Announcements:

1. Announcements

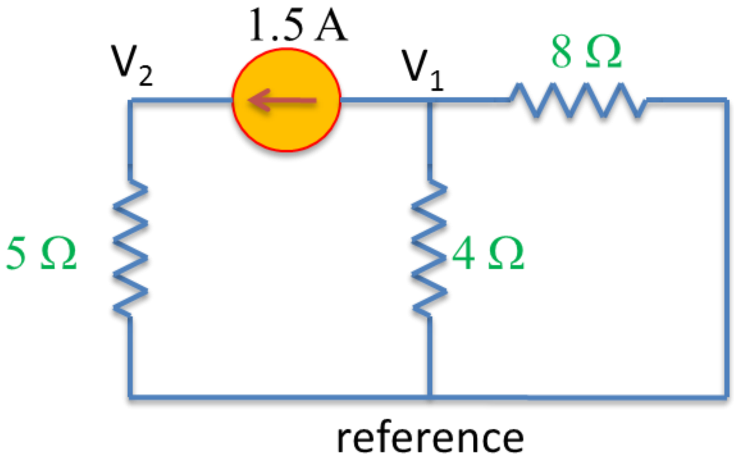
EECS 70A: Network Analysis

Lecture 6

Nodal Analysis(Review)

Based on KCL, Use node voltages as circuits variables.

1. Define a reference node.
2. Label remaining nodes. (n-1 nodes)
3. Apply KCL + ohm to all nodes and supernodes
    1. Express all I’s in terms of v’s
4. Apply KVL to loops with voltage source
5. Solve the n-1 simultaneous equations, to find V’s
6. Use Ohm’s law to find the currents.

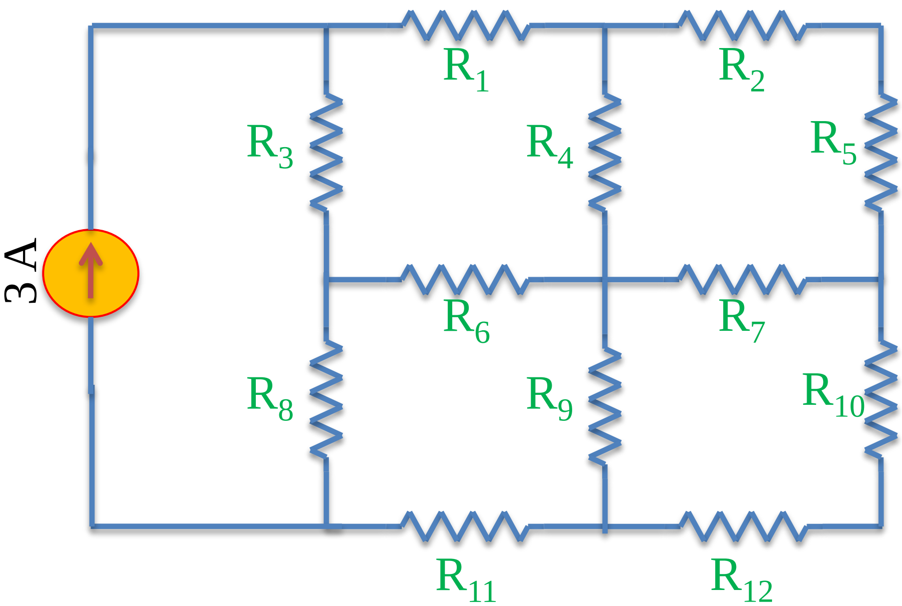


$$V1 = \frac{\begin{vmatrix} \frac{3}{2} & \frac{1}{5} \\ -\frac{3}{2} & 0 \end{vmatrix}}{\begin{vmatrix} 0 & \frac{1}{5} \\ \frac{3}{8} & 0 \end{vmatrix}} = -4\ V$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

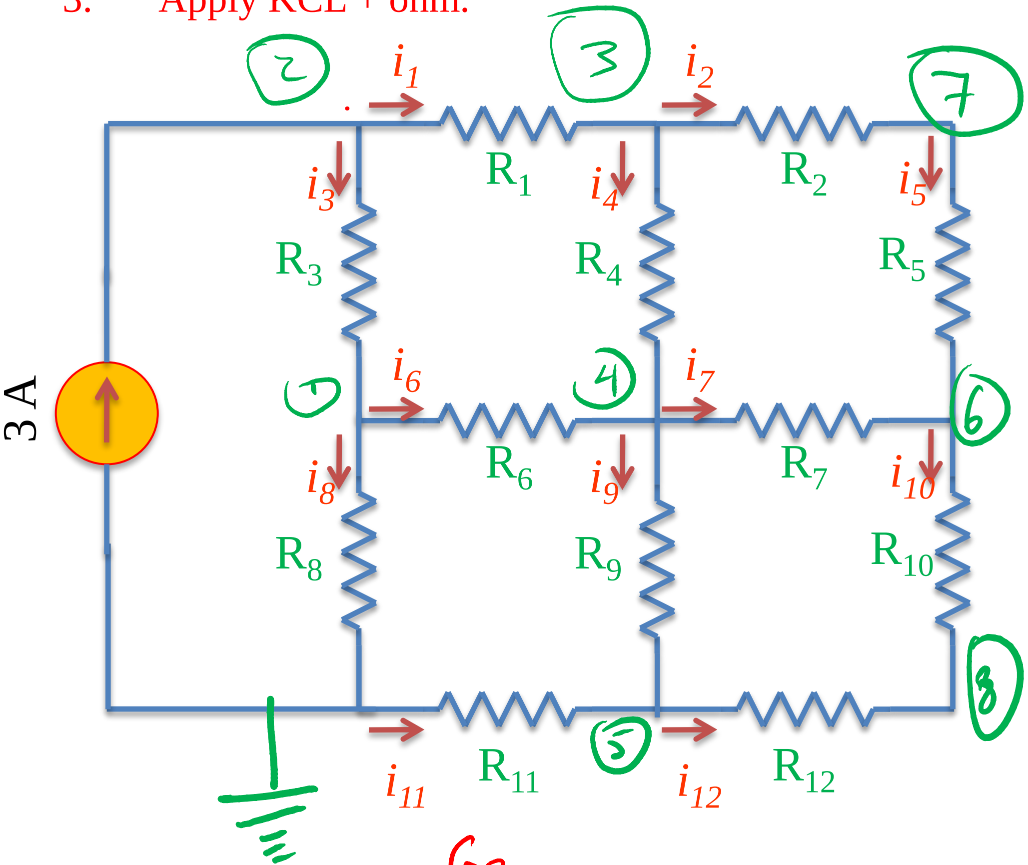
$$V_1 = \frac{\frac{3}{2} \times 0 - \left(-\frac{3}{2}\right)\left(\frac{1}{5}\right)}{0 \times 0 - \left(\frac{1}{5}\right)\left(\frac{3}{8}\right)} = -4\ V$$

Example Nodal Problem (detailed solution)



