Announcements:

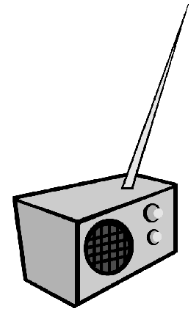
1. Announcement

EECS 70A: Network Analysis

Lecture 13

Wireless Communications

Broadcast Radio:



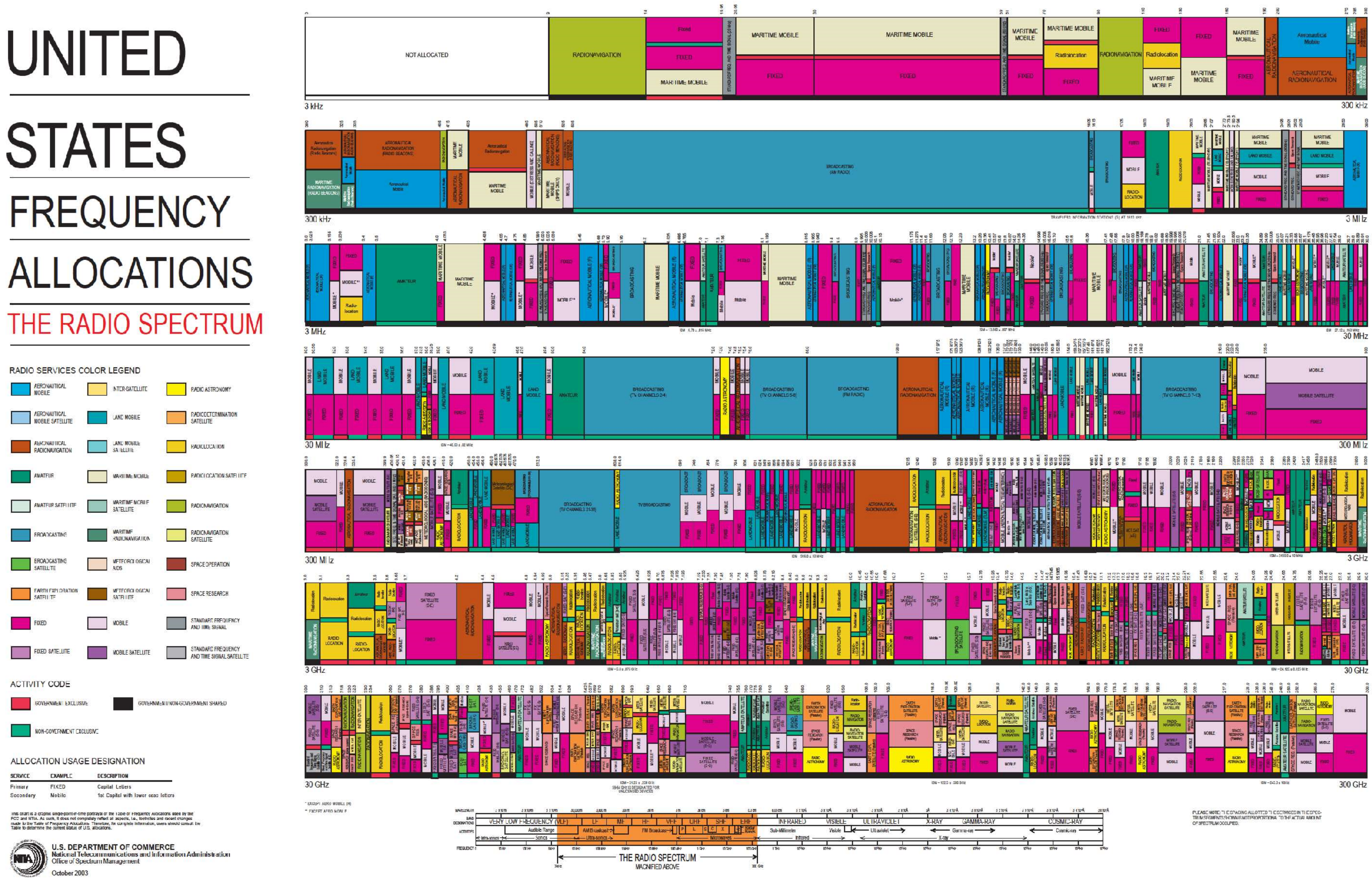
Telecom:

Internet:

3G data:

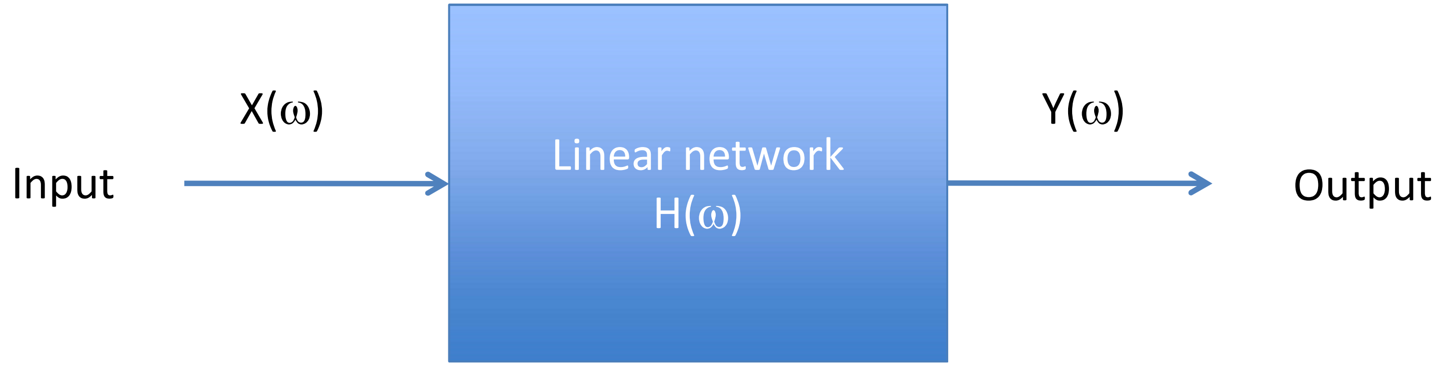
All use sine waves (phasors) as way to describe signals and circuits.

