EECS 70A: Network Analysis

Midterm review

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Topics covered

- KCL, KVL
- Nodal analysis
- Mesh analysis
- Thevenin/Norton theorem

Nodal Analysis(Review)

Based on KCL, use node voltages as circuits variables.

1. Define a reference node.
2. Label remaining nodes. (n-1 nodes)
3. Apply KCL + ohm to all nodes and supernodes (e.g. V_1, V_2, V_3, …)
   Express all i's in terms of v's
4. Apply KVL to the voltage source
   If one end of voltage source connected to ground, don't need to
5. Solve the n-1 simultaneous equations, to find V's
6. Use Ohm's law to find the currents.

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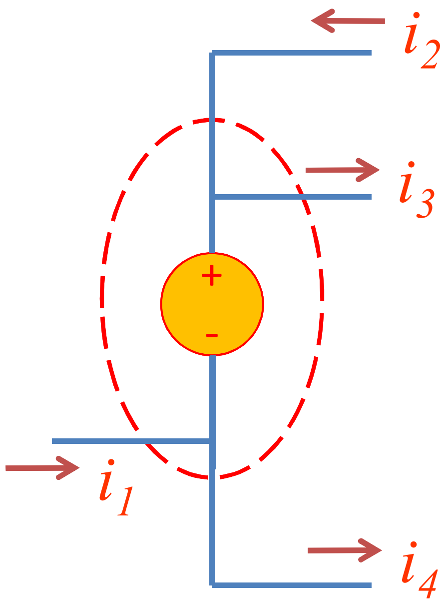
"Supernode"

A node with a voltage source in it…



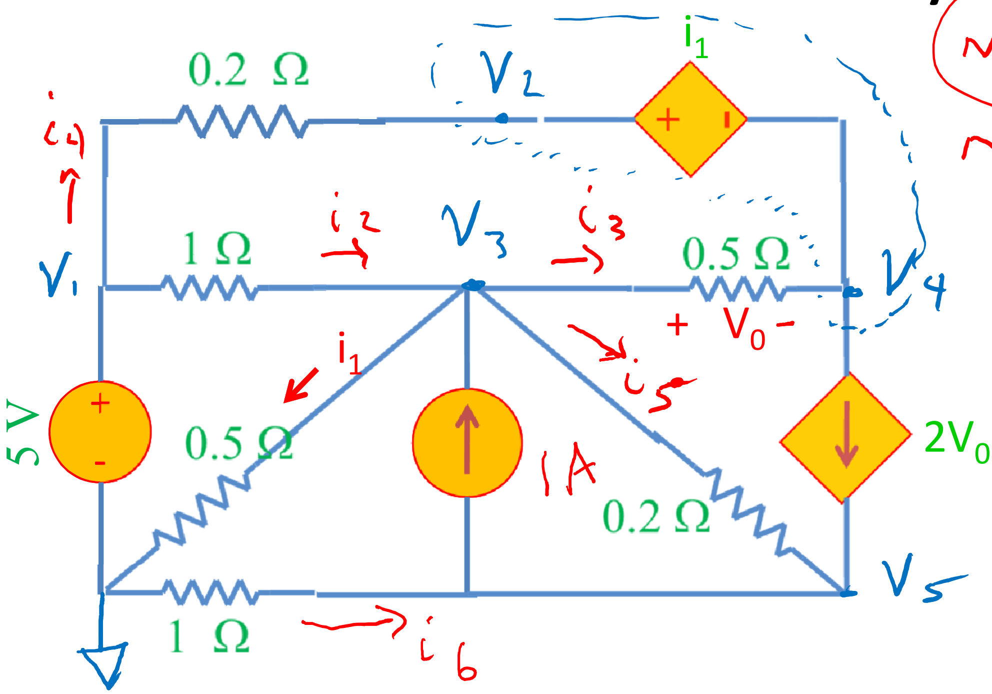
KCL: IN=OUT

$$i_1+i_2=i_3+i_4$$

Must define a supernode if a voltage source appears when doing nodal analysis…
(unless one end of voltage source connected to reference node)

1. Define a reference node.
2. Label remaining nodes.
3. Apply KCL + ohm to all nodes and supernodes
4. Apply KVL to loop with voltage source

Nodal Analysis-Example



N1: $V_1 = 5V$

N2:

