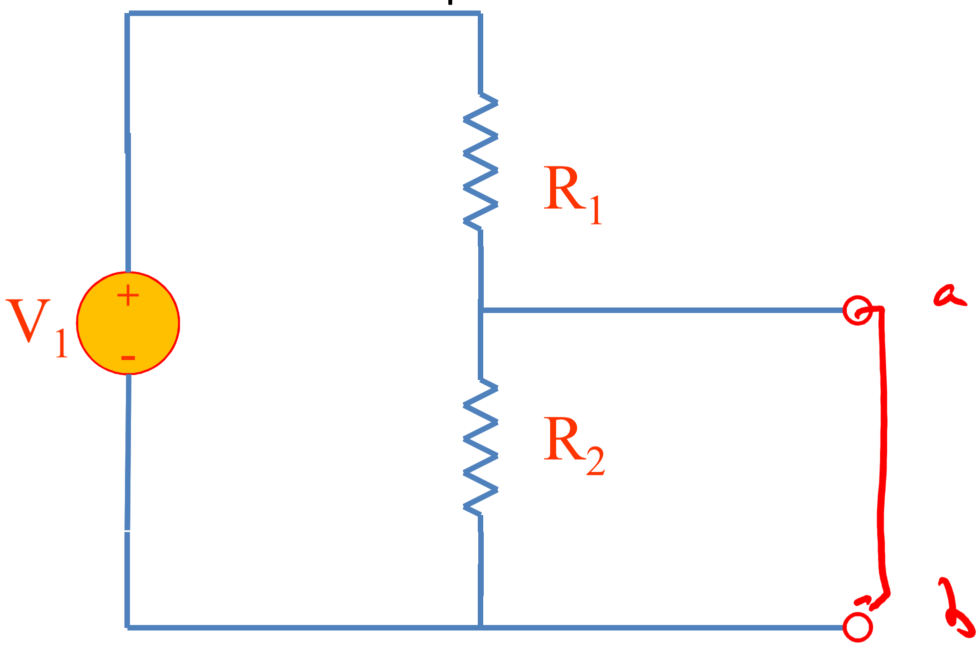
$$R_{source} >> R_{load}$$

We say R_{load} *"loads down"* the source.

IMPORTANT!

Example

Find Thevenin equivalent circuit:

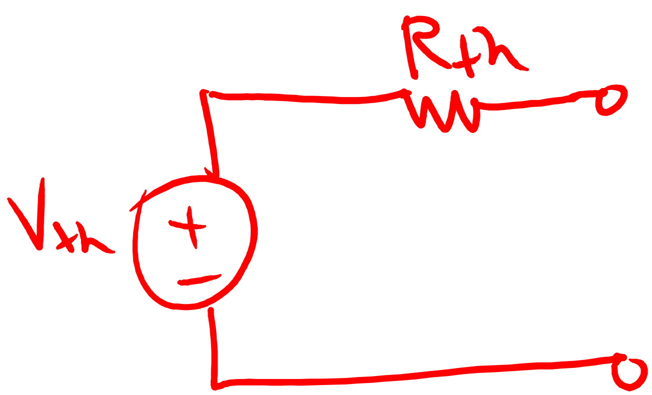


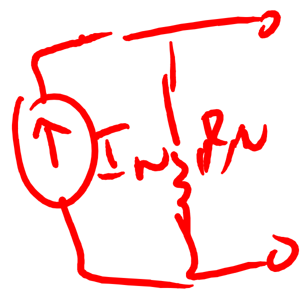
1) $V_{th} = V_{ab}$ (open)

$$= V_1 \frac{R_2}{R_1+R_2}$$

2) $$R_{th} = \frac{V_{th}}{I_{short\ circuit}}$$

$$= \frac{V_1 \frac{R_2}{R_1+R_2}}{I_{short\ circuit}}$$

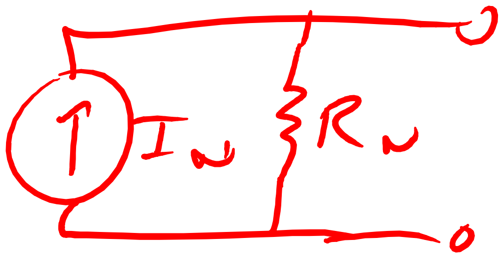




$$= \frac{\frac{\cancel{V_1} R_2}{R_1+R_2}}{\frac{\cancel{V_1}}{R_1}} = \frac{R_1 R_2}{R_1+R_2}$$

