Announcements:

1. HW
2. etc

# EECS 70A: Network Analysis

# *Phasors*

Fourrier

$$f(t) = \mathrm{Re}\left[\sum_n a_n e^{j\omega_n t}\right]$$

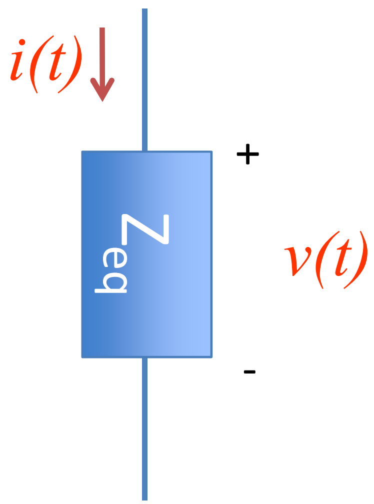
$$\rightarrow \int a(\omega)\, d\omega\, e^{j\omega t}$$

$$v(t) = \mathrm{Re}\left(\underbrace{\mathbf{V}}_{\text{voltage phasor}} e^{j\omega t}\right)$$

$$i(t) = \mathrm{Re}\left(\underbrace{\mathbf{I}}_{\text{current phasor}} e^{j\omega t}\right)$$

$$\mathbf{V} = \mathbf{I}\, Z$$

# Conversion procedures




Given i(t) find v(t):

$$i(t) \rightarrow \mathbf{I} \rightarrow \mathbf{V} = \mathbf{I}\, Z_{eq} \rightarrow v(t)$$

Given v(t) find i(t):

$$v(t) \rightarrow \mathbf{V} \rightarrow \mathbf{I} = \mathbf{V} / Z_{eq} \rightarrow i(t)$$

# Conversion procedures

$$\mathrm{i(t)} \rightarrow \mathbf{I}$$

$$\mathrm{v(t)} \rightarrow \mathbf{V}$$

$$\mathbf{I} \rightarrow \mathrm{i(t)}$$

$$\mathbf{V} \rightarrow \mathrm{v(t)}$$

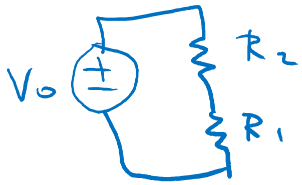
$$i(t) = I_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{I} = I_m e^{j\phi}$$

$$v(t) = V_0 \cos(\omega t) \Rightarrow V = V_0$$

$$\underline{v(t) = V_m \cos(\omega t + \phi)} \Rightarrow \underline{\mathbf{V} = V_m e^{j\phi}}$$

$$\underline{i(t) = \mathrm{Re}\left(\mathbf{I}\, e^{j\omega t}\right)}$$

$$v(t) = \mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right)$$




$V_{R_1} = V_0 \frac{R_1}{R_1+R_2}$

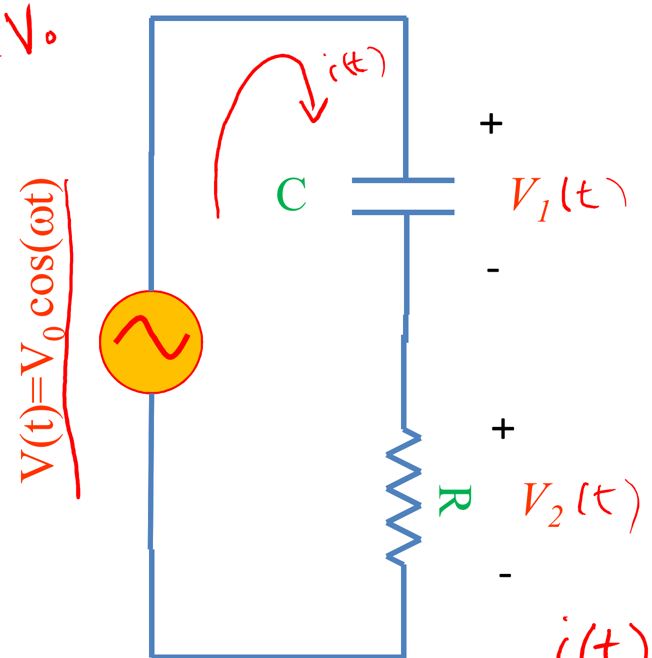
# Example problem #3

$t$ time

$\tau = RC$

Find i(t), $V_1$(t), $V_2$(t) for this circuit: (instructor)