Frequency Allocations



http://www.ntia.doc.gov/osmhome/allochrt.PDF

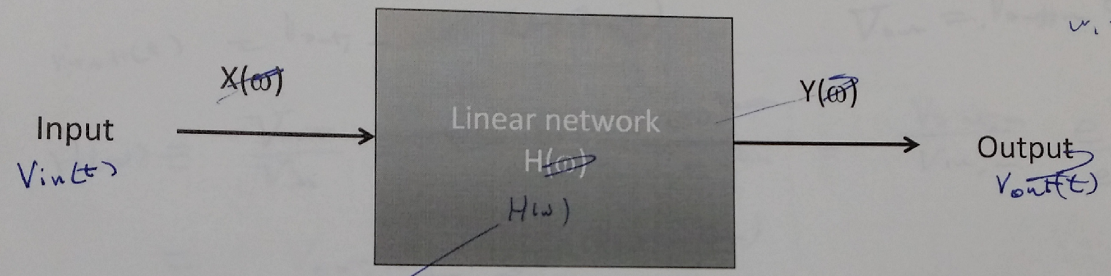
"Transfer function"



Significance of Transfer Function H(ω)

During lecture computer crashed so a few of these pages are scans of my written notes that were presented in lecture.

"Transfer Function"



$V_{in} = \sum a_n e^{i\omega n}$

$V_{out} = \sum c_n e^{i\omega n}$

ω, two ... out

Could define $\frac{V_{out}(t)}{V_{in}(t)}$. Too complicated for arbitrary circuits

Instead define $\frac{V_{out}}{V_{in}}$ assume periodic functions ie

$V_{in}(t) = V_{in,m} \cos(\omega t + \phi_{in})$

$V_{out}(t) = V_{out,m} \cos(\omega t + \phi_{out})$

"transfer function"

$H(\omega) \equiv \frac{V_{out}}{V_{in}}$

Note: $\frac{V_{out}(t)}{V_{in}(t)}$ and $\frac{V_{out}}{V_{in}}$ will depend on ω.

will do problems to find H(ω).

Significance of H(ω)

$v_{in}(t) = V_{in,m} \cos(\omega t + \phi_{in})$ $\qquad V_{in} = V_{in,m} e^{j\phi_{in}}$

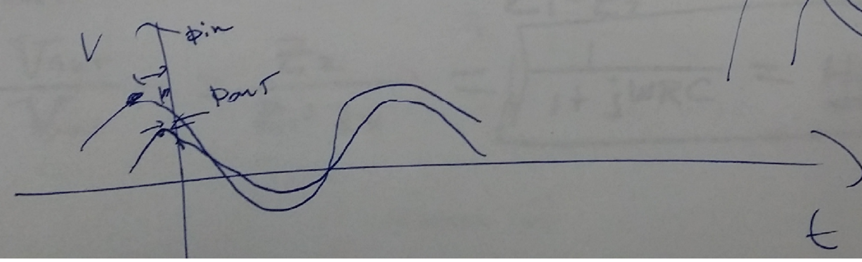
$v_{out}(t) = V_{out,m} \cos(\omega t + \phi_{out})$ $\qquad V_{out} = V_{out,m} e^{j\phi_{out}}$

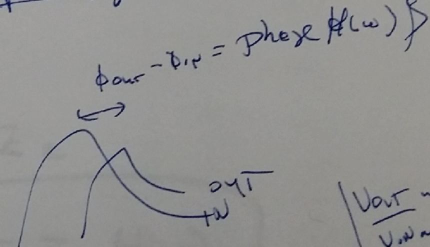
$$H(\omega) \equiv \frac{V_{out}}{V_{in}} = \frac{V_{out,m} e^{j\phi_{out}}}{V_{in,m} e^{j\phi_{in}}} = \frac{V_{out,m}}{V_{in,m}} \frac{e^{j\phi_{out}}}{e^{j\phi_{in}}}$$

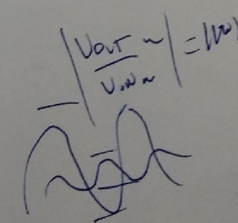
$$= \frac{V_{out,m}}{V_{in,m}} e^{j(\phi_{out} - \phi_{in})}$$

$|H(\omega)| = \dfrac{V_{out,m}}{V_{in,m}}$ tells us important info about amplitude of output signal.