$$= R_1 \| R_2$$

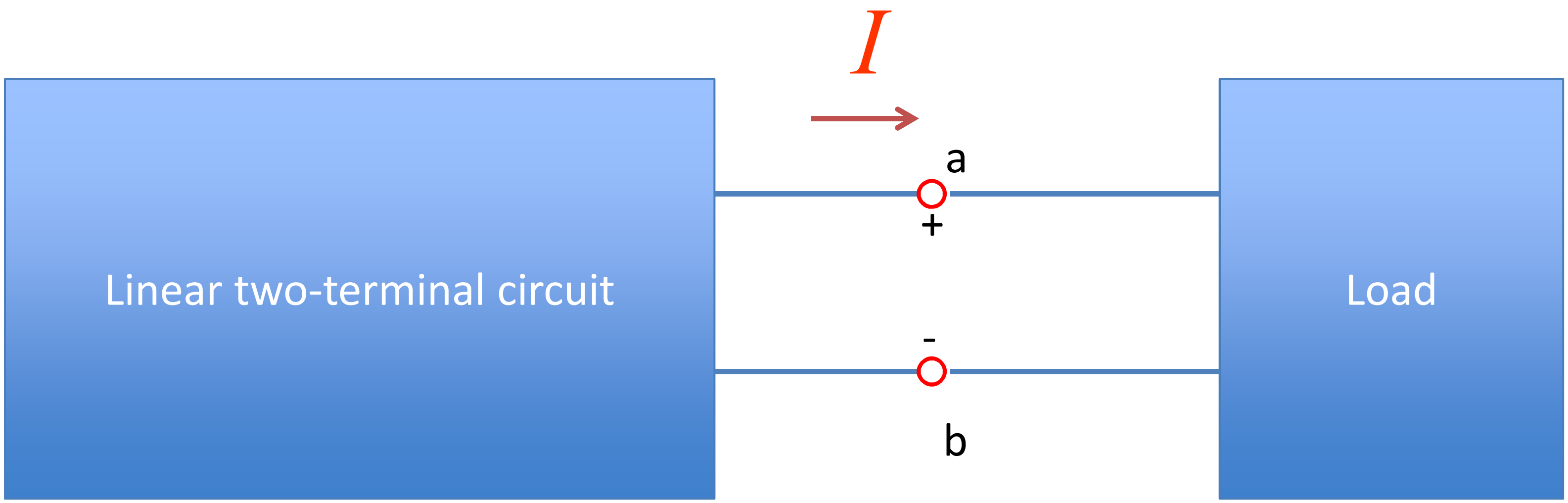
EX PART 2 FIND NORTON EQ CIRC.



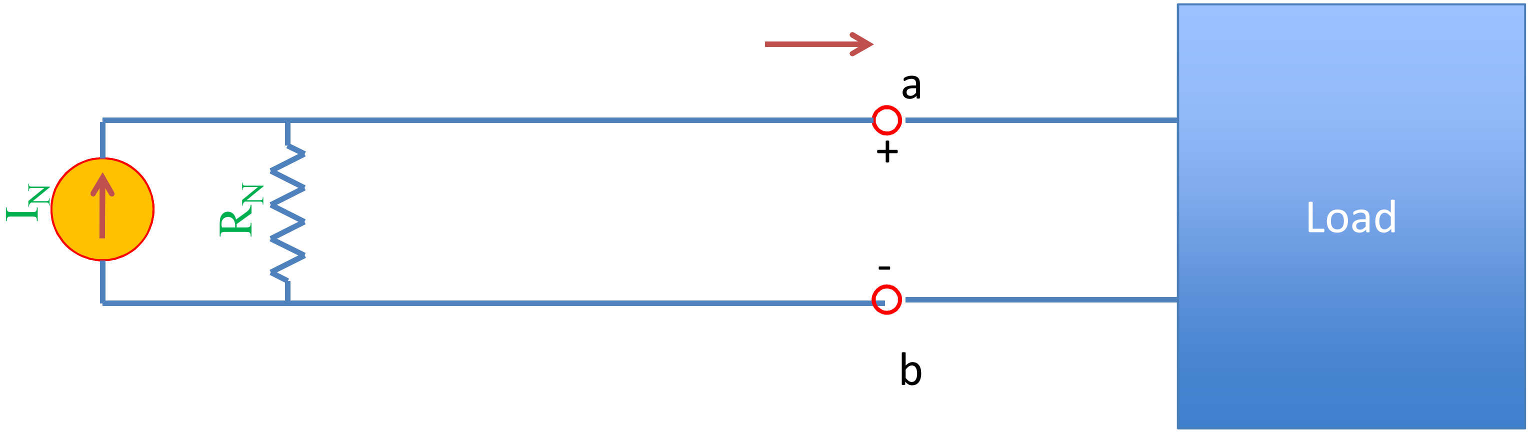
$$R_N = R_1 \| R_2 \; (= R_{th})$$

$$I_N = \frac{V_{th}}{R_N} = \frac{V_1 \frac{\cancel{R_2}}{\cancel{R_1+R_2}}}{\frac{R_1 \cancel{R_2}}{\cancel{R_1+R_2}}} = \frac{V_1}{R_1}$$