Same circuit: Nodal analysis

1. Define a reference node.
2. Label remaining nodes.
3. Apply KCL + ohm.



① IN = OUT $i_3 = i_6 + i_8$

$$\frac{V_2 - V_1}{R_3} = \frac{V_1 - V_4}{R_6} + \frac{V_1 - 0}{R_8}$$

$G_3 = \frac{1}{R_3}$ $G_6 = \frac{1}{R_6}$ etc

G_3

$(G_3 - G_6 - G_8) V_1 + (G_3) V_2 + (0) V_3 + (G_6) V_4 + (0) V_5 + (0) V_6$
$+ (0) V_7 + (0) V_8 = (0)$

$$-(G_3+G_6+G_8)V_1 + G_3V_2 + 0V_3 + G_6V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

$$-G_3V_1 + (G_1+G_3)V_2 + (-G_1)V_3 + 0V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 3$$

$$0V_1 + G_1V_2 + (-G_1-G_2-G_4)V_3 + (G_2)V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

$$G_8V_1 + 0V_2 + G_4V_3 + -(G_6+G_4+G_9+G_7)V_4 + G_9V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

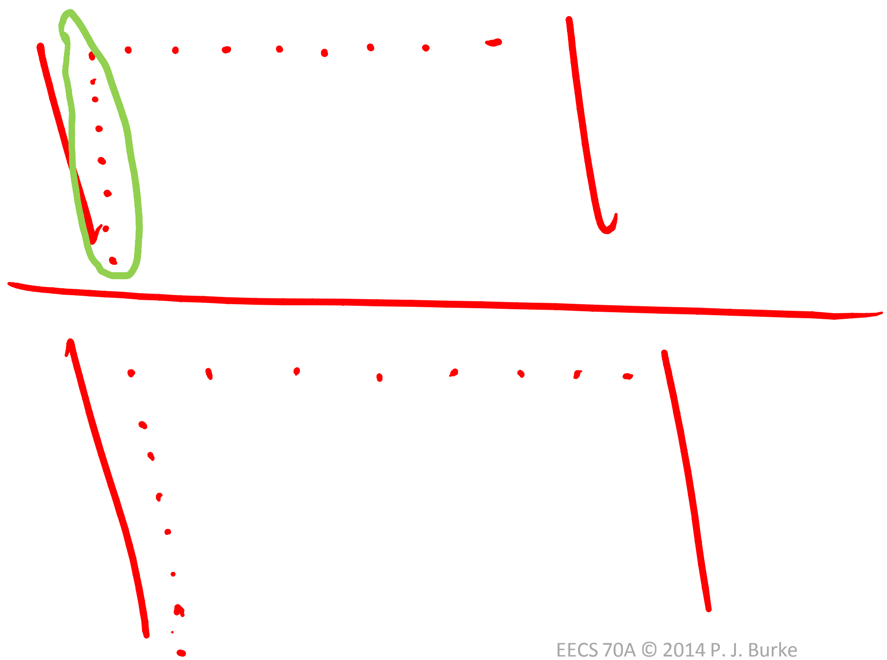
$$0V_1 + 0V_2 + 0V_3 + (-G_9)V_4 + (G_9+G_{12}+G_{11})V_5 + 0V_6 + 0V_7 + (-G_{12})V_8 = 0$$

$$0V_1 + 0V_2 + 0V_3 + G_7V_4 + 0V_5 + (-G_5-G_7-G_{10})V_6 + G_5V_7 + G_{10}V_8 = 0$$

$$0V_1 + 0V_2 + G_2V_3 + 0V_4 + 0V_5 + G_5V_6 + (-G_2-G_5)V_7 + 0V_8 = 0$$

$$0V_1 + 0V_2 + 0V_3 + 0V_4 + G_{12}V_5 + G_{10}V_6 + (-G_{12}-G_{10})V_8 = 0$$

$V_1 =$



LHS coeffs.

$$-(G_3+G_6+G_8)V_1 + G_3V_2 + 0V_3 + G_6V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

$$-G_3V_1 + (G_1+G_3)V_2 + (-G_1)V_3 + 0V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 3$$

$$0V_1 + G_1V_2 + (-G_1-G_2-G_4)V_3 + (G_2+G_4)V_4 + 0V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

$$G_8V_1 + 0V_2 + G_4V_3 - (G_6+G_4+G_9+G_7)V_4 + G_9V_5 + 0V_6 + 0V_7 + 0V_8 = 0$$

$$0V_1 + 0V_2 + 0V_3 + (-G_9)V_4 + (G_9+G_{12}+G_{11})V_5 + 0V_6 + 0V_7 + (-G_{12})V_8 = 0$$

$$0V_1 + 0V_2 + 0V_3 + G_7V_4 + 0V_5 + (-G_5-G_7-G_{10})V_6 + G_5V_7 + G_{10}V_8 = 0$$

$$0V_1 + 0V_2 + G_2V_3 + 0V_4 + 0V_5 + G_5V_6 + (-G_2-G_5)V_7 + 0V_8 = 0$$

$$0V_1 + 0V_2 + 0V_3 + 0V_4 + G_{12}V_5 + G_{10}V_6 + (-G_{12}-G_{10})V_8 = 0$$

$$V_1 = \frac{\left| \; N \; \right|}{\left| \; D \; \right|}$$

LHS coeffs.

$$
A = \begin{pmatrix}
G_3+G_6+G_8 & -G_3 & 0 & -G_6 & 0 & 0 & 0 & 0 \\
-G_3 & G_1+G_3 & -G_1 & 0 & 0 & 0 & 0 & 0 \\
\vdots & & & & & & & \\
0 & 0 & 0 & 0 & G_{12} & G_{10} & 0 & -G_{12}-G_{10}
\end{pmatrix}
\begin{matrix} (1) \\ (2) \\ \vdots \\ (8) \end{matrix}
$$

$V_1 \quad V_2 \quad V_3$

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 3 & 4 \end{vmatrix} = -1 \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} + 0 \begin{vmatrix} & \\ & \end{vmatrix} - 0 \begin{vmatrix} & \\ & \end{vmatrix}$$