Phase $H(\omega)$ = phase shift out vs in

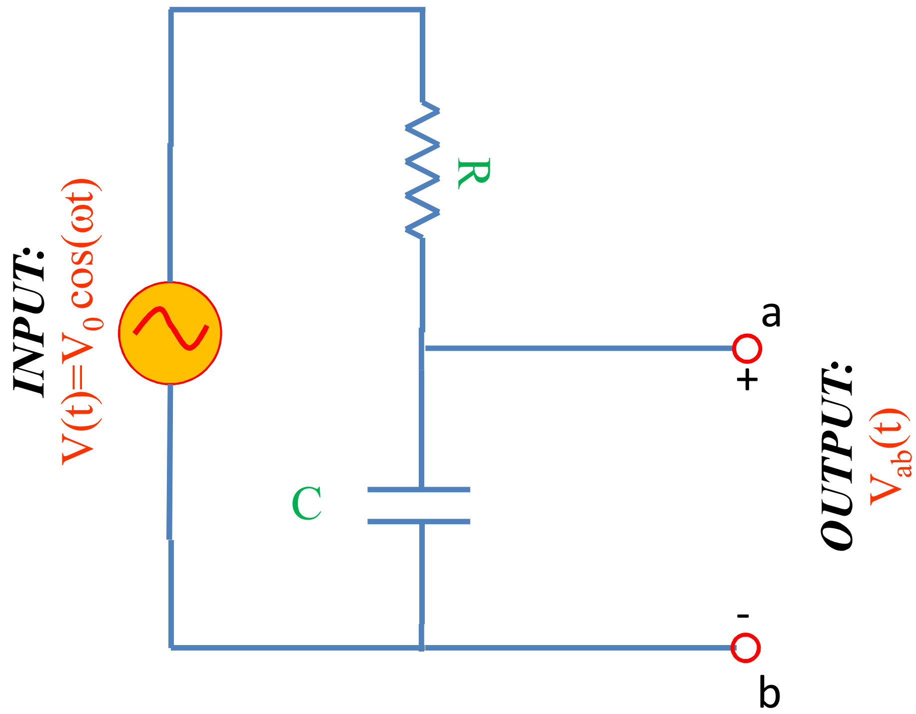


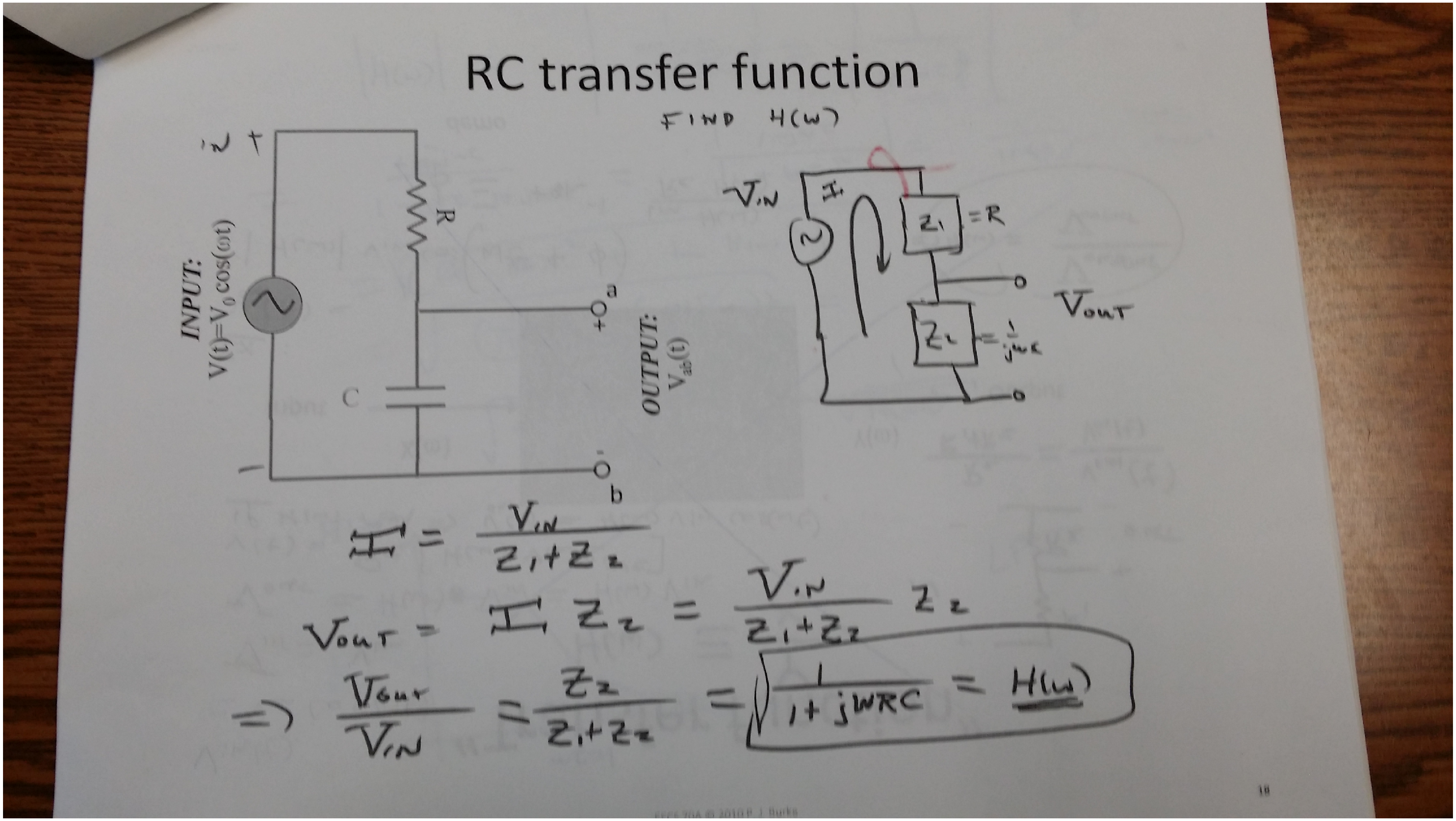




RC transfer function (Low pass filter)