Norton's Theorem

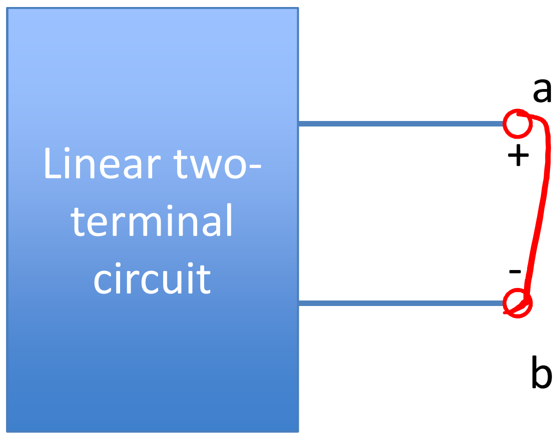


Equivalent to:



Finding V_{th}, R_{th}

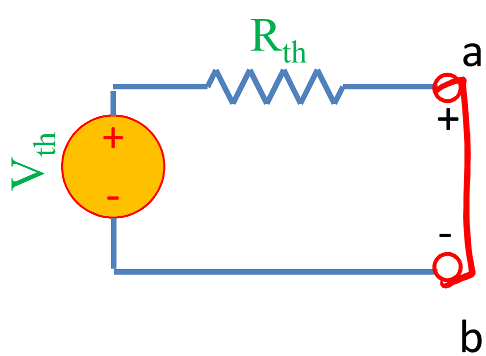
Once you find R_{th}, V_{th} easy to find I_N, V_N



Open (i.e. nothing connected to a, b)

$V_{ab} = V_{th} = I_N R_N$ (*)

Equivalent to:

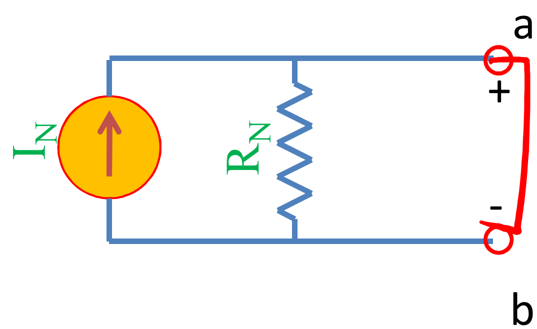


(Equivalent to:)

consider a short to b

$I_{short\ circuit} =$ (**)

$\frac{V_{th}}{R_{th}} = I_N$

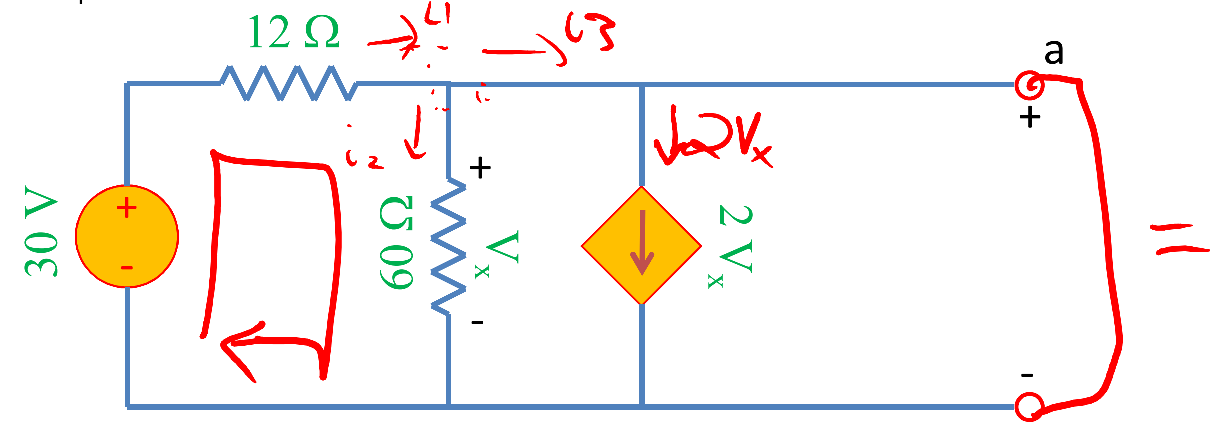


$R_N = R_{th}$

$I_N = V_{th}/R_{th}$

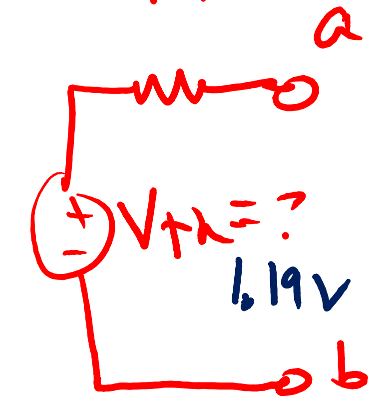
Example

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



0.476 Ω

$R_{th} = ?$



1) Find V_{ab}

$V_{ab} = V_x$ KCL $i_1 = i_2 + i_3$ (*)