N3: IN = OUT

$i_2 + 1A = i_3 + i_1 + i_5$

$$\frac{V_1 - V_3}{1} + 1A = \frac{V_3 - V_4}{0.5} + \frac{V_3 - 0}{0.5} + \frac{V_3 - V_5}{0.2}$$

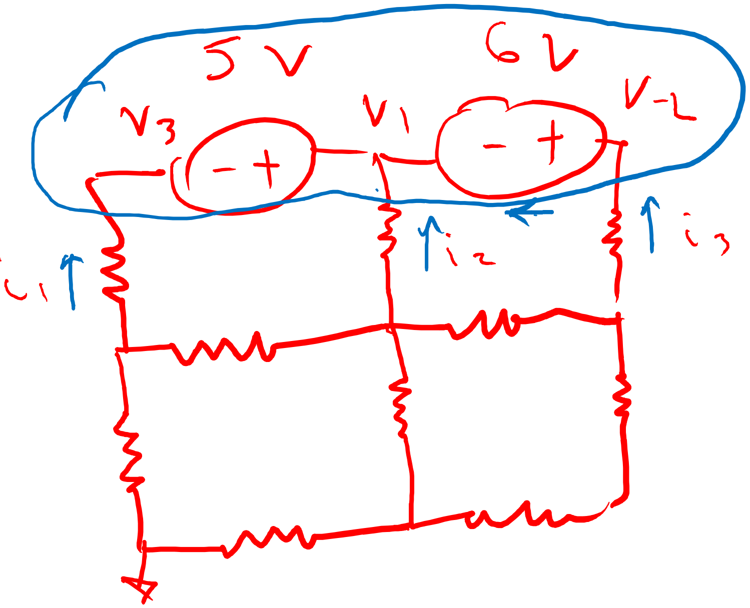
N4

N5 IN = OUT

$$\underbrace{2V_0}_{(V_3 - V_4)} + \frac{V_3 - V_5}{0.2} + \frac{0 - V_5}{1} = 1A$$

SN: $i_4 + i_3 = 2V_0$ $\quad \frac{V_1 - V_2}{0.2} + \frac{V_3 - V_4}{0.5} = 2(V_3 - V_4)$