Find H(ω)




RC transfer function
FIND H(w)
INPUT:
V(t)=V0cos(ωt)
R
C
a
b
OUTPUT:
Vab(t)
VIN
I
Z1 = R
Z2 = 1/jwRC
VOUT
I = VIN / (Z1+Z2)
VOUT = I Z2 = VIN / (Z1+Z2) Z2
=> VOUT / VIN = Z2 / (Z1+Z2) = 1 / (1+jWRC) = H(w)

Magnitude RC Transfer Function H(ω)

$$H(\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega\tau}$$

$$|H(\omega)| = \left|\frac{1}{1+j\omega\tau}\right| = \left|\frac{1}{1+j\omega\tau}\cdot\frac{1-j\omega\tau}{1-j\omega\tau}\right| = \left|\frac{1-j\omega\tau}{1+(\omega\tau)^2}\right|$$

$$= \frac{1}{1+(\omega\tau)^2}\left|1-j\omega\tau\right| = \frac{1}{1+(\omega\tau)^2}\sqrt{(1)^2+(-\omega\tau)^2}$$

$$= \boxed{\frac{1}{\sqrt{1+(\omega\tau)^2}} = H(\omega)}$$

$$|H(\omega)|\Big|_{\omega\tau=1} = ???$$

$\tau \equiv RC$

"cutoff frequency"

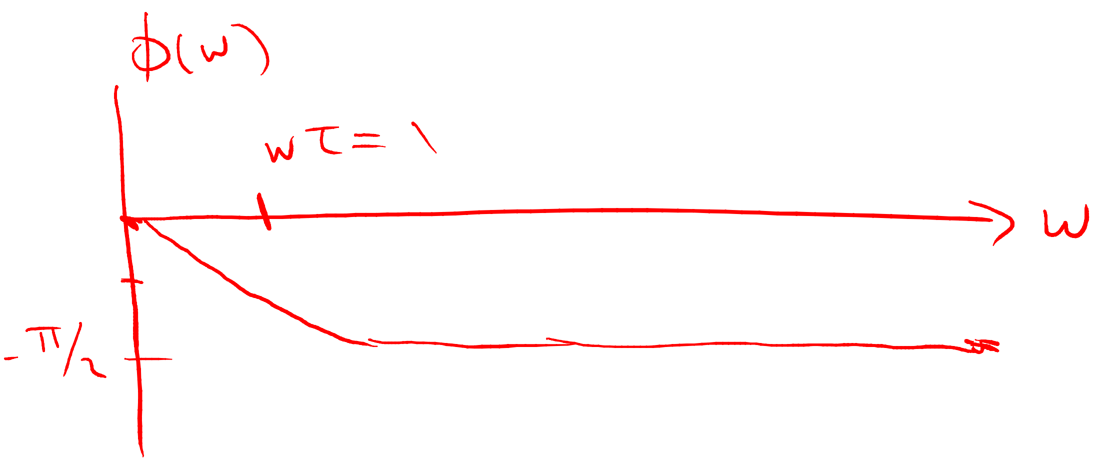
Phase of RC Transfer Function H(ω)

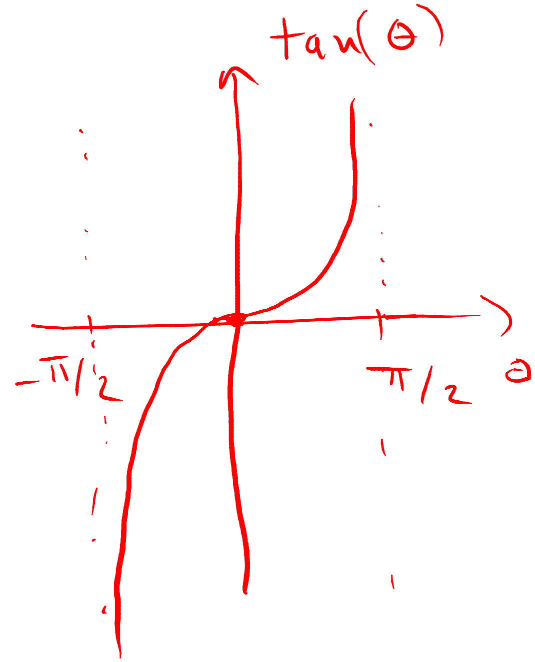
$$H(\omega) = \frac{1}{1+j\omega\tau} = \frac{1}{\sqrt{1+(\omega\tau)^2}}(1-j\omega\tau)$$

$$\phi = \tan^{-1}\frac{\text{Im } H(\omega)}{\text{Re } H(\omega)} = \tan^{-1}(-\omega\tau) = -\tan(\omega\tau)$$