KVL $-30 + i_1 12 + V_x \Rightarrow i_1 = \frac{30 - V_x}{12}$

$i_2 = \frac{V_x}{60}$ $i_3 = 2V_x$

$\frac{30 - V_x}{12} = \frac{V_x}{60} + 2V_x$

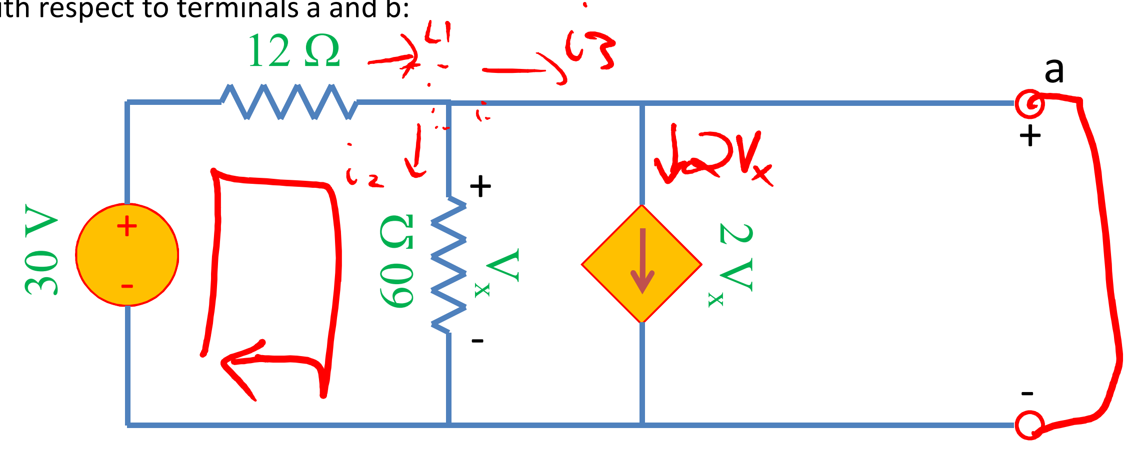
$V_x = 1.19\ V = V_{th}$

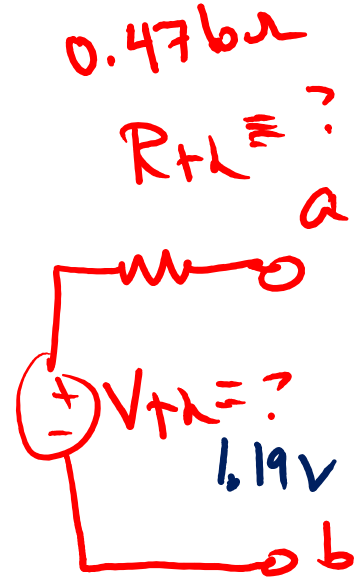
$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{1.19V}{2.5A} = 0.476\ \Omega$

2) Find $I_{short\ circuit} = \frac{30V}{12\Omega} = 2.5\ A$

Example

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:





1) Find Vab

$V_{ab} = V_x$

KCL $i_1 = i_2 + i_3$ (*)