KVL @ Vsource $\quad V_2 - V_4 = i_1 = \frac{V_3 - 0}{0.5}$

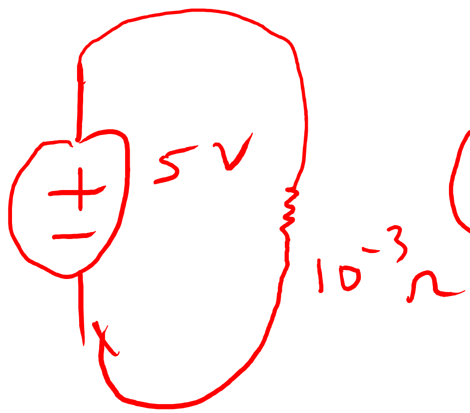


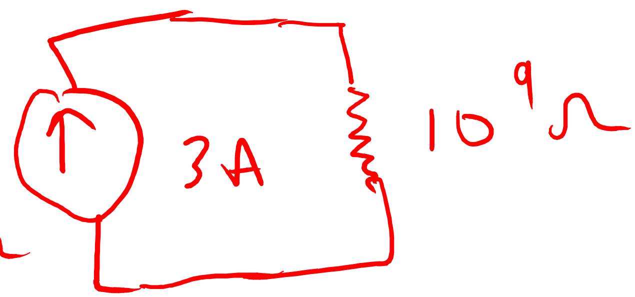
IN = OUT

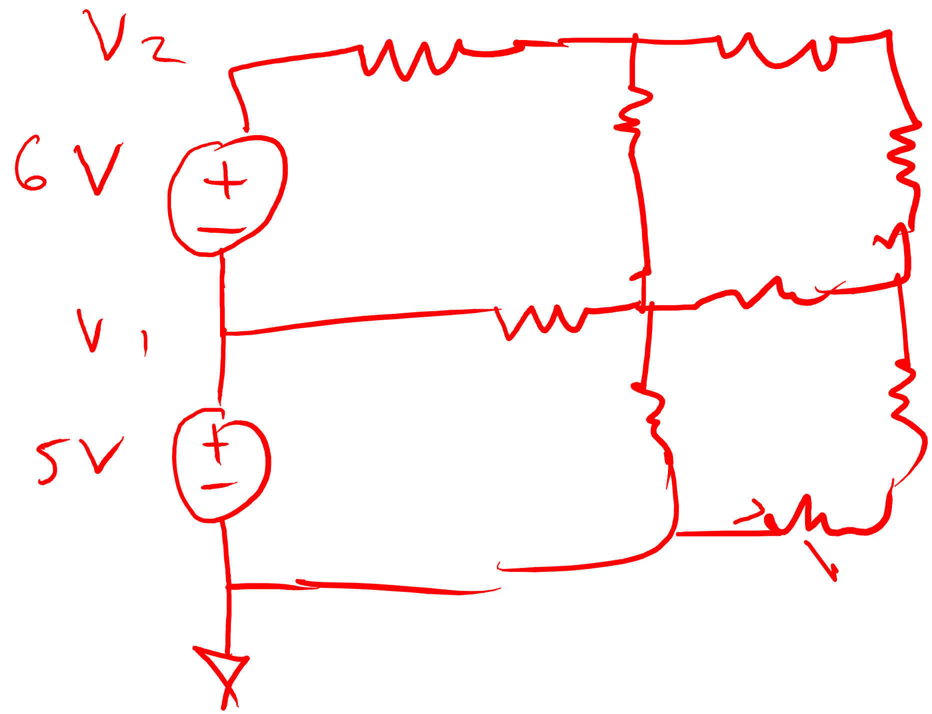
$V_1 - V_3 = 5$

$V_2 - V_1 = 6V$

$V_2 - V_3 = 5 + 6V$







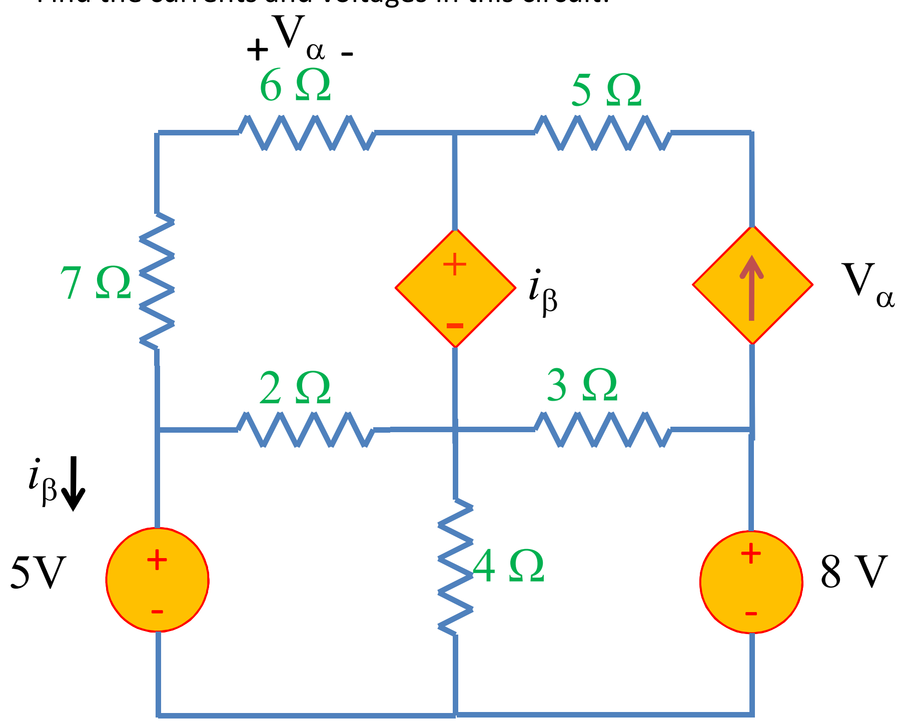
$V_1 = 5V$

$V_2 = 11V$

$V_2 - V_1 = 6V$

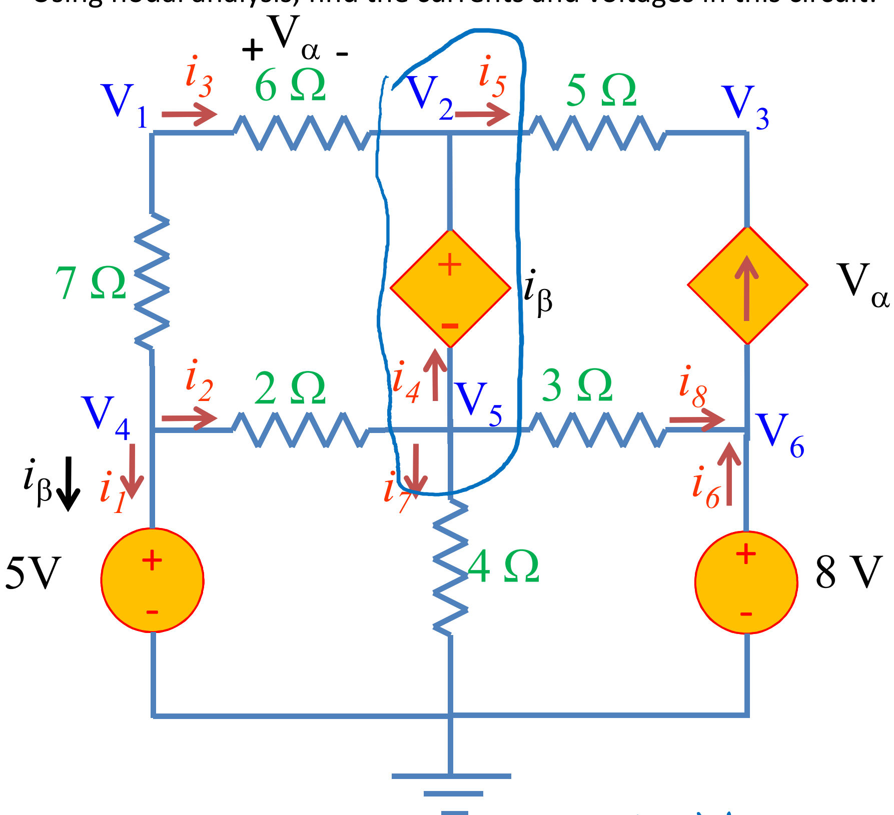
Nodal analysis example

Find the currents and voltages in this circuit:



Nodal analysis example

Using nodal analysis, find the currents and voltages in this circuit:



N1: IN = OUT $\quad i_3 = i_3$

$$\frac{V_4-V_1}{7} = \frac{V_1-V_2}{6}$$

N2: SN

N3: IN = OUT $\quad i_5 + V_\alpha = 0$

$$\frac{V_2-V_3}{5} + V_1 - V_2 = 0$$

N4: $V_4 = 5$ (lucky)

N5: SN

N6: $V_6 = 8$

SN: IN = OUT $\quad i_3 + i_2 = i_5 + i_8 + i_7$

$$\frac{V_1-V_2}{6} + \frac{V_4-V_5}{2} = \frac{V_2-V_3}{5} + \frac{V_5-V_6}{3} + \frac{V_5-0}{4}$$

KVL @ Vsource: $\boxed{V_2 - V_5 = i_\beta} = -i_2 - i_3$

$$= -\left(\frac{V_4-V_5}{2}\right) - \left(\frac{V_4-V_1}{7}\right)$$

$$\_\_ V_1 + \_\_ V_2 + \_\_ V_3 + \_\_ V_4 + \_\_ V_5 + \_\_ V_6 = \_\_$$

N1 $\quad -\frac{1}{7} + \frac{1}{6} \quad -\frac{1}{6} \quad 0 \quad \frac{1}{7} \quad 0 \quad 0 \quad = \quad 0$

N3

N4

N 6

SN

KVL@ SN

6X6 Solve $V_1 - V_6$.

then solve $i_1 - i_8$

Mesh analysis summary

Based on KVL, use mesh currents as circuit variables.

1. Assign mesh currents i_1, i_2, … i_n
2. Apply KVL + Ohm's law to each mesh
3. Supermesh (if there is a current source present):

- CASE 1: current source only in one mesh.
  - Already have the current for that mesh => no need to write KVL for that mesh
- CASE 2: current source exits between two meshes. => create a supermesh
  - Apply KVL to the supermesh
  - Apply KCL to a node in the branch where two meshes intersect