$\omega \to 0 \quad \omega\tau \to 0 \quad \phi \to -\tan^{-1}(0) = 0$

$\omega \to \infty \quad \omega\tau \to \infty \quad \phi \to -\tan^{-1}(\infty) = -\pi/2$





Decibels

$$|H(\omega)| = \frac{V_{out,m}}{V_{in,m}}$$

$$\log x^n = n \log x$$

$$dB \equiv 10 \log \frac{P_{out}}{P_{in}} = 10 \log \left[\frac{V_{out,m}^2/R}{V_{in,m}^2/R} \right] = 10 \log \left(\frac{V_{out,m}}{V_{in,m}} \right)^2$$

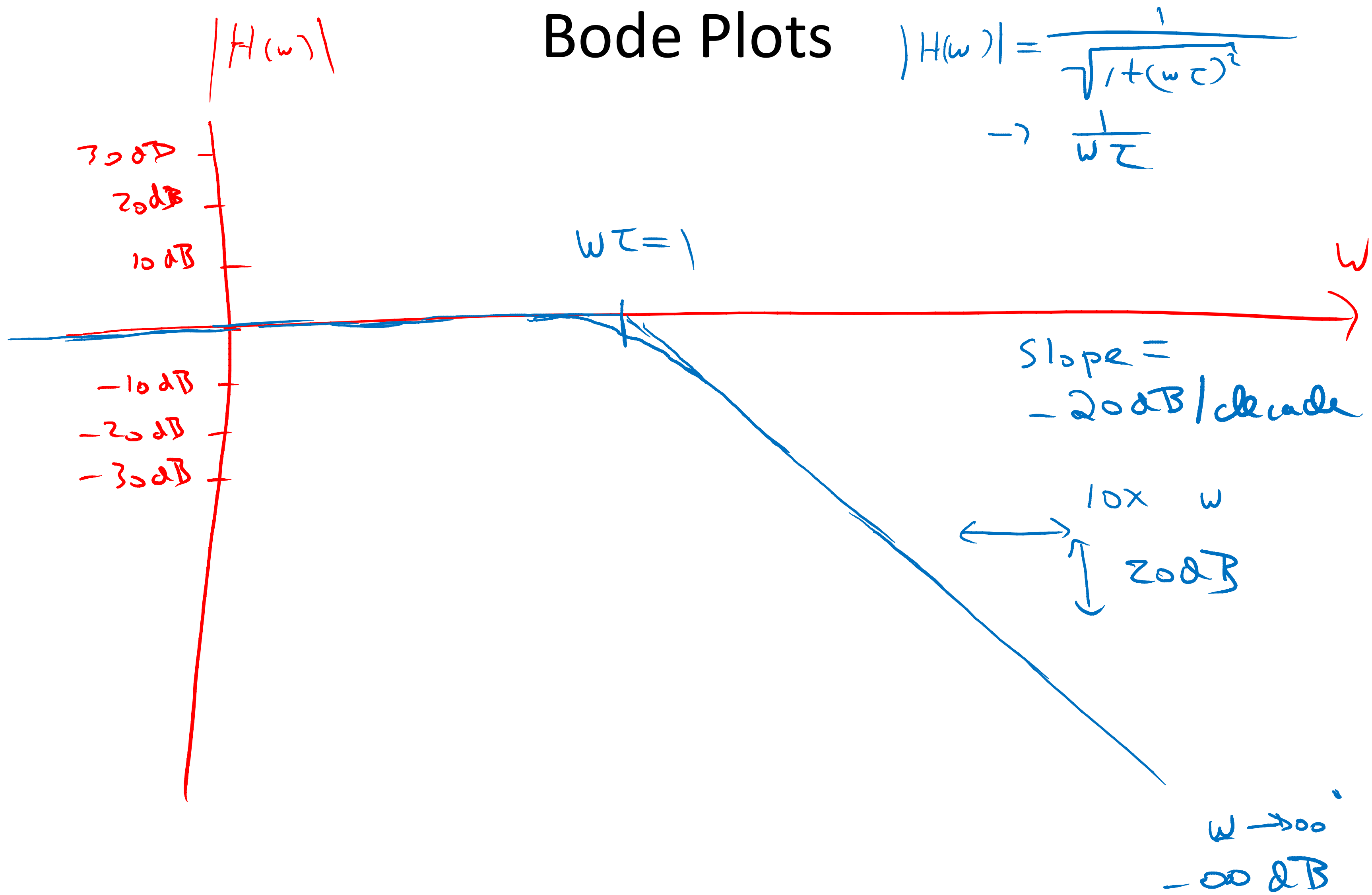
$$= 20 \log \frac{V_{out,m}}{V_{in,m}}$$