KVL $-30 + i_1 12 + V_x \Rightarrow i_1 = \frac{30 - V_x}{12}$

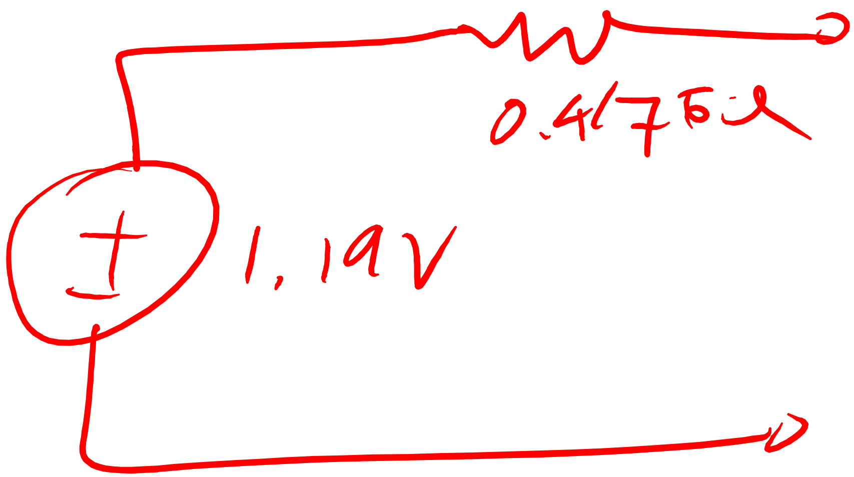
$i_2 = \frac{V_x}{60}$ $i_3 = 2V_x$

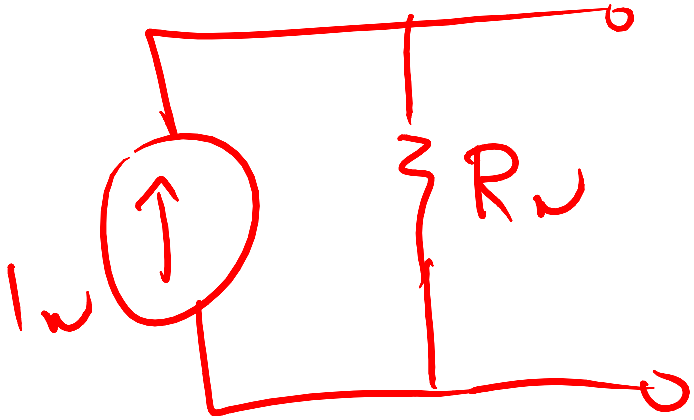
$\frac{30 - V_x}{12} = \frac{V_x}{60} + 2V_x$

$V_x = 1.19 \checkmark = V_{th}$

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{1.19V}{2.5A} = 0.476\,\Omega$

2) Find Ishort circuit $= \frac{30v}{12\Omega} = 2.5A$

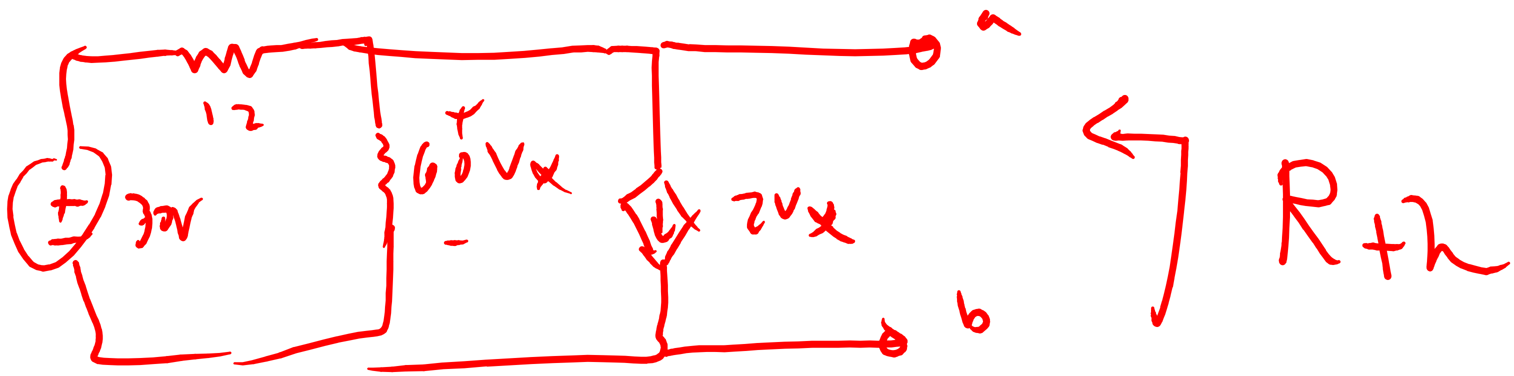




$$R_N = 0.476\,\Omega$$

$$I_N = \frac{V_{th}}{R_N}$$

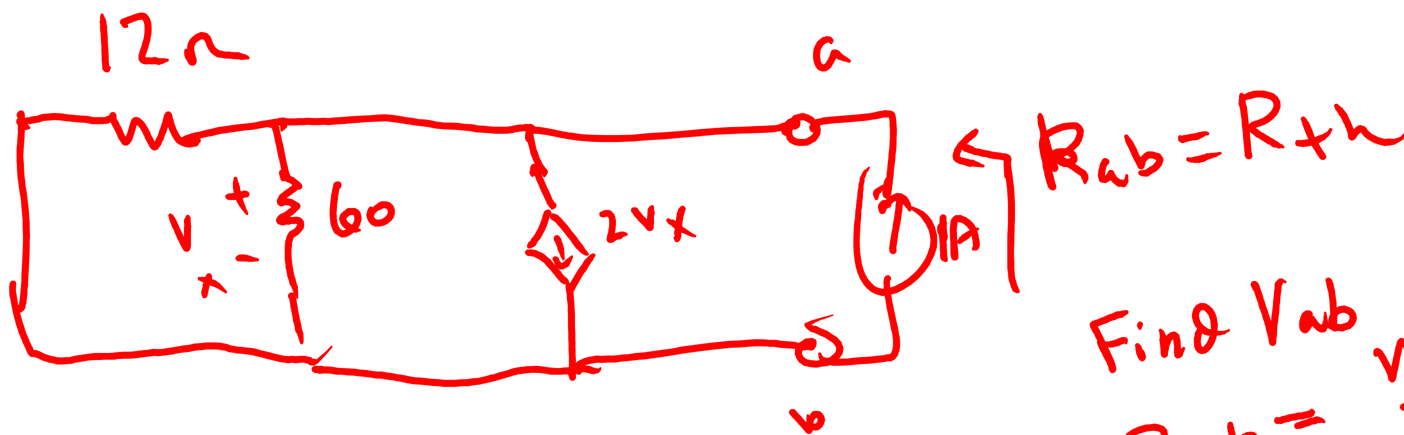
$$= \frac{1.19\,V}{0.476\,\Omega} = 2.5\,A$$