1. Solve the equations for i_1 ,i_2 ,…i_n
   (e.g. using Kramer's rule)
2. Then solve for voltages using Ohm's law

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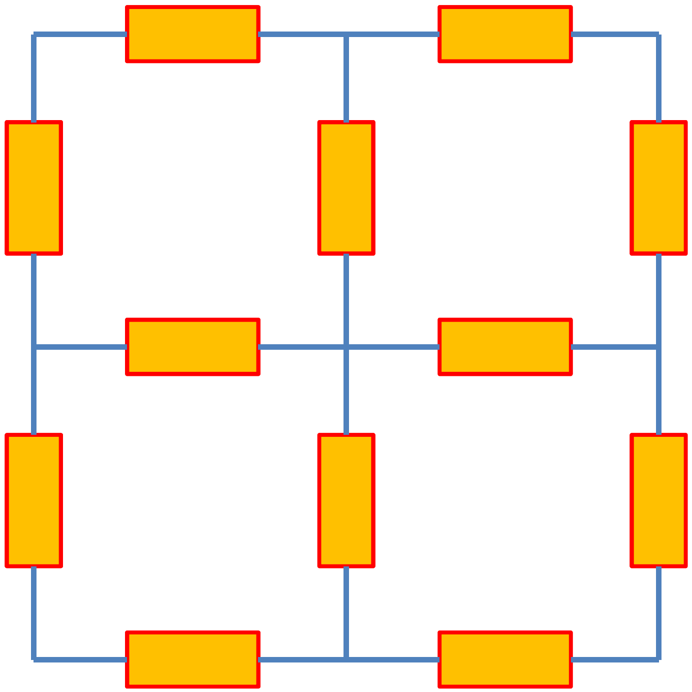
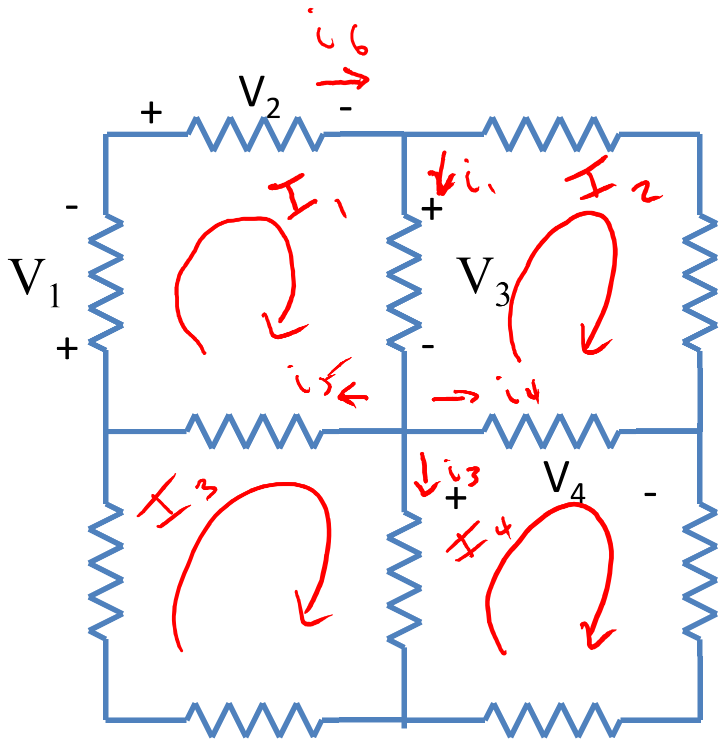
Mesh Analysis-Introduction
What is a Mesh?



- A loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loops within it.

Mesh Analysis-Introduction

Mesh Current vs. Element Current



- The current through a mesh is known as mesh current.
- Direction of the mesh current is arbitrary-conventionally assumed to be clockwise.
- The current through an element can be the same as mesh current or the subtraction of two mesh currents.