$\frac{V_{out,m}}{V_{in,m}}$	LINEAR	dB
	1	$20 \log 1 = 0$
	10	$20 \log 10 = 20$
	0.1	$20 \log 0.1 = 20 \log 10^{(-1)}$

$$= (-1)\, 20 \log 10$$

$$= -20$$

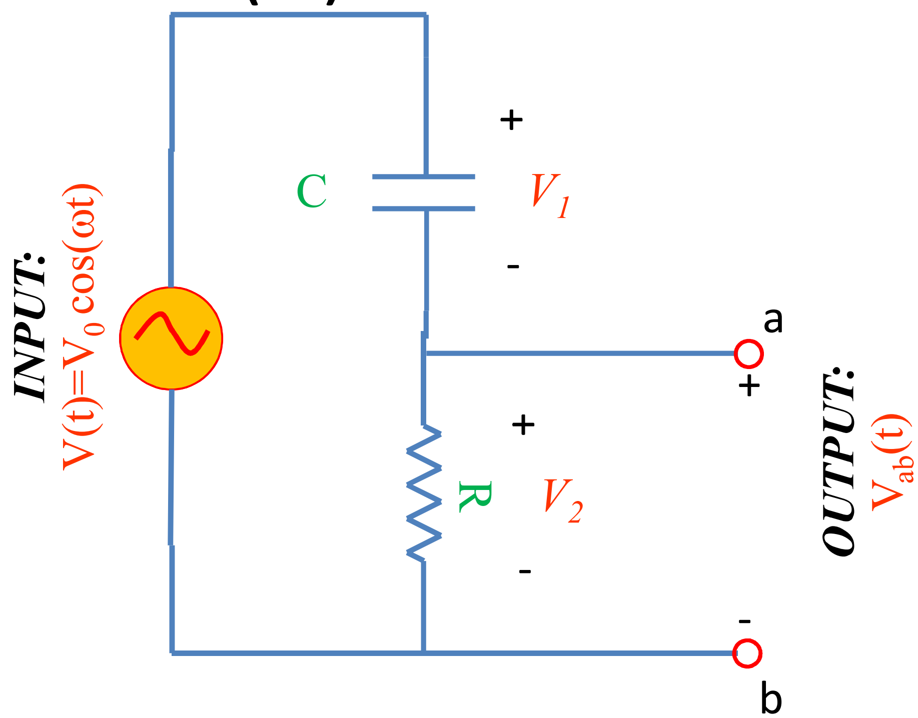
Bode Plots

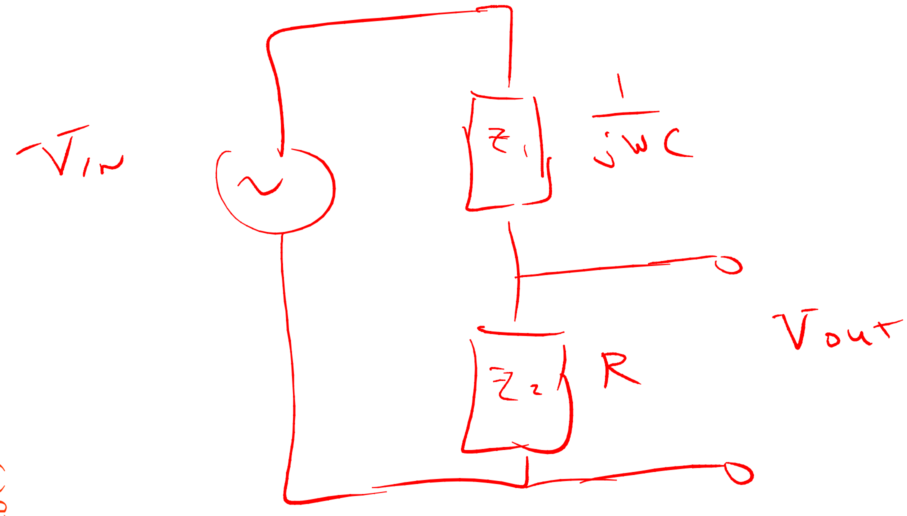


|H(ω)|
|H(ω)| = 1 / √(1+(ωτ)²)
→ 1/(ωτ)
30dB
20dB
10 dB
-10 dB
-20 dB
-30dB
ωτ=1
ω
Slope =
-20dB/decade
10x ω
20dB
ω→∞
-∞ dB

RC transfer function (High pass filter)

Find H(ω)





$$\frac{V_{out}}{V_{IN}} = \frac{Z_2}{Z_1+Z_2}$$

$$= \frac{R}{\frac{1}{j\omega C}+R}\cdot\frac{j\omega C}{j\omega C} = \frac{j\omega RC}{1+j\omega RC} = \boxed{\frac{j\omega\tau}{1+j\omega\tau} = H(\omega)}$$

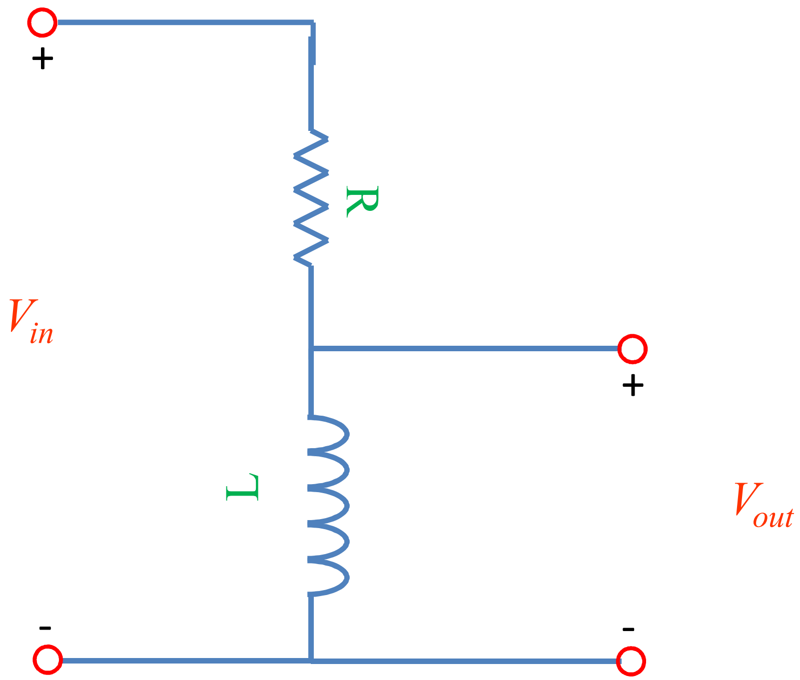
$|H(\omega)|$ $\quad \omega \to 0 \quad 0$