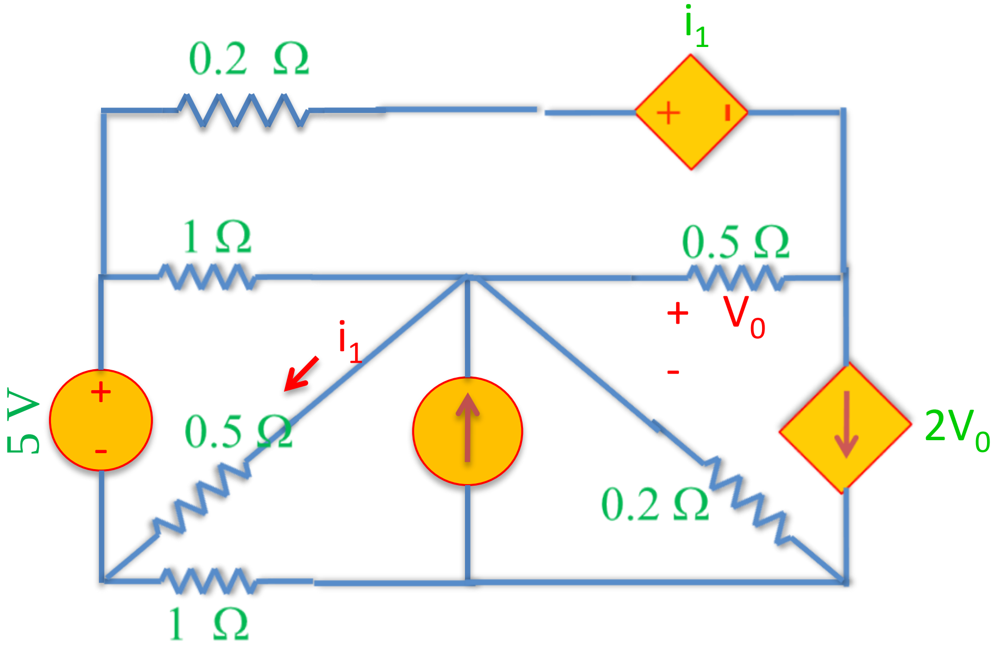
$$= -(4-6) = +2$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & e & h \end{vmatrix} = a \begin{vmatrix} e & F \\ e & h \end{vmatrix} - b \begin{vmatrix} d & F \\ g & h \end{vmatrix} + c \begin{vmatrix} d & e \\ g & e \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & e & h \end{vmatrix} = -d \begin{vmatrix} b & c \\ e & h \end{vmatrix} + e \begin{vmatrix} a & c \\ g & h \end{vmatrix} - F \begin{vmatrix} a & b \\ g & e \end{vmatrix}$$

a

Nodal Analysis-Example

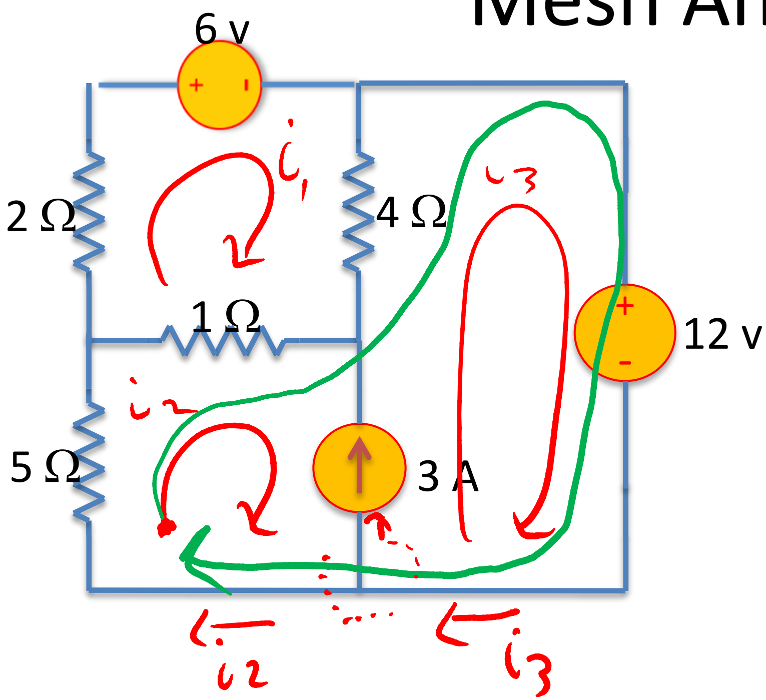


Mesh Analysis(Review)

Based on KVL, use mesh currents as circuits variables.

1. Assign mesh currents i_1 ,i_2 ,…i_n
   A. Create supermesh if current source
2. Apply KVL+ Ohm's law to each mesh KCL w/ curr. source
3. Solve the equations for mesh currents i_1 ,i_2 ,…i_n
4. Find voltage drops

Mesh Analysis- Example



KVL mesh 1

$$2i_1 + 6 + (i_1 - i_3)4 + (i_1 - i_2)1 = 0$$

KVL supermesh

$$i_2 5 + (i_2 - i_1)1 + (i_3 - i_1)4 + 12 = 0$$

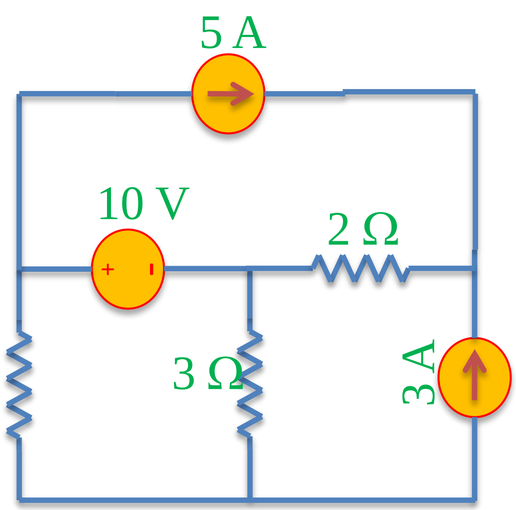
~~$i_2 = 3 + i_3$~~

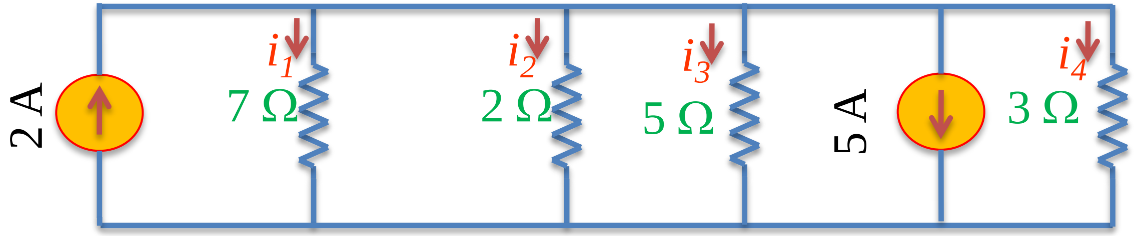
$$i_3 = 3A + i_2$$

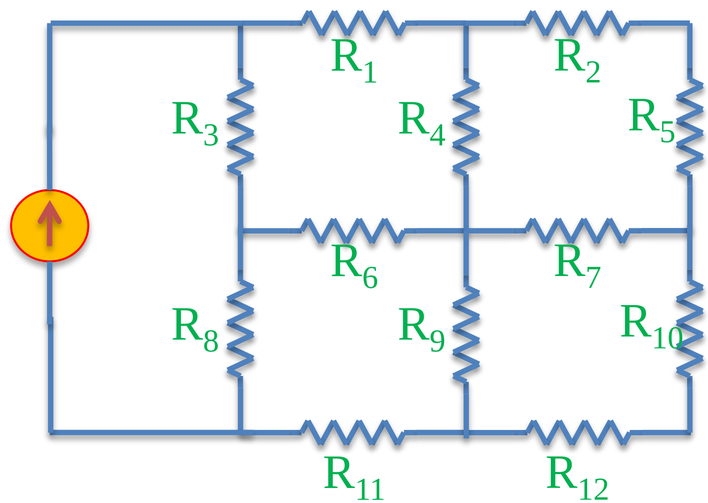
Nodal Versus Mesh Analysis

- The method that results in fewer number of equations is more suitable.
  - Mesh analysis for networks with many series connected elements
  - Nodal Analysis for networks with many parallel connected elements