Find R_{ab} when all indep. sources turned off

Volt sources → shorts

curr sources → open



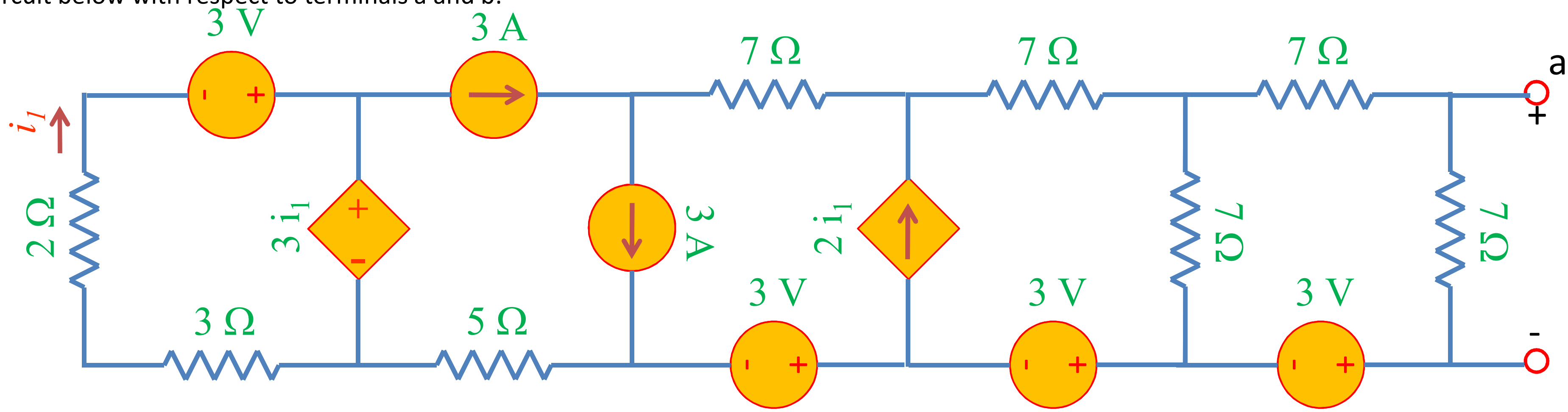
Find V_{ab}

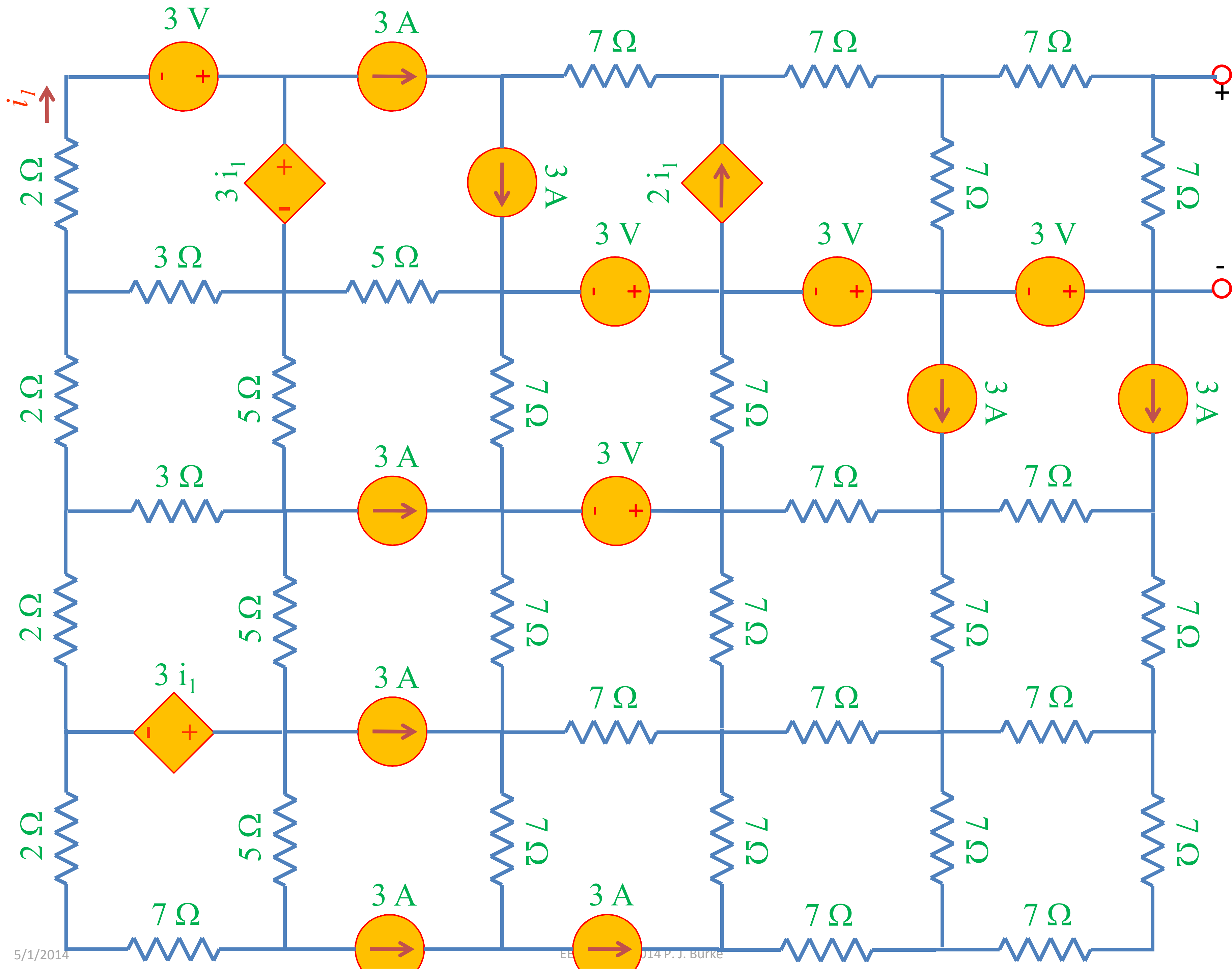
$R_{ab} = \frac{V_{ab}}{1A}$

2

"Baby" monster problem

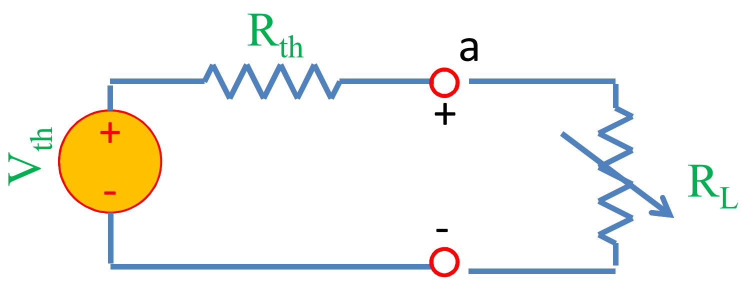
Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



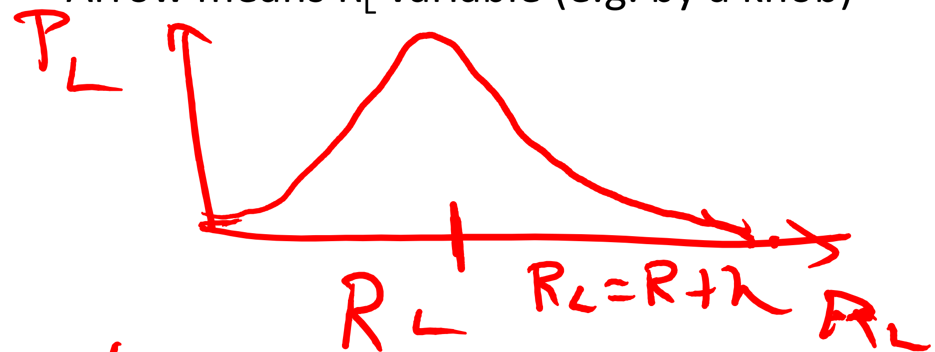


3 V
3 A
7 Ω
7 Ω
7 Ω
a
+
i1
2 Ω
3 i1
3 A
2 i1
7 Ω
7 Ω
3 Ω
5 Ω
3 V
3 V
3 V
-
b
2 Ω
5 Ω
7 Ω
7 Ω
3 A
3 A
3 Ω
3 A
3 V
7 Ω
7 Ω
2 Ω
5 Ω
7 Ω
7 Ω
7 Ω
7 Ω
3 i1
3 A
7 Ω
7 Ω
7 Ω
2 Ω
5 Ω
7 Ω
7 Ω
7 Ω
7 Ω
7 Ω
3 A
3 A
7 Ω
7 Ω

