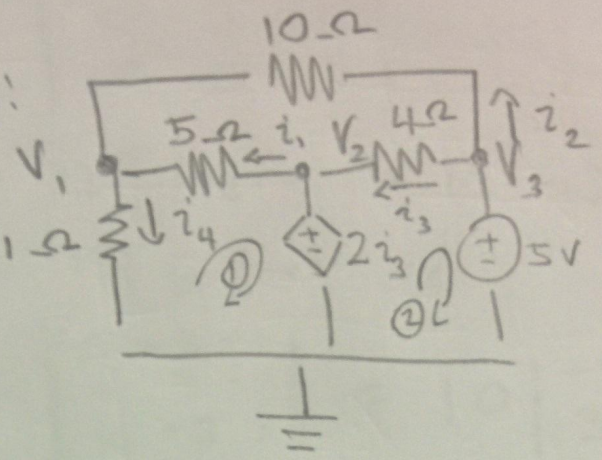
1) Given:



Determine: All node voltages

Solve:

Node $i_1 + i_2 = i_4 \Rightarrow \frac{V_2 - V_1}{5} + \frac{V_3 - V_1}{10} = \frac{V_1}{1}$

$2V_2 - 2V_1 + V_3 - V_1 = 10V_1$

$13V_1 - 2V_2 - V_3 = 0$

Loop ①

$-i_4 - 5i_1 + 2i_3 = 0$

$-\frac{V_1}{1} - 5\left(\frac{V_2 - V_1}{5}\right) + 2\left(\frac{V_3 - V_2}{4 \cdot 2}\right) = 0$

$-2V_1 - 2V_2 + 2V_1 + V_3 - V_2 = 0$

$0V_1 + 3V_2 - V_3 = 0$

Loop ②

$-2i_3 - 4i_3 + 5 = 0$

$3 \cdot 6\left[\frac{V_3 - V_2}{4 \cdot 2}\right] = 5$

$0V_1 + 3V_3 - 3V_2 = 10$

$$\begin{bmatrix} 13 & -2 & -1 \\ 0 & 3 & -1 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$D = \begin{vmatrix} 13 & -2 & -1 \\ 0 & 3 & -1 \\ 0 & -3 & 3 \end{vmatrix} = 13 \begin{vmatrix} 3 & -1 \\ -3 & 3 \end{vmatrix} + 0 + 0$$

$$= 13[(3 \cdot 3) - (-3 \cdot -1)] = 13[9 - 3]$$

$$= 13[6] = 78$$

$$A = \begin{vmatrix} 0 & -2 & -1 \\ 0 & 3 & -1 \\ 10 & -3 & 3 \end{vmatrix} = 10 \begin{vmatrix} -2 & -1 \\ 3 & -1 \end{vmatrix} + 0 + 0$$

$$= 10[(-2 \cdot -1) - (3 \cdot -1)] = 10[2 + 3]$$

$$= 50$$

$$B = \begin{vmatrix} 13 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 10 & 3 \end{vmatrix} = 13 \begin{vmatrix} 0 & -1 \\ 10 & 3 \end{vmatrix} + 0 + 0$$

$$= 13[(0 \cdot 3) - (-1 \cdot 10)] = 13[0 + 10]$$

$$= 130$$

$$C = \begin{vmatrix} 13 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & -3 & 10 \end{vmatrix} = 13 \begin{vmatrix} 3 & 0 \\ -3 & 10 \end{vmatrix} + 0 + 0$$

$$= 13[(3 \cdot 10) - (-3 \cdot 0)] = 13[30 - 0]$$

$$= 390$$

$$V_1 = \frac{A}{D} = \frac{50}{78} = 0.641025\ V$$

$$V_2 = \frac{B}{D} = \frac{130}{78} = 1.666666\ V$$

$$V_3 = \frac{C}{D} = \frac{390}{78} = 5\ V$$

$$i_1 = \frac{V_2 - V_1}{5} = \frac{1.666666\,V + 0.1641025\,V}{5} = 0.205127\,A$$

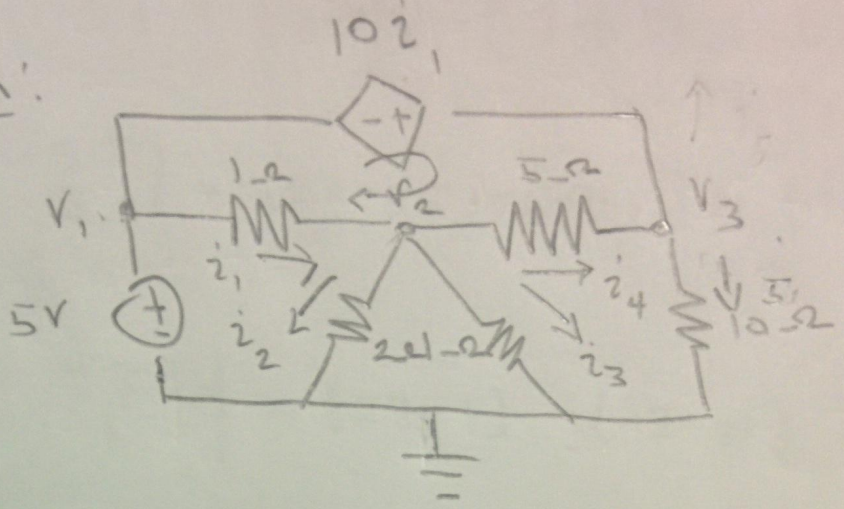
$$i_2 = \frac{V_3 - V_1}{10} = \frac{5\,V + 0.1641025\,V}{10} = 0.435897\,A$$

$$i_3 = \frac{V_3 - V_2}{4} = \frac{5\,V + 1.666666\,V}{4} = 0.8333335\,A$$

$V_1 =$	0.641 V	$i_1 =$	0.205A
$V_2 =$	1.67 V	$i_2 =$	0.436A
$V_3 =$	5 V	$i_3 =$	0.833A
		$i_4 =$	0.641A

$$i_4 = \frac{V_1}{1} = 0.1641025$$

2] Given:



Determine: All voltages & currents

Solve: Nodal analysis

ground $V_1 = 5\ V$

node $i_1 = i_2 + i_3 + i_4$

$$\frac{V_1 - V_2}{1} = \frac{V_2}{2} + \frac{V_2}{1} + \frac{V_2 - V_3}{5}$$

$$10V_1 - 10V_2 = 5V_2 + 10V_2 + 2V_2 - 2V_3$$

$$10V_1 - 27V_2 + 2V_3 = 0$$

But $V_1 = 5\ V \quad \therefore \quad 27V_2 - 2V_3 = 50$

loop: $-10i_1 - 5i_4 - i_1 = 0$

$$11i_1 + 5i_4 = 0$$

$$11\left(\frac{V_1 - V_2}{1}\right) + 5\left(\frac{V_2 - V_3}{5}\right) = 0$$

$$11V_1 - 11V_2 + V_2 - V_3 = 0$$

$$11V_1 - 10V_2 - V_3 = 0$$

But $V_1 = 5\ V \quad \therefore \quad 10V_2 + V_3 = 55$

$$\begin{bmatrix} 27 & -2 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 55 \end{bmatrix}$$

$V_1 = 5\ V$

$V_2 = 3.404\ V$

$V_3 = 20.95\ V$

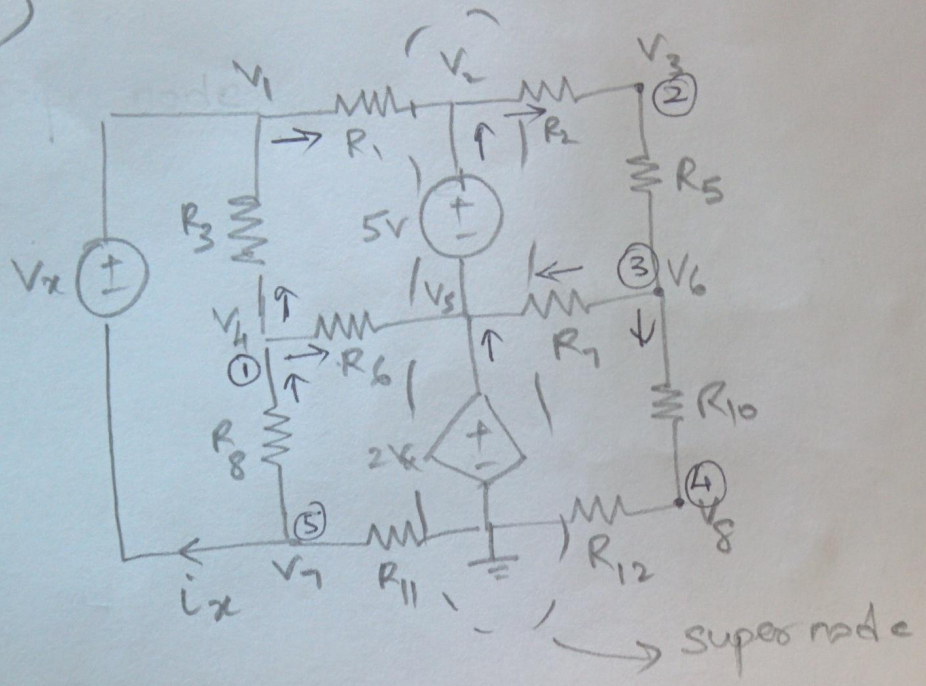
$$i_1 = \frac{V_1 - V_2}{1} = 1.596\ A$$

$$i_2 = \frac{V_2}{2} = 1.702\ A$$

$$i_3 = \frac{V_2}{1} = 3.404\ A$$

$$i_4 = \frac{V_2 - V_3}{5} = -3.509\ A$$

P3)



① super node

$$\frac{V_1 - V_2}{R_1} + \frac{V_8}{R_{12}} + \frac{V_4 - V_5}{R_6} + \frac{V_6 - V_5}{R_7} = \frac{V_7}{R_{11}} + \frac{V_2 - V_3}{R_2}$$

$$V_1\left(\frac{1}{R_1}\right) + V_2\left(\frac{-1}{R_1} - \frac{1}{R_2}\right) + V_3\left(\frac{1}{R_2}\right) + V_4\left(\frac{1}{R_6}\right) + V_5\left(\frac{-1}{R_6} - \frac{1}{R_7}\right)$$

$$+ V_6\left(\frac{1}{R_7}\right) + V_7\left(\frac{-1}{R_{11}}\right) + V_8\left(\frac{1}{R_{12}}\right) = 0 \quad \rightarrow *$$

② node ①

$$\frac{V_7 - V_4}{R_8} = \frac{V_4 - V_1}{R_3} + \frac{V_4 - V_5}{R_6}$$

$$V_1\left(\frac{-1}{R_3}\right) + V_4\left(\frac{1}{R_3} + \frac{1}{R_6} + \frac{1}{R_8}\right) + V_5\left(\frac{-1}{R_6}\right) + V_7\left(\frac{-1}{R_8}\right) = 0$$

$\hookrightarrow$ #

node ②

$$\frac{V_2 - V_3}{R_2} = \frac{V_3 - V_6}{R_5}$$

$$V_2\left(\frac{1}{R_2}\right) + V_3\left(-\frac{1}{R_2} - \frac{1}{R_5}\right) + V_6\left(\frac{1}{R_5}\right) = 0 \rightarrow *$$

node ③

$$\frac{V_3 - V_6}{R_5} = \frac{V_6 - V_5}{R_7} + \frac{V_6 - V_8}{R_{10}}$$

$$V_3\left(\frac{1}{R_5}\right) + V_6\left(-\frac{1}{R_5} - \frac{1}{R_7} - \frac{1}{R_{10}}\right) + V_5\left(\frac{1}{R_7}\right) + V_8\left(\frac{1}{R_{10}}\right) = 0 \rightarrow *$$

node ④

$$\frac{V_4 - V_8}{R_{10}} = \frac{V_8}{R_{12}}$$

$$\frac{V_4}{R_{10}} + V_8\left(-\frac{1}{R_{10}} - \frac{1}{R_{12}}\right) = 0 \rightarrow *$$

Also, we know

$V_5 = 2V_x$ or $V_x = \frac{V_5}{2}$

$V_2 - V_5 = 5 \rightarrow *$

$V_1 - V_7 = V_x$ or $V_1 - V_7 - \frac{V_5}{2} = 0 \rightarrow *$

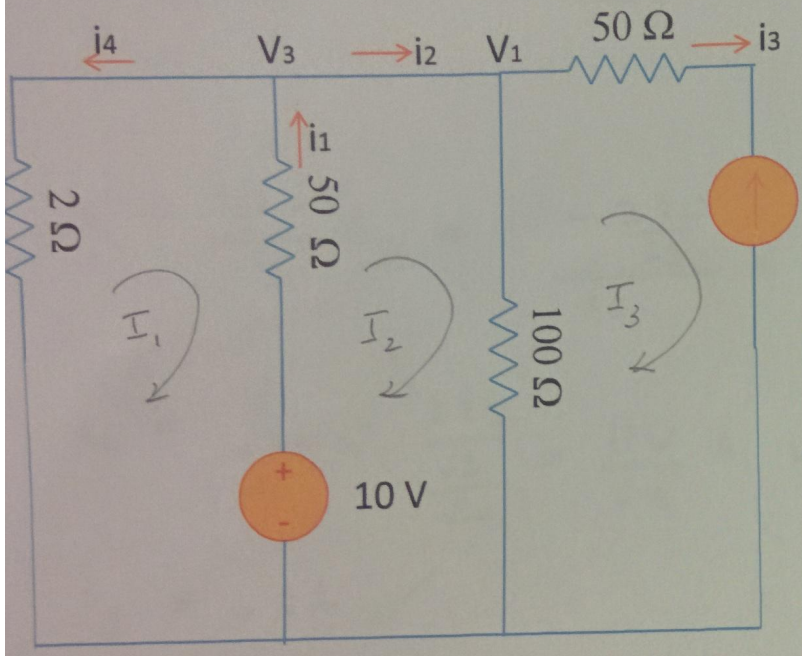
node ⑤

$$\frac{V_1}{R_{11}} + \frac{V_7}{R_8} + V_4\left(\frac{1}{R_8}\right) = -i_x$$

$$
\begin{bmatrix}
\frac{1}{R_1} & -\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \frac{1}{R_2} & \frac{1}{R_6} & -\left(\frac{1}{R_6}+\frac{1}{R_7}\right) & \frac{1}{R_7} & \frac{-1}{R_{11}} & \frac{1}{R_{12}} \\
-\frac{1}{R_3} & 0 & 0 & \left(\frac{1}{R_3}+\frac{1}{R_6}+\frac{1}{R_8}\right) & -\frac{1}{R_6} & 0 & -\frac{1}{R_8} & 0 \\
0 & \frac{1}{R_2} & -\left(\frac{1}{R_2}+\frac{1}{R_5}\right) & 0 & 0 & \frac{1}{R_5} & 0 & 0 \\
0 & 0 & \frac{1}{R_5} & 0 & \frac{1}{R_7} & -\left(\frac{1}{R_5}+\frac{1}{R_7}+\frac{1}{R_{10}}\right) & 0 & \frac{1}{R_{10}} \\
0 & 0 & 0 & \frac{1}{R_{10}} & 0 & 0 & 0 & -\left(\frac{1}{R_{10}}+\frac{1}{R_{12}}\right) \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1/5 & 0 & -1 & 0 \\
\frac{1}{R_{11}} & 0 & 0 & -\frac{1}{R_8} & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}
$$

* Since i_x is not given, Answer is in terms of i_x.

lem 4: use mesh analysis to find all the currents and node voltages.



① $2I_1 + 50(I_1 - I_2) + 10V = 0$

② $50(I_2 - I_1) + 100(I_2 - I_3) - 10 = 0$

③ $I_3 = -2A$

$+ \quad 52I_1 - 50I_2 = -10V$

$* \quad -50I_1 + 150I_2 - 100I_3 = 10V$

$* \quad I_3 = -2A$

$$\begin{bmatrix} 52 & -50 & 0 \\ -50 & 150 & -100 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \\ -2 \end{bmatrix}$$

$I_1 = \frac{-110}{53}$ $\quad I_2 = -\frac{519}{265}$

$I_3 = -2$

$= I_2 - I_1 = \frac{31}{265} A$

$= I_2 = -\frac{519}{265} A$

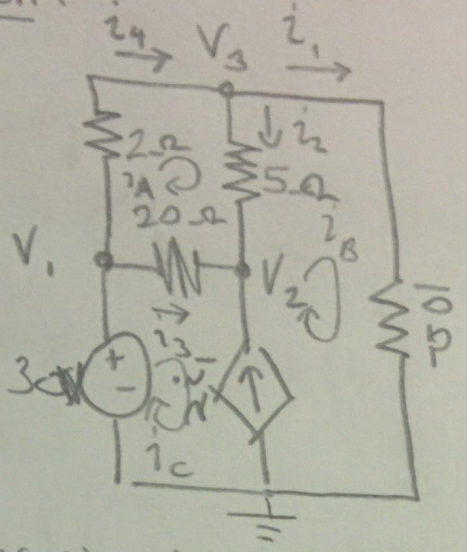
$i_4 = \frac{110}{53} A$

$i_3 = -2A$

$V_3 = V_1 = \frac{220}{53} V$

$V_3 = 2(i_4) = 2\left(\frac{110}{53}\right) A = \frac{220}{53} V$

5] Given:



use mesh analysis

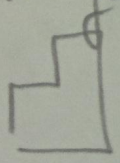
Determine: currents & voltages

Solve:

Mesh A: $2i_A + 5(i_A - i_B) + 20(i_A - i_C) = 0$

$27i_A - 5i_B - 20i_C = 0$

Mega Mesh:



$-30 + 20(i_C - i_A) + 5(i_B - i_A) + 10i_B = 0$

$-25i_A + 15i_B + 20i_C = 30$

Node $2i_1 = i_B - i_C$ $\quad i_1 = i_B$

$2i_B = i_B - i_C$

$i_B - i_C = 0$

$$\begin{bmatrix} 27 & -5 & -20 \\ -5 & 3 & 4 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 27 & -5 & -20 \\ -5 & 3 & 4 \\ 0 & 1 & -1 \end{vmatrix} = -64$$

$$A = \begin{vmatrix} 0 & -5 & -20 \\ 6 & 3 & 4 \\ 0 & 1 & -1 \end{vmatrix} = -150$$

$$B = \begin{vmatrix} 27 & 0 & -20 \\ -5 & 6 & 4 \\ 0 & 0 & -1 \end{vmatrix} = -162$$

$$C = \begin{vmatrix} 27 & -5 & 0 \\ -5 & 3 & 6 \\ 0 & 1 & 0 \end{vmatrix} = -162$$

$i_A = \frac{A}{D} = 2.34375\ A$

$i_B = \frac{B}{D} = 2.53125\ A$

$i_C = \frac{C}{D} = 2.53125\ A$

$i_1 = i_B = 2.53125$

$i_2 = i_A - i_B = 2.34375 - 2.53125 = -0.1875$

$i_3 = i_C - i_A = 2.53125 - 2.34375 = 0.1875$

$i_4 = i_A = 2.34375\ A$

$V_1 = 30\ V$

$i_3 = \frac{V_1 - V_2}{20} \Rightarrow V_2 = V_1 - 20 i_3$

$= 30 - 20(0.1875)$

$= 26.25\ V$

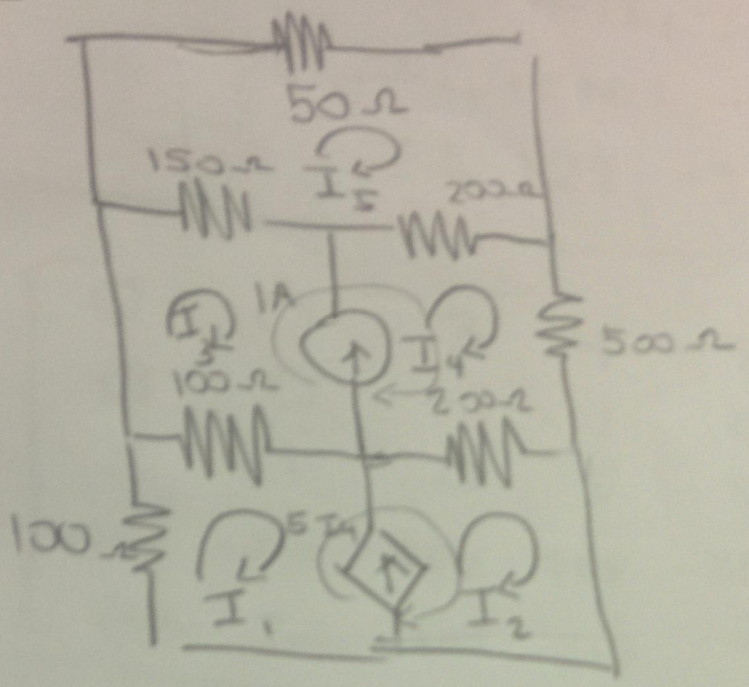
$i_2 = \frac{V_3 - V_2}{5} \Rightarrow V_3 = 5 i_2 + V_2$

$= 5(-0.1875) + 26.25\ V$

$= 25.3125$

$i_1 = 2.53A$	$V_1 = 30\ V$
$i_2 = -0.188A$	$V_2 = 26.25\ V$
$i_3 = 0.188A$	$V_3 = 25.3\ V$
$i_4 = 2.34\ A$	

6



Determine: Mesh Matrix

Solve:

Mesh 5: $50I_5 + 200(I_5 - I_4) + 150(I_5 - I_3) = 0$

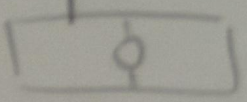
$-150I_3 - 200I_4 + 400I_5 = 0$

$3I_3 + 4I_4 - 8I_5 = 0$

↑ $1A = I_4 - I_3 \Rightarrow -I_3 + I_4 = 1$

◇↑ $5I_4 = I_2 - I_1 \Rightarrow I_1 - I_2 + 5I_4 = 0$

Mega Mesh

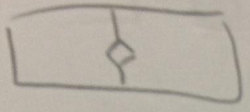


$150(I_3 - I_5) + 200(I_4 - I_5) + 500I_4$

$+ 200(I_4 - I_2) + 100(I_3 - I_1) = 0$

$-100I_1 - 200I_2 + 250I_3 + 900I_4 - 350I_5 = 0$

Mega Mesh



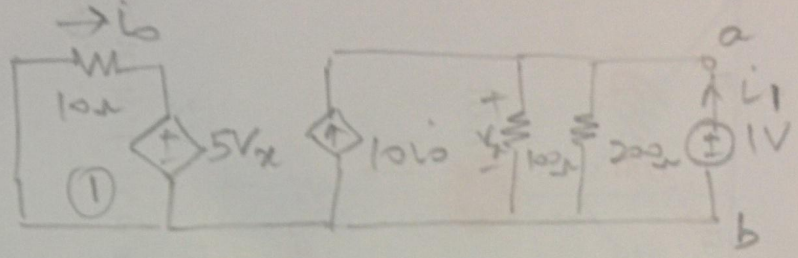
$$100I_1 + 100(I_1 - I_3) + 200(I_2 - I_4) = 0$$

$$200I_1 + 200I_2 - 100I_3 - 200I_4 = 0$$

$$2I_1 + 2I_2 - I_3 - 2I_4 = 0$$

$$\begin{bmatrix} 0 & 0 & 3 & 4 & -8 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 \\ -100 & -200 & 250 & 900 & -350 \\ 200 & 200 & -100 & -200 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

P7) To find R_{TH},



$V_x = 1\,V$

∴ In loop ①,

$-5V_x = 10 i_o \Rightarrow i_o = -0.5$

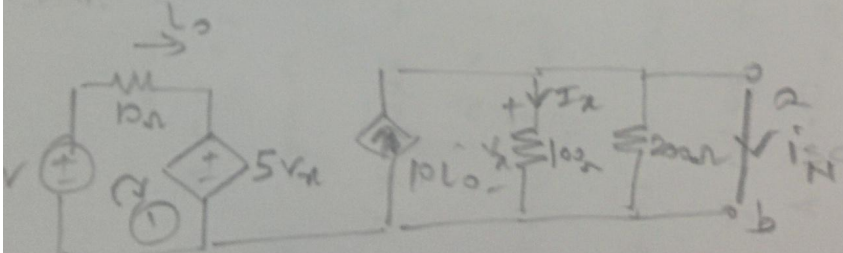
@ node 'a',

$10 i_o + i_1 = \frac{1}{100} + \frac{1}{200}$

$i_1 = 5.015\ A$

$\therefore R_{TH} = \frac{1}{i_1} = \frac{1}{5.015}\ \Omega$

i_N



$\Rightarrow 10 i_o = i_N$

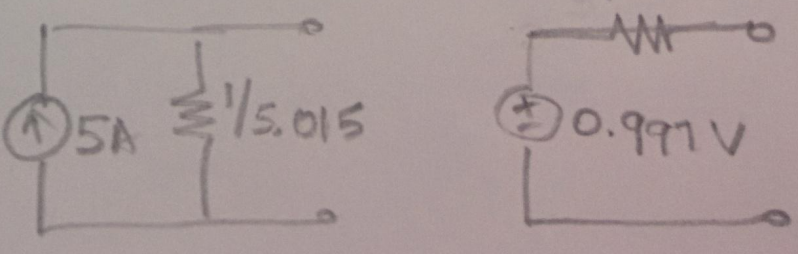
or $V_x = 0$

KVL in loop 1

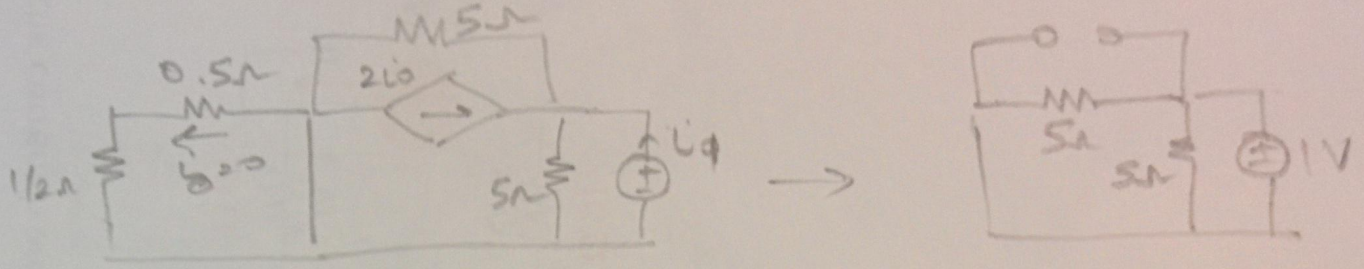
$\therefore -5 + 10 i_o + 5V_x = 0$

$i_o = \frac{5}{10} = 0.5\ A$

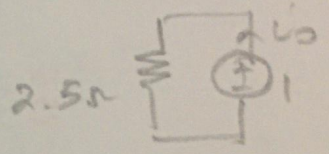
$\therefore i_N = 5\ A$



R_{TH}: Ignore independent sources

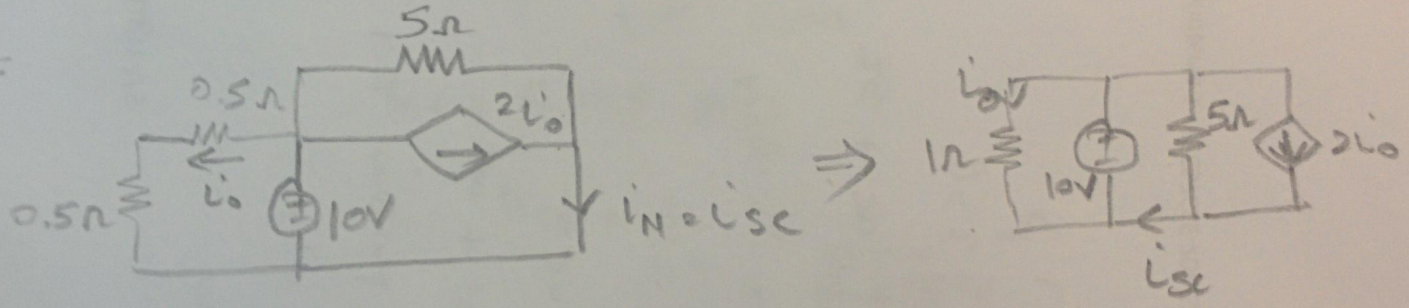


$i_0 = 0$



$$i_0 = 1/2.5$$

$$R_N = R_{TH} = 1/i_0 = 2.5\ \Omega$$



$$i_0 = \frac{10}{1} = 10\ A$$

$$i_{sc} = \frac{10}{5} + 2i_0$$

$$= \frac{10}{5} + 2 \times 10$$

$$= 12\ A$$