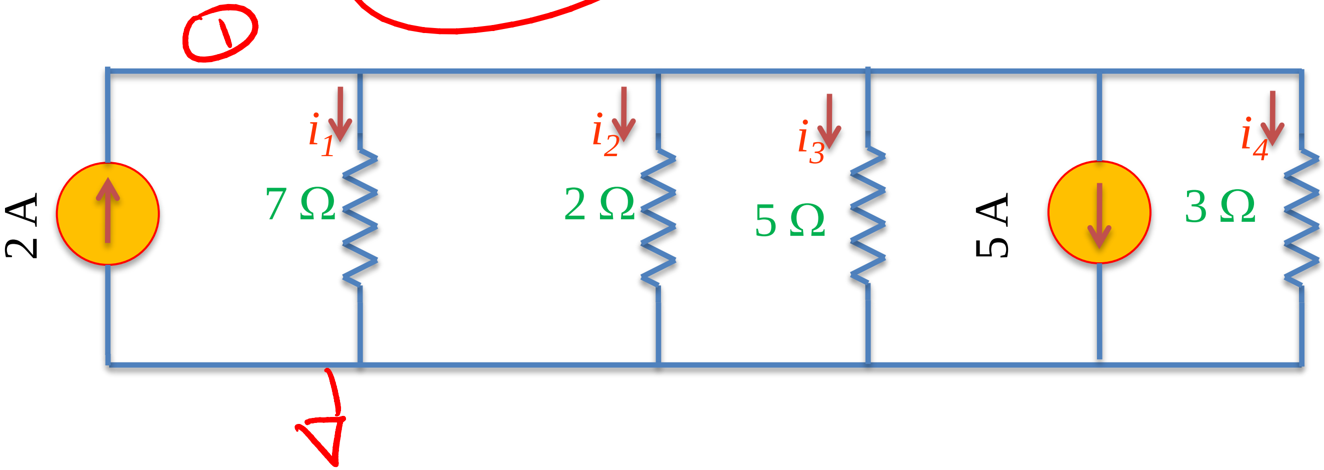
But also depends on the type of the sources.







Nodal vs. mesh analysis?



IN = OUT @ NODE 1

$$2 = i_1 + i_2 + i_3 + 5 + i_4$$

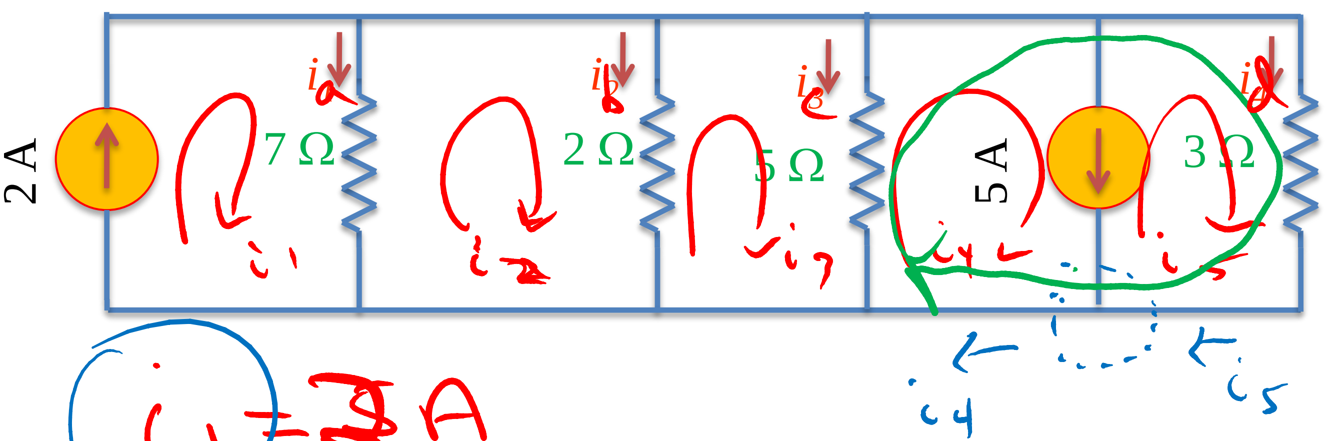
$$2 = \frac{V_1}{7} + \frac{V_1}{2} + \frac{V_1}{5} + 5 + \frac{V_1}{3}$$

$$V_1 = -2.5\ V$$

$$i_4 = -\frac{2.5}{3}\ A$$

$$i_1 = \frac{-2.5}{7}\ A \qquad i_2 = \frac{-2.5}{2}\ A \qquad i_3 = -\frac{2.5}{5}\ A$$

Nodal vs. mesh analysis?



$i_a = i_1 - i_2$

$i_b = i_2 - i_3$

$i_c = i_3 - i_4$

$i_d = i_5$

$i_1 = 2A$

①

② $7(i_2 - i_1) + 2(i_2 - i_3) = 0$

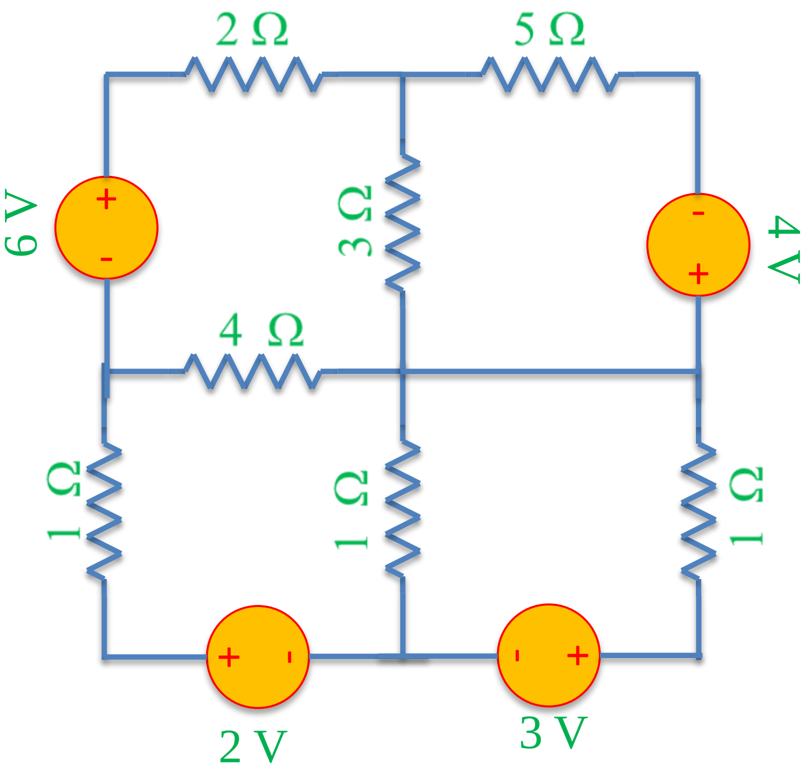
③ $2(i_3 - i_2) + 5(i_3 - i_4) = 0$

④ $5(i_4 - i_3) + 3(i_5) = 0$

KCL IN = out $5A + i_5 = i_4$

5 eq 5 unknowns. Solve!