$\omega \to \infty \quad 1$

Example problem

$\tau = L/R$

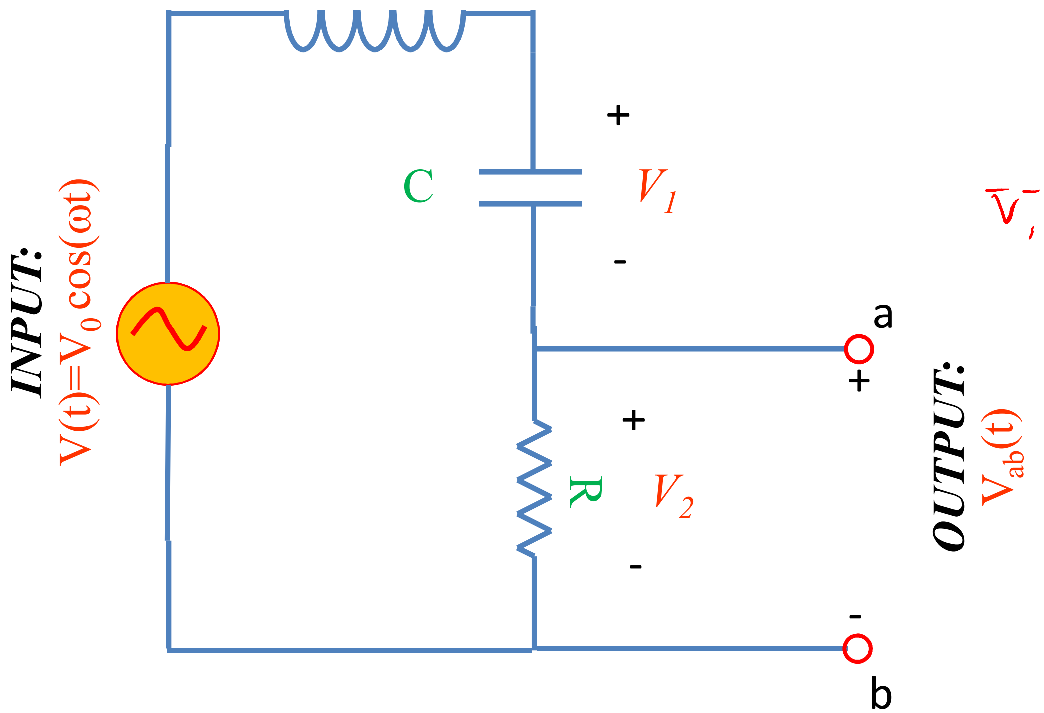
Find H(ω) for this circuit, then sketch the magnitude of H(ω) vs ω: (students)

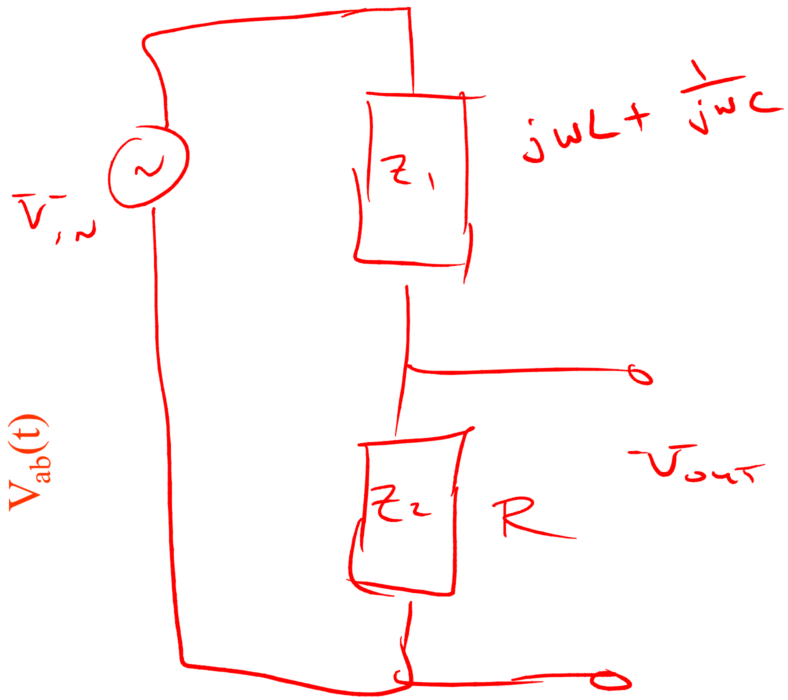


$$H(\omega) = \frac{j\omega L}{R + j\omega L} \cdot \frac{1/R}{1/R}$$

$$= \frac{j\omega \tau}{1 + j\omega \tau}$$

Band pass filter (RLC)



