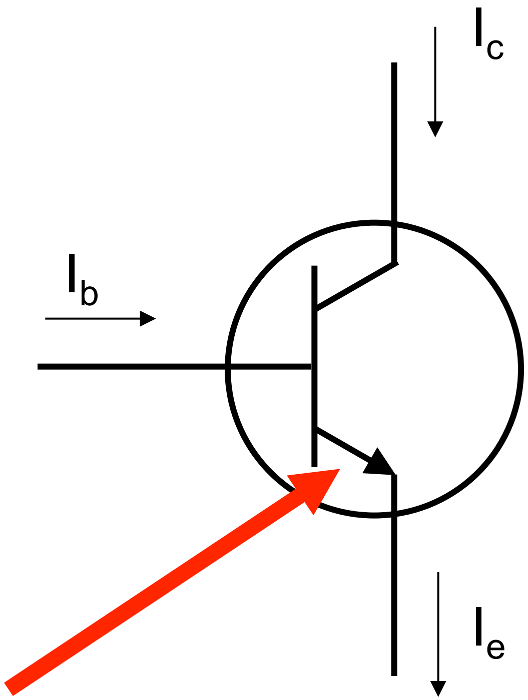
10^{th} week lectures: Benchmarking Nanoelectronics

Transistor AC properties

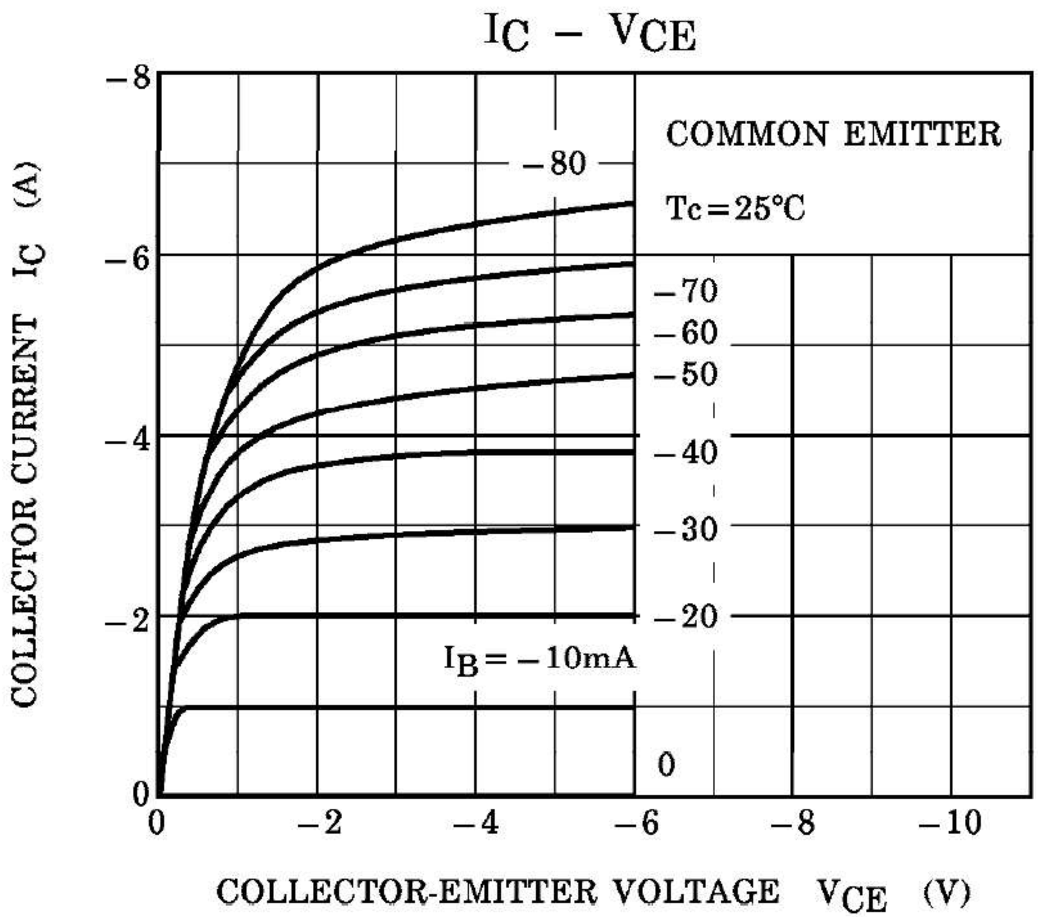
"Normal active" bias



Like a diode.

- E-B forward bias ($V_b > V_e$)
- C-B reverse bias ($V_c > V_b$)
- $I_{ce} = 100\ I_{be} = bI_{be}$

Global dc properties

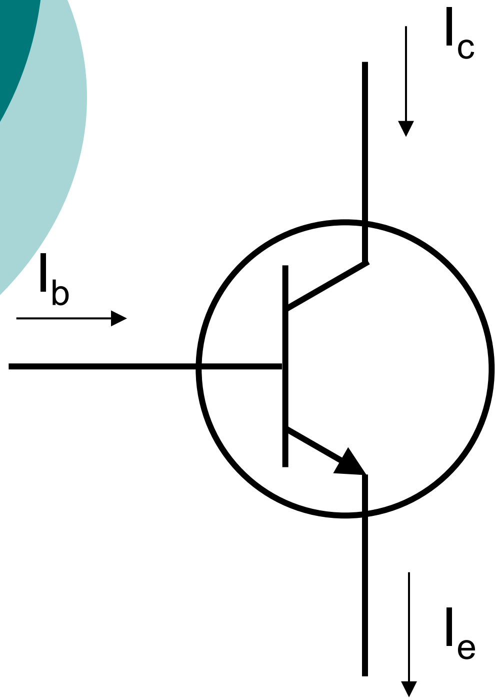


Note
Early effect.

It is assumed you know this, so it is rare to see on data sheets!

http://www.toshiba.com/taec/components/Datasheet/2SA1244DS.pdf

ac properties: notation



$$I_E \text{ dc}$$

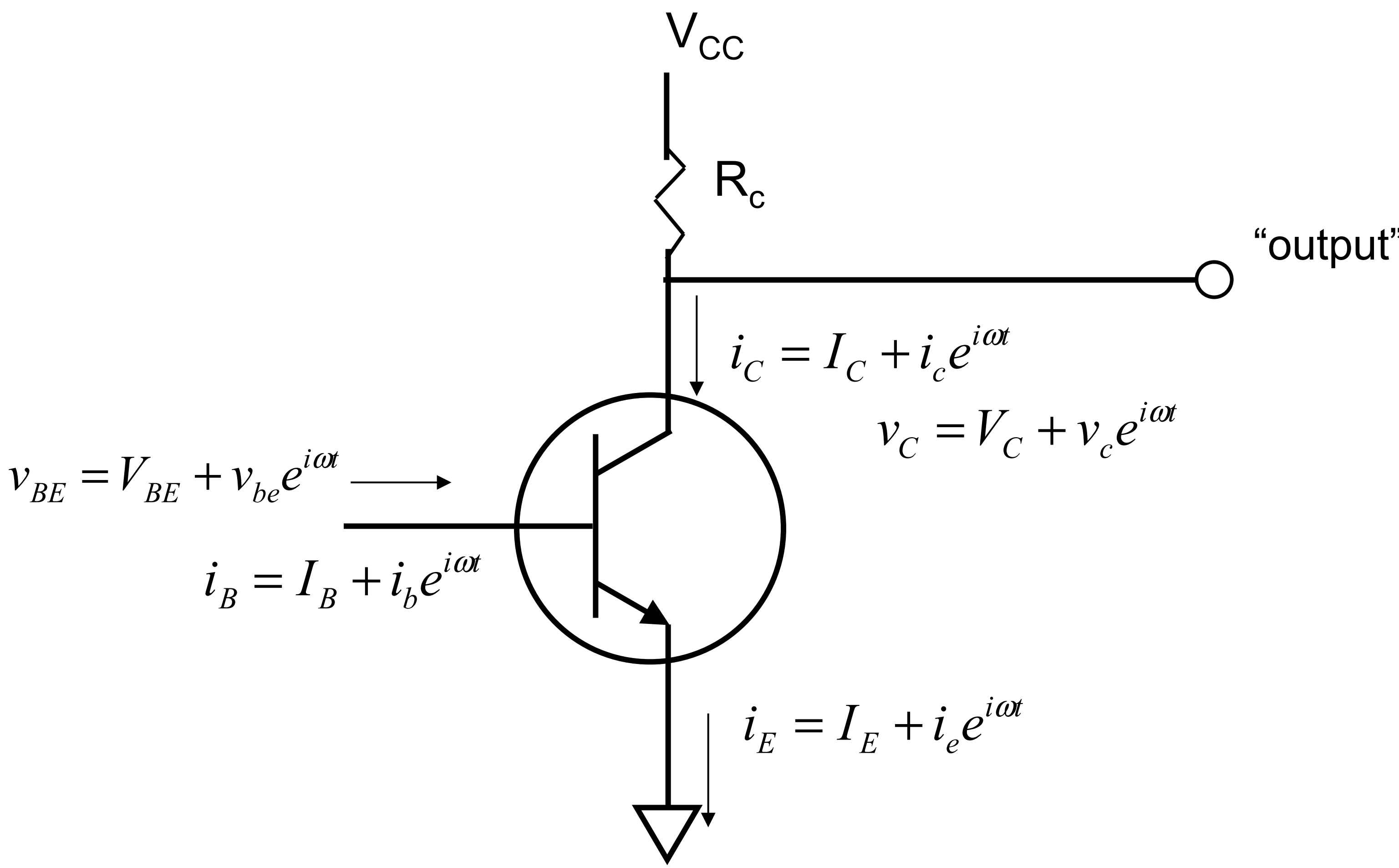
$$i_e \text{ ac}$$

$$i_E \text{ total} = I_E + i_e$$

We will use equivalent circuit #1 (implicitly).

ac properties

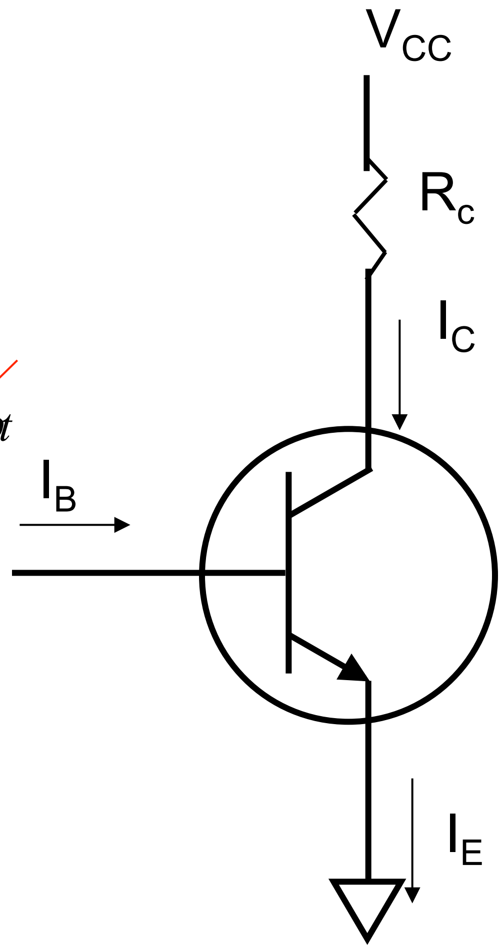
common-emitter configuration



Note: three terminal device has three-terminal equivalent ac circuit.

dc analysis

common-emitter configuration



$$v_{BE} = V_{BE} + v_{be}e^{i\omega t}$$

0 for now

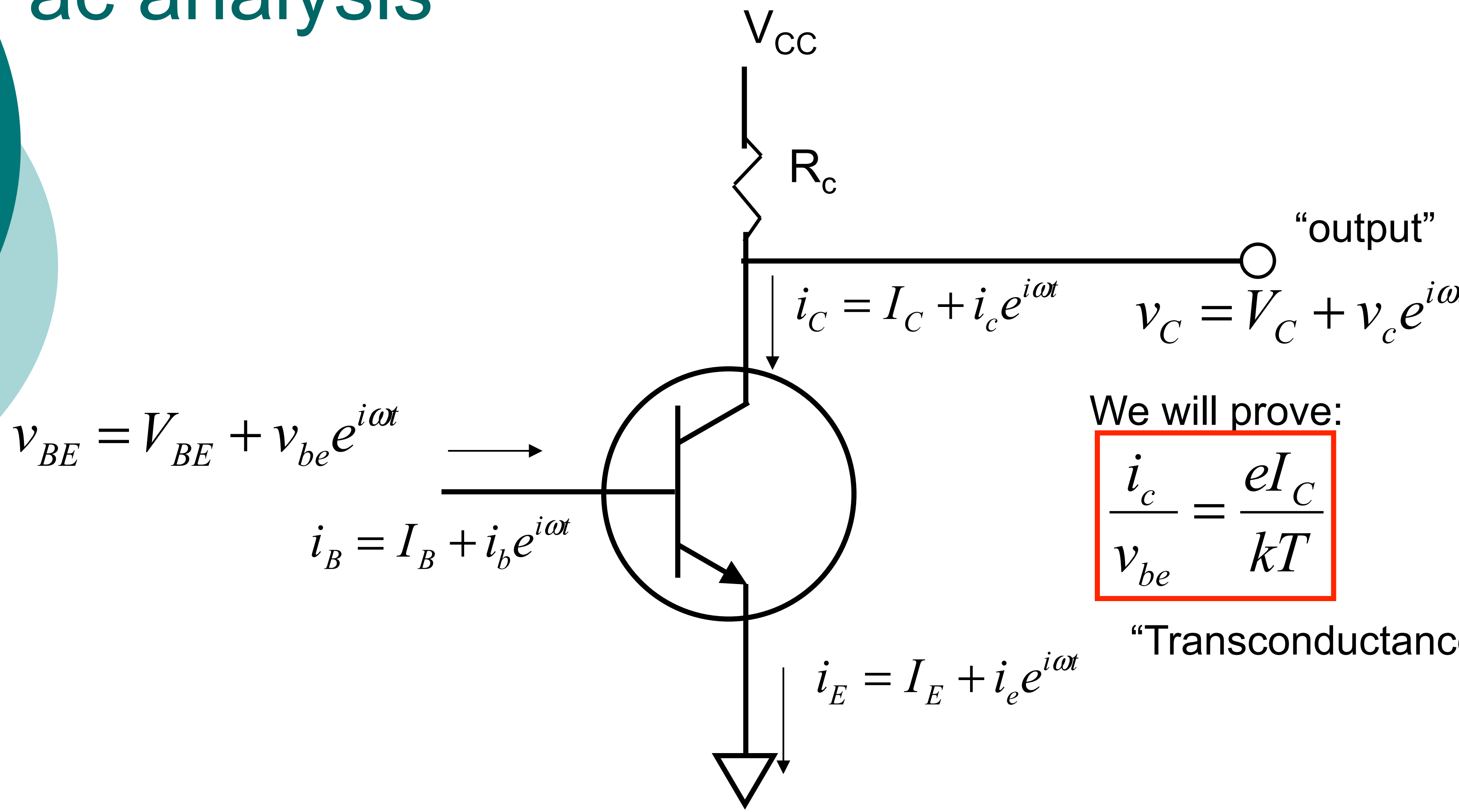
$$I_C = I_S e^{eV_{BE}/kT}$$

$$I_E = I_C / \alpha$$

$$I_B = I_C / \beta$$

$$V_C = V_{CE} = V_{CC} - I_C R_C$$

ac analysis



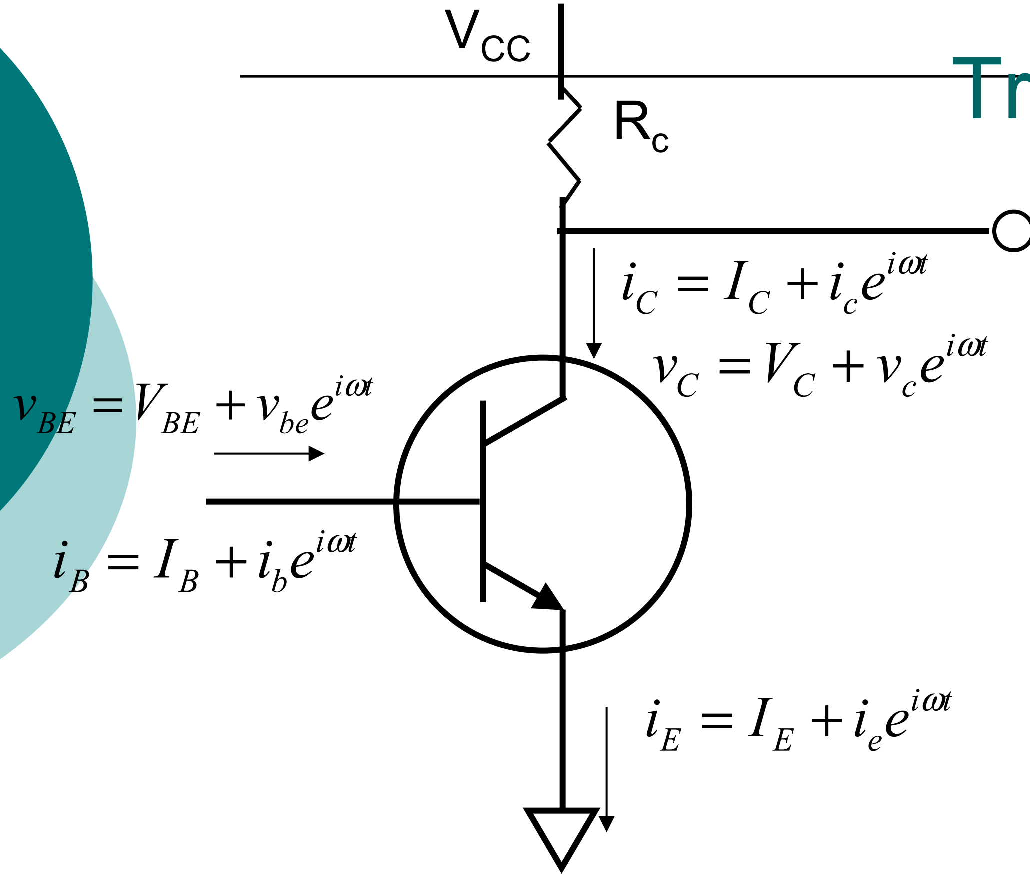
We will prove:

$$\frac{i_c}{v_{be}} = \frac{eI_C}{kT}$$

"Transconductance"

Note: three terminal device has three-terminal equivalent ac circuit.