Nodal vs. Mesh Analysis



MESH

4 unknowns

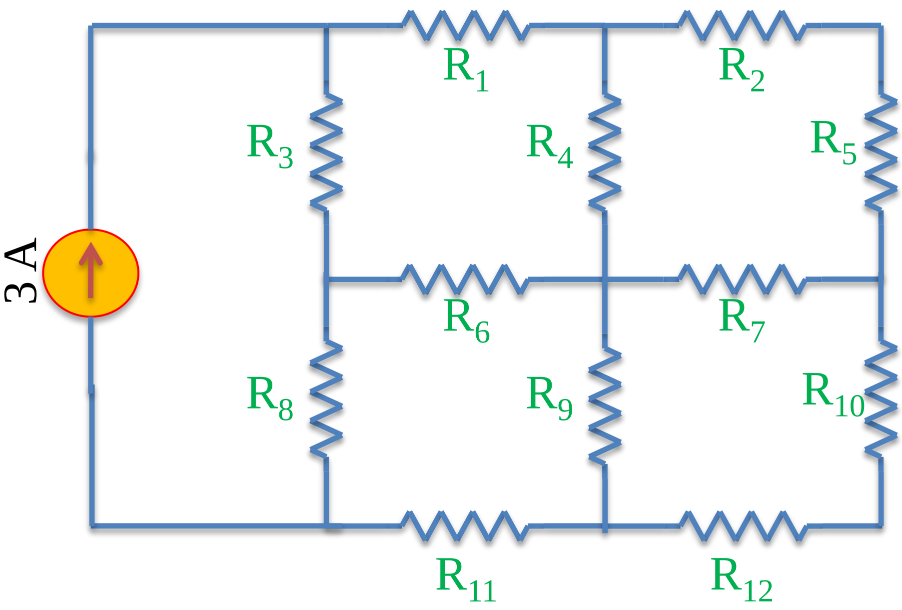
4 eqns.

