1) In graphene, we have a linear relationship between energy and momentum:

$$E = v_F k = v_F \sqrt{(k_x)^2 + (k_y)^2} = v_F \sqrt{\left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_x}\right)^2}$$

Derive the density of states vs. energy in graphene.

2) Now imagine you have a graphene nanoribbon. L_y is small. Calculate the density of states vs. energy of a 1d graphene nanoribbon.

3) Which metal would you pick for a low resistance contact to n-type charge carriers in a semiconducting carbon nanotube: Large work function, or small work function? Why?

$$U = \frac{|z_{21} - z_{12}|^2}{4[\mathrm{Re}(z_{11}) \cdot \mathrm{Re}(z_{22}) - \mathrm{Re}(z_{12}) \cdot \mathrm{Re}(z_{21})]}$$

$$U = \frac{|y_{21} - y_{12}|^2}{4[\mathrm{Re}(y_{11}) \cdot \mathrm{Re}(y_{22}) - \mathrm{Re}(y_{12}) \cdot \mathrm{Re}(y_{21})]}$$

$$z' = z/R_o. \quad y' = yR_o. \quad h_{11}' = h_{11}/R_o; \; h_{12}' = h_{12}; \; h_{21}' = h_{21}; \; h_{22}' = h_{22}R_o.$$

$$s_{11} = \frac{(z_{11}' - 1)(z_{22}' + 1) - z_{12}' \cdot z_{21}'}{(z_{11}' + 1)(z_{22}' + 1) - z_{12}' \cdot z_{21}'}$$

$$s_{12} = \frac{2 z_{12}'}{(z_{11}' + 1)(z_{22}' + 1) - z_{12}' \cdot z_{21}'}$$

$$s_{21} = \frac{2 z_{21}'}{(z_{11}' + 1)(z_{22}' + 1) - z_{12}' \cdot z_{21}'}$$

$$s_{22} = \frac{(z_{11}' + 1)(z_{22}' - 1) - z_{12}' \cdot z_{21}'}{(z_{11}' + 1)(z_{22}' + 1) - z_{12}' \cdot z_{21}'}$$

$$y_{11}' = \frac{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12} \cdot s_{21}}$$

$$y_{12}' = \frac{-2 s_{12}}{(1 + s_{11})(1 + s_{22}) - s_{12} \cdot s_{21}}$$

$$y_{21}' = \frac{-2 s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12} \cdot s_{21}}$$

$$y_{22}' = \frac{(1 + s_{11})(1 - s_{22}) + s_{12} \cdot s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12} \cdot s_{21}}$$

$$z_{11}' = \frac{(1 + s_{11})(1 - s_{22}) + s_{12} \cdot s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} \cdot s_{21}}$$

$$z_{12}' = \frac{2 s_{12}}{(1 - s_{11})(1 - s_{22}) - s_{12} \cdot s_{21}}$$

$$z_{21}' = \frac{2 s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} \cdot s_{21}}$$

$$z_{22}' = \frac{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} \cdot s_{21}}$$

$$h_{11}' = \frac{(1 + s_{11})(1 + s_{22}) - s_{12} \cdot s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}$$

$$h_{12}' = \frac{2 s_{12}}{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}$$

$$h_{21}' = \frac{2 s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}$$

$$h_{22}' = \frac{(1 - s_{11})(1 - s_{22}) - s_{12} \cdot s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} \cdot s_{21}}$$

$$s_{11} = \frac{(1 - y_{11}')(1 + y_{22}') + y_{12}' \cdot y_{21}'}{(1 + y_{11}')(1 + y_{22}') - y_{12}' \cdot y_{21}'}$$

$$s_{12} = \frac{-2 y_{12}'}{(1 + y_{11}')(1 + y_{22}') - y_{12}' \cdot y_{21}'}$$

$$s_{21} = \frac{-2 y_{21}'}{(1 + y_{11}')(1 + y_{22}') - y_{12}' \cdot y_{21}'}$$

$$s_{22} = \frac{(1 + y_{11}')(1 - y_{22}') + y_{12}' \cdot y_{21}'}{(1 + y_{11}')(1 + y_{22}') - y_{12}' \cdot y_{21}'}$$

$$y_{11} = \frac{z_{22}}{z_{11} z_{22} - z_{21} z_{12}}$$

$$y_{12} = \frac{-z_{12}}{z_{11} z_{22} - z_{21} z_{12}}$$

$$y_{21} = \frac{-z_{21}}{z_{11} z_{22} - z_{21} z_{12}}$$

$$y_{22} = \frac{z_{11}}{z_{11} z_{22} - z_{21} z_{12}}$$

$$z_{11} = \frac{y_{22}}{y_{11} y_{22} - y_{21} y_{12}}$$

$$z_{12} = \frac{-y_{12}}{y_{11} y_{22} - y_{21} y_{12}}$$

$$z_{21} = \frac{-y_{21}}{y_{11} y_{22} - y_{21} y_{12}}$$

$$z_{22} = \frac{y_{11}}{y_{11} y_{22} - y_{21} y_{12}}$$

$$h_{11} = \frac{z_{11} z_{22} - z_{21} z_{12}}{z_{22}}$$

$$h_{12} = \frac{z_{12}}{z_{22}}$$

$$h_{21} = -\frac{z_{21}}{z_{22}}$$

$$h_{22} = \frac{1}{z_{22}}$$

$$s_{11} = \frac{(h_{11}' - 1)(1 + h_{22}') - h_{12}' \cdot h_{21}'}{(1 + h_{11}')(1 + h_{22}') - h_{12}' \cdot h_{21}'}$$

$$s_{12} = \frac{2 h_{12}'}{(1 + h_{11}')(1 + h_{22}') - h_{12}' \cdot h_{21}'}$$

$$s_{21} = \frac{-2 h_{21}'}{(1 + h_{11}')(1 + h_{22}') - h_{12}' \cdot h_{21}'}$$

$$s_{22} = \frac{(1 + h_{11}')(1 - h_{22}') + h_{12}' \cdot h_{21}'}{(1 + h_{11}')(1 + h_{22}') - h_{12}' \cdot h_{21}'}$$

$$y_{11} = \frac{1}{h_{11}}$$

$$y_{12} = -\frac{h_{12}}{h_{11}}$$

$$y_{21} = \frac{h_{21}}{h_{11}}$$

$$y_{22} = \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{11}}$$

$$z_{11} = \frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}}$$

$$z_{12} = \frac{h_{12}}{h_{22}}$$

$$z_{21} = -\frac{h_{21}}{h_{22}}$$

$$z_{22} = \frac{1}{h_{22}}$$

$$h_{11} = \frac{1}{y_{11}}$$

$$h_{12} = -\frac{y_{12}}{y_{11}}$$

$$h_{21} = \frac{y_{21}}{y_{11}}$$

$$h_{22} = \frac{y_{11} y_{22} - y_{21} y_{12}}{y_{11}}$$

4) For the circuit below, find the y-matrix (5 points each matrix element). Next, find h_{21} completely (5 points). Find Mason's gain U in terms of the circuit elements (5 points). Find f_T and f_{Max} (5 points each). **CIRCLE YOUR ANSWERS.**