Transconductance



$$v_{BE} = V_{BE} + v_{be} e^{i\omega t}$$

$$i_C = I_S e^{e v_{BE}/kT}$$

$$= I_S e^{e(V_{BE} + v_{be} e^{i\omega t})/kT}$$

$$= I_S e^{e V_{BE}/kT} e^{e v_{be} e^{i\omega t}/kT}$$

$$= I_C e^{e v_{be} e^{i\omega t}/kT}$$

$$e^x \approx 1 + x \text{ for small x}$$

$$i_C \approx I_C \left(1 + \frac{e v_{be}}{kT} e^{i\omega t} \right)$$

But

$$i_C = I_C + i_c e^{i\omega t}$$

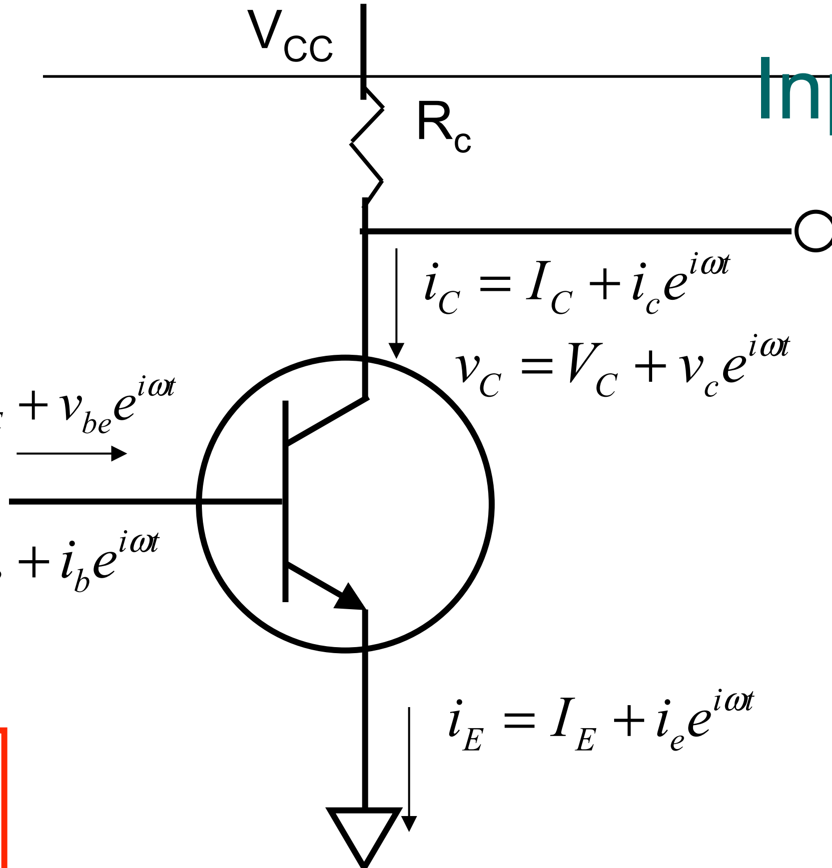
So:

$$i_c = I_C \frac{e v_{be}}{kT} \Rightarrow \frac{i_c}{v_{be}} = \frac{e I_C}{kT}$$

Typical number is 40 mA/V.

g_m

Input impedance



$$i_C = I_C + i_c e^{i\omega t}$$

$$v_C = V_C + v_c e^{i\omega t}$$

$$v_{BE} = V_{BE} + v_{be} e^{i\omega t}$$

$$i_B = I_B + i_b e^{i\omega t}$$

$$i_E = I_E + i_e e^{i\omega t}$$

$$\frac{v_{be}}{i_b} = ?$$

$$i_B = \frac{i_C}{\beta} = \frac{I_C + i_c e^{i\omega t}}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} I_C \frac{e v_{be}}{kT} e^{i\omega t}$$

But

$$i_B = I_B + i_b e^{i\omega t}$$

So:

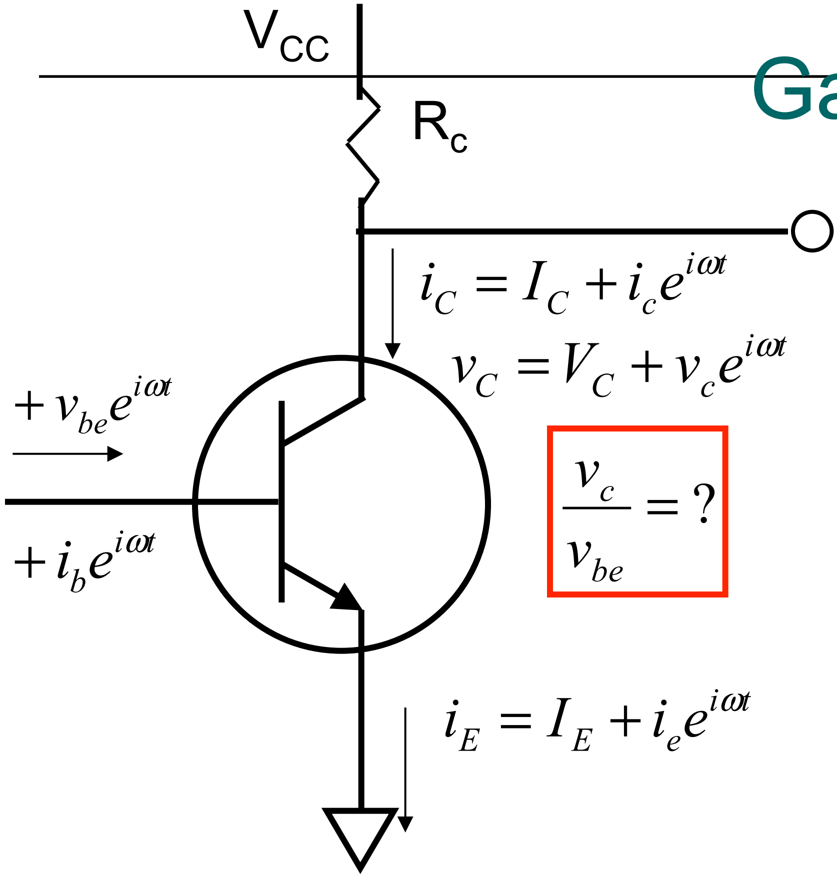
$$i_b = \frac{1}{\beta} I_C \frac{e v_{be}}{kT} = \frac{g_m}{\beta} v_{be}$$

So:

$$\frac{v_{be}}{i_b} = \frac{g_m}{\beta}$$

What is typical input impedance?

Gain



$$v_{BE} = V_{BE} + v_{be}e^{i\omega t}$$

$$i_B = I_B + i_b e^{i\omega t}$$

$$i_C = I_C + i_c e^{i\omega t}$$

$$v_C = V_C + v_c e^{i\omega t}$$

$$i_E = I_E + i_e e^{i\omega t}$$

$$\frac{v_c}{v_{be}} = ?$$

$$v_C = V_{CC} - i_C R_C$$
$$= V_{CC} - \left(I_C + i_c e^{i\omega t}\right) R_C$$
$$= \left(V_{CC} - I_C R_C\right) + i_c R_C e^{i\omega t}$$
$$= V_C + i_c R_C e^{i\omega t}$$

But

$$v_C = V_C + v_c e^{i\omega t}$$

So:

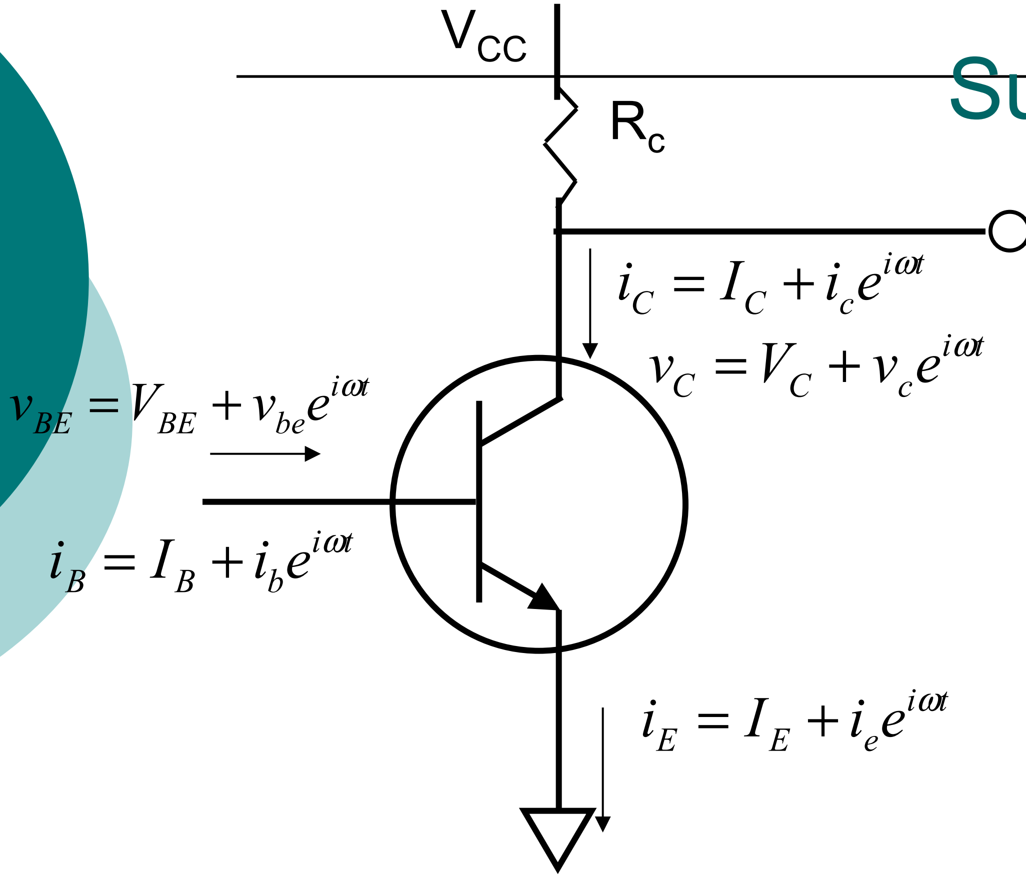
$$v_C = -i_c R_C = -g_m v_{be} R_C$$

So:

$$\frac{v_c}{v_{be}} = g_m R_C$$

What is typical gain?

Summary



$$i_b = \frac{g_m}{\beta} v_{be}$$ input imp.

$$i_c = \frac{eI_C}{kT} v_{be}$$ transcond.

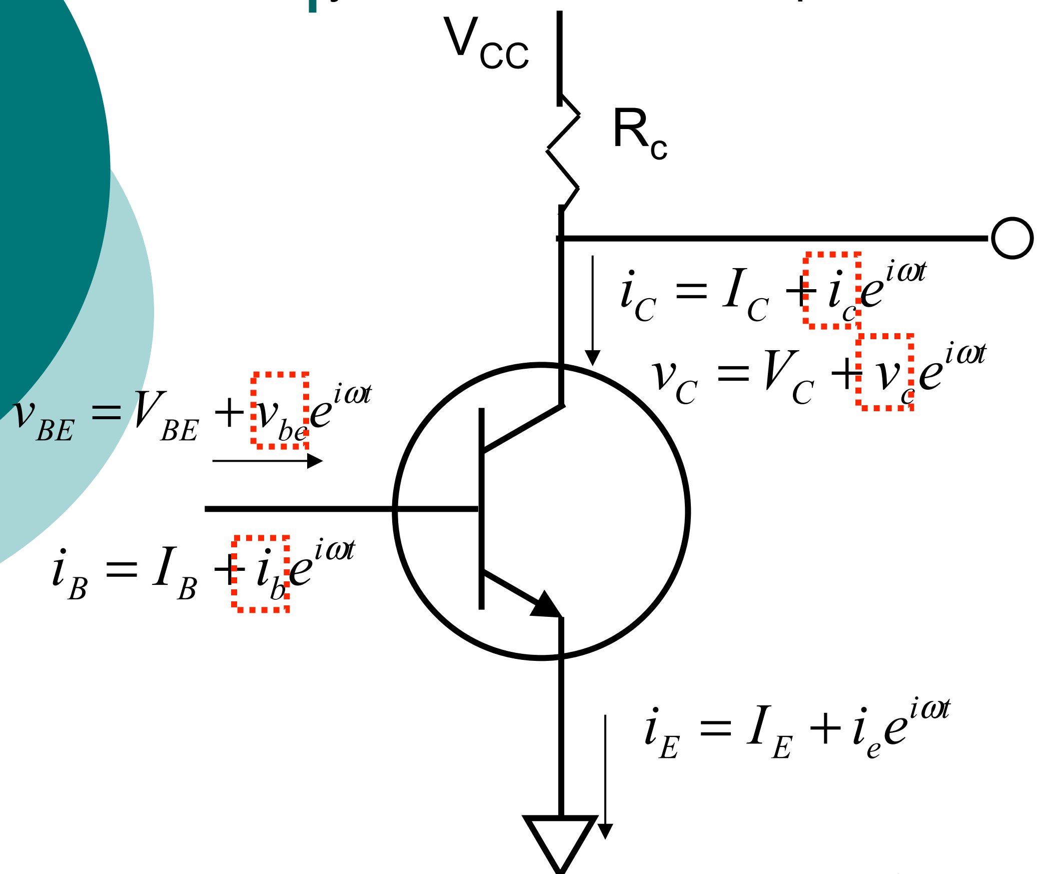
In *matrix* form:

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

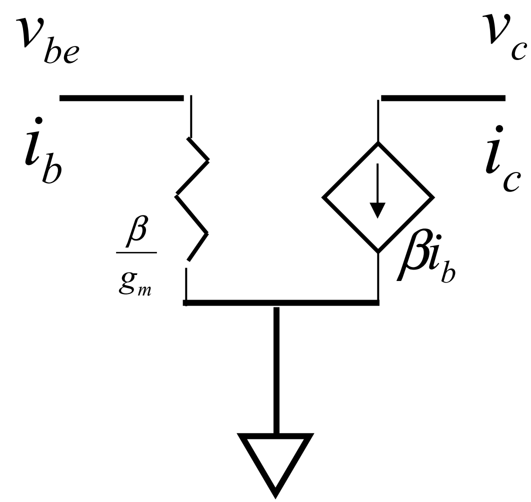
Admittance (Y) *matrix*

AC equivalent circuit:

If we are only interested in ac components, life can be simplified:



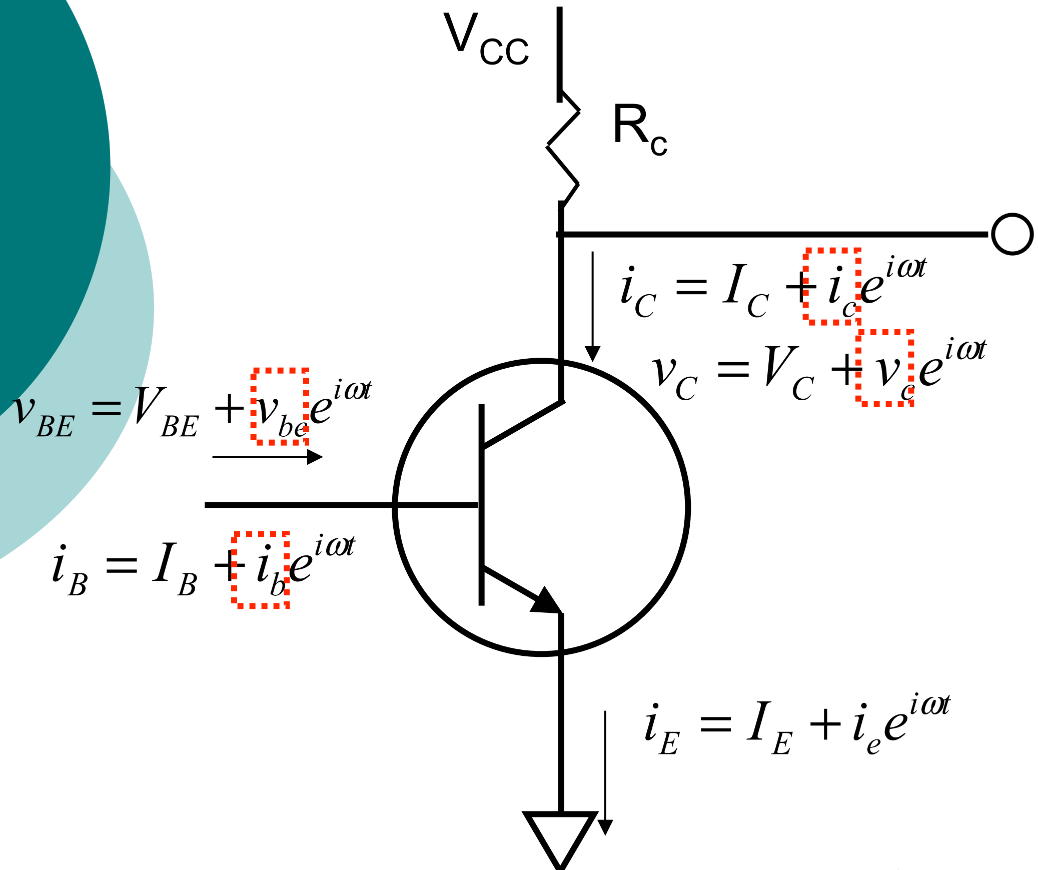
Hybrid p model:



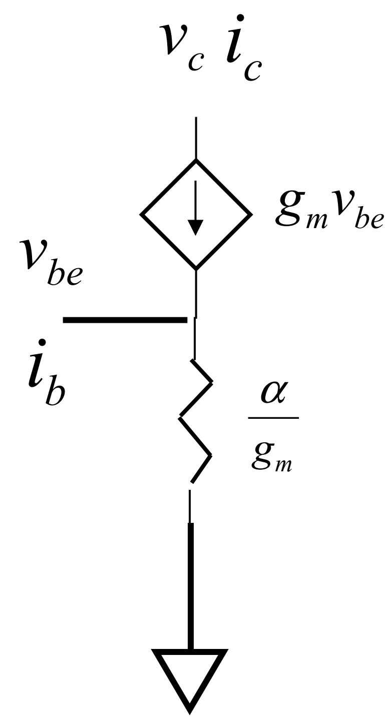
$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

T model

If we are only interested in ac components, life can be simplified:



T model:



$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

Rules for ac analysis

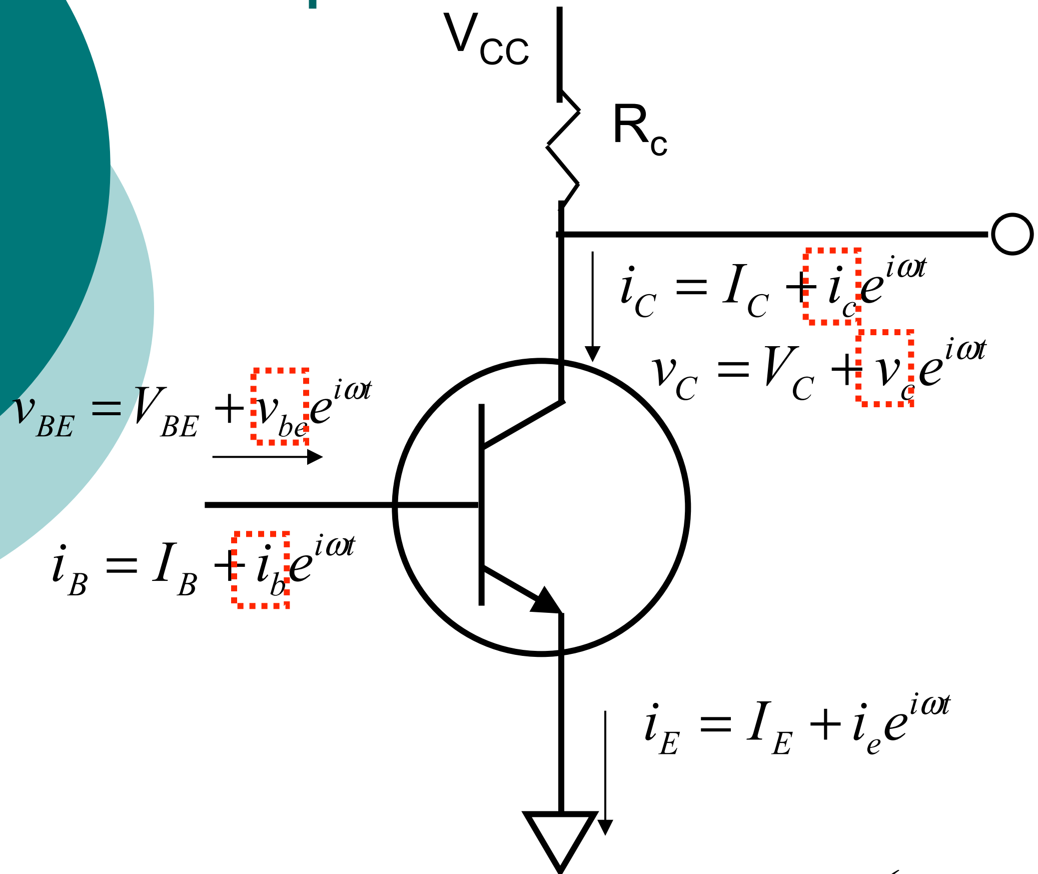
- From complete circuit, calculate dc currents and voltages
- For ac analysis only:
  - dc voltage source -> short circuit
  - dc current source -> open circuit
- Replace transistor with p or T-model
- Now solve (simplified) ac circuit