NODAL

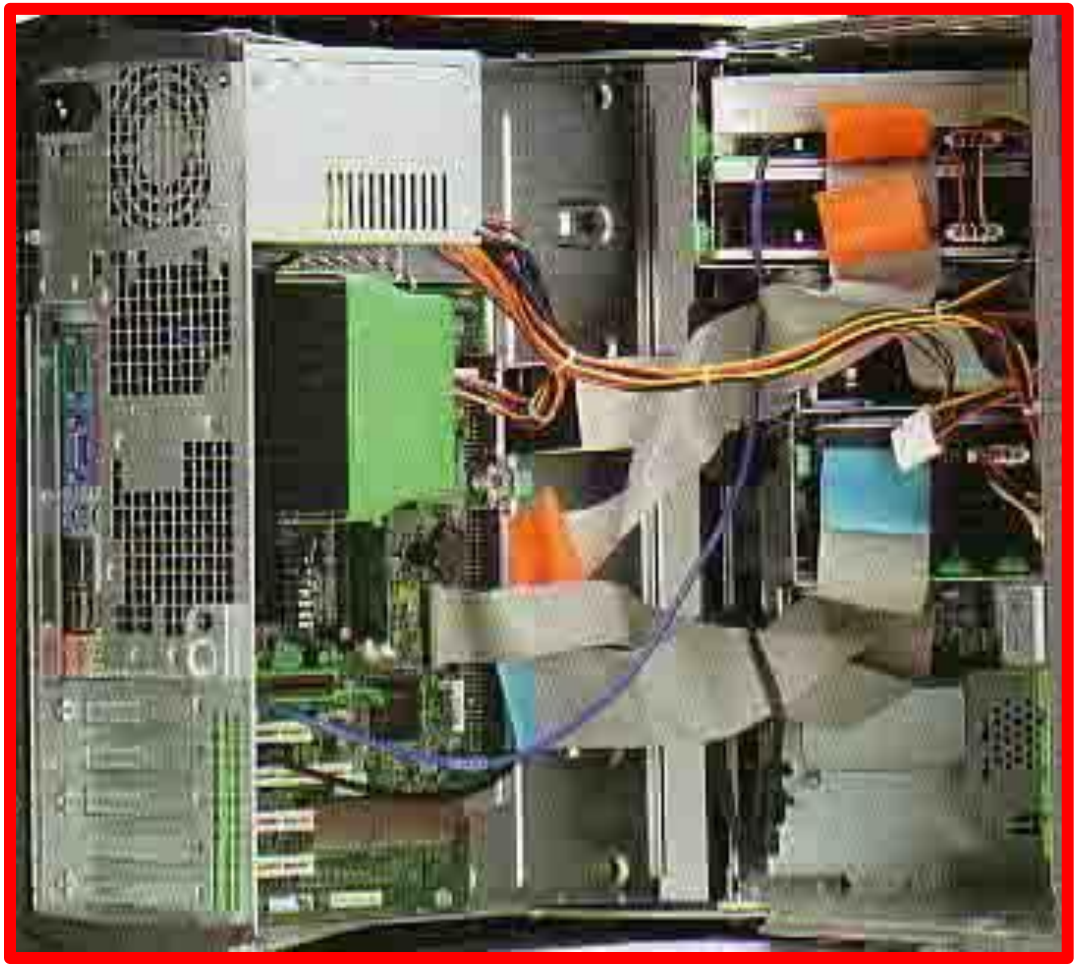
9 NODES

8 eqn. 8 unknown

Nodal vs. Mesh Analysis

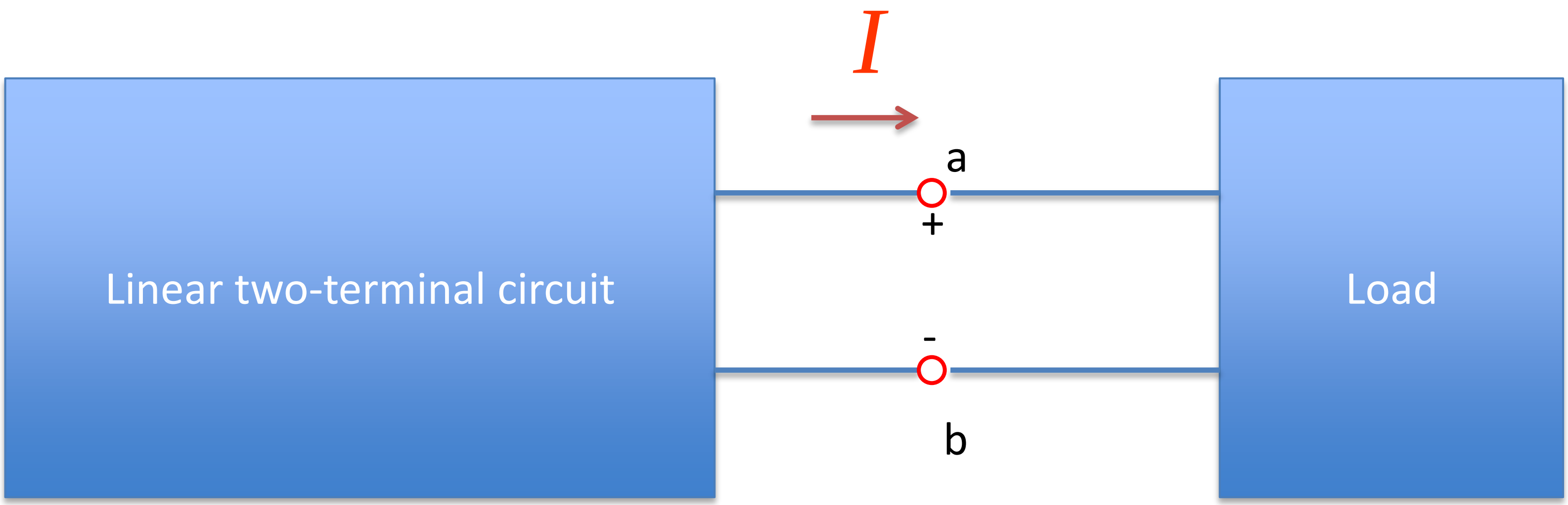


Compartmentalization: Need for simplicity

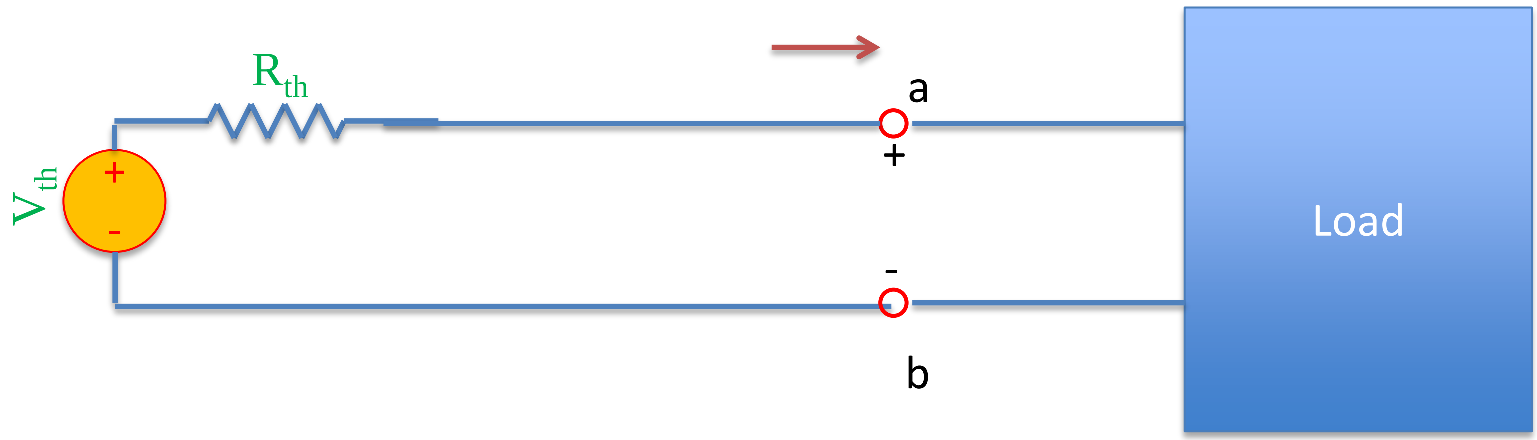


Power brick image.
And ask class to show their own…
Demo: Computer?

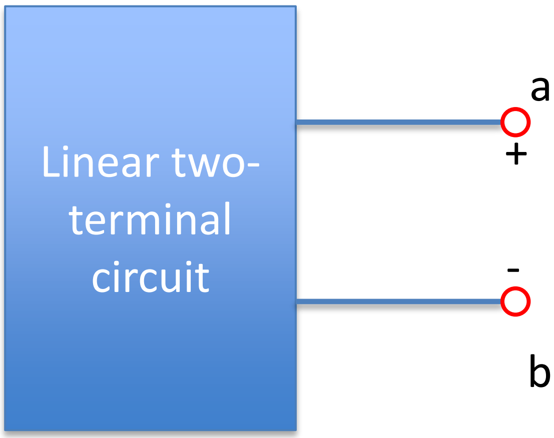
Thevenin's Theorem



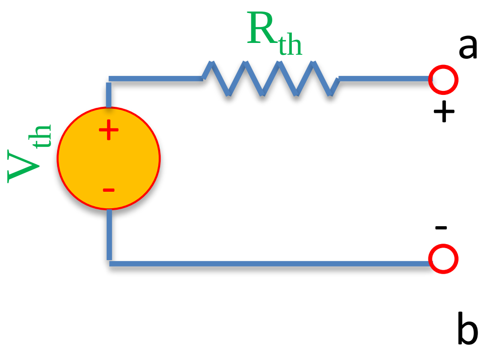
Equivalent to:



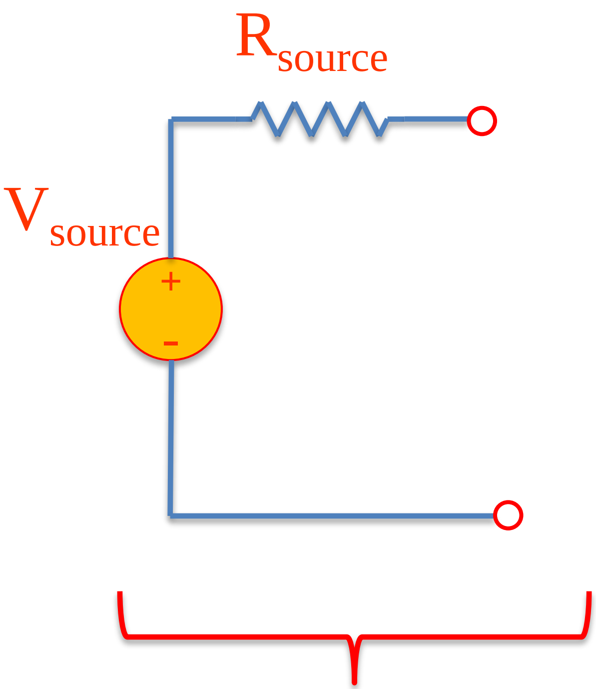
Finding V_{th}, R_{th}



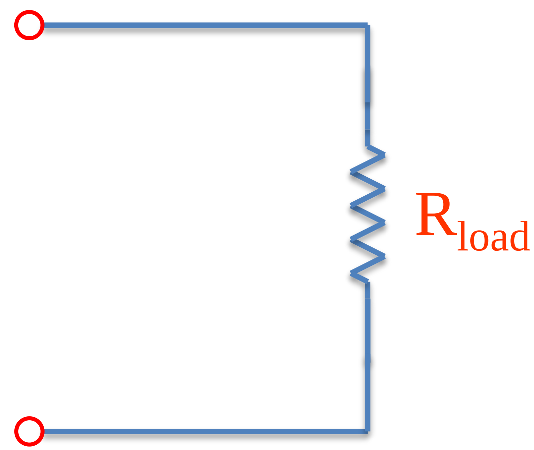
Equivalent to:



Source/load



Thevenin Thm:
Any circuit can be represented by this equivalent circuit.



