

$$\begin{aligned}
 Z &= R \parallel \frac{1}{j\omega C} \\
 &= \frac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}
 \end{aligned}$$

By voltage division rule,

$$V_{out} = \frac{Z}{Z + j\omega L} \cdot V_{in}$$

$$\begin{aligned}
 \therefore H(\omega) &= \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{Z}{Z + j\omega L} = \frac{R}{1 + j\omega RC} \\
 &= \frac{R}{\frac{R}{1 + j\omega RC} + j\omega L}
 \end{aligned}$$

$$= \frac{R}{R + j\omega L + (j\omega)^2 RLC}$$

$$= \frac{R}{R + j\omega L - \omega^2 RLC}$$

$$2. \quad H(\omega) = \frac{1}{1+j\omega\tau_1} \cdot \frac{1}{1+j\omega\tau_2} \quad (\tau_2 > \tau_1)$$

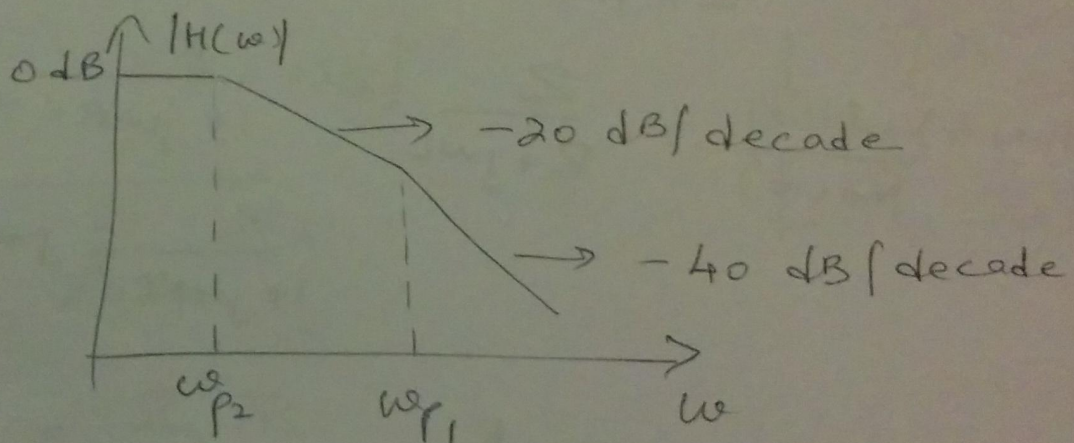
$$\text{poles at } |j\omega\tau_1| = 1 \Rightarrow \omega_{p1} = \frac{1}{\tau_1}$$

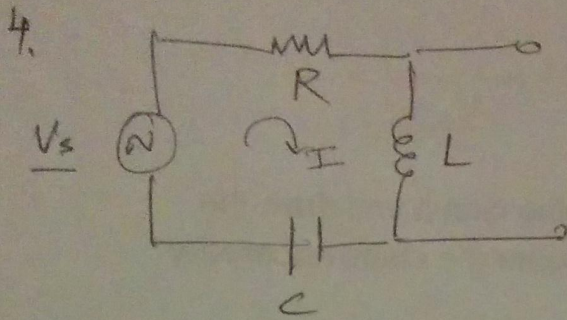
$$|\omega\tau_2| = 1 \Rightarrow \omega_{p2} = \frac{1}{\tau_2}$$

since, $\tau_2 > \tau_1$

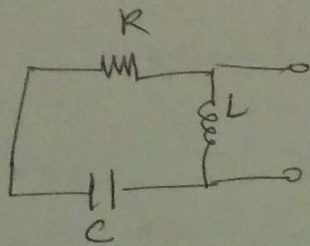
$$\omega_{p2} < \omega_{p1}$$

$$|H(0)| = 1 \quad \text{or} \quad 0 \text{ dB}$$





R_{TH}



$$Z_{TH} = \left(R + \frac{1}{j\omega C} \right) \parallel (j\omega L)$$

$$= \left(R + \frac{1}{j\omega C} \right) (j\omega L)$$

$$R + \frac{1}{j\omega C} + j\omega L$$

$$= \frac{[j\omega RC + 1] (j\omega L)}{j\omega RC + 1 + (j\omega)^2 CL}$$

$$= \frac{-\omega^2 RCL + j\omega L}{1 + j\omega RC - \omega^2 CL}$$

$$= \frac{-\omega^2 RCL + j\omega L}{1 + j\omega RC - \omega^2 CL}$$

$$= \frac{-\omega^2 RCL + j\omega L}{1 + j\omega RC - \omega^2 CL}$$

$$\underline{V_{TH}} = V_{oc}$$

$$I = \frac{V_s}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{(j\omega C) V_s}{1 + j\omega RC + (j\omega)^2 LC}$$

$$= \frac{(j\omega C) V_s}{1 + j\omega RC + (j\omega)^2 LC}$$

$$V_{TH} = Z_L \cdot I = (j\omega L) I$$

$$= \frac{(j\omega)^2 LC \cdot V_s}{1 + j\omega RC + (j\omega)^2 LC}$$

$$= \frac{(j\omega)^2 LC \cdot V_s}{1 + j\omega RC + (j\omega)^2 LC}$$

$$5. a) \omega_0 z = 1 \Rightarrow \omega_0 = 1/z$$

$$\omega_1 = 0.01 z^{-1} = 0.01 \omega_0$$

$$\omega_2 = 10 z^{-1} = 10 \omega_0$$

from Bode Plot,

$$|H(\omega_1)| = 0 \text{ dB} = 1 \text{ V}$$

$$|H(\omega_2)| = -20 \text{ dB} = 10^{-20/20} = 10^{-1} = 0.1 \text{ V}$$

$$\angle H(\omega_1) = 0^\circ$$

$$\angle H(\omega_2) = -90^\circ$$

$$\begin{aligned} \therefore V_{out}(t) &= \text{Re} \left[|H(\omega_1)| e^{j\angle H(\omega_1)} e^{j\omega_1 t} + |H(\omega_2)| e^{j\angle H(\omega_2)} e^{j\omega_2 t} \right] \\ &= \text{Re} \left[1 \cdot e^{0} \cdot e^{j\omega_1 t} + 1 \times 0.1 e^{+90^\circ} \cdot e^{j\omega_2 t} \right] \\ &= \cos \omega_1 t + 0.1 \cos(\omega_2 t - 90^\circ) \end{aligned}$$

b)

$$\omega_1 = 0.1 \omega_0$$

$$\omega_2 = 100 \omega_0$$

from Bode Plot,

$$|H(\omega_1)| = 0 \text{ dB} = 1 \text{ V}$$

$$|H(\omega_2)| = -40 \text{ dB} = 10^{-40/20} = 0.01$$

$$\angle H(\omega_1) = 0$$

$$\angle H(\omega_2) = -90^\circ$$

$$\begin{aligned} \therefore V_{out}(t) &= \text{Re} \left(1 \cdot e^{0} \cdot e^{j\omega_1 t} + 1 \times 0.01 e^{-j90^\circ} \cdot e^{j\omega_2 t} \right) \\ &= \cos \omega_1 t + 0.01 \cos(\omega_2 t - 90^\circ) \end{aligned}$$