Next

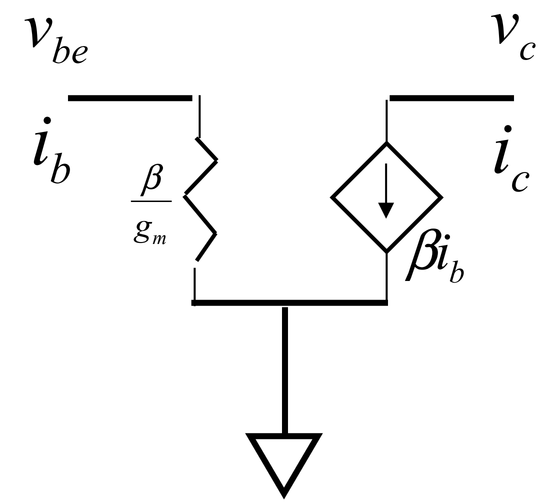
- Generalized y-parameters
- not just common emitters
- Capacitances
- y-parameters from doping profile
- Definition of f_T

AC equivalent circuit:

If we are only interested in ac components, life can be simplified:



Hybrid π model:



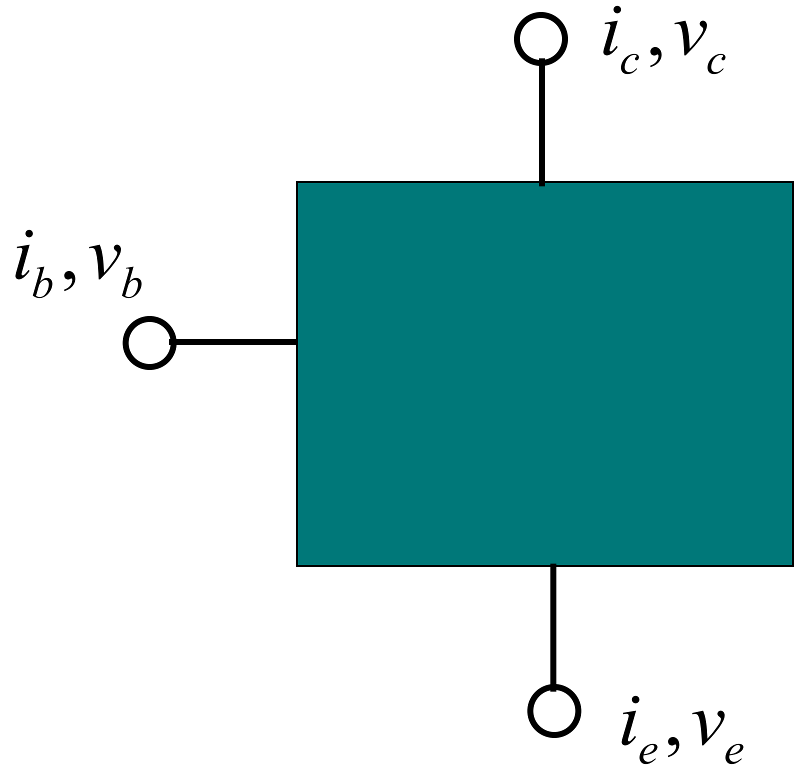
$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

Discuss easy interpretation of π model.

General admittance matrix

Last lecture, we had emitter grounded.
Called common emitter configuration:

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$



In general:

$$\begin{pmatrix} i_e \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \begin{pmatrix} v_e \\ v_b \\ v_c \end{pmatrix}$$

Y-matrix has 9 elements, but once you know 4 you know them all because:

$$i_e = i_b + i_c$$

and:

$$v_{cb} + v_{be} = v_{ce}$$

See book about details procedure to get 9 parameters from only 4.

Three configurations:

Common emitter configuration (v_e=0):

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} y_{bb} & y_{bc} \\ y_{cb} & y_{cc} \end{pmatrix} \begin{pmatrix} v_b \\ v_c \end{pmatrix} = [y]_e \begin{pmatrix} v_b \\ v_c \end{pmatrix}$$

Common base configuration (v_b=0):

$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} y_{ee} & y_{ec} \\ y_{ce} & y_{cc} \end{pmatrix} \begin{pmatrix} v_e \\ v_c \end{pmatrix} = [y]_b \begin{pmatrix} v_e \\ v_c \end{pmatrix}$$

← Easiest to calculate from doping profile.

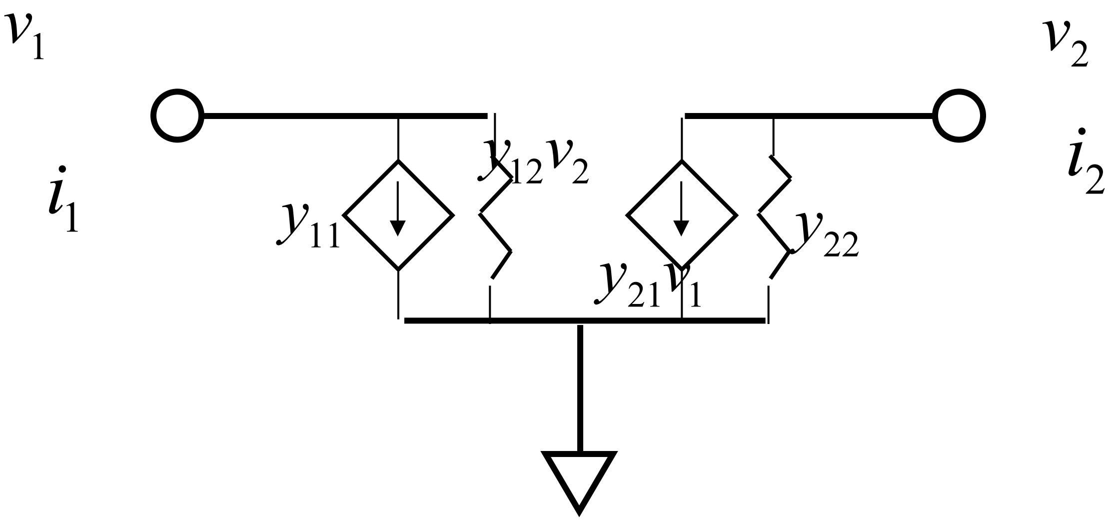
Common collector configuration (v_c=0):

$$\begin{pmatrix} i_b \\ i_e \end{pmatrix} = \begin{pmatrix} y_{bb} & y_{be} \\ y_{eb} & y_{ee} \end{pmatrix} \begin{pmatrix} v_b \\ v_e \end{pmatrix} = [y]_c \begin{pmatrix} v_b \\ v_e \end{pmatrix}$$

Generalized p model:

Regardless of which configuration you use, the following p model applies:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

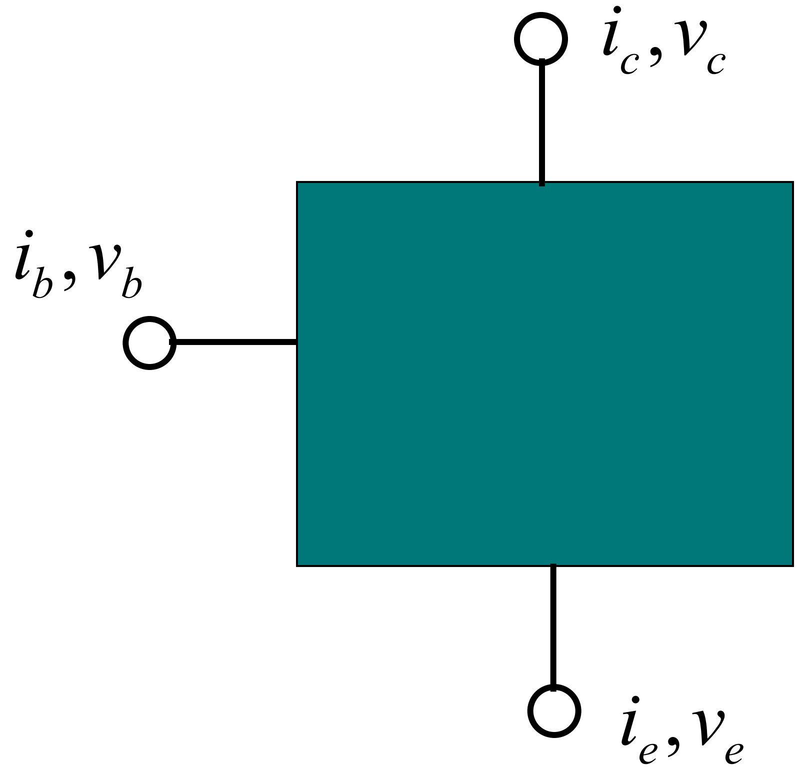


Common emitter: 1=base, 2=collector
Common base: 1=emitter, 2=collector
Common collector: 1=base, 2= emitter

You might be used to V=IR

General impedance matrix

$$\begin{pmatrix} v_e \\ v_b \\ v_c \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} i_e \\ i_b \\ i_c \end{pmatrix}$$



Y-matrix has 9 elements, but once you know 4 you know them all because:

h matrix:

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

Common emitter: 1=base, 2=collector
Common base: 1=emitter, 2=collector
Common collector: 1=base, 2= emitter

Note: In general, matrix elements depend on dc currents, dc voltages, and frequency. Spec. sheet (or model) will provide the matrix elements as a table vs. frequency, usually for only one bias current.

Common emitter h matrix:

$$\begin{pmatrix} v_b \\ i_c \end{pmatrix} = \begin{pmatrix} h_{11e} & h_{12e} \\ h_{21e} & h_{22e} \end{pmatrix} \begin{pmatrix} i_b \\ v_c \end{pmatrix}$$

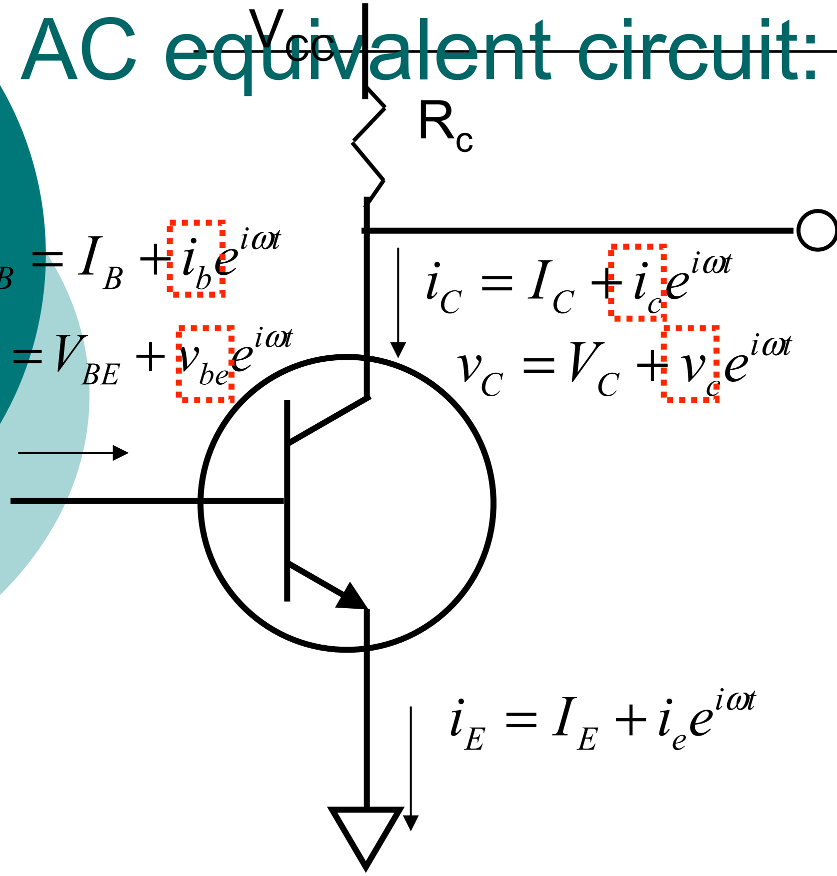
- Early effect:
  Collector voltage changes current gain (b).
- b depends on frequency *and* collector voltage.
- How do we define frequency at which b = 1?
- At v_c=0. This *is* h_{21e}

$$i_c = h_{21e} i_b + h_{22e} v_c \rightarrow h_{21e} i_b$$

- We define f_T such that:

$$\left| h_{21e} \right| (f_T) = 1$$

AC equivalent circuit:



$$i_B = I_B + i_b e^{i\omega t}$$

$$v_{BE} = V_{BE} + v_{be} e^{i\omega t}$$

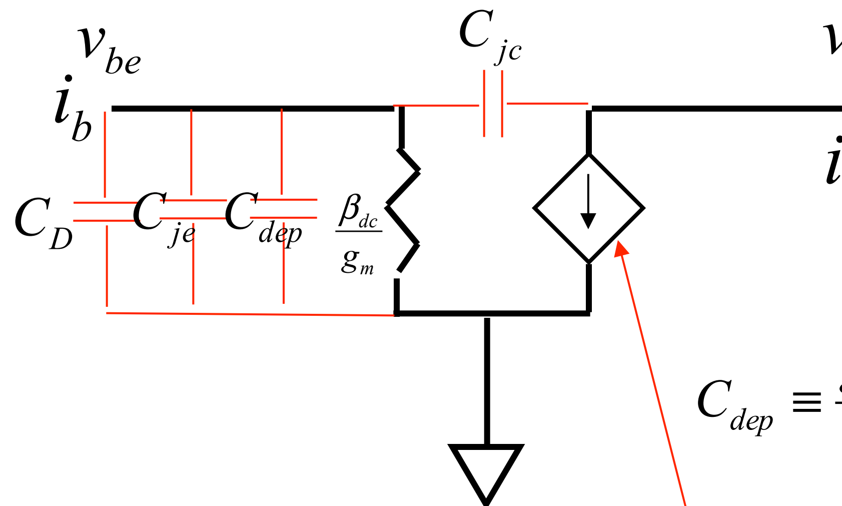
$$i_C = I_C + i_c e^{i\omega t}$$

$$v_C = V_C + v_c e^{i\omega t}$$

$$i_E = I_E + i_e e^{i\omega t}$$

Hybrid p model:

Red is new for ac:

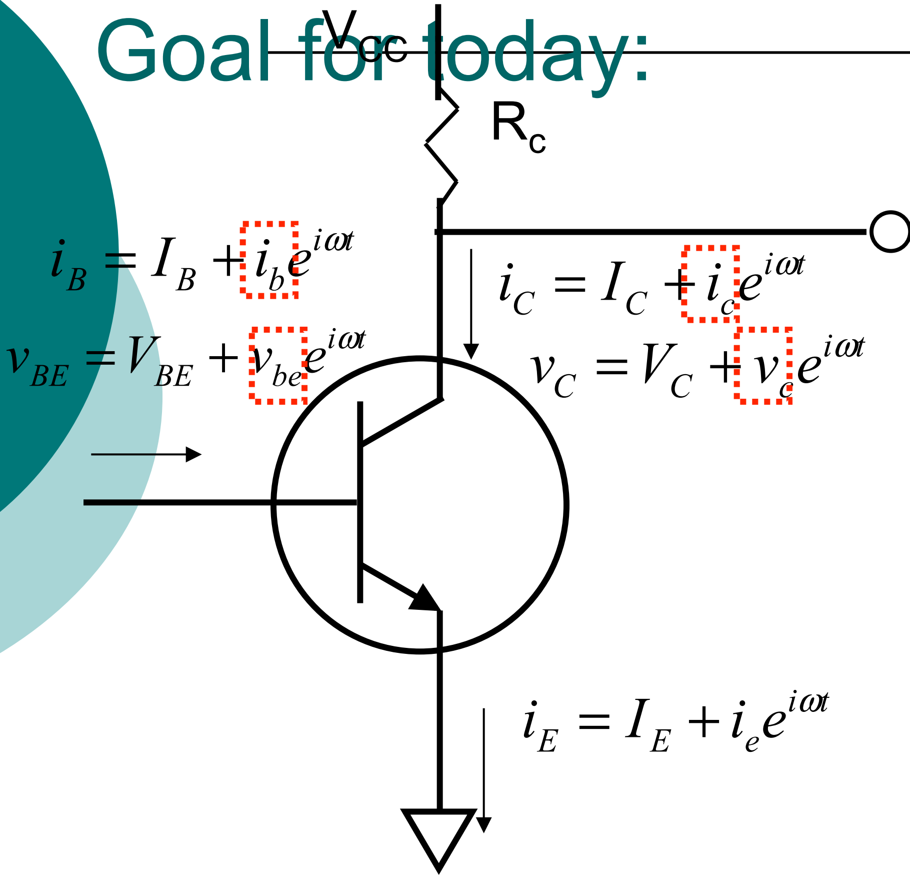


$$C_{dep} \equiv \frac{g_e \tau_M}{2}$$

$$\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i\frac{\omega}{3\omega_0}\right] e^{\omega\tau_M/2} v_{be}$$

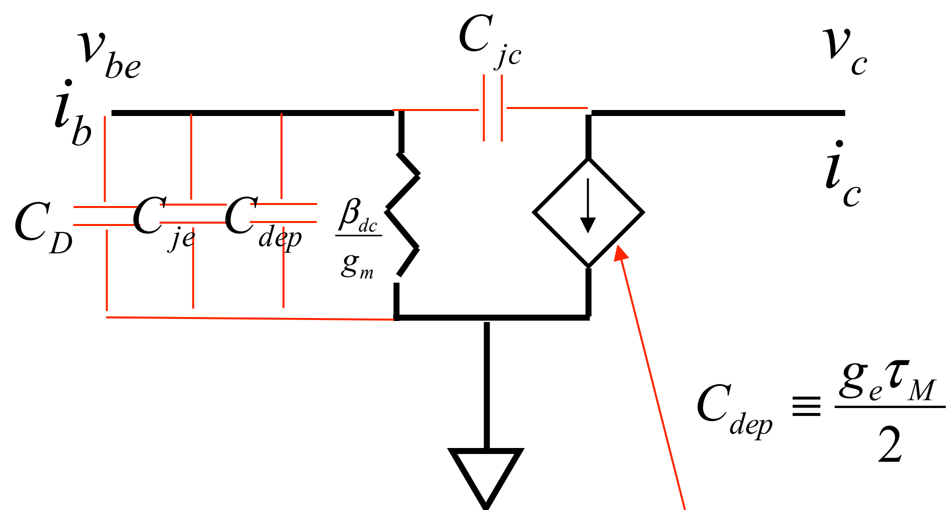
- Circuit model good only for low frequencies
- At high frequencies computer must be used!
- That concludes our derivation of intrinsic HBT behavior.
- Next will include parasitics, and discuss f_T, f_{max}