$\tilde{V} = V_0$

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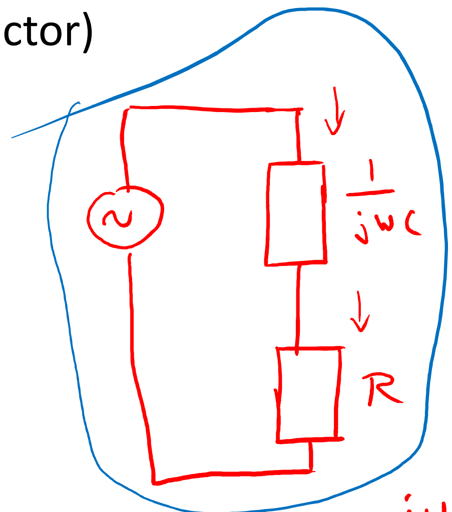


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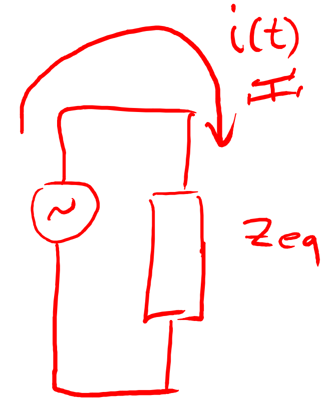



$$Z_{eq} = \left(R + \frac{1}{j\omega C}\right)\frac{j\omega C}{j\omega C} = \frac{1+j\omega RC}{j\omega C}$$

$$\tilde{V} = Z_{eq}\tilde{I} \Rightarrow \tilde{I} = \frac{\tilde{V}}{Z_{eq}} = \frac{V_0}{R+\frac{1}{j\omega C}}$$

$$i(t) = Re\left[\tilde{I} e^{j\omega t}\right] = Re\left[\frac{V_0}{R+\frac{1}{j\omega C}} e^{j\omega t}\right] \ldots$$

$$i(t) = Re\left[V_0 \frac{R}{R} \frac{j\omega C}{1+j\omega RC} e^{j\omega t}\right] = Re\left[\frac{V_0}{R} \frac{j\omega RC}{1+j\omega RC} e^{j\omega t}\right]$$

$$= \frac{V_0}{R} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega\tau}\right)\right)$$

$$i(t) = Re\left[\frac{V_0}{R}\frac{j\omega RC}{1+j\omega RC}e^{j\omega t}\right] = \frac{V_0}{R}Re\left[\frac{j\omega\tau}{1+j\omega\tau}e^{j\omega t}\right]$$

$$= \frac{V_0}{R}Re\left[re^{j\phi}e^{j\omega t}\right]$$

$$= \frac{V_0}{R}Re\left[r\,e^{j(\omega t+\phi)}\right]$$

$$= \frac{V_0}{R}r\cos(\omega t+\phi)$$

$$\frac{j\omega\tau}{1+j\omega\tau} = re^{j\phi}$$

$$\frac{j\omega\tau}{(1+j\omega\tau)}\frac{1-j\omega\tau}{(1-j\omega\tau)} = \frac{j\omega\tau+(\omega\tau)^2}{1+(\omega\tau)^2}$$

$$= \omega\tau\frac{j+\omega\tau}{1+(\omega\tau)^2} = \frac{\omega\tau}{1+(\omega\tau)^2}(j+\omega\tau)$$

$$e^u e^v = e^{u+v}$$

$$i(t) = \frac{V_0}{R}Re\left[\frac{\omega\tau}{1+(\omega\tau)^2}(j+\omega\tau)e^{j\omega t}\right] =$$

