1) (a) $v(t) = 2\cos(\omega t + \pi/12)$

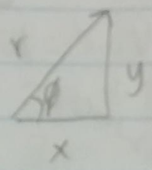
$v(t) = V_m \cos(\omega t + \phi)$

$V_m = 2 \quad \phi = \pi/12$

$V_m = r = 2$

$\boxed{V = 2e^{\frac{\pi}{12}j}}$

$V = x + yj$? $V = re^{j\phi}$?



$x = r\cos\phi = 2\cos\frac{\pi}{12} = 1.932$

$y = r\sin\phi = 2\sin\frac{\pi}{12} = 0.518$

$\boxed{V = 1.932 + j(0.518)}$

(b) $i(t) = 4\sin(\omega t + \pi/8)$

$i(t) = I_m \cos(\omega t + \phi)$

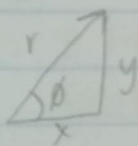
$\theta = \frac{\pi}{8}$

$\phi = \frac{\pi}{8} - \frac{\pi}{2} = \frac{\pi}{8} - \frac{4\pi}{8} = -\frac{3\pi}{8}$

$r = I_m = 4$

$\boxed{I = 4e^{(-3\pi/8)j}}$

$I = x + jy$? $I = re^{j\phi}$?



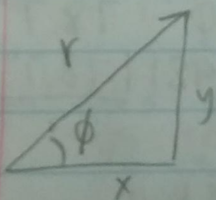
$x = r\cos\phi = 4\cos(-\frac{3\pi}{8}) = 1.531$

$y = r\sin\phi = 4\sin(-\frac{3\pi}{8}) = -3.696$

$\boxed{I = 1.531 - 3.696j}$

(c) $V = 2 + 1j = 2 + j$ $\quad v(t) = ?$

$x = 2 \quad y = 1$



$x^2 + y^2 = r^2$

$2^2 + 1^2 = r^2$

$4 + 1 = r^2$

$r = \sqrt{5}$

$\tan\phi = \frac{y}{x} = \frac{1}{2}$

$\phi = 26.57°$

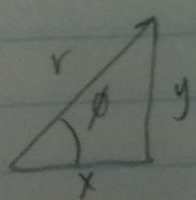
$v(t) = V_m \cos(\omega t + \phi)$

$V_m = r = \sqrt{5}$

$\boxed{v(t) = \sqrt{5}\cos(\omega t + 26.57°)}$

(d) $I = 4 + 2j$ $\quad i(t) = ?$

$x = 4 \quad y = 2$



$x^2 + y^2 = r^2$

$4^2 + 2^2 = r^2$

$16 + 4 = r^2$

$r = \sqrt{20} = 2\sqrt{5}$

$\tan\phi = \frac{y}{x} = \frac{2}{4} = \frac{1}{2}$

$\phi = 26.57°$

$i(t) = I_m \cos(\omega t + \phi)$

$I_m = r = 2\sqrt{5}$

$\boxed{i(t) = 2\sqrt{5}\cos(\omega t + 26.57°)}$

P2

Given: $I(t) = 2\cos(\underline{\underline{10}}t + 30^\circ)$, $\omega = 10\,\text{rad/s}$

$$\underline{\underline{\dot{I} = 2\angle 30^\circ}}$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{(10\,\text{rad/s})(100\times10^{-3}\,\text{F})} = -j\,\Omega$$

$$Z_{eq} = 5\,\Omega \parallel -j\,\Omega$$

$$Z_{eq} = \left(\frac{1}{5} - \frac{1}{j}\right)^{-1} = 0.9805\angle -78.69^\circ$$

$$V = I \cdot Z_{eq} = 2\angle 30^\circ \cdot 0.9805\angle -78.69^\circ$$

$$\underline{\underline{V = 1.961\angle -48.69^\circ}}$$

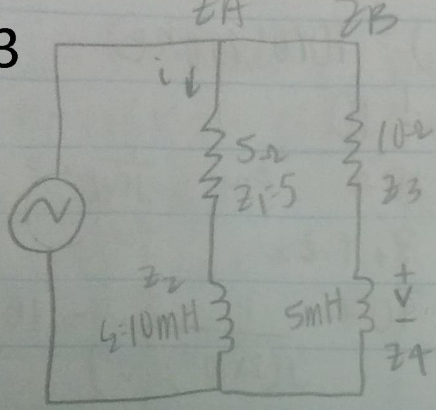
$$i = \frac{V}{R} = \frac{1.961\angle -48.69^\circ}{5}$$

$$\underline{i = 0.3922\angle -48.69^\circ}$$

$$\boxed{V(t) = 1.961\cos(10t - 48.69^\circ)}$$

$$\boxed{i(t) = 0.3922\cos(10t - 48.69^\circ}$$

3



$V_s(t) = 20\cos(100t)$ $\quad \omega = 100$

$V_s = 20\angle 0^\circ = 20$

$Z_1 = 5\Omega \quad Z_2 = j\omega(10mH) \quad Z_3 = 10\Omega \quad Z_4 = j\omega(5mH)$

$Z_A = 5 + j\omega(10\times10^{-3}) = 5 + j(100)(10\times10^{-3})$

$Z_A = 5 + j$

$I_{1A} = V_s / Z_A = 20/(5+j)$

$= \frac{20(5-j)}{(5+j)(5-j)} = \frac{100-20j}{25+1} = \frac{100-20j}{26}$

$I_{1A} = 3.846 - 0.769j$

$(3.846^2) + (-0.769)^2 = r^2$ $\quad \tan\phi = \frac{(-0.769)}{3.846}$ $\quad \phi = -11.31^\circ$

$r = 3.921$

$\boxed{i(t) = 3.921\cos(100t - 11.31^\circ)\ A}$

$Z_B = 10 + j\omega(5\times10^{-3}) = 10 + j(100)(5\times10^{-3}) = 10 + 0.5j$

$V_s = I_B Z_B$

$I_B = \frac{V_s}{Z_B} = \frac{20}{(10+0.5j)}\frac{(10-0.5j)}{(10-0.5j)} = \frac{200-10j}{100.25} = 1.995 - 0.0998j$

$V_3 = I_B Z_3 = (1.995 - 0.0998j)(10) = 19.95 - 0.998j$

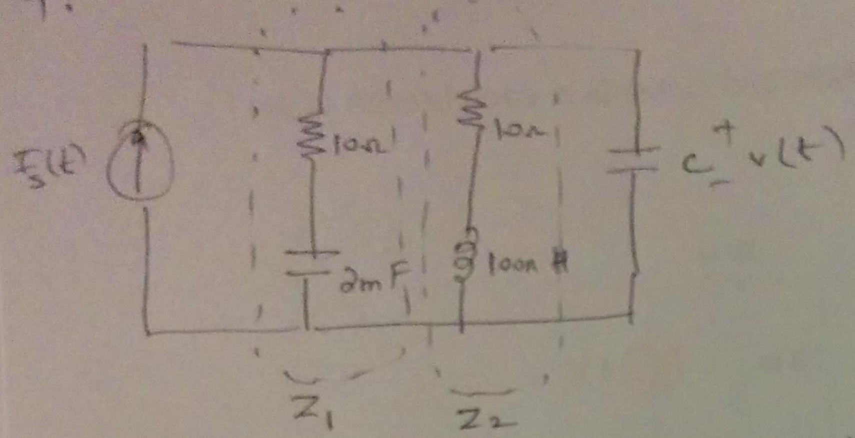
$V_s - V_3 = 20 - 19.95 + 0.998j = 0.05 + 0.998j$

$0.05^2 + 0.998^2 = r^2$ $\quad \tan\phi = \frac{0.998}{0.05}$ $\quad \phi = 87.132^\circ$

$r = 0.999$

$\boxed{v(t) = 0.999\cos(100t + 87.1^\circ)}$

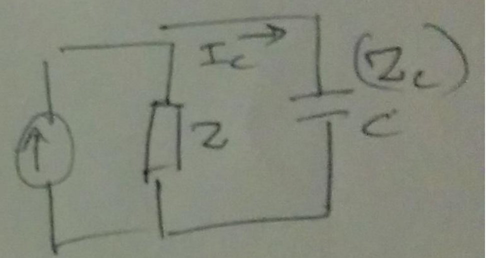
4.



$Z_1 = 10 + \dfrac{1}{j10^6 \times 2m} = 10 + j(500\mu)$

$Z_2 = 10 + j10^6 \times 100n = 10 + j0.1$

$Z = Z_1 \| Z_2 = 5 + 0.025j$

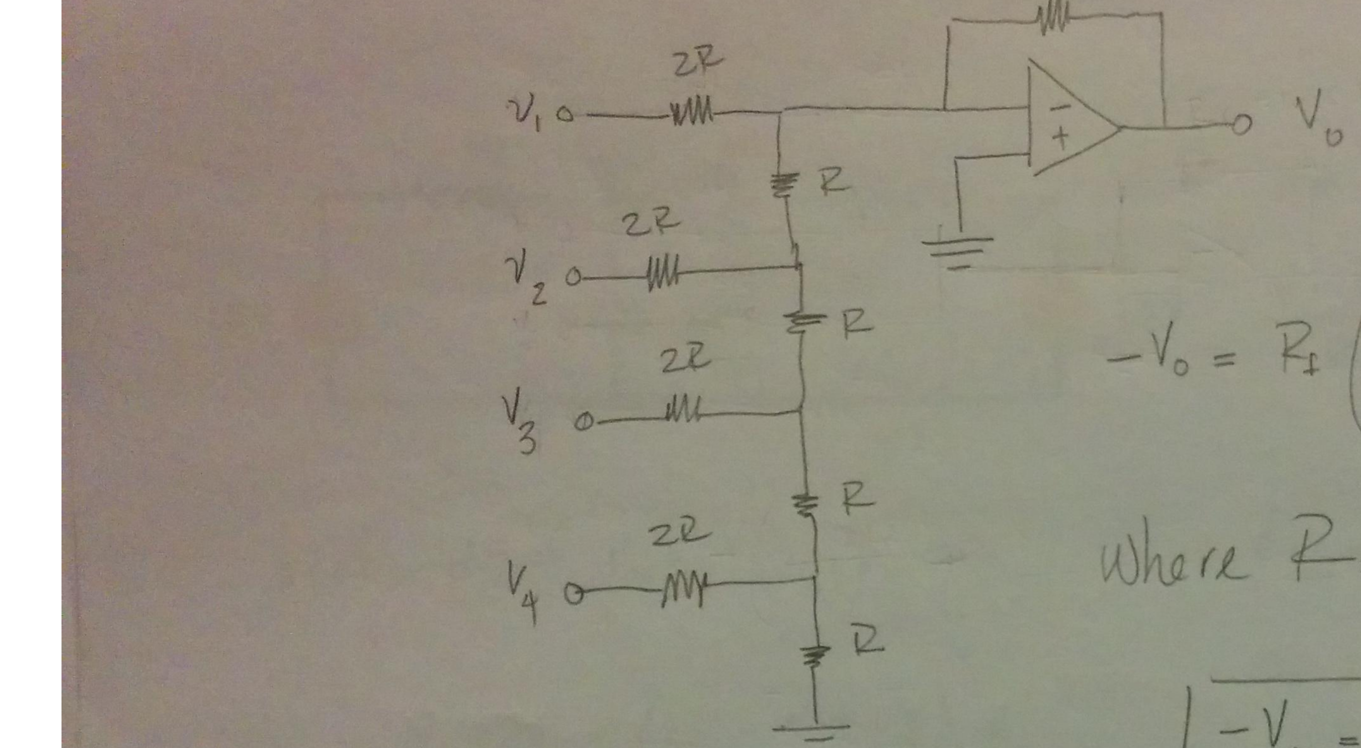


$\underline{I_c} = \dfrac{Z}{Z + Z_c} \cdot \underline{I_s}$

$\underline{V} = Z_c \cdot \underline{I_c}$

$= \dfrac{Z_c \cdot Z}{Z + Z_c} \cdot \underline{I_s}$

Problem 6. Design a 4 - bit (4 input) D/A converter using an op-amp and 9 resistors. Let R_f = 10 kΩ and R = 5kΩ. Find the output voltage for the input V_i = [1 0 0 1].



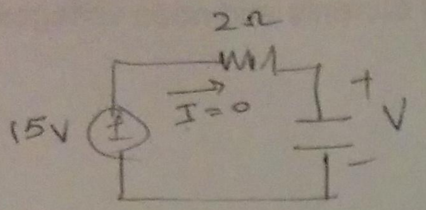
$$-V_o = R_f\left(\frac{V_1}{2R} + \cancel{\frac{V_2}{4R}} + \cancel{\frac{V_3}{8R}} + \frac{V_4}{16R}\right)$$

where $R = 5\,k\Omega$

$$\boxed{-V_o = 1.125\ V}$$

$$\boxed{V_o = -1.125\ V}$$

7. At $t<0$, the capacitor is completely charged. Hence, is represented by

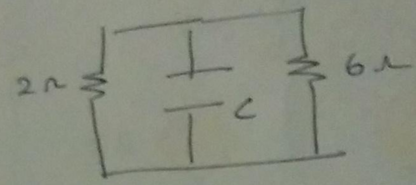


$\therefore$ $V(0^-) = 15V$

$V(0^+) = 15V$

At $t>0$, the thevenin resistance connected to the capacitor is

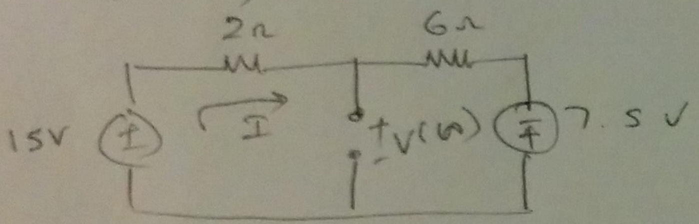
$R_{TH} = 2 \| 6 = 1.5\,\Omega$



$\therefore$ The time-constant

$\tau = R_{th} C = 1.5 \times \frac{1}{3} = 0.5$ s

At $t = \infty$, the circuit is,



$-15 + 2I + 6I - 7.5 = 0$

$8I = 22.5$

$I = 2.8125$

$\therefore$ $v(\infty) = 15 - 2I = 9.375$

$\therefore$ $v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$

$\therefore$ $v(t) = (9.375 + 5.625 e^{-2t})$ for $t>0$,

at $t = 0.5$

$v(t) = 7.63$ V

Problem 5

using KVL: $V_s = V_1 + V_2$

$$V_2 = iR$$

$$i = C\frac{dV_1}{dt}$$

therefore $V_s = RC\cdot\frac{dV_1}{dt} + V_1 \Rightarrow \frac{dV_1}{dt} + \frac{V_1}{RC} = \frac{V_0 \cos wt}{RC}$ (*)

The solution should obey the form of $A\cos wt + B\sin wt$

Assume $V_1 = A\cos wt + B\sin wt$

Then $\frac{dV_1}{dt} = -Aw\cos wt + Bw\sin wt$, substitute into (*)

$$-Aw\sin wt + Bw\cos wt + \frac{A}{RC}\cos wt + \frac{B}{RC}\sin wt = \frac{V_0}{RC}\cos wt$$

$$\left(Bw + \frac{A}{RC}\right)\cos wt + \left(\frac{B}{RC} - Aw\right)\sin wt = \frac{V_0}{RC}\cos wt$$

Compare the coefficients.

$$\begin{cases} Bw + \frac{A}{RC} = \frac{V_0}{RC} \\ \frac{B}{RC} - Aw = 0 \Rightarrow B = AwRC \end{cases}$$

$$AwRC\cdot w + \frac{A}{RC} = \frac{V_0}{RC} \Rightarrow A = \frac{V_0}{RC\left(w^3RC + \frac{1}{RC}\right)} = \frac{V_0}{w^2R^2C^2+1}$$

$$B = AwRC = \frac{wRC}{w^2R^2C^2+1}\cdot V_0$$

$$V_1 = A\cos wt + B\sin wt$$

$$= \frac{V_0}{1+w^2R^2C^2}\cos wt + \frac{wRC}{w^2R^2C^2+1}V_0\sin wt$$

$$= \sqrt{\left(\frac{V_0}{1+w^2R^2C^2}\right)^2 + \left(\frac{V_0 wRC}{1+w^2R^2C^2}\right)^2}\cos(wt+\varphi)$$

where $\varphi = \operatorname{atan}\left(\frac{-\frac{wRC V_0}{w^2R^2C^2+1}}{\frac{V_0}{w^2R^2C^2+1}}\right) = \operatorname{atan}(-wRC)$

$$\therefore V_1 = \frac{V_0}{\sqrt{1+w^2R^2C^2}}\cos(wt+\varphi), \quad \varphi = \operatorname{atan}(-wRC)$$

$$V_2 = V_S - V_1$$

$$= V_0 \cos\omega t - \frac{V_0}{\sqrt{1+\omega^2R^2C^2}} \cos(\omega t + \varphi)$$

where $\varphi = \operatorname{atan}(-\omega RC)$