Goal for today:



Hybrid p model:

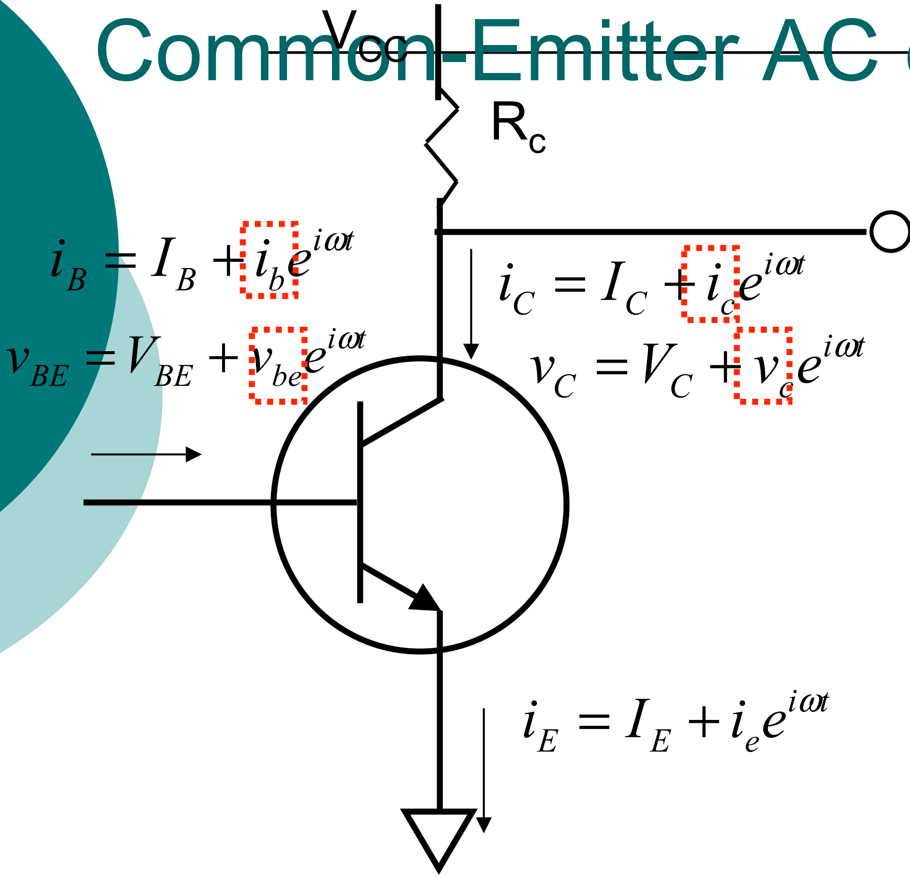
Red is new for ac:



$$\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i\frac{\omega}{3\omega_0}\right] e^{\omega\tau_M/2} v_{be}$$

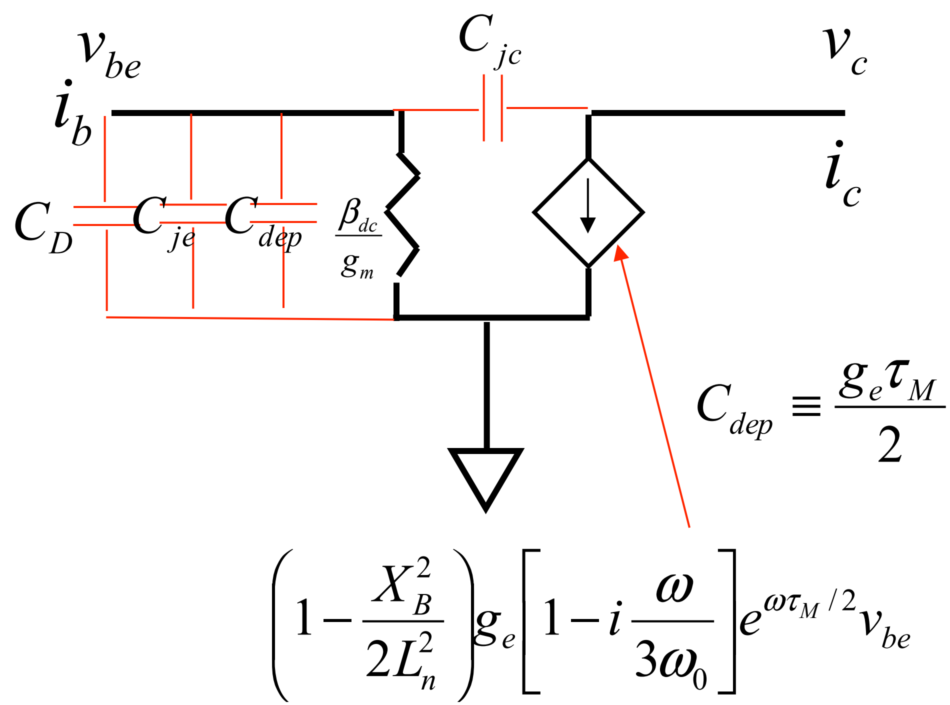
Parasitics
f_T, f_{max}

Common-Emitter AC equivalent circuit:



Hybrid p model:

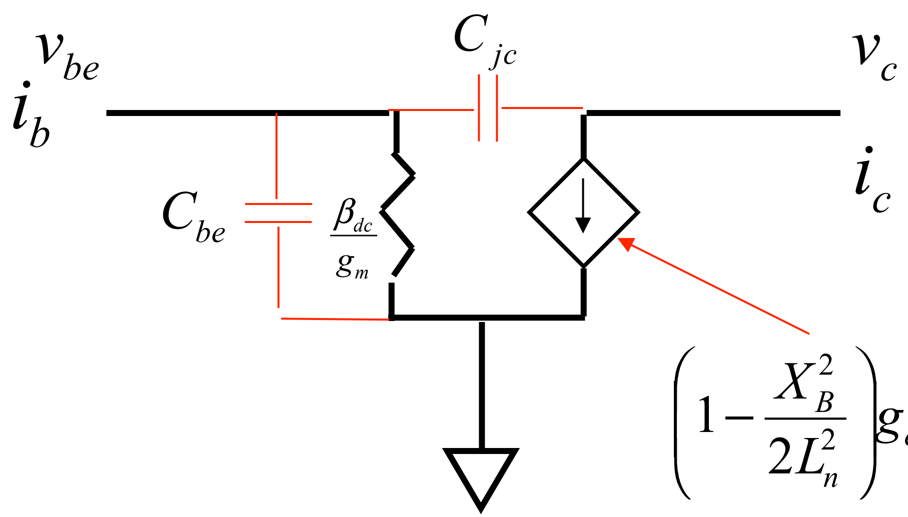
Red is new for ac:



- Circuit model good only for low frequencies
- At high frequencies computer must be used!

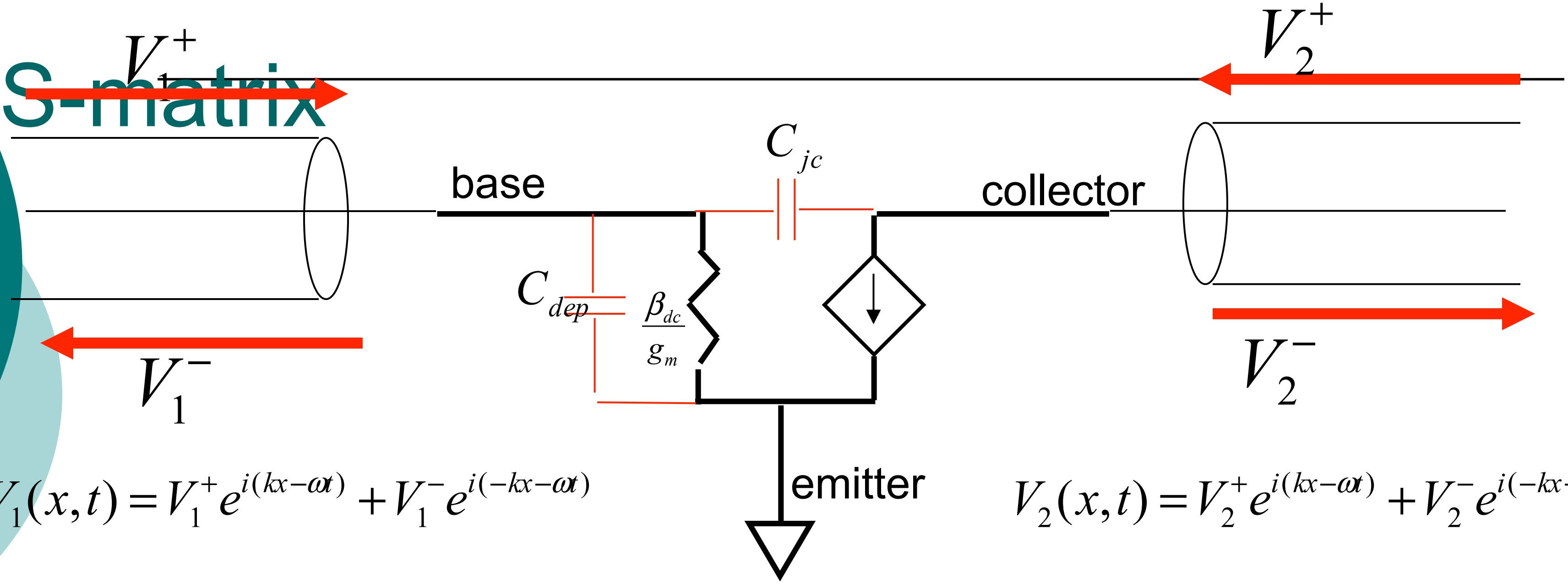
Hybrid p model:

simplified:



$$\left(1-\frac{X_B^2}{2L_n^2}\right)g_e\left[1-i\frac{\omega}{3\omega_0}\right]e^{\omega\tau_M/2}v_{be}$$

S-matrix



$$V_1(x,t) = V_1^+ e^{i(kx-\omega t)} + V_1^- e^{i(-kx-\omega t)}$$

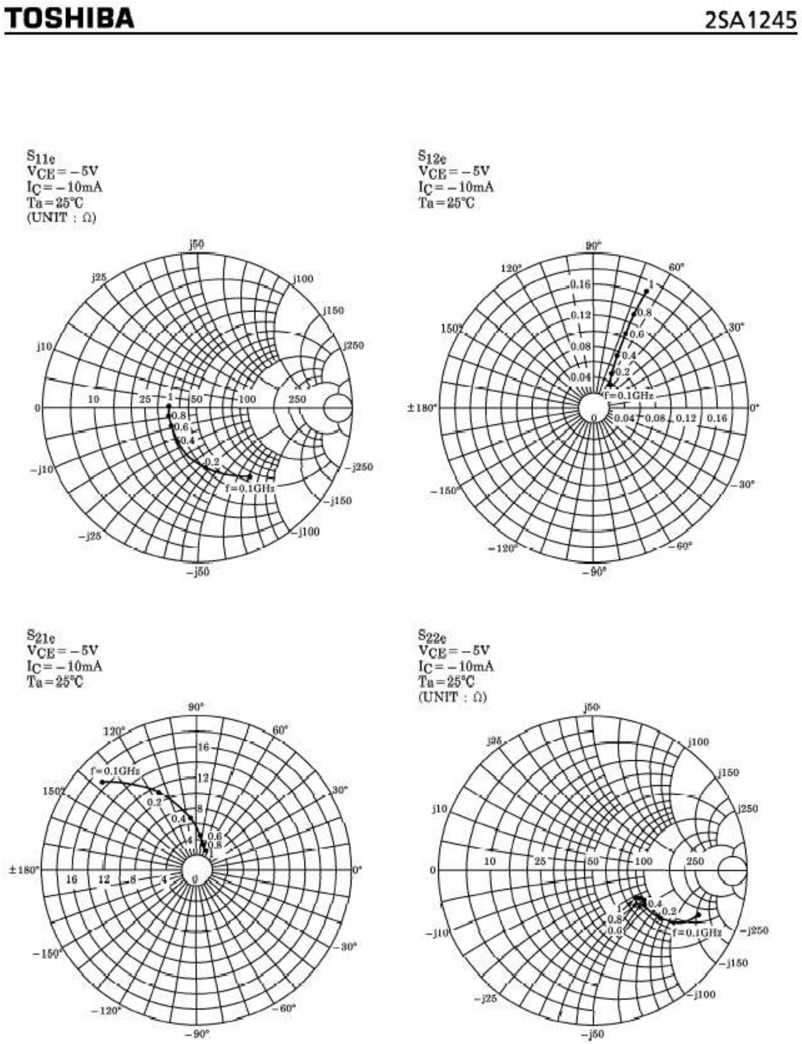
$$V_2(x,t) = V_2^+ e^{i(kx-\omega t)} + V_2^- e^{i(-kx-\omega t)}$$

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

S-parameters



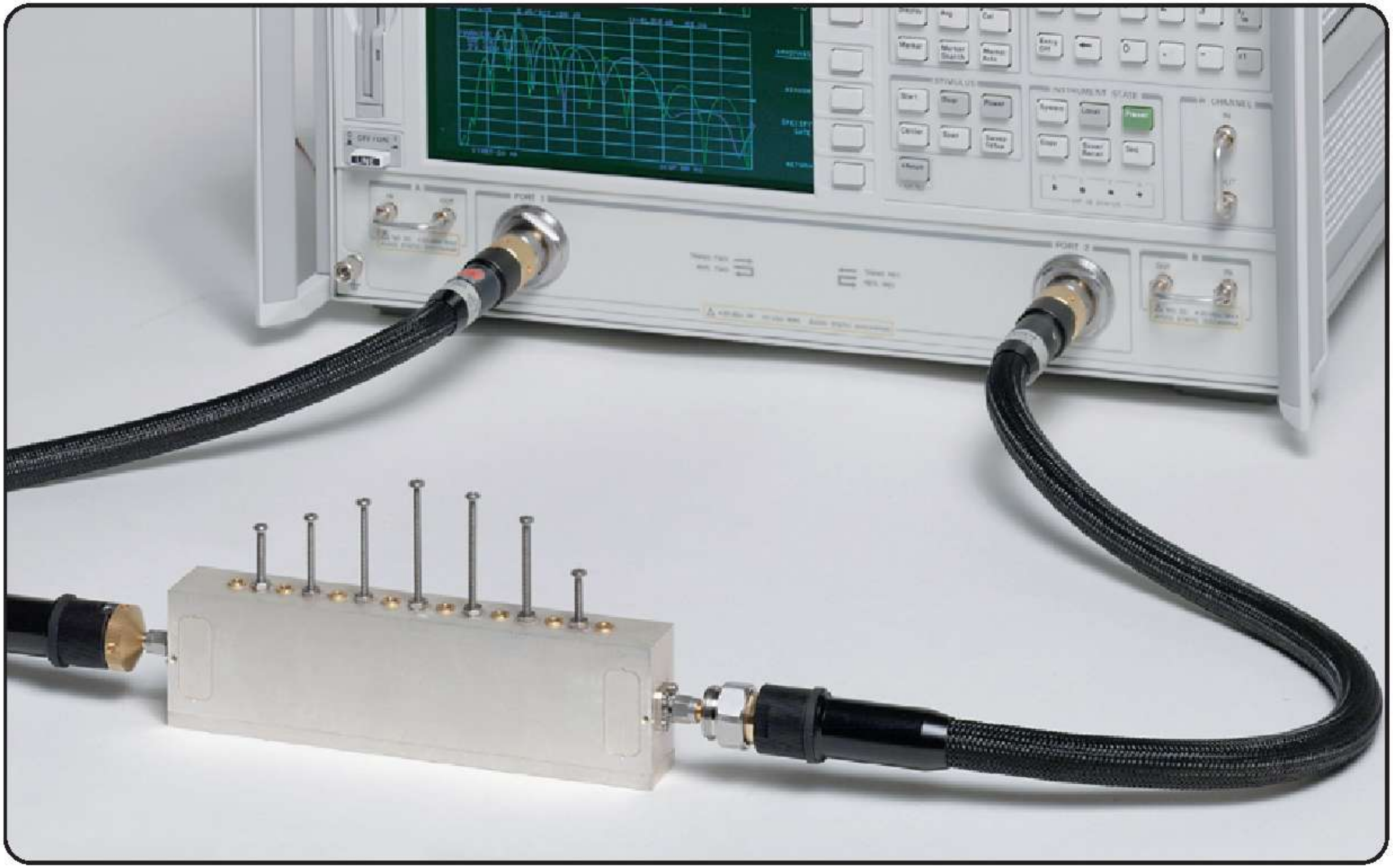
This is what you see on data sheets.
Related to input impedance, output impedanc
and gain vs. frequency.
=> Need to discuss ac performance.

Summary of parameters

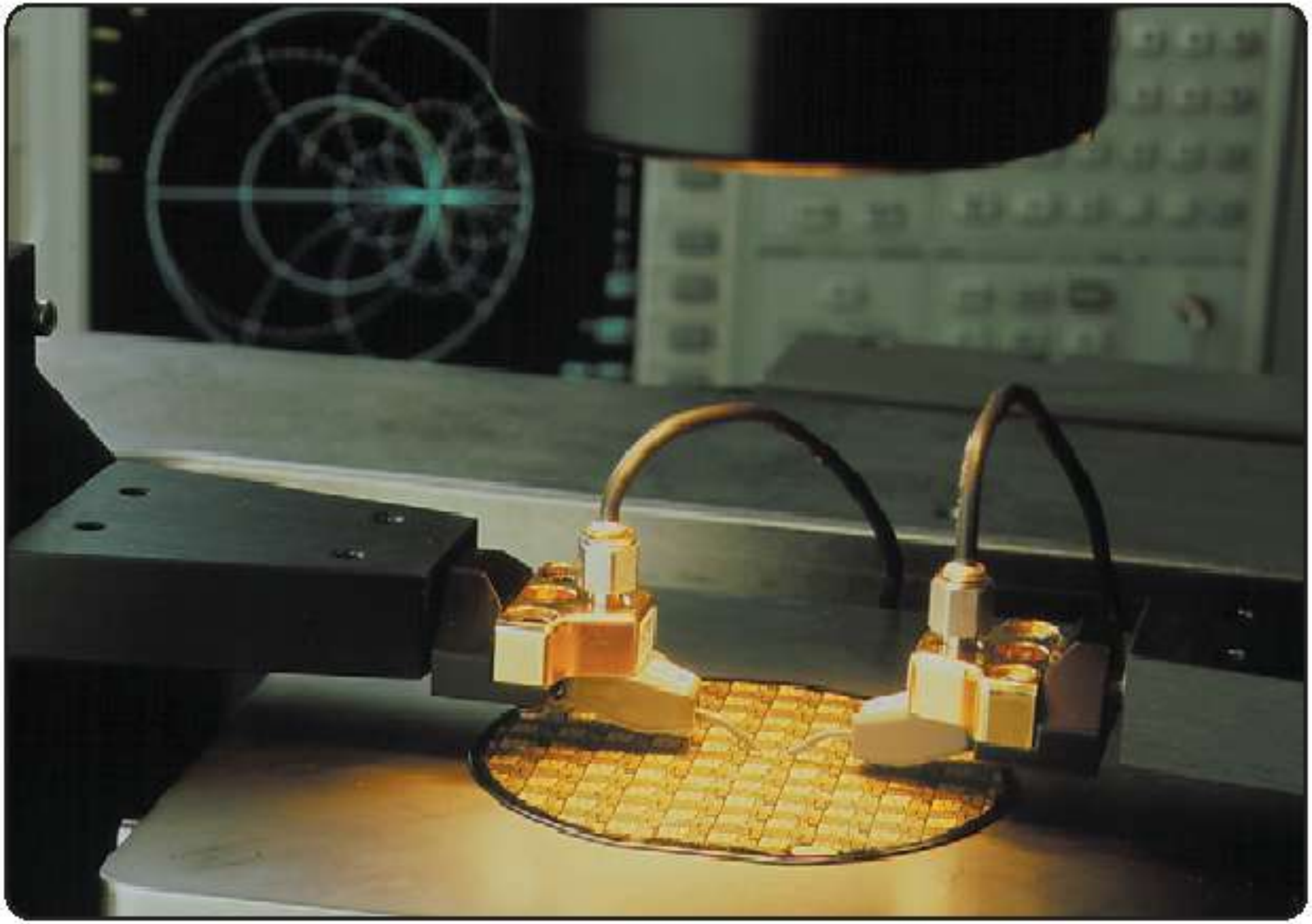
- Impedance matrix (V=IR -> V=IZ)
- Admittance matrix (I=YV)
- h-matrix (combination)
- ABCD matrix (combination)
- S-matrix (microwave reflections and transmissions)

"If you know one, then you know them all…"
See Liu, page 249 for conversions.