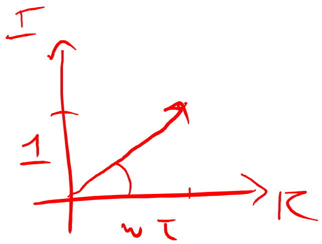
$$= \frac{V_0}{R}\frac{\omega\tau}{1+(\omega\tau)^2}Re\left[(j+\omega\tau)e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{1+(\omega\tau)^2}Re\left[(?)e^{j(?)}e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{1+(\omega\tau)^2}Re\left[\sqrt{1+(\omega\tau)^2}\,e^{j\tan^{-1}\frac{1}{\omega\tau}}e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}Re\left[e^{j\tan^{-1}\frac{1}{\omega\tau}}e^{j\omega t}\right]$$

$$u \equiv \omega\tau + j = (?)\, e^{j(?)}$$




$$= \sqrt{Re(u)^2 + Im(u)^2}\; e^{j \tan^{-1} \frac{Im(u)}{Re(u)}}$$

$$= \sqrt{1 + (\omega\tau)^2}\; e^{j \tan^{-1} \frac{1}{\omega\tau}}$$

# DISCUSSION

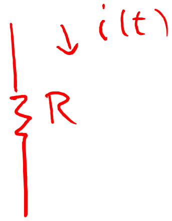
$$U = \frac{a + jb}{c + jd} \qquad a \; b \; c \; d \text{ real}$$

$Re(U)$

Also:

write $a + jb$ as $\_\_\_\_\_ \, e^{j\phi}$

$V_2(t) = ?$



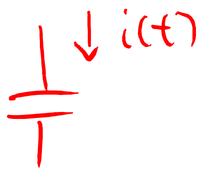

$\tilde{I} \rightarrow \tilde{V}_2 = \tilde{I} Z_R = \tilde{I} R$

$$V_2(t) = \mathrm{Re}\left[\tilde{V}_2 e^{j\omega t}\right] = \mathrm{Re}\left[\tilde{I} R e^{j\omega t}\right]$$

$$= R \ \mathrm{Re}\left[\tilde{I} e^{j\omega t}\right] =$$

$$V_2(t) = \cancel{R} \ \frac{V_0}{\cancel{R}} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos\left(\omega t + \tan^{-1} \frac{1}{\omega\tau}\right)$$

$V_1(t) = ?$




$\tilde{I} \rightarrow \tilde{V}_1 = \tilde{I} Z_C = \tilde{I} \frac{1}{j\omega C}$

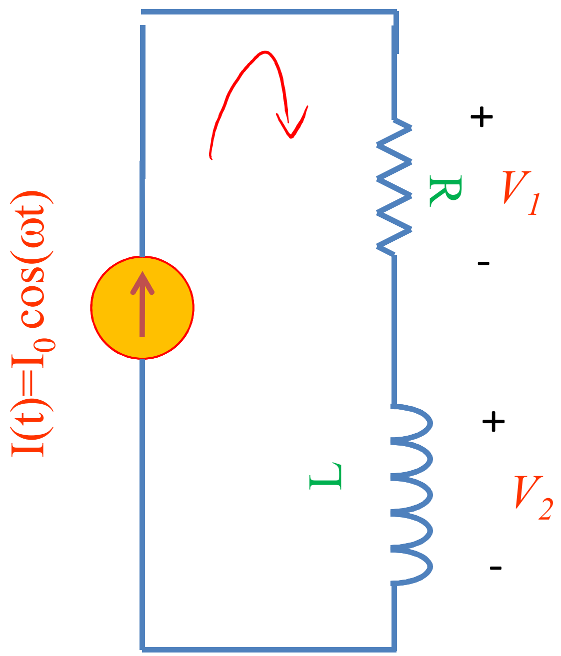
$$V_1(t) = \mathrm{Re}\left(\tilde{V}_1 e^{j\omega t}\right) = \mathrm{Re}\left[\tilde{I} \frac{1}{j\omega C} e^{j\omega t}\right]$$

$$= \frac{V_0 \omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos\left(\omega t + \left(-\tan^{-1} \frac{1}{\omega\tau}\right)\right)$$

