Note: This HW is a "toy model" of graphene.
Does not get at 1) Valley degeneracy 2) 2D–1D nanoribbon effects

1) In graphene, we have a linear relationship between energy and momentum:

$$E = \hbar v_F k = \hbar v_F \sqrt{(k_x)^2 + (k_y)^2} = \hbar v_F \sqrt{\left(\frac{n_x \pi}{L_x}\right)^2 + \left(\frac{n_y \pi}{L_x}\right)^2} \quad \Rightarrow k = \frac{1}{v_F} E$$

↑ typo

$$\Rightarrow \frac{dk}{dE} = \frac{1}{v_F}$$

Derive the density of states vs. energy in graphene.

2) Now imagine you have a graphene nanoribbon. L_y is small. Calculate the density of states vs. energy of a 1d graphene nanoribbon.

$$E = \hbar v_F \sqrt{k_x^2 + k_{y0}^2} \qquad k_{y0} = \frac{\pi}{L_y}$$

1) $D(E)\,dE = D(k)\,dk$

$$D(k)\,dk = \left[\frac{1 \text{ state}}{(\pi/L)^2} \times 2 \text{ (spin)}\right] \times \text{area of disk of radius } k \text{ in } k\text{-space } k_x > 0\ k_y > 0$$

$$= \left[\frac{1}{(\pi/L)^2} \times 2\right] 2\pi k\,dk\,\frac{1}{4}$$

$$= L^2 \frac{1}{\pi} k\,dk = L^2 \frac{1}{\pi} \frac{E}{\hbar v_F}\,dk$$

$$\Rightarrow D(k) = L^2 \frac{1}{\pi} k = L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F}$$

$$D(E) = D(k)\frac{dk}{dE} = \boxed{L^2 \frac{1}{\pi} E \frac{1}{\hbar v_F} \frac{1}{\hbar v_F} = D(E)}$$

$$\boxed{\rho(E) = \frac{E}{\hbar^2 \pi v_F^2}}$$

2) $E = \hbar v_F k$

$D(E)\,dE = D(k)\,dk$

$$D(k)\,dk = \left[\frac{1}{\pi/L} \times 2 \text{ (spin)}\right] \times \text{distance in } k\text{-space between } k, k+dk$$

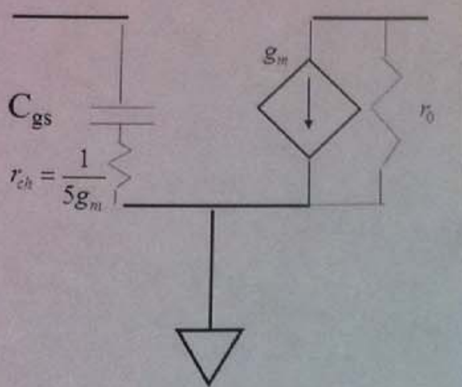
$$= \frac{2}{\pi/L} \times dk \Rightarrow D(k) = L\frac{2}{\pi}$$

$$D(E) = D(k)\frac{dk}{dE} = L\frac{2}{\pi}\frac{1}{v_F} \Rightarrow \boxed{\rho(E) = \frac{2}{\pi}\frac{1}{v_F}}$$

3)Which metal would you pick for a low resistance contact to n-type charge carriers in a semiconducting carbon nanotube: Large work function, or small work function? Why?

n-type means we want Fermi energy of contact in the conduction band, i.e. closer to the vacuum energy than away… So we want a LOW work function metal to make low contact resistance to the n-type branch….

3) For the circuit below, find the y-matrix. Next, fin[d ...] terms of the circuit elements. Find f_T and f_{Max}.



$$U = \frac{|y_{21} - y_{12}|^2}{4[Re(y_{11})Re(y_{22}) - Re(y_{12})Re(y_{21})]}$$

$$= \frac{g_m^2}{4 \frac{1}{r_o} Re\left[\frac{j\omega C_{gs} 5g_m}{(5g_m + j\omega C_{gs})}\right]}$$

$$= \frac{g_m^2 r_o}{4} \Big/ Re\left(\frac{j\omega C_{gs} 5g_m (5g_m - j\omega C_{gs})}{(5g_m + j\omega C_{gs})(5g_m - j\omega C_{gs})}\right)$$

$$= \frac{g_m^2 r_o}{4} \Big/ Re\left(\frac{j\cdots + (\omega C_{gs})^2 5g_m}{(5g_m)^2 + (\omega C_{gs})^2}\right)$$

$$= \frac{g_m^{\cancel{2}} r_o}{4} \frac{(5g_m)^2 + (\omega C_{gs})^2}{(\omega C_{gs})^2 5g_m}$$ 10pts

$$\boxed{U = \frac{g_m r_o}{20\omega^2 C_{gs}^2}(25g_m^2 + \omega^2 C_{gs}^2)}$$

CORRECTION

$$U = \frac{g_m r_o}{20\omega^2 C_{gs}^2}(25 g_m^2 + \omega^2 C_{gs}^2)$$

$$U\big|_{f_{MAX}} = 1$$

$$1 = \frac{g_m r_o}{20 C_{gs}^2 \omega_{MAX}^2}(25 g_m^2 + \omega_{MAX}^2 C_{gs}^2)$$

$$1 - \frac{g_m r_o}{20} = \frac{25}{20} g_m r_o \frac{g_m^2}{C_{gs}^2 \omega_{MAX}^2}$$

$$\omega_{MAX}^2 = \frac{25}{20} g_m r_o \frac{g_m^2}{C_{gs}^2 \omega_{MAX}^2} \Big/ 1 - \frac{g_m r_o}{20}$$

$$\omega_{MAX} = 5 \frac{g_m}{C_{gs}} \sqrt{\frac{g_m r_o}{20 - g_m r_o}} = 2\pi f_{MAX}$$