Measurement techniques



Measurement techniques



Cost (*rough* estimates)

- 10 GHz: $50,000
- 20 GHz: $70,000
- 40 GHz: $90,000
- 110 GHz: $250,000
- > 110 GHz: very expensive

For cost and difficulty reasons, parameters of transistor not always measure all the way up to f_T, but extrapolated.

These are only estimates. Contact vendor for actual prices.

Example

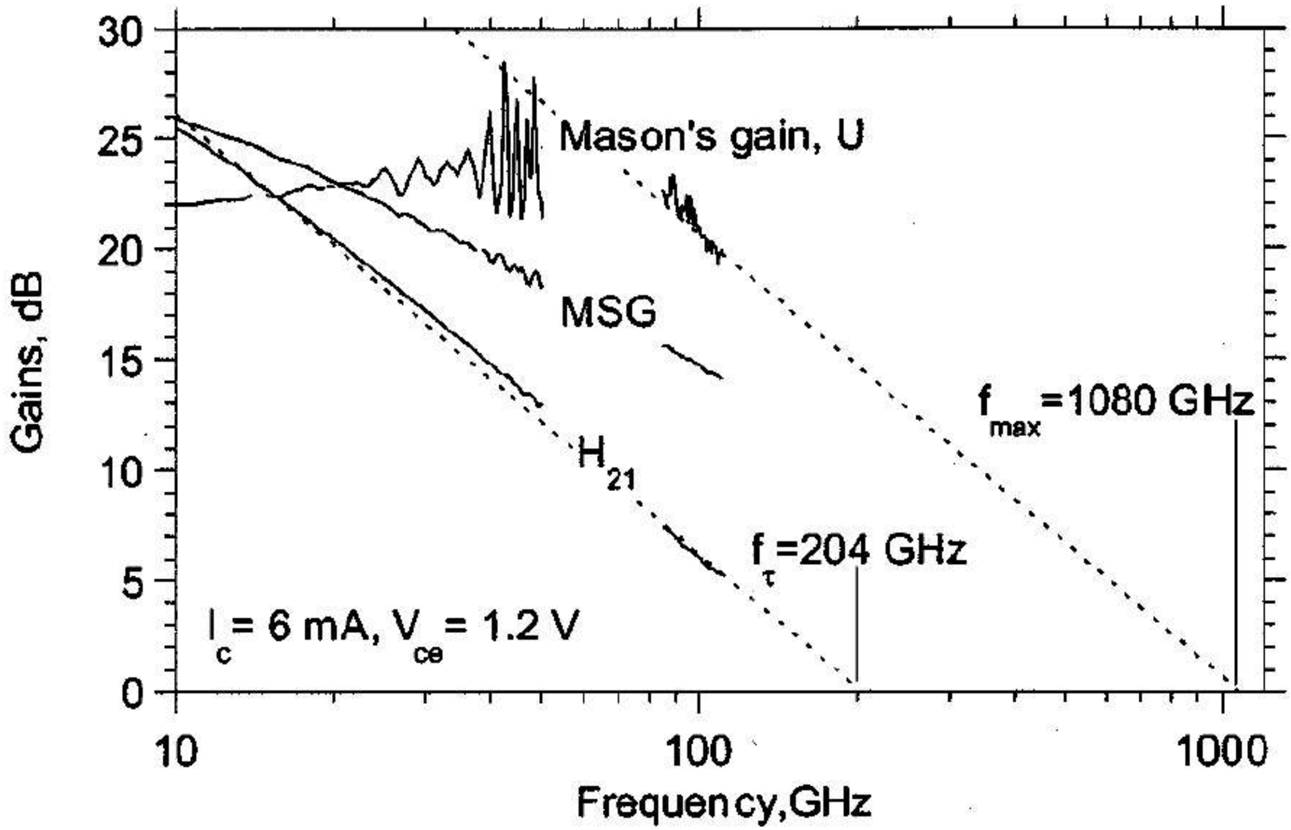
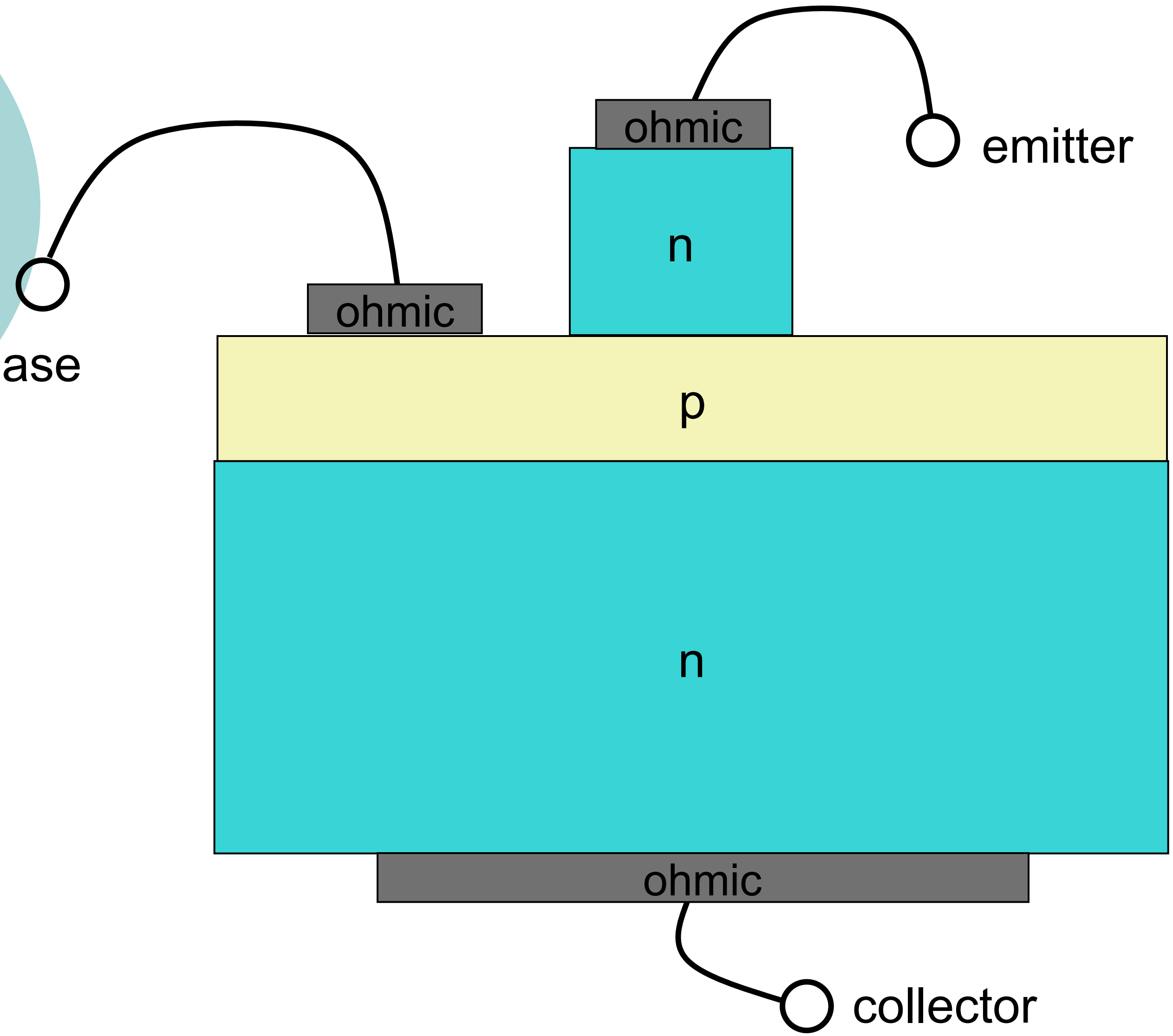


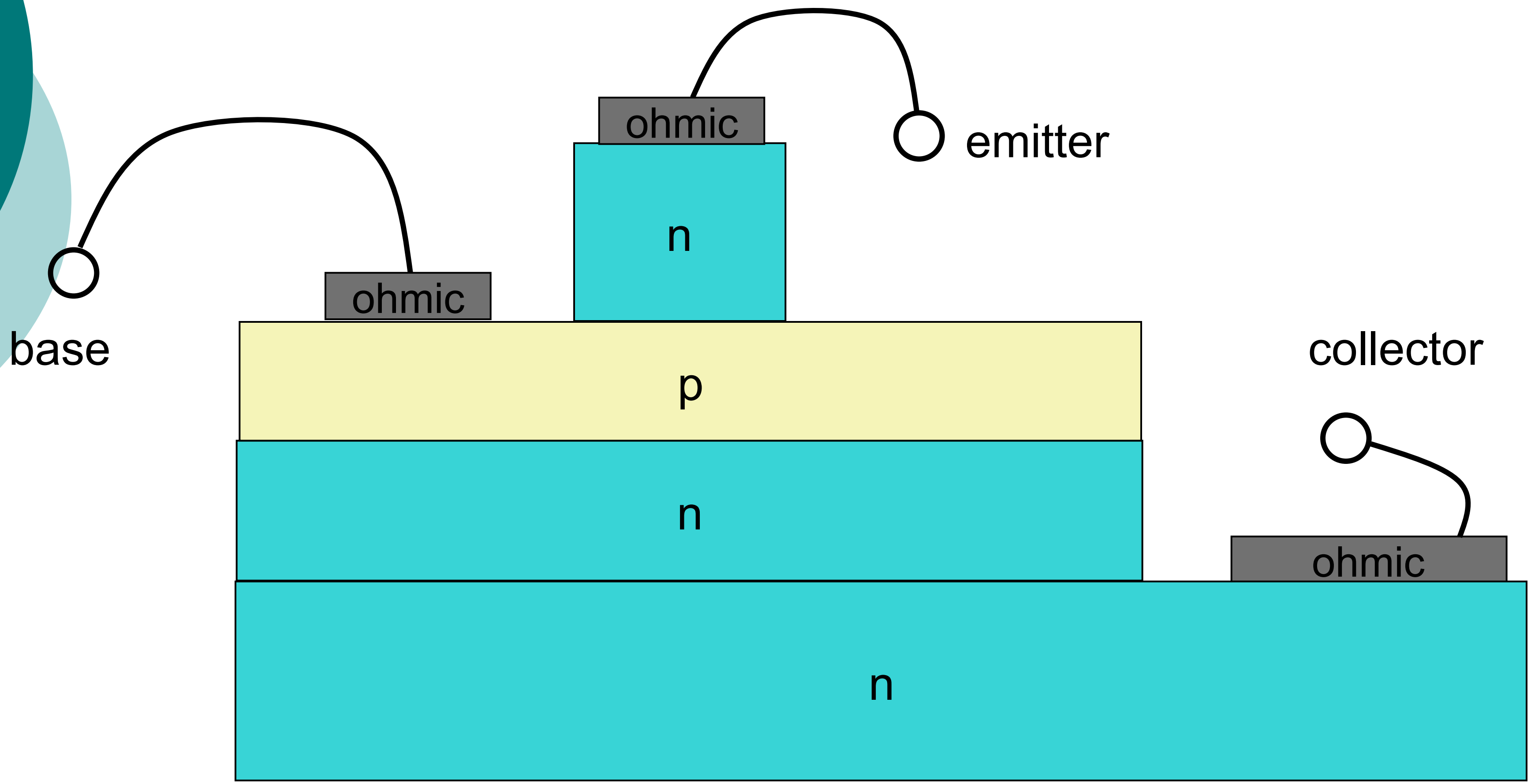
Fig. 14. Gains of a 0.4 μm $\times$ 6 μm emitter and 0.7 μm $\times$ 10 μm collector HBT fabricated using electron-beam lithography. Theoretical -20 dB/decade (H_{21}, U) gain slopes are indicated. The device exhibits an *extrapolated* 1.08 THz $f_{\max}$.

From Rodwell, et al, TRANSACTIONS ON ELECTRON DEVICES 48 (11): 2606-2624 IEEE NOV 2001

Parasitics:



Parasitics:



Ohmic contact

Specific contact resistance typically 10^{-6} ohm-micron^2
(Discuss on board.)

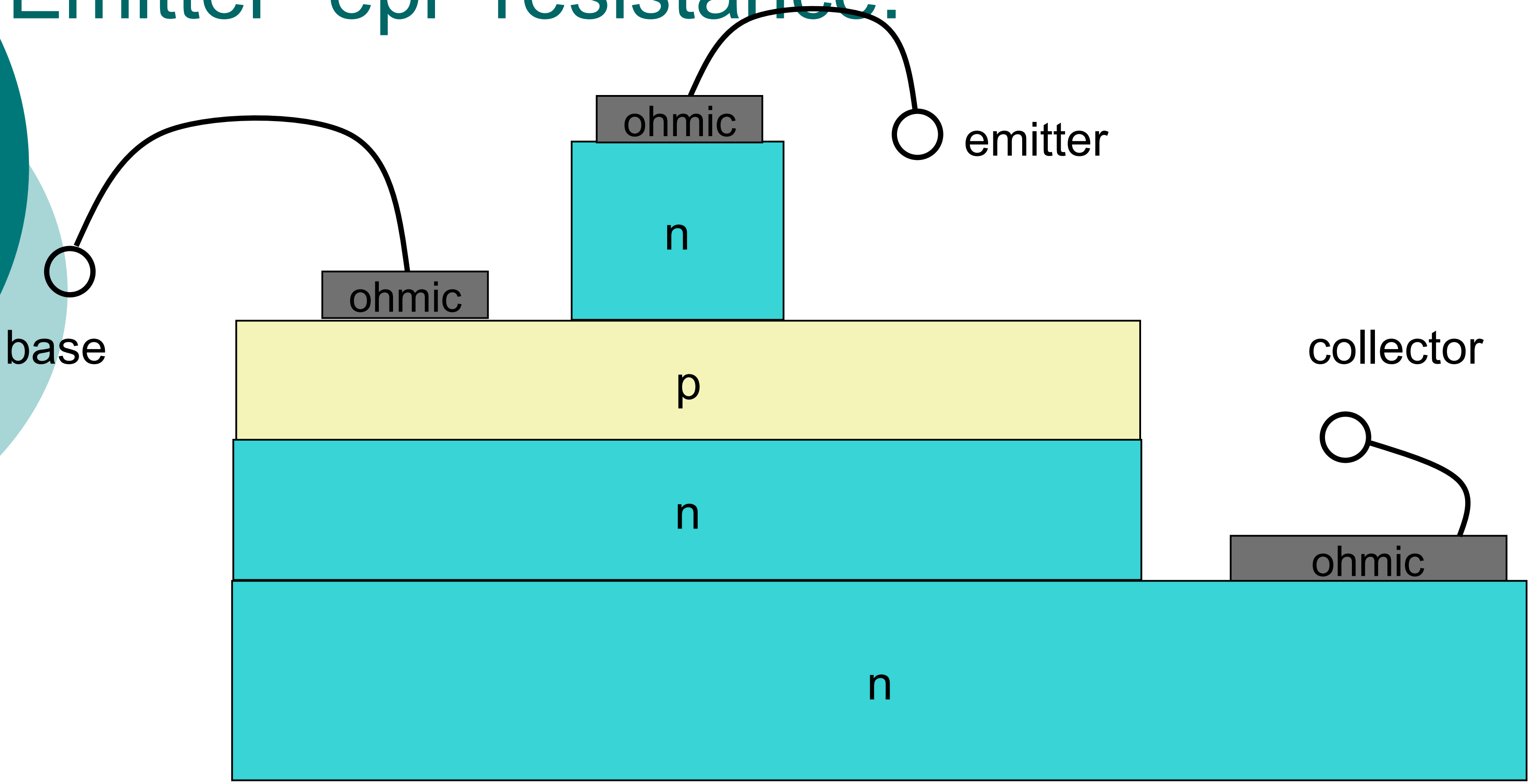
$$R_{EE} = \frac{\rho_{\sigma E}}{A_E} = \frac{\rho_{\sigma E}}{L_E W_E}$$

For a distributed contact, thinks are a little more complicated.
(Draw distributed RC network on board, discuss.)
A solution is:

$$R_{BB} = \frac{\sqrt{R_{SHB}\rho_{\sigma B}}}{L_E} \coth\left(W_B \sqrt{\frac{R_{SHB}}{\rho_{\sigma B}}} \right)$$

R_{SHB} is R per square (discuss)