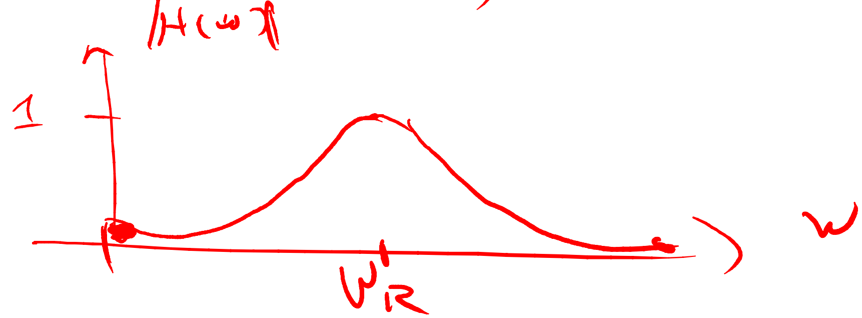


$$H(\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{R}{R + j\frac{1}{\omega}\left(\omega^2 L - \frac{1}{C}\right)}$$

$$\omega^2 L - \frac{1}{C} = 0$$

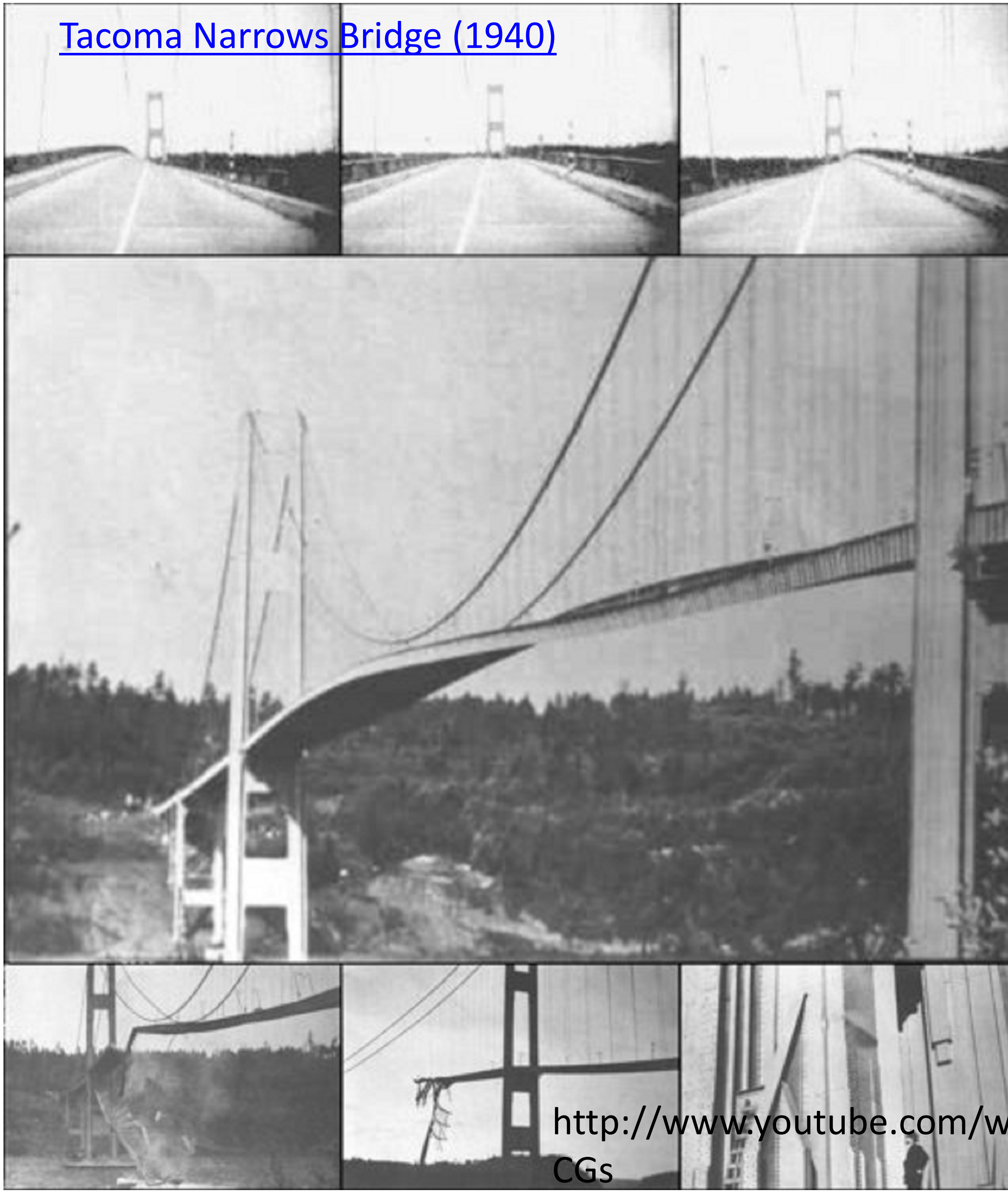
$$\omega_R = \sqrt{\frac{1}{LC}}$$



http://www.peachparts.com/shopforum/showthread.php?t=256624

Resonance

Tacoma Narrows Bridge (1940)



Nov 7, 1940

http://www.youtube.com/watch?v=3mclp9QmCGs