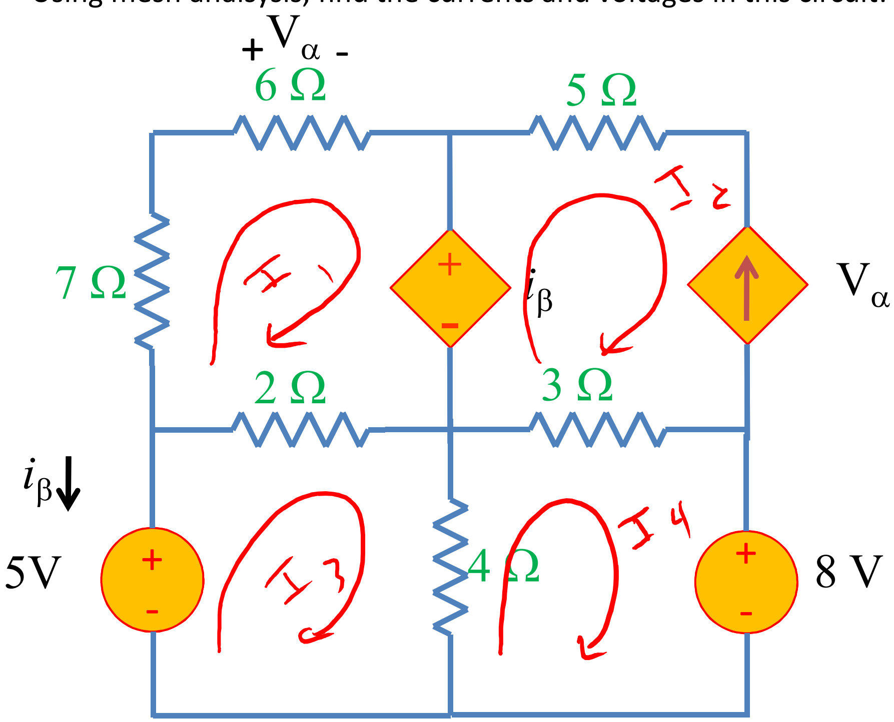
$i_1 = I_1 - I_2$

$i_4 = I_4 - I_2$

$i_6 = I_1$

Mesh analysis example

Using mesh analsysis, find the currents and voltages in this circuit:



M1

$I_1 7\Omega + I_1 6 + i_\beta + (I_1 - I_3)2$

$-I_3$

M2 SM BUT LUCKY (TRICK)

$I_2 = -V_\alpha$

$= -(I_1 6)$

M3 $-5 + 2(I_3 - I_1)$

$+ 4(I_3 - I_4) = 0$

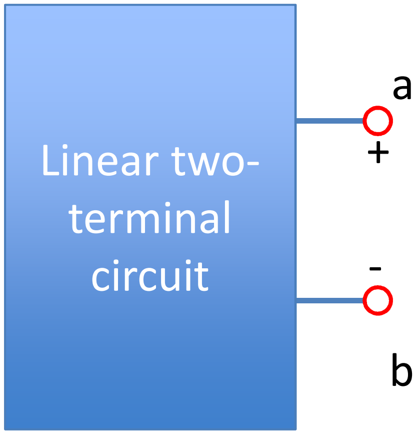
M4 $(I_4 - I_3)4 +$

$3(I_4 - I_2) + 8 = 0$

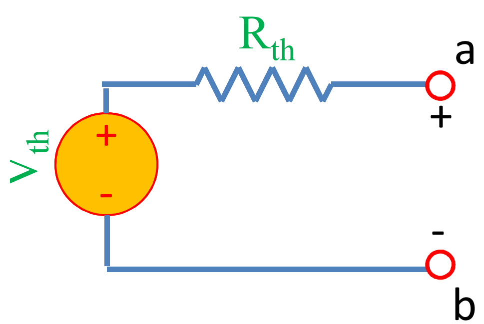
4 EQ 4 UNKNOWNS

FIND $I_1 I_2 I_3 I_4$

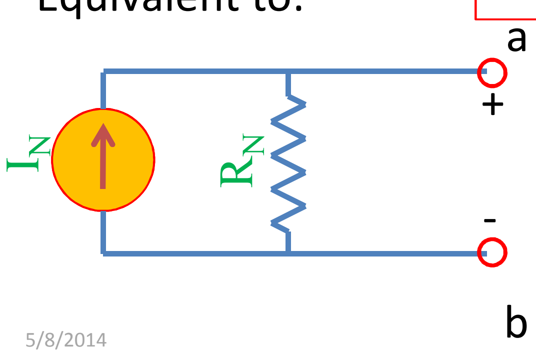
Thevenin, Norton Theorems:



Equivalent to:



Equivalent to:



Thevenin:

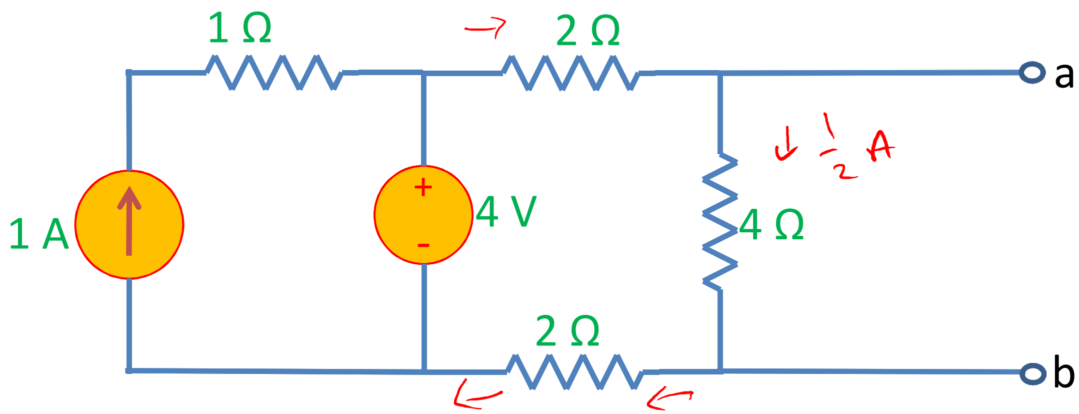
1. **Calculating V_{th}:**
   Connect nothing to a-b. Calculate voltage. This is V_{th}.
2. **Calculating R_{th}:**
   <u>Method 1:</u>
   Connect terminal a to b (short).
   Calculate the current from a to b. This is call $I_{short\ circuit}$.
   $R_{th}=V_{th}/I_{short\ circuit}$.
   <u>Method 2:</u>
   Find the input resistance looking into terminals a-b after all the independent sources have been turned off.
   (Voltage sources become shorts, current sources become opens.)
   Trick (if dependent sources present):
   Apply a 1 A current source to terminals a-b, find V_{ab}
   $R_{th} = V_{ab}/1A$.

Norton:

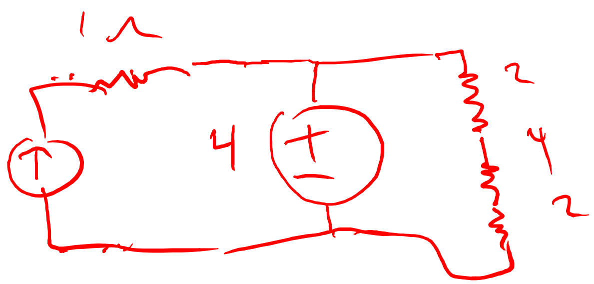
1. **Calculating R_N:**
   $R_N=R_{th}$ (same method as 2 above)
2. **Calculating I_N:**
   $I_N=V_{th}/R_{th}$
   OR:
   $I_N=I_{short\ circuit}$

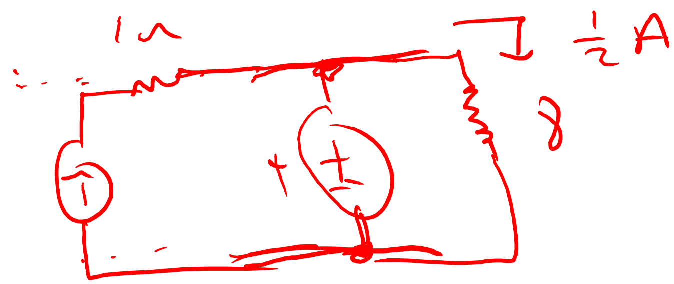
Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:

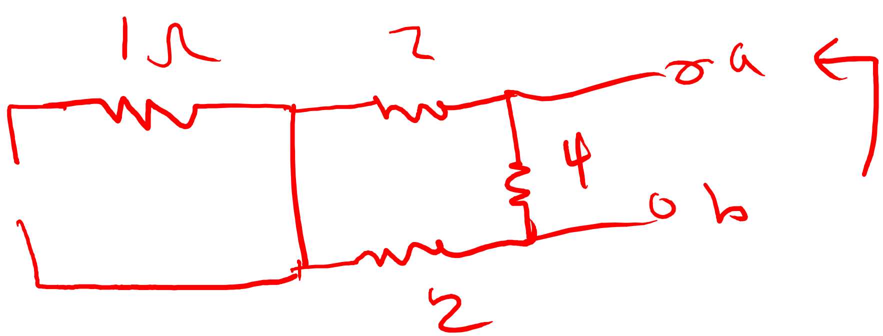
Example problem



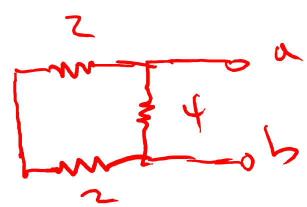
$$v_{ab} = 2 \checkmark$$
$$= V_{th}$$







$$R_{ab} = 2$$



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