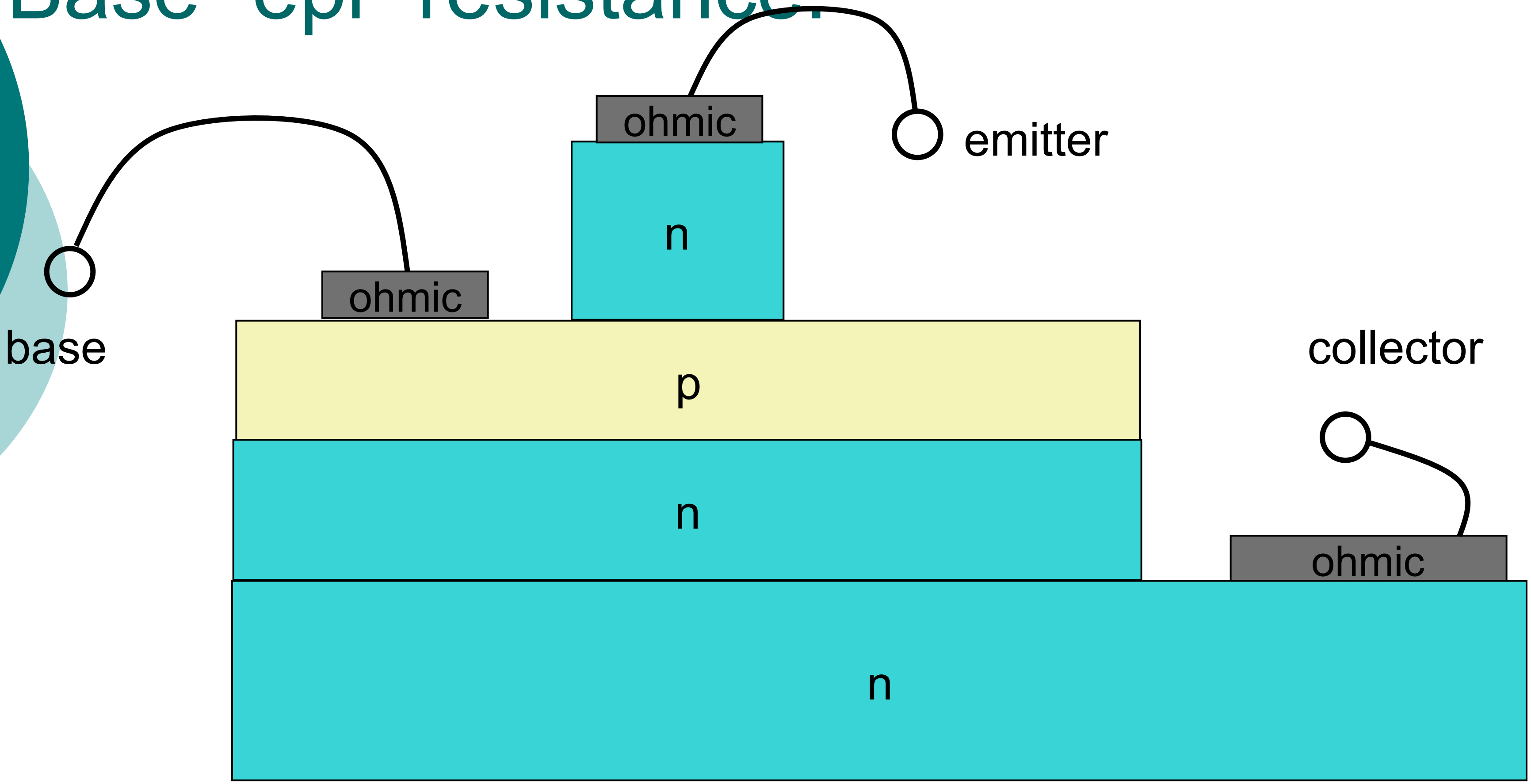
Emitter "epi" resistance:



$$R_{E(epi)} = \rho_{E(epi)} \frac{X_{E(epi)}}{L_E W_E}$$

(discuss)

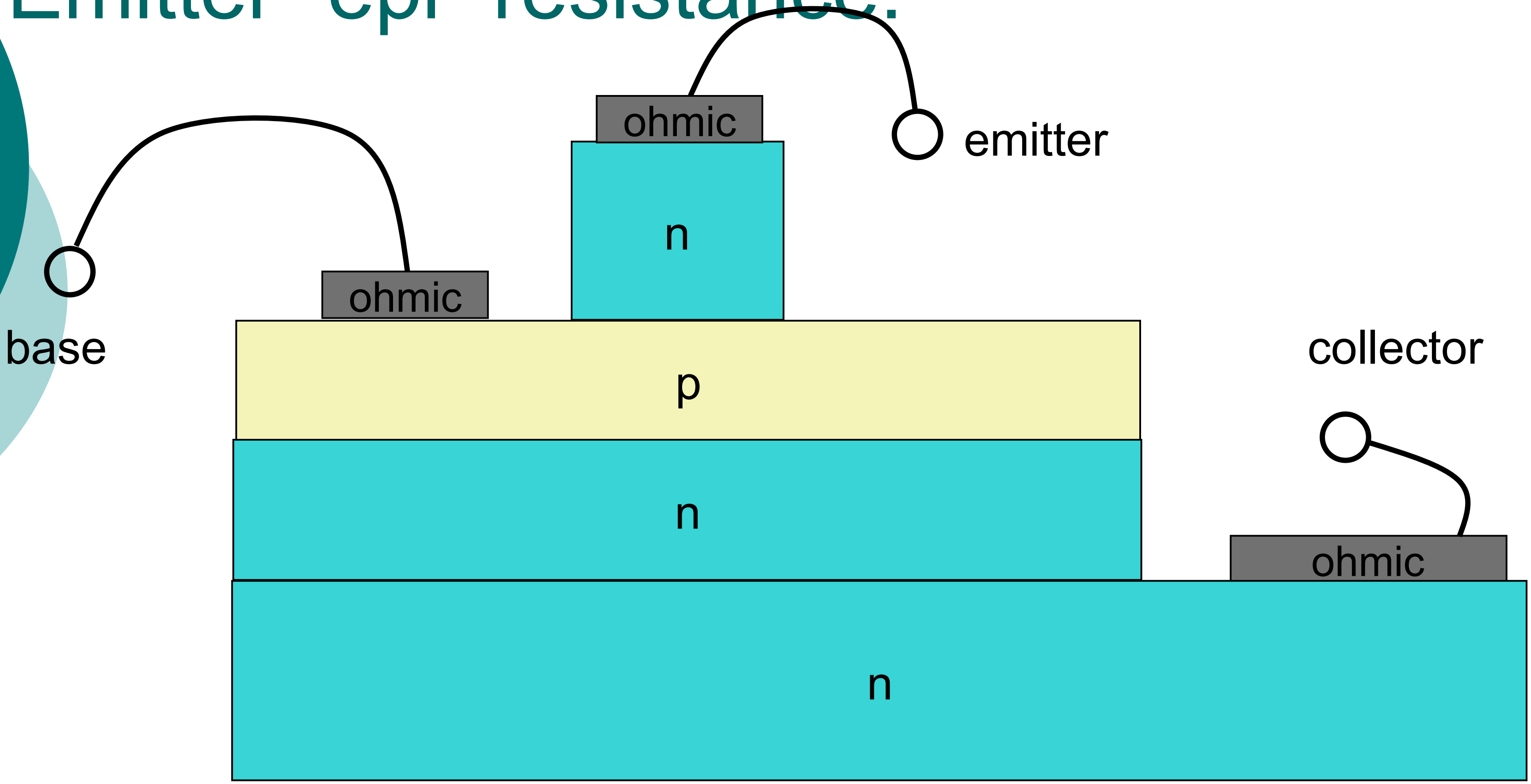
Base "epi" resistance:



$$R_{Bx(epi)}$$

(discuss)

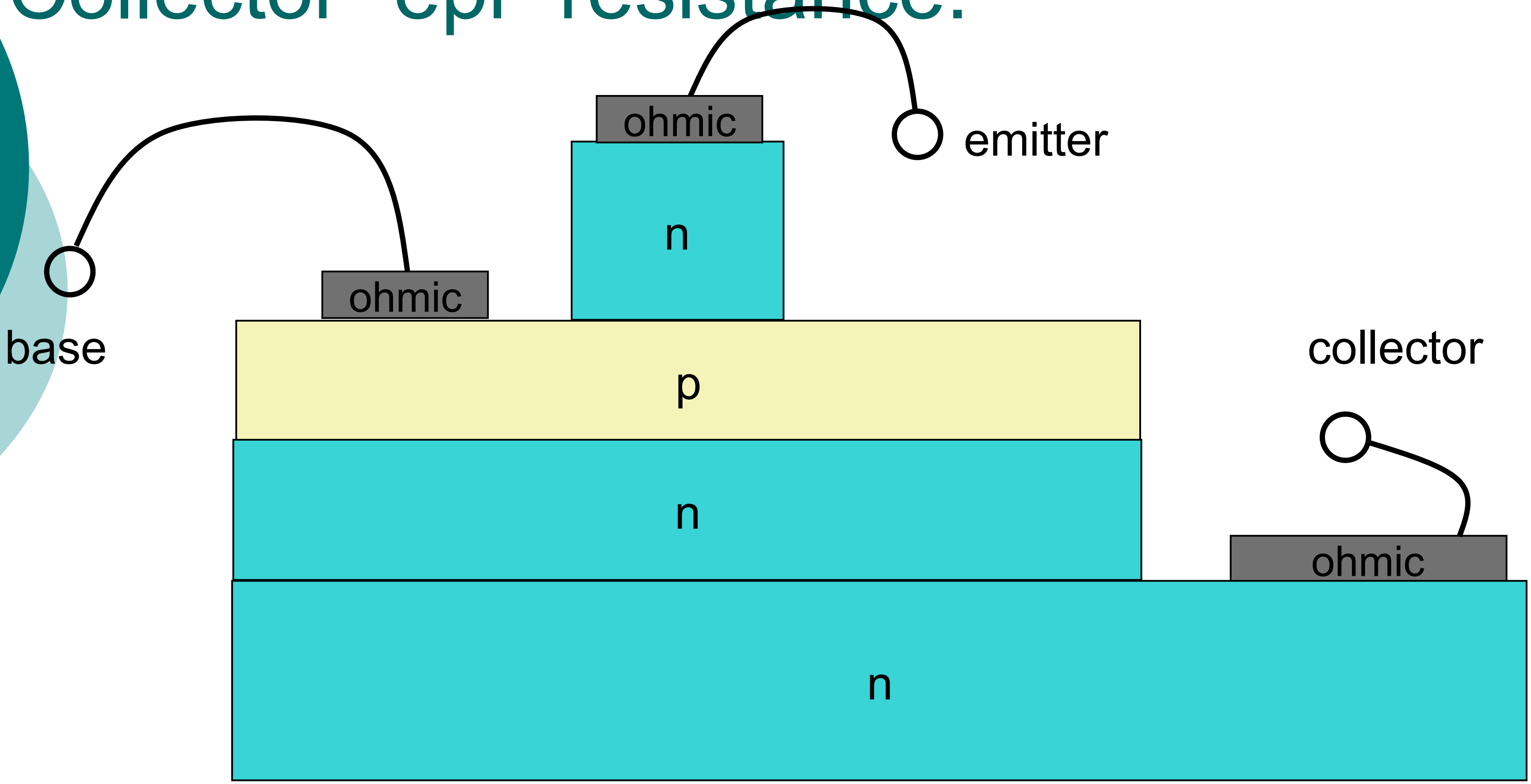
Emitter "epi" resistance:



$$R_{E(epi)} = \rho_{E(epi)} \frac{X_{E(epi)}}{L_E W_E}$$

(discuss)

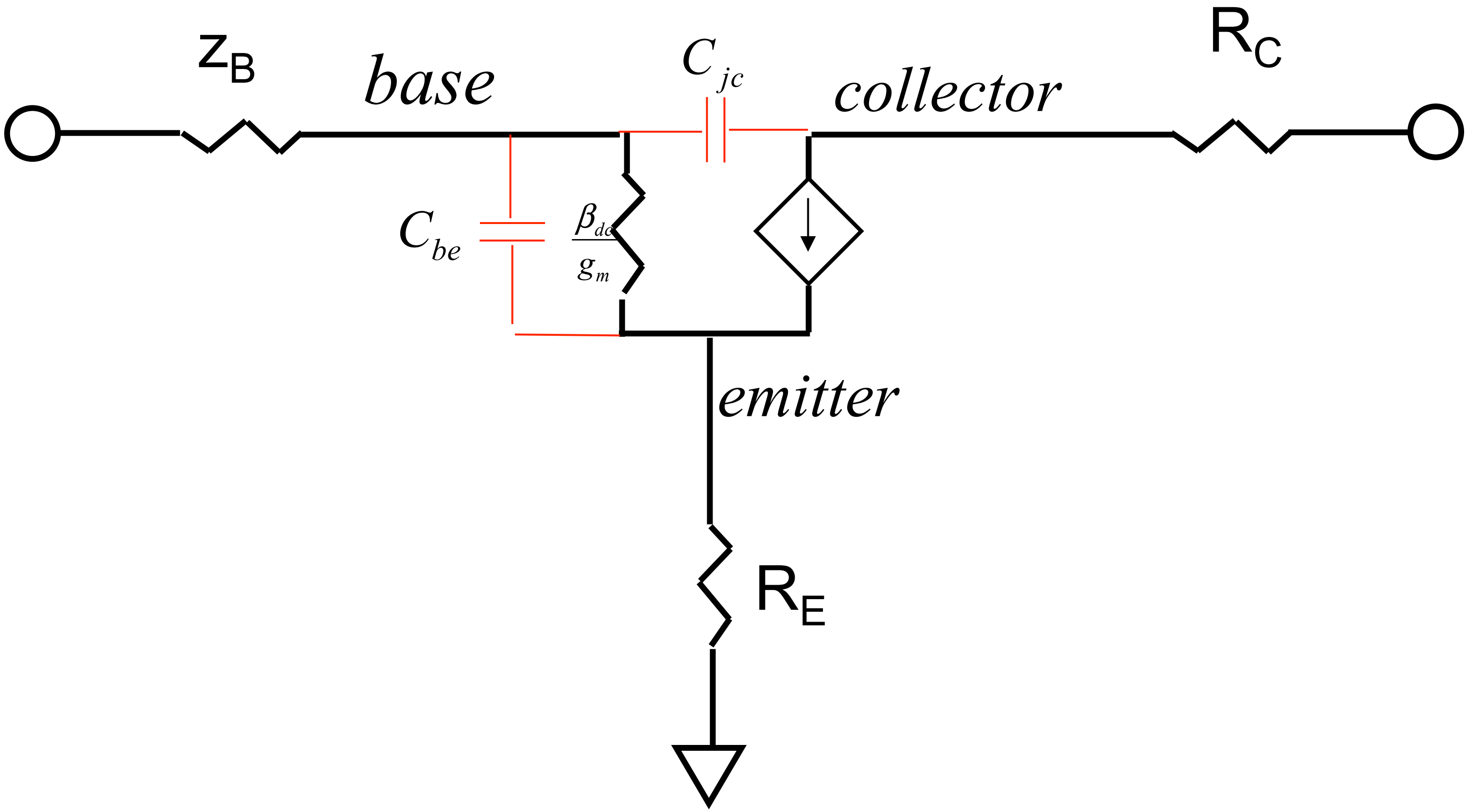
Collector "epi" resistance:



$$R_{C(epi)} = \rho_{C(epi)} \frac{X_{C(epi)}}{L_E W_E}$$

(discuss)
(also discuss spreading effect)

Parasitics: In summary



Total parasitics include contact, epi, and metal layer resistance.
Sometimes inductance also added in.

f_T:

$$\begin{pmatrix} v_b \\ i_c \end{pmatrix} = \begin{pmatrix} h_{11e} & h_{12e} \\ h_{21e} & h_{22e} \end{pmatrix} \begin{pmatrix} i_b \\ v_c \end{pmatrix}$$

- Early effect:
  Collector voltage changes current gain (b).
- b depends on frequency *and* collector voltage.
- How do we define frequency at which b = 1?
- At v_c=0. This *is* h_{21e}

$$i_c = h_{21e} i_b + h_{22e} v_c \rightarrow h_{21e} i_b$$

- We define f_T such that:

$$\left| h_{21e} \right| (f_T) = 1$$

f_T

$$f_T = \frac{1}{2\pi\tau_{ec}}$$

$$\tau_{ec} = \tau_e + \tau_b + \tau_{sc} + \tau_c$$

Emitter charging time

$$\tau_e = \frac{kT}{eI_C}\left(C_{je} + C_{jc}\right)$$

Time to charge up junction capacitors.

Base transit time

$$\tau_e = \frac{X_B^2}{2D_{nB}}$$

Time to charge up base minority carriers.
Or: time to diffuse from emitter to collector.
(Built in field helps a lot here.)

Space-charge transit time

$$\tau_e = \frac{X_{dep}}{2v_{sat}}$$

Or: time to *drift* through space charge of base-collector junction.

Collector charging time

$$\tau_c = (R_E + R_C) \cdot C_{jc}$$

Time to charge collector junction capacitor through parasitic resistors.

f_T:

$$\begin{pmatrix} v_b \\ i_c \end{pmatrix} = \begin{pmatrix} h_{11e} & h_{12e} \\ h_{21e} & h_{22e} \end{pmatrix} \begin{pmatrix} i_b \\ v_c \end{pmatrix}$$

"It can be shown that…"

$$h_{21e} = \frac{\alpha_{T0}}{(1-\alpha_{T0}) + i(f / f_T)}$$

$$\alpha_{T0} \equiv 1 - \frac{X_B^2}{2L_n^2}$$

Discuss rolloff, low frequency value.

f_{max}:

In real circuits, we do not want to short circuit the output!
Unilateral power gain: if impedance matching network is set up so that there is no reverse transmission (S_{12}=0), in that case the power gain is called the *unilateral power gain*.

"It can be shown that…"

$$U = \frac{\left|z_{21} - z_{12}\right|^2}{4\left[\mathrm{Re}(z_{11})\,\mathrm{Re}(z_{22}) - \mathrm{Re}(z_{12})\,\mathrm{Re}(z_{21})\right]}$$

"It can be shown that…"

$$U = \frac{\alpha_{T0}{}^2 \omega_T}{4\,\mathrm{Re}(z_b) C_{jc} \omega^2}$$

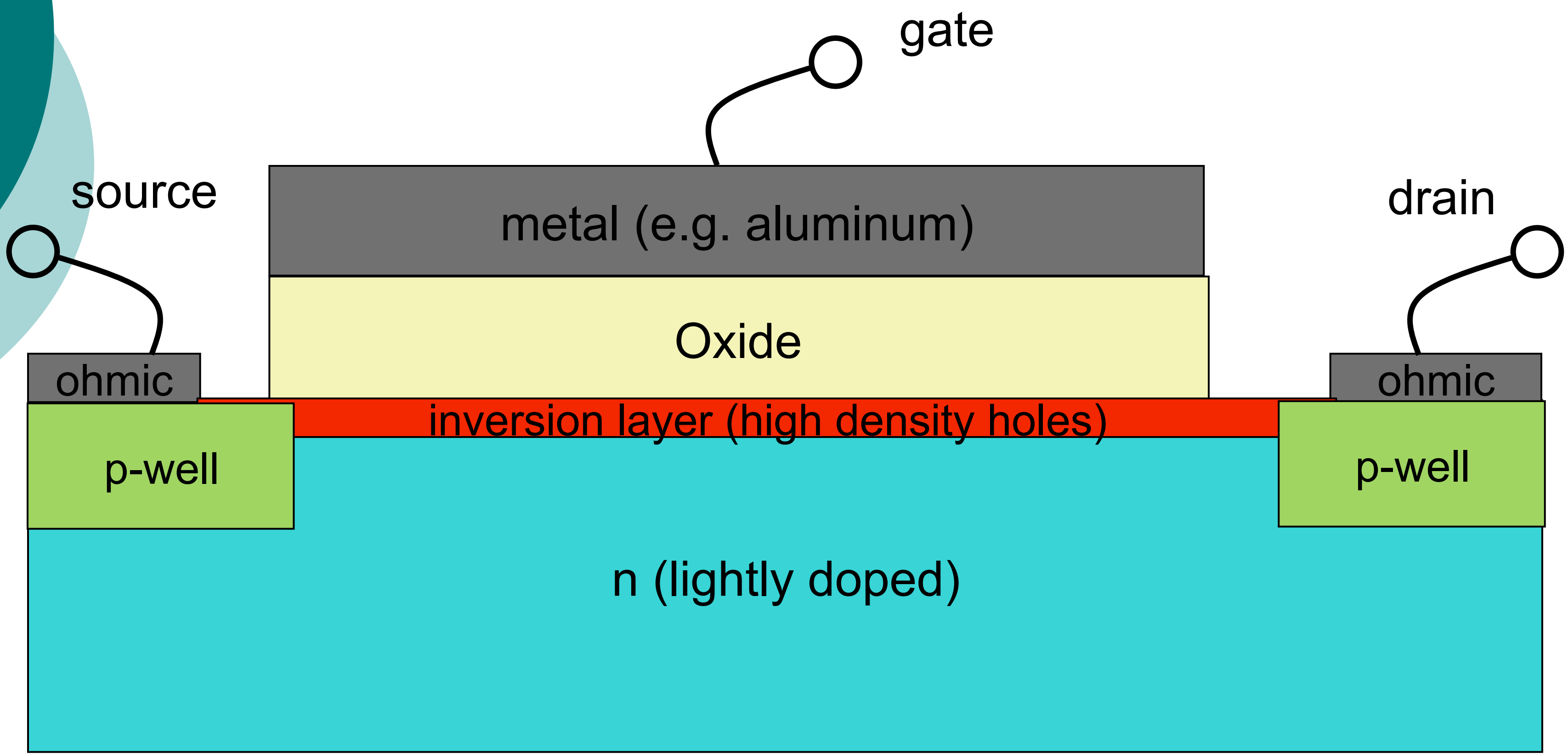
$$f_{\max} = \sqrt{\frac{f_T}{8\pi r_b C_{jc}}}$$

Discuss r_b dependence, want heavily doped base. Need for *H*BT.

Field Effect Devices

- MOSFET
- JFET
- MESFET
- HEMT

Si MOSFET:



- Gate has high input resistance (10^{12} W)
- Si covered in ECE 277A.
- No oxide for GaAs, so need different type of device.

Non-oxide transistors

- JFET: Junction Field Effect Transistor
- MESFET: Metal Electron Semiconductor Field Effect Transistor
- HEMT: High Electron Mobility Transistor
  - Also called:
  - MODFET Modulation Doped Field Effect Transistor
  - TEGFET Two-Dimensional Electron Gas Field Effect Transistor
  - pHEMT Pseudomorphic HEMT
  - HFET Heterojunction Field Effect Transistor

Integrating:

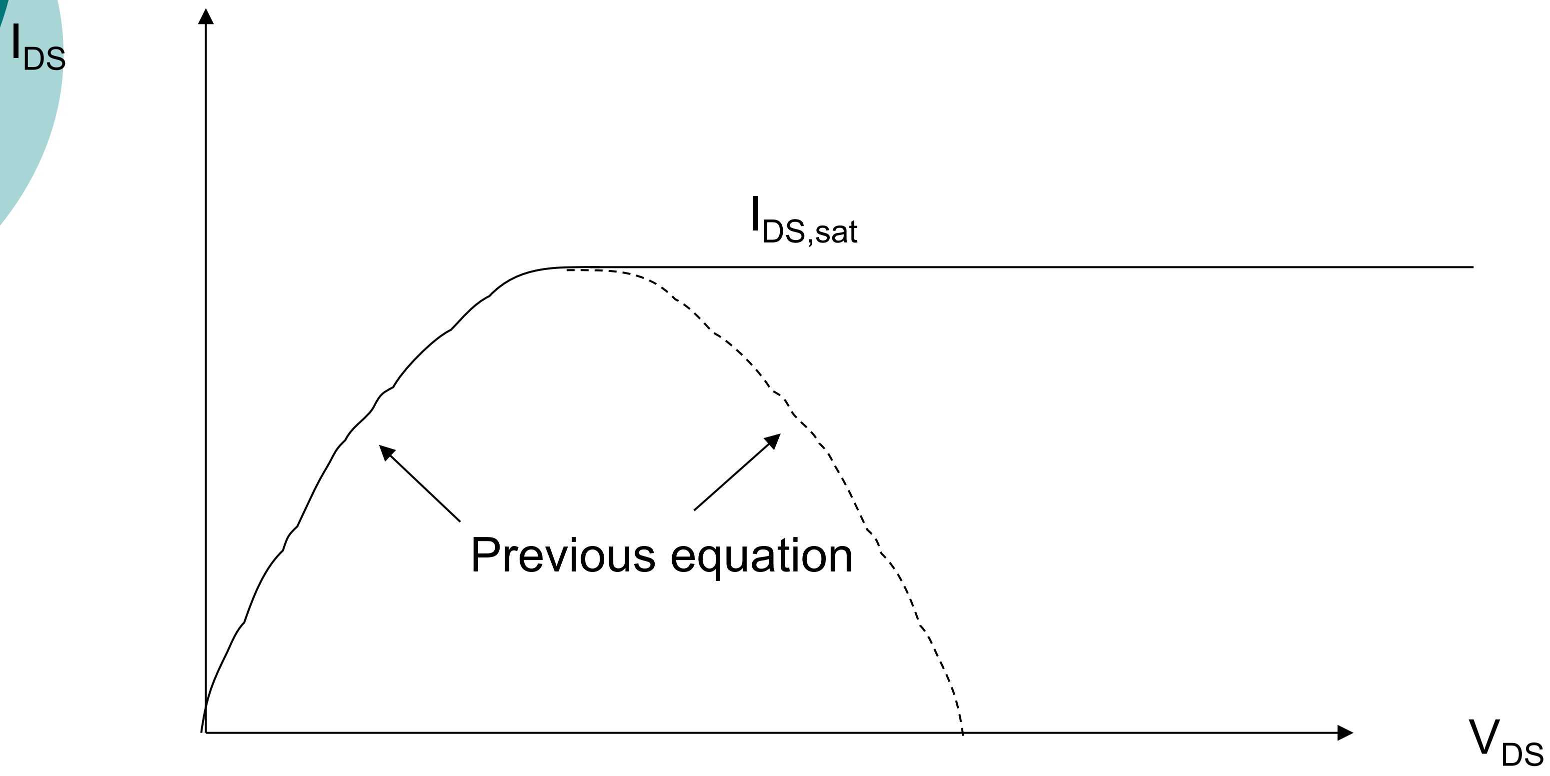
$$I_D = \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$

$$\int_0^L I_D dx = \int_0^L \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W dx$$

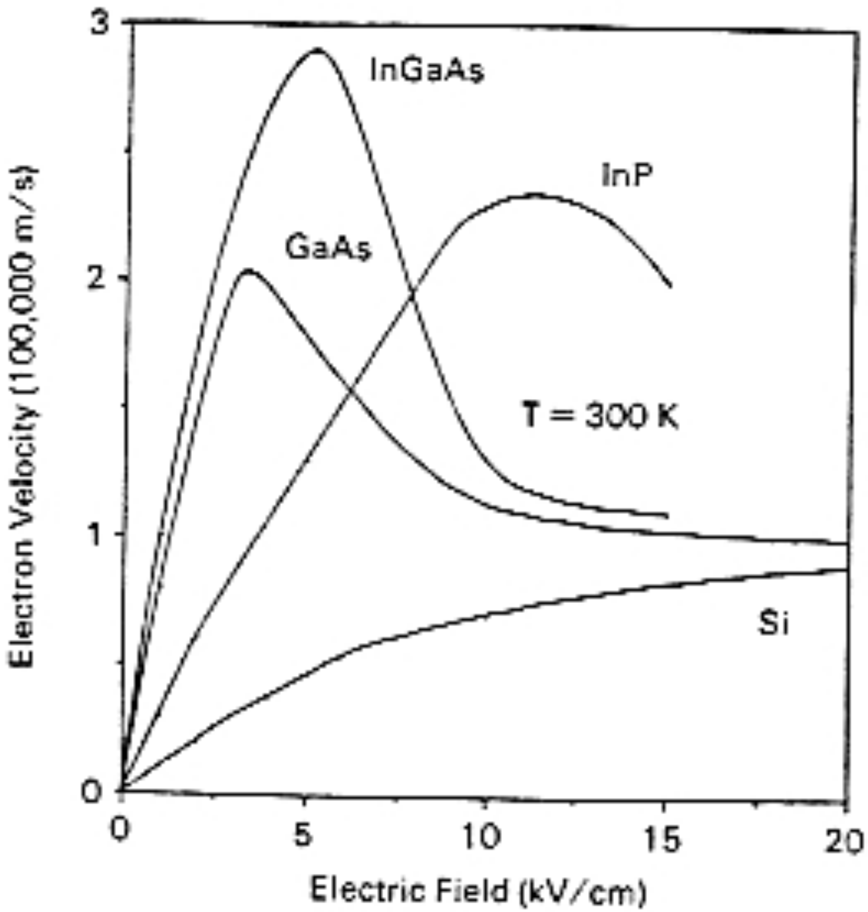
$$= \int_{V_{CS}(0)}^{V_{CS}(L)} \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot \partial V_{CS}(x) \cdot W = doable$$

$$I_D = \frac{W \cdot \mu \cdot C_{ox}}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^{\ 2}}{2}\right]$$

FET I-V curves



Velocity saturation



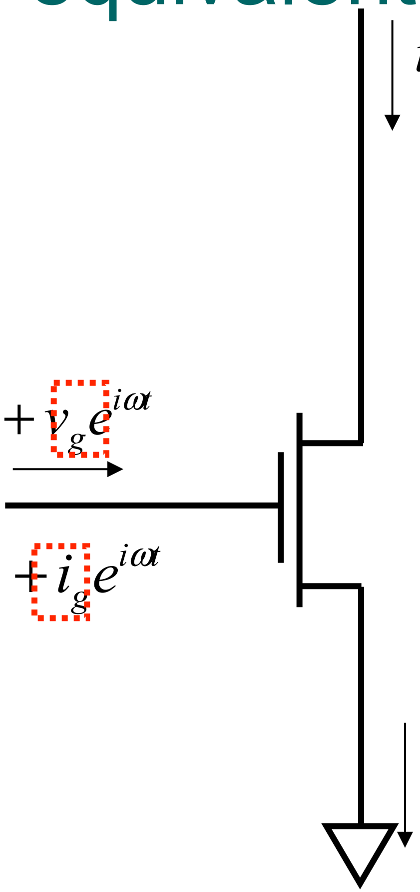
$$v = \mu \cdot E$$

$$\rightarrow v_{sat}$$

From Shur, Physics of Semiconductor Devices

FET AC properties

AC equivalent circuit:



$$i_D = I_D + i_d e^{i\omega t}$$

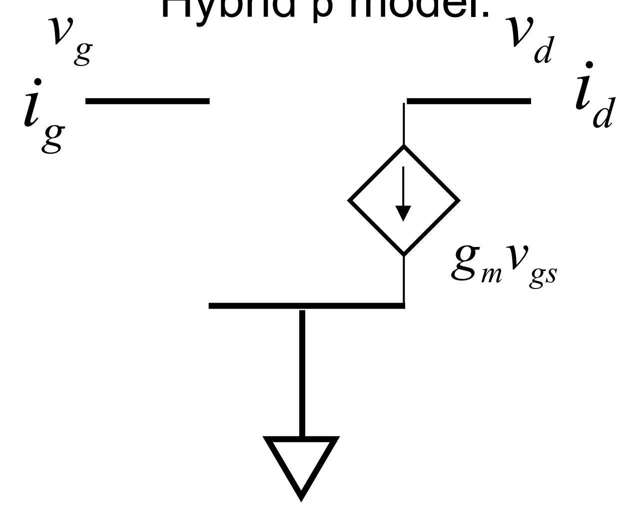
$$v_D = V_D + v_d e^{i\omega t}$$

$$v_G = V_G + v_g e^{i\omega t}$$

$$i_G = I_G + i_g e^{i\omega t}$$

$$i_S = I_S + i_s e^{i\omega t}$$

Hybrid p model:



$$\begin{pmatrix} i_g \\ i_d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ g_m & 0 \end{pmatrix} \begin{pmatrix} v_g \\ v_d \end{pmatrix}$$

This is the common-source Y-matrix. You can get all the matrices from it.

What about capacitances?

Book sticks to long channel devices, well below transit time frequency.

Modern devices are short and f_T *is* the transit time frequency.

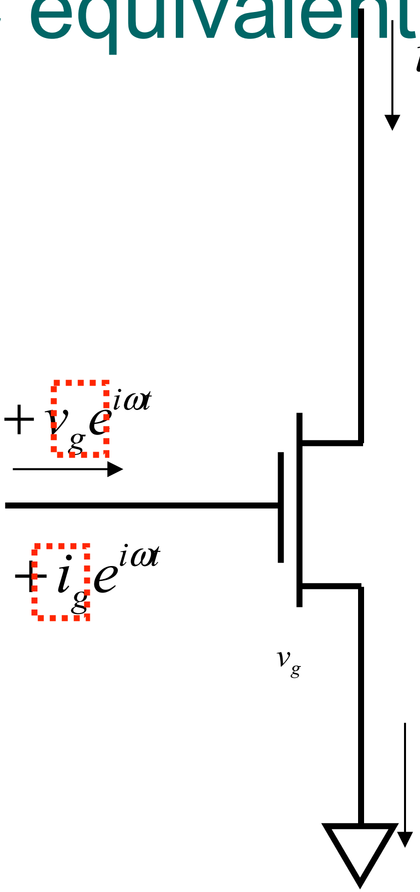
AC equivalent circuit:

$$i_D = I_D + i_d e^{i\omega t}$$

$$v_D = V_D + v_d e^{i\omega t}$$

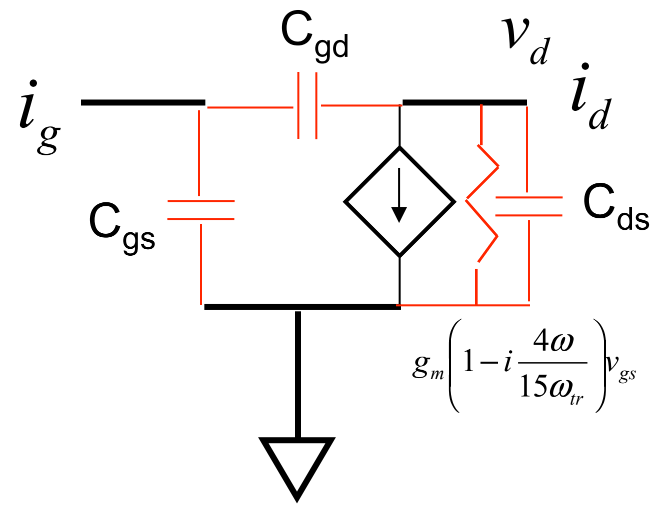
$$v_G = V_G + v_g e^{i\omega t}$$

$$i_G = I_G + i_g e^{i\omega t}$$



$$i_S = I_S + i_s e^{i\omega t}$$

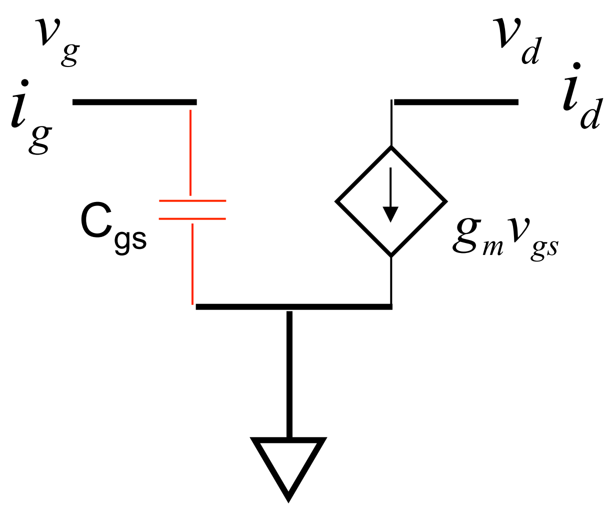
Hybrid p model:



C_{gs} dominates.

f_T

Hybrid p model:



When current flowing through capacitor is equal to $g_m v_{gs}$

then the frequency is f_T.

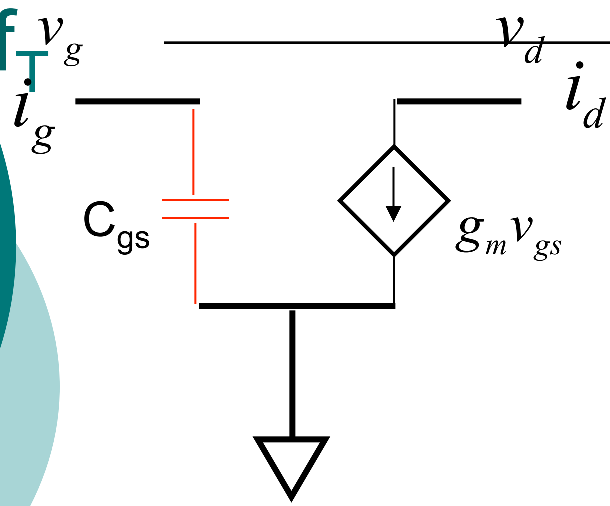
$$i_g = v_{gs}(\omega C_{gs}) \qquad i_d = g_M v_{gs}$$

At f_T

$$g_M v_{gs} = v_{gs}(\omega_T C_{gs})$$

$$\Rightarrow \omega_T = \frac{g_M}{C_{gs}} \Rightarrow f_T = \frac{g_M}{2\pi C_{gs}}$$

f_T



$$f_T = \frac{g_M}{2\pi C_{gs}}$$

In HW#6, you will prove for the long-channel device:

$$g_M = \frac{W\mu C'_{ox}}{L}\left(V_{GS} - V_T\right)$$

$$C'_{ox} \sim C_{gs}/(LW)$$

$$f_T \rightarrow \frac{1}{2\pi}\frac{\mu\left(V_{GS} - V_T\right)}{L^2}$$

For a short-channel device,

$$g_M = v_{sat} W C'_{ox}$$

$$f_T \rightarrow \frac{v_{sat}}{2\pi L} = \frac{1}{2\pi\tau_{tr}}$$

So book model is only good for frequencies much less than f_T.