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Case 1:

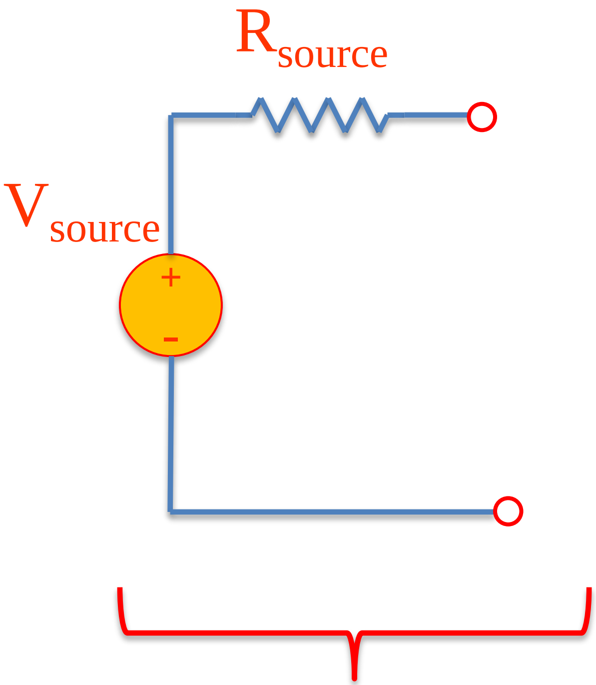
$$R_{load} >> R_{source}$$

Case 2:

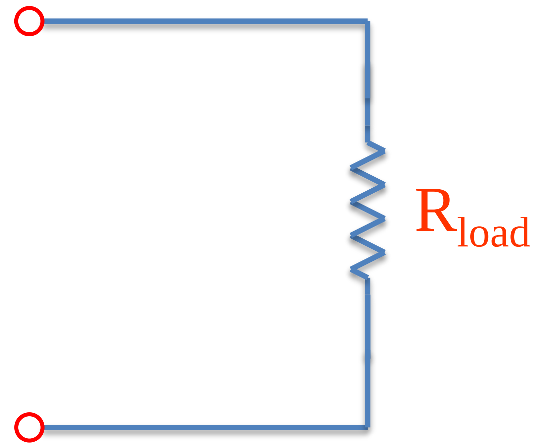
$$R_{source} >> R_{load}$$

We say R_{load} "*loads down*" the source.

Source/load



Thevenin Thm:
<u>Any</u> circuit can be represented by this equivalent circuit.



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Case 1:

$$R_{load} >> R_{source}$$

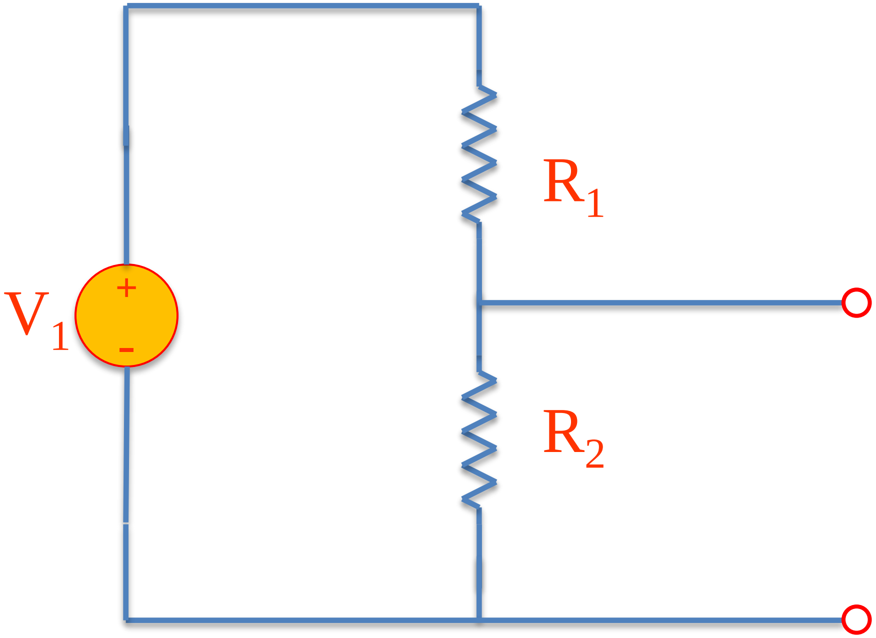
Case 2:

$$R_{source} >> R_{load}$$

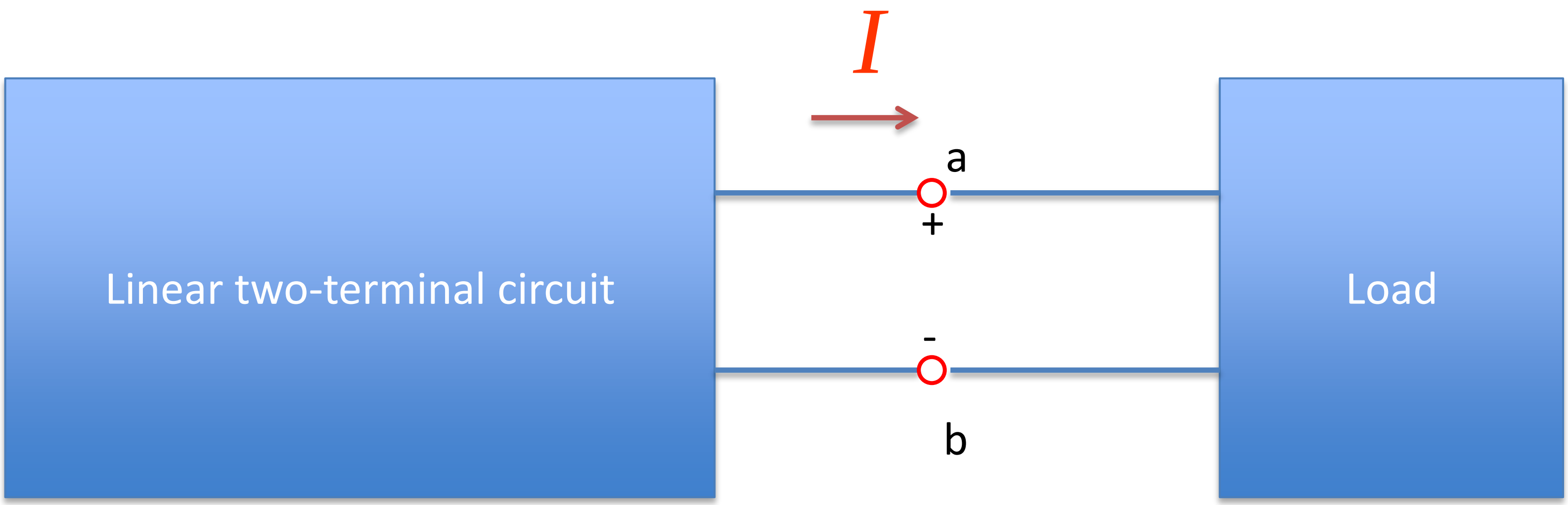
We say R_{load} "*loads down*" the source.