"after some work"

# Example problem #4

Find i(t), $V_1$(t), $V_2$(t) for this circuit: (students)




$$V_1(t) = R\,i(t) = R I_0 \cos(\omega t)$$

$$V_2(t) = \mathrm{Re}\left[V_2 e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\cancel{I}\, j\omega L\, e^{j\omega t}\right]$$

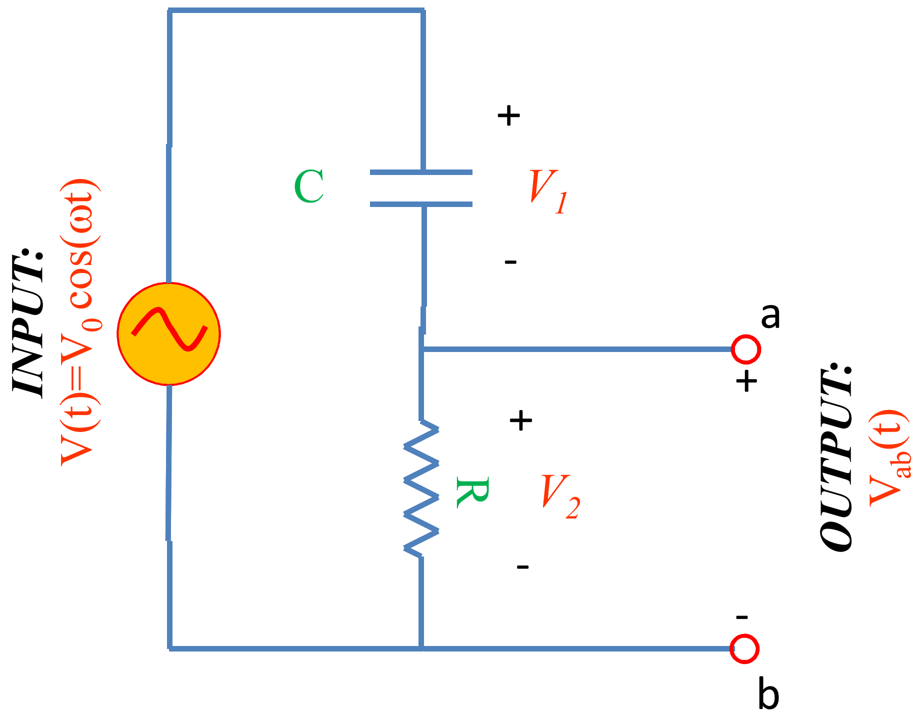
$$= \mathrm{Re}\left[I_0\, j\omega L\, e^{j\omega t}\right]$$

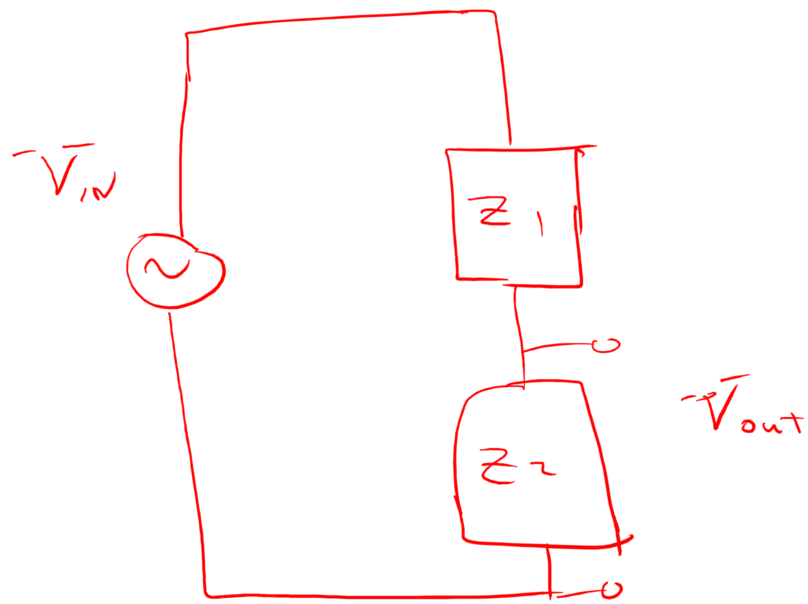
$$= I_0 L\omega\, \mathrm{Re}\left[j e^{j\omega t}\right]$$