Power



Arrow means R_L variable (e.g. by a knob)

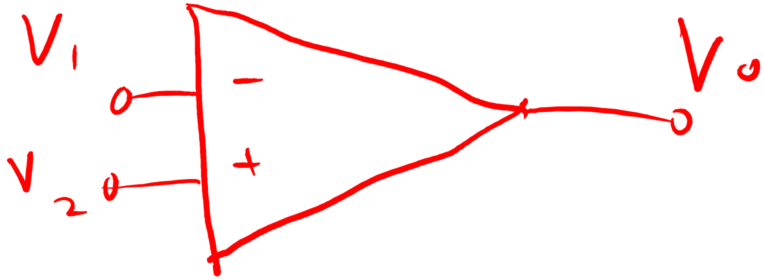


Power delivered to load = ?

Voltage across R_L $= V_{th} \dfrac{R_L}{R_L + R_{th}}$

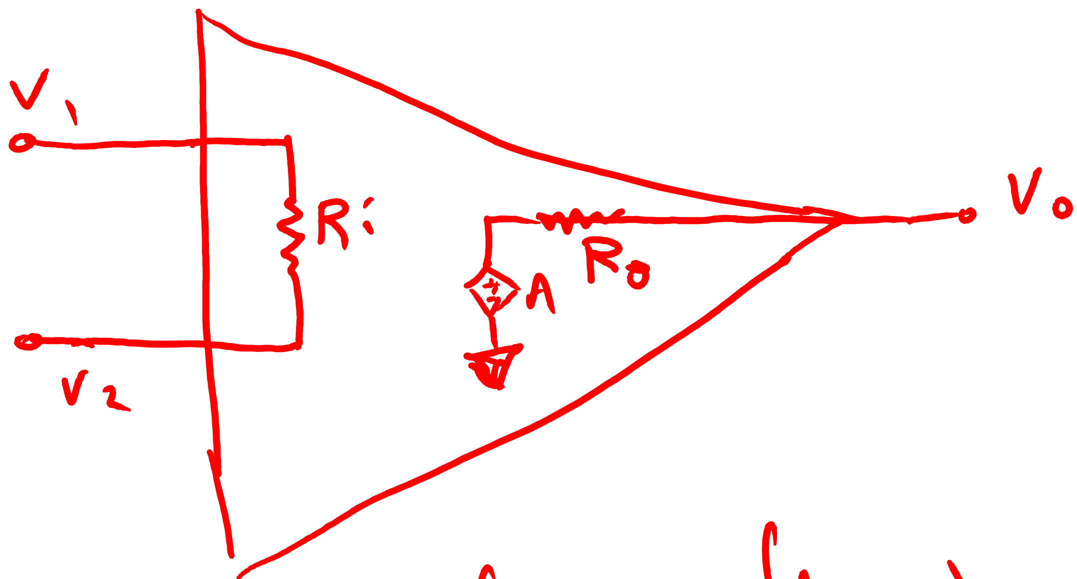
Power delivered to load $= V_L \times I_L$

$$= V_{th} \frac{R_L}{R_L + R_{th}} \frac{V_{th}}{R_L + R_{th}} = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$



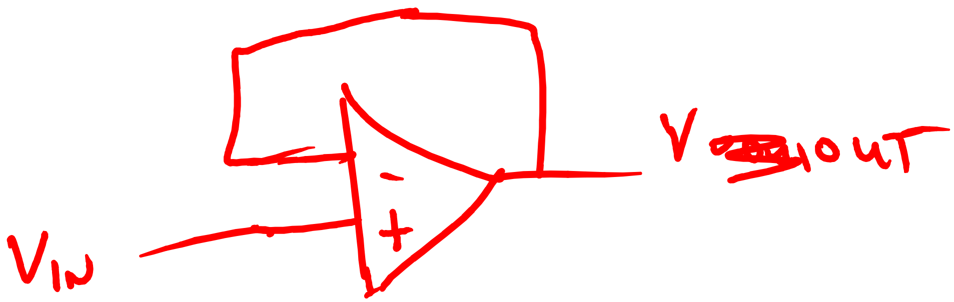
OPAMP

	IDEAL	REAL
$R_i =$	∞	$10^6 - 10^{12}\,\Omega$
$R_o =$	0	$1 - 100\,\Omega$
$A =$	∞	$10^4 - 10^6$



$$V_o = A\,(V_2 - V_1)$$

Voltage Follower



$$V_{OUT} = V_{IN}$$

IDEAL