Example

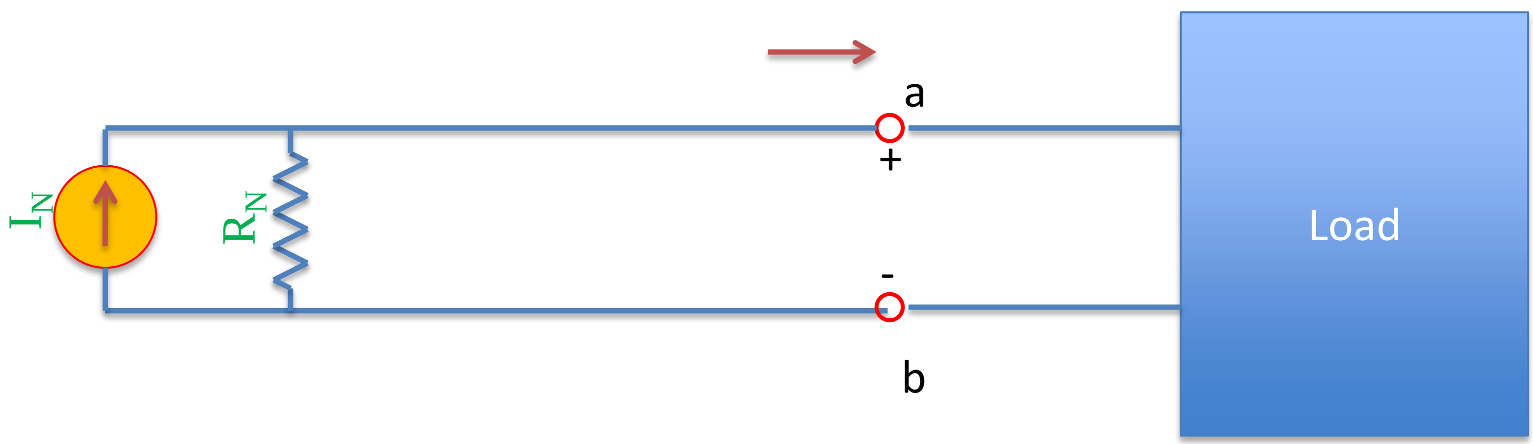
Find Thevenin equivalent circuit:



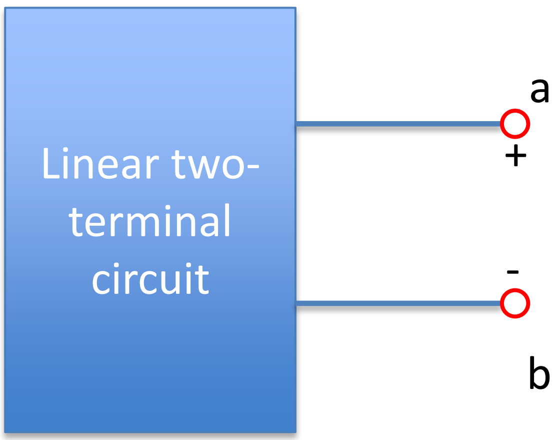
Norton's Theorem



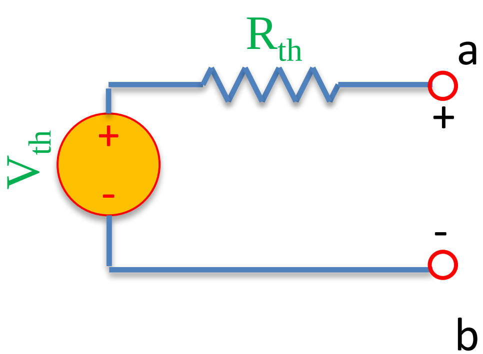
Equivalent to:



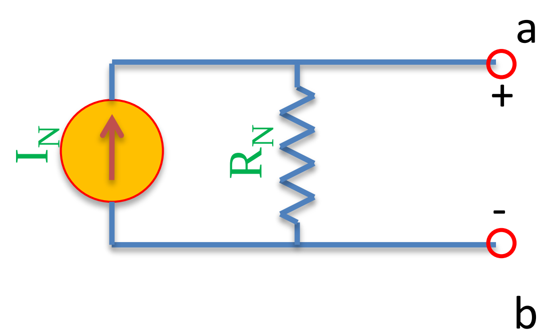
Finding V_{th}, R_{th}



Equivalent to:

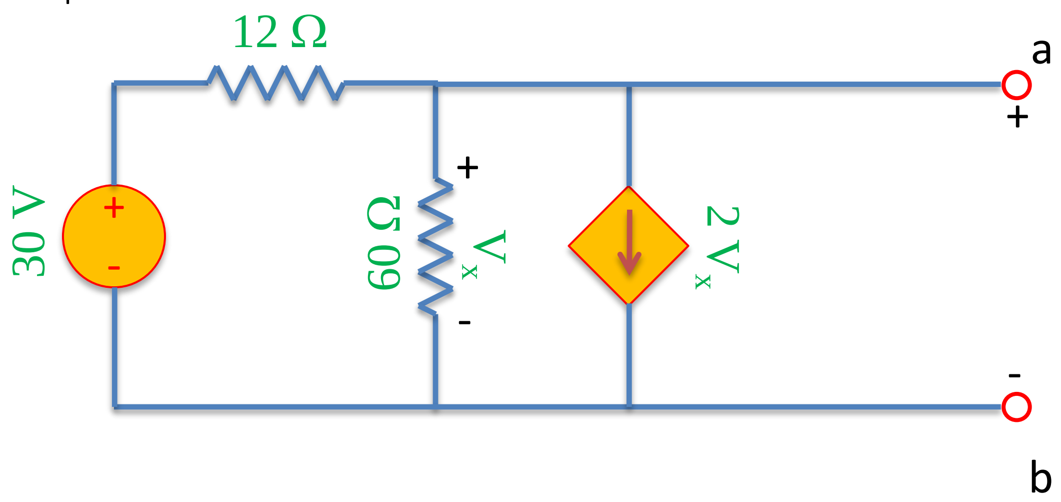


Equivalent to:



Example

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



“Baby” monster problem

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b: