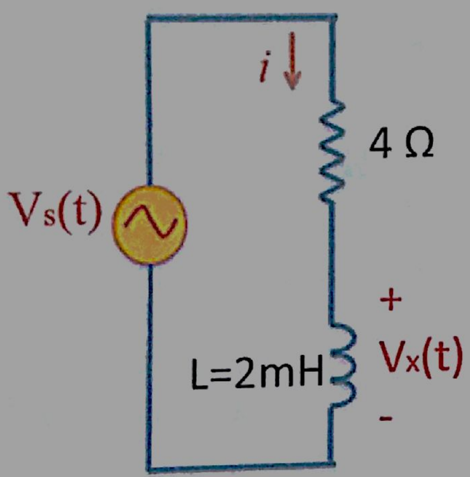
PROBLEM 1 : (25 Points)

Given $V_s(t) = 2Cos(2000t + 45^\circ)$. Find $V_x(t)$ and $I(t)$ and the corresponding phasors.



$$\underline{V_s} = 2\angle 45^\circ$$

$$Z_L = j\omega L = j(2000)(2m) = j4\Omega$$

$$\underline{V_x} = \frac{Z_L \cdot \underline{V_s}}{Z_L + 4} = \frac{j4 \cdot 2\angle 45^\circ}{4+4j} = 1.414\angle 90^\circ$$

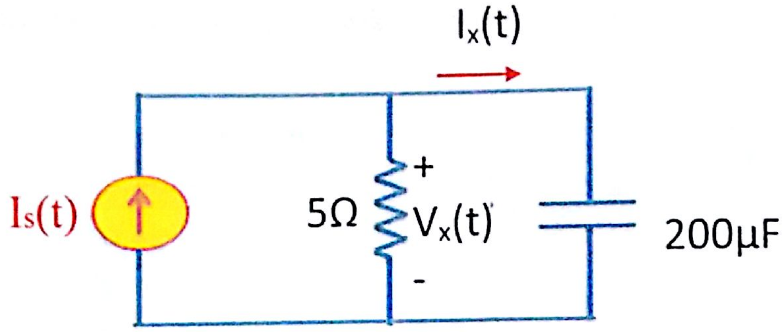
$$V_x(t) = 1.414\, Cos(2000t + 90^\circ)$$

$$\underline{I} = \frac{\underline{V_s}}{4+4j} = \frac{2\angle 45^\circ}{4+4j} = \frac{2\angle 45^\circ}{4\sqrt{2}\angle 45^\circ} = \frac{1}{2\sqrt{2}}$$

$$I(t) = \frac{1}{2\sqrt{2}} Cos(2000t)$$

PROBLEM 2. (25 Points)

Given $I_s(t) = 10\cos(1000t + 15^\circ)$. Find $V_x(t)$ and $I_x(t)$ and the corresponding phasors.



$$\underline{I_s} = 10\angle 15^\circ$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j1000 \times 2000\mu} = -5j\ \Omega$$

$$\underline{I_x} = \frac{R}{R + Z_c}\,\underline{I_s} = \frac{5 \cdot 10\angle 15^\circ}{5 + 5j} = \frac{5 \cdot 10\angle 15^\circ}{5\sqrt{2}\angle 45^\circ}$$

$$= \frac{10}{\sqrt{2}}\angle 60^\circ$$

$$I_x(t) = \frac{10}{\sqrt{2}}\cos(1000t + 60^\circ)$$

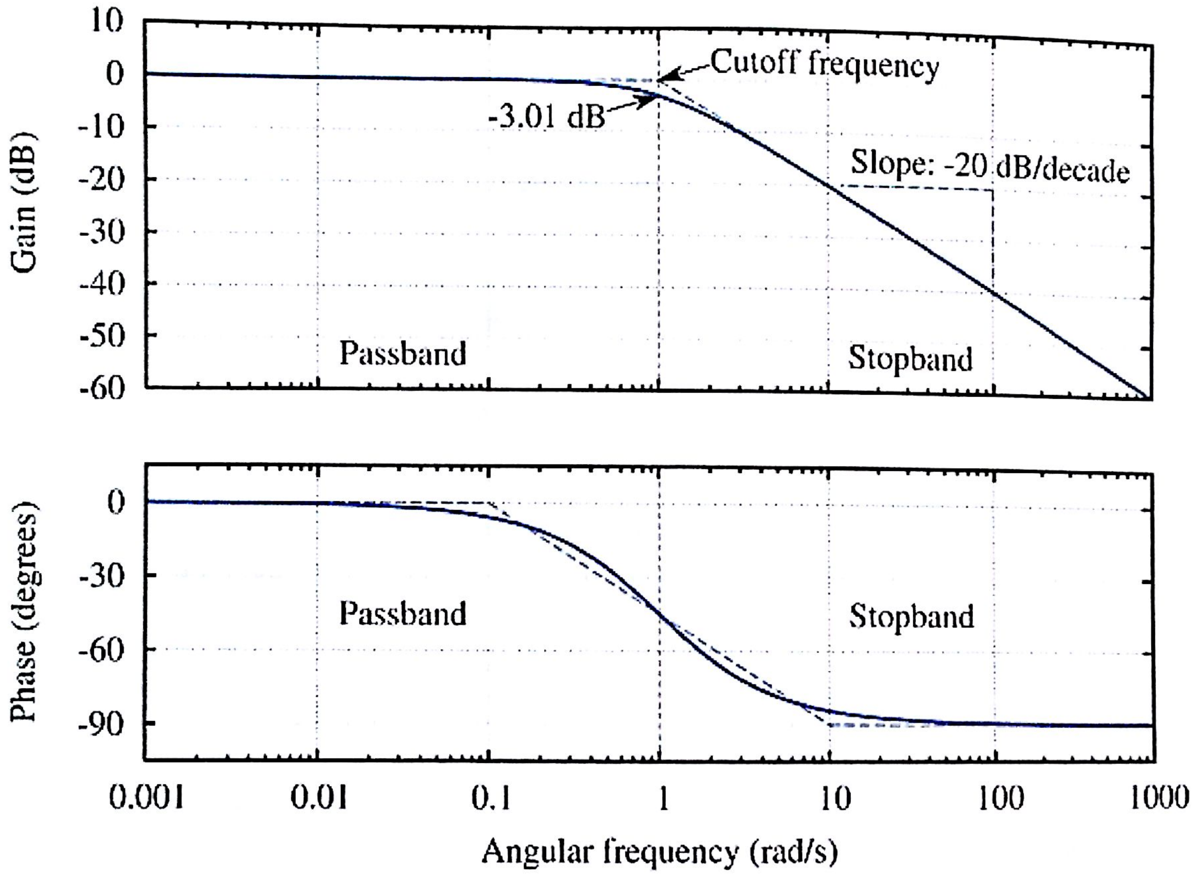
$$\underline{V_x} = Z_c \cdot \underline{I_x} \quad [Z_c \text{ and } R \text{ are in parallel}]$$

$$= -5j \cdot \frac{10}{\sqrt{2}}\angle 60^\circ = \frac{50}{\sqrt{2}}\angle -30^\circ$$

$$V_x(t) = \frac{50}{\sqrt{2}}\cos(1000t - 30^\circ)$$

Problem 3 : (25 Points)

Given the Bode plot of a transfer function . Find the output voltages for each of the input signals. Note : Use the approximation (given by dashed line) when looking up the value in the figure.



a) $V_{in}(t) = 4\cos(\omega t + 30^\circ)$, when $\omega = 0.001$ rad/s.Find $V_{out}(t)$.

b) $V_{in}(t) = 2\cos(\omega t + 60^\circ)$, when $\omega = 0.01$ rad/s.Find $V_{out}(t)$.

c) $V_{in}(t) = 10\cos(\omega t + 45^\circ)$, when $\omega = 1$ rad/s.Find $V_{out}(t)$.

d) $V_{in}(t) = 4\cos(\omega t)$, when $\omega = 10$ rad/s.Find $V_{out}(t)$.

e) $V_{in}(t) = \cos(\omega t + 0^\circ)$, when $\omega = 100$ rad/s.Find $V_{out}(t)$.

$\underline{V_{out}} = |H(j\omega)| e^{j\phi} \underline{V_{in}}$.

a) $|H(j\omega)| = 0$ dB $= 1$ V ; $\underline{V_{in}} = 4 \angle 30°$

$\phi = 0°$

$\underline{V_{out}} = 1 \cdot e^{j0} \cdot 4 \angle 30°$

$= 4 \angle 30°$

$V_{out}(t) = 4 \cos(\omega t + 30°)$

b) $|H(j\omega)| = 0$ dB, $\phi = 0$; $\underline{V_{in}} = 2 \angle 60°$

$\underline{V_{out}} = 1 e^{j0} \cdot 2 \angle 60° = 2 \angle 60°$

$V_{out}(t) = 2 \cos(\omega t + 60°)$

c) $\underline{V_{in}} = 10 \angle 45°$; $|H(j\omega)| = 0$ dB $= 1$ V ; $\phi = -45°$

$\underline{V_{out}} = 1 e^{-j45°} \cdot 10 \angle 45° = 10 \angle 0°$

$V_{out}(t) = 10 \cos(\omega t)$

d) $\underline{V_{in}} = 4 \angle -10°$, $|H(j\omega)| = -20$ dB $= 10^{-20/10} = 0.01$

$\phi = -90°$

$\underline{V_{out}} = 0.01 e^{-j90°} \cdot 4 \angle -10° = 0.04 \angle -100°$

$V_{out}(t) = 0.04 \cos(\omega t - 100)$

e) $\underline{V_{in}} = 1 \angle 0°$; $|H(j\omega)| = -40$ dB $= 10^{-40/10} = 100\mu$

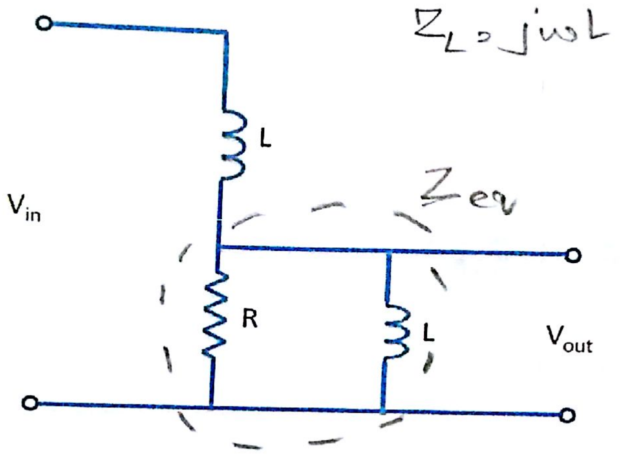
$\phi = -90°$

$\underline{V_{out}} = 100\mu \, e^{-j90°} \cdot 1 = 100\mu \angle -90°$

$V_{out}(t) = 100\mu \cos(\omega t - 90°)$

PROBLEM 4 : (25Points)

Find the transfer function and sketch the magnitude of H(ω).



$$Z_{eq} = R \| Z_L$$

$$= R \| j\omega L$$

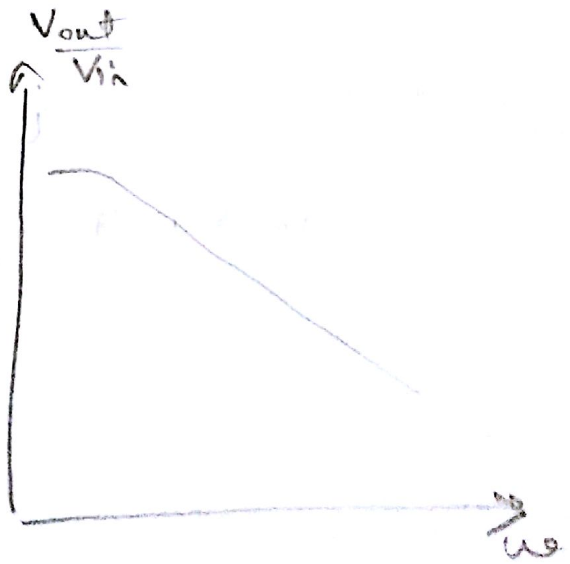
$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{j\omega L}$$

$$Z_{eq} = \frac{j\omega L R}{R + j\omega L}$$

$$V_{out} = \frac{Z_{eq}}{Z_L + Z_{eq}} \cdot V_{in}$$

$$= \frac{\dfrac{j\omega L R}{R + j\omega L}}{j\omega L + \dfrac{j\omega L R}{R + j\omega L}} = \frac{j\omega L R \cdot V_{in}}{j\omega L R + j\omega L R + (j\omega L)^2}$$

$$\frac{V_{out}}{V_{in}} = \frac{j\omega L R}{2 j\omega L R - \omega^2 L^2}$$



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