$$= I_0 L\omega$$

$$= -I_0 L\omega \sin(\omega t)$$

# High pass filter



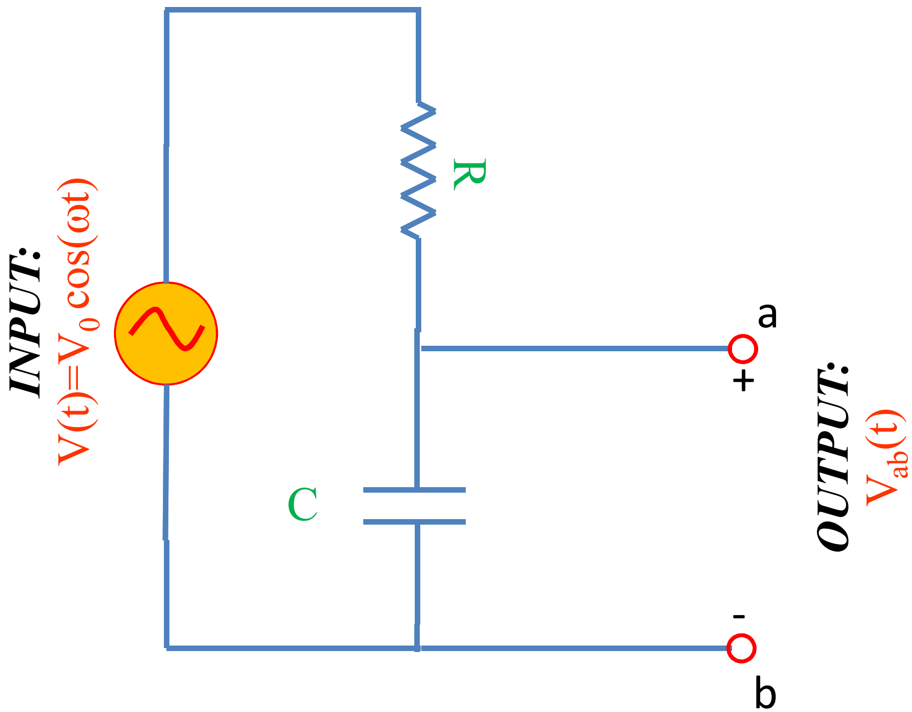




$$\tilde V_{out} = \tilde V_{IN} \frac{Z_2}{Z_1+Z_2} = \tilde V_{IN} \frac{R}{R+\frac{1}{j\omega C}}$$

$$= \tilde V_{IN} \frac{j\omega CR}{1+j\omega RC} = \tilde V_{IN} \frac{j\omega\tau}{1+j\omega\tau}$$

As $\underset{\omega \to 0}{\omega \to \infty}$ $\tilde V_{OUT} = \tilde V_{IN} \Rightarrow V_{out}(t) = V_{in}(t)$

# Low pass filter




$V_c = I_c \frac{1}{j\omega C}$

# Complex numbers

Euler's relationship:

$$e^{j\phi} = \cos\phi + j\sin\phi$$

*<u>Memorize</u>*

Need to be able to manipulate complex numbers, e.g. given:

$$u = \frac{A + jB}{C + jD}$$

(A,B,C,D all real)

- Find Re(u), Im(u)
- Express u as x+jy, $re^{i\phi}$
- Find Re(u $re^{j\omega t}$)

Did some of this in office hours..