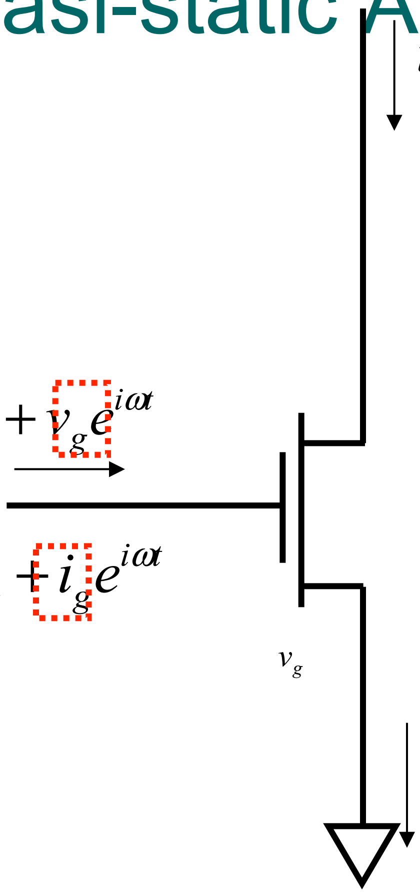
Quasi-static AC equivalent circuit:

$$i_D = I_D + i_d e^{i\omega t}$$

$$v_D = V_D + v_d e^{i\omega t}$$

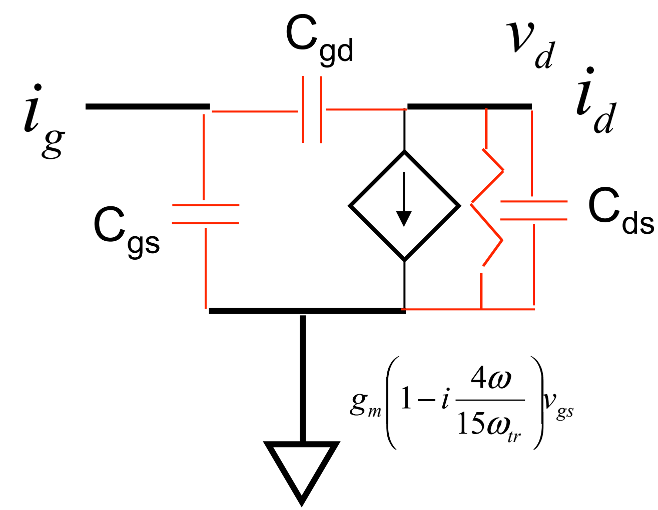
$$v_G = V_G + v_g e^{i\omega t}$$

$$i_G = I_G + i_g e^{i\omega t}$$



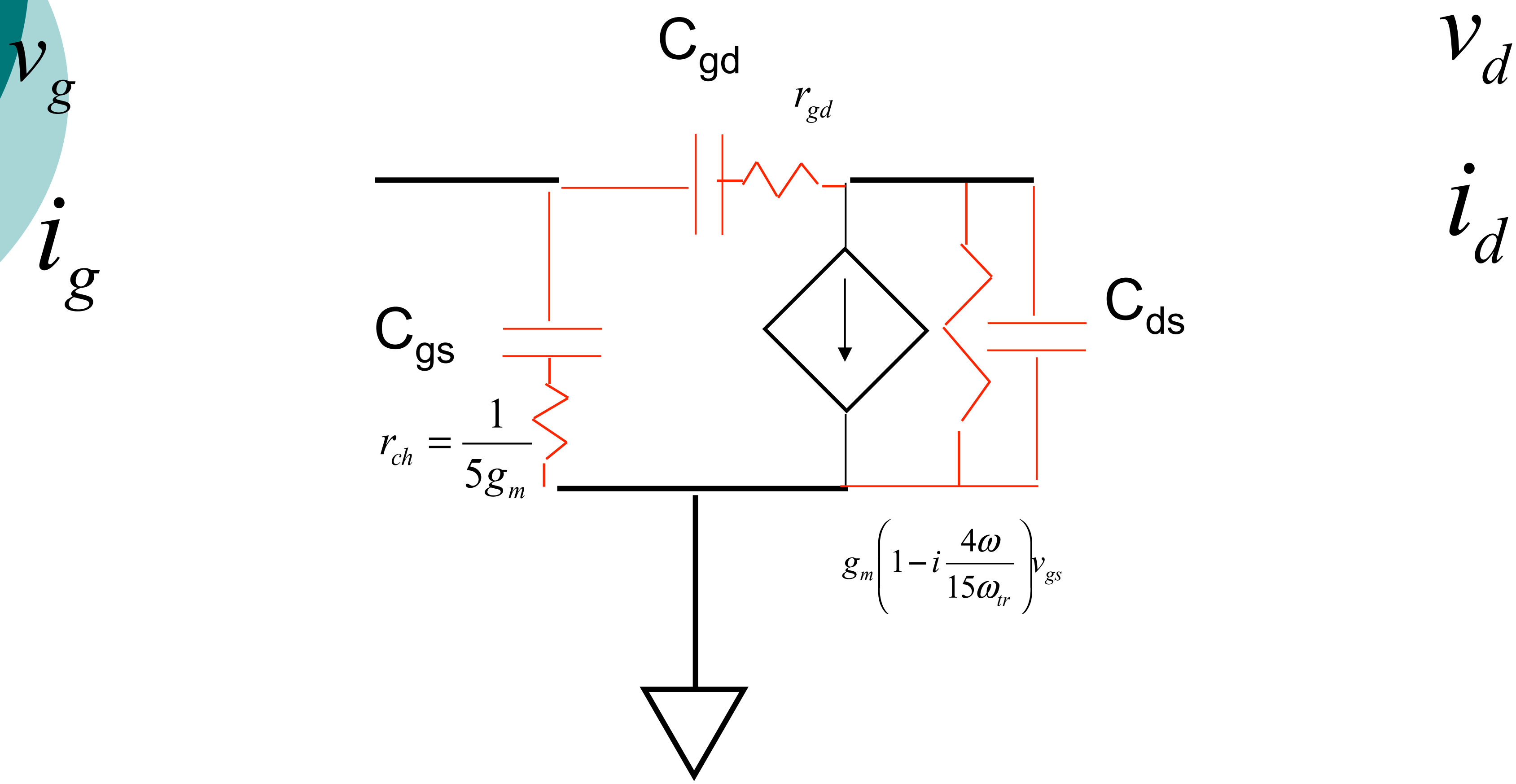
$$i_S = I_S + i_s e^{i\omega t}$$

Hybrid p model:



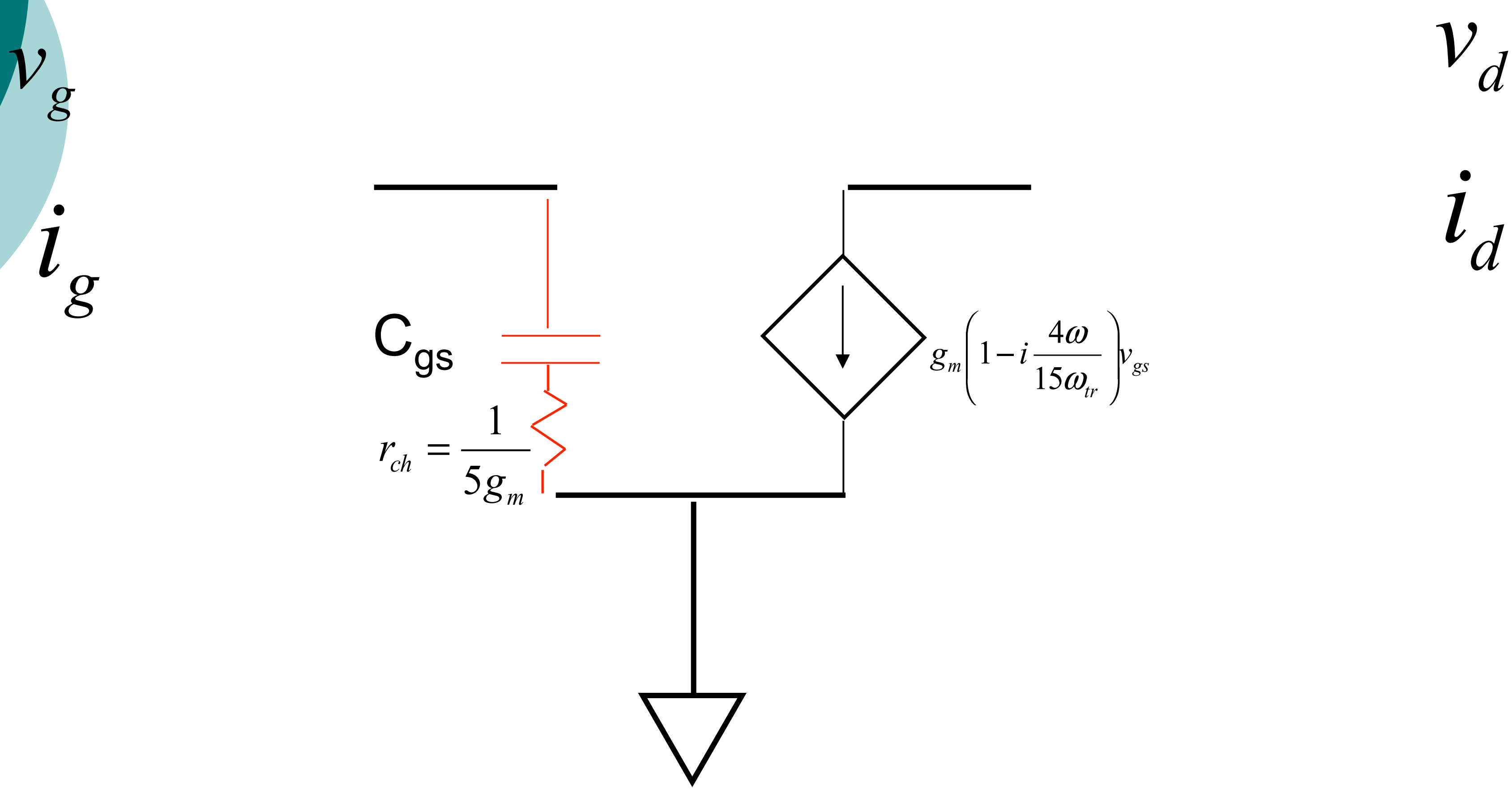
Non-quasi static model:

Hybrid p model:



Non-quasi static model in saturation:

Hybrid p model:



Parasitics: Gate resistance:

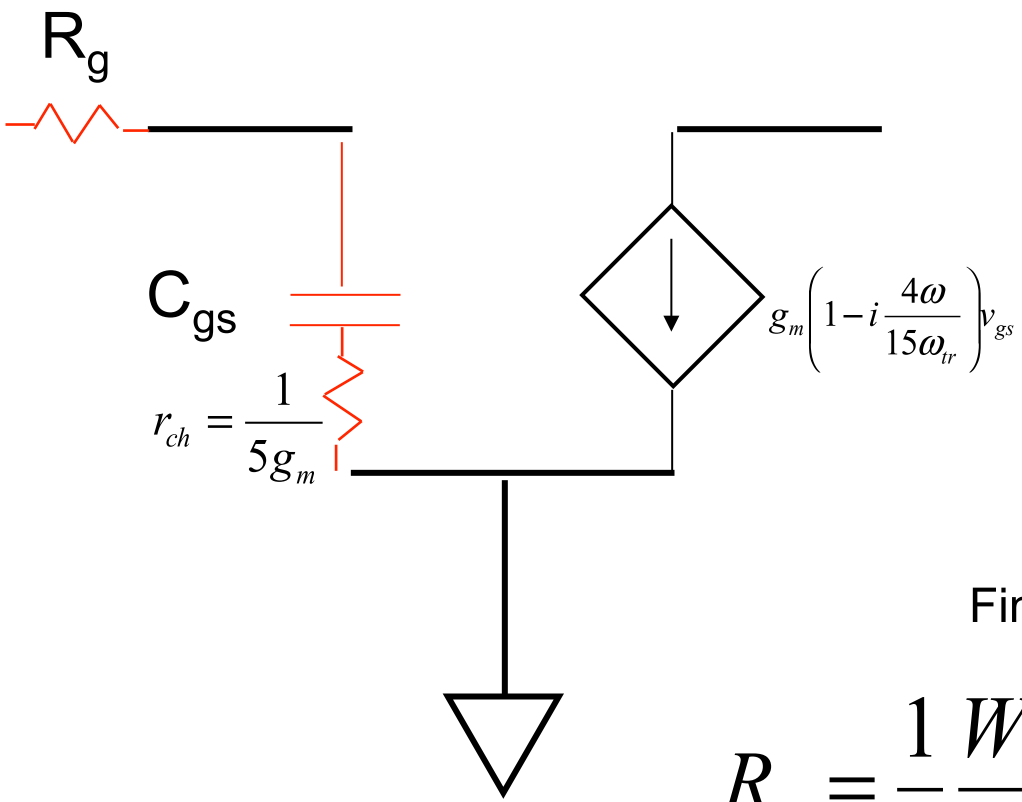
$$R_g = \frac{1}{3}\frac{W}{L} R_{square}$$

v_g

i_g

v_d

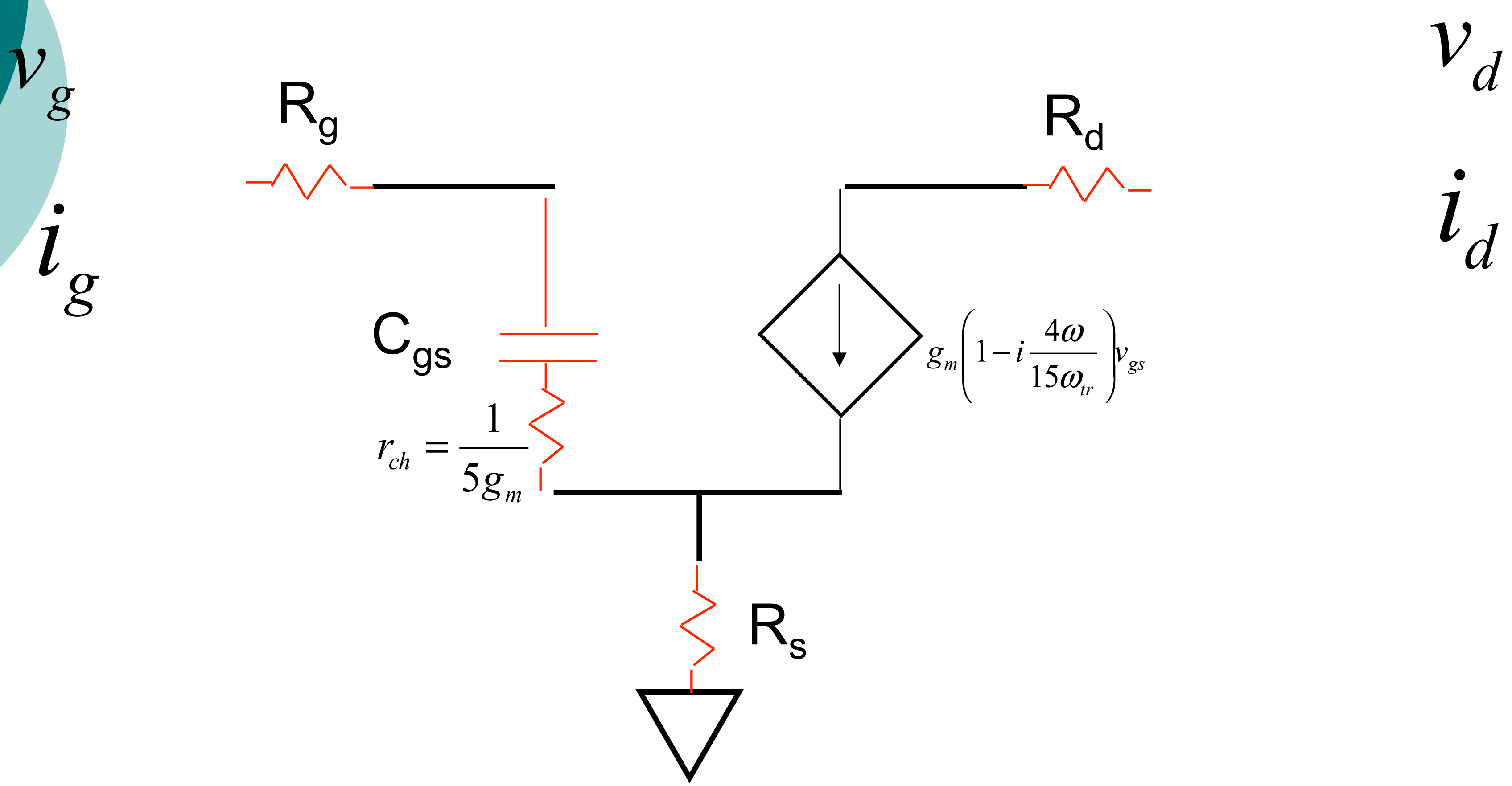
i_d



Fingers:

$$R_g = \frac{1}{3}\frac{W}{L} R_{square} \frac{1}{N}$$

Parasitics: Source/Draina resistance:

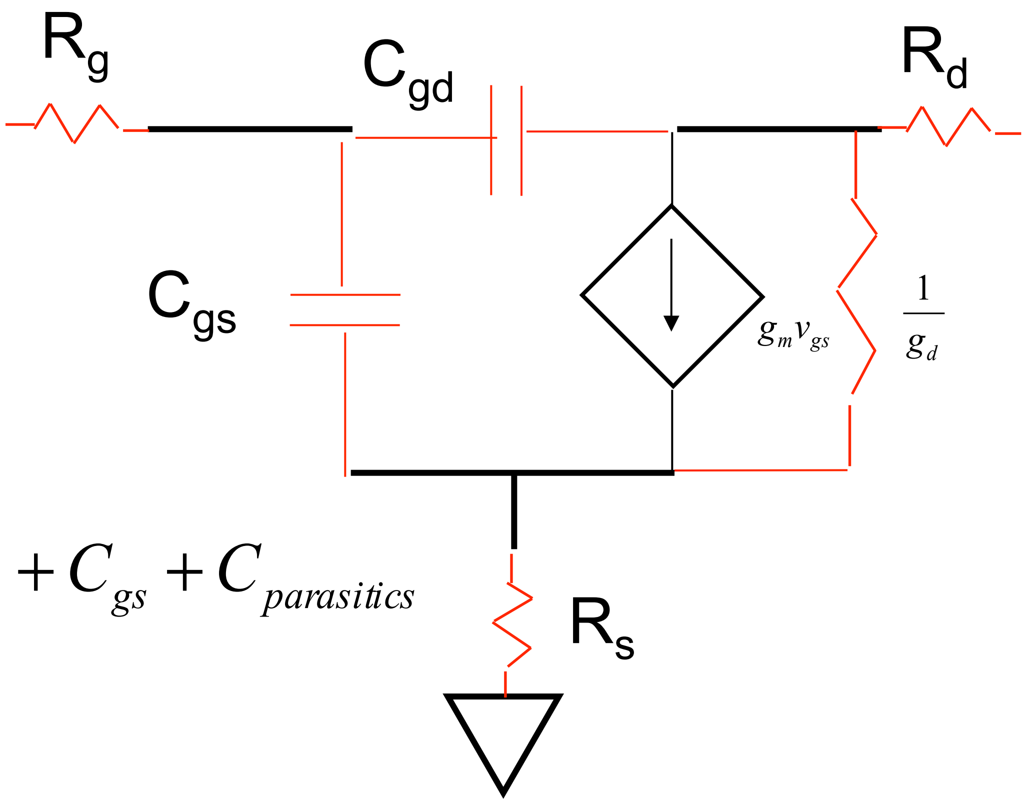


f_T

$$\frac{1}{2\pi f_T} = \frac{C_{gg,t}}{g_m} + \frac{C_{gg,t}}{g_m}(R_S + R_D)g_d + (R_S + R_D)C_{gd,t}$$

v_g

i_g



v_d

i_d

$$C_{gg,t} \equiv C_{gd} + C_{gs} + C_{parasitics}$$

f_{MAX}

$$f_{MAX} = \sqrt{\frac{f_T}{8\pi R_G C_{gd,t}\left[1+\left(\frac{2\pi f_T}{C_{gd,t}}\right)\Psi\right]}}$$

$$\Psi \equiv (R_S + R_D)\frac{C_{gg,t}{}^2 g_d^2}{g_m^2} + (R_S + R_D)\frac{C_{gd,t}C_{gg,t}g_d}{g_m} + \frac{C_{gg,t}{}^2 g_d}{g_m^2}$$

f_{max} helped by fingers.
f_T not helped by fingers.
fMax sometimes larger, sometimes smaller than f_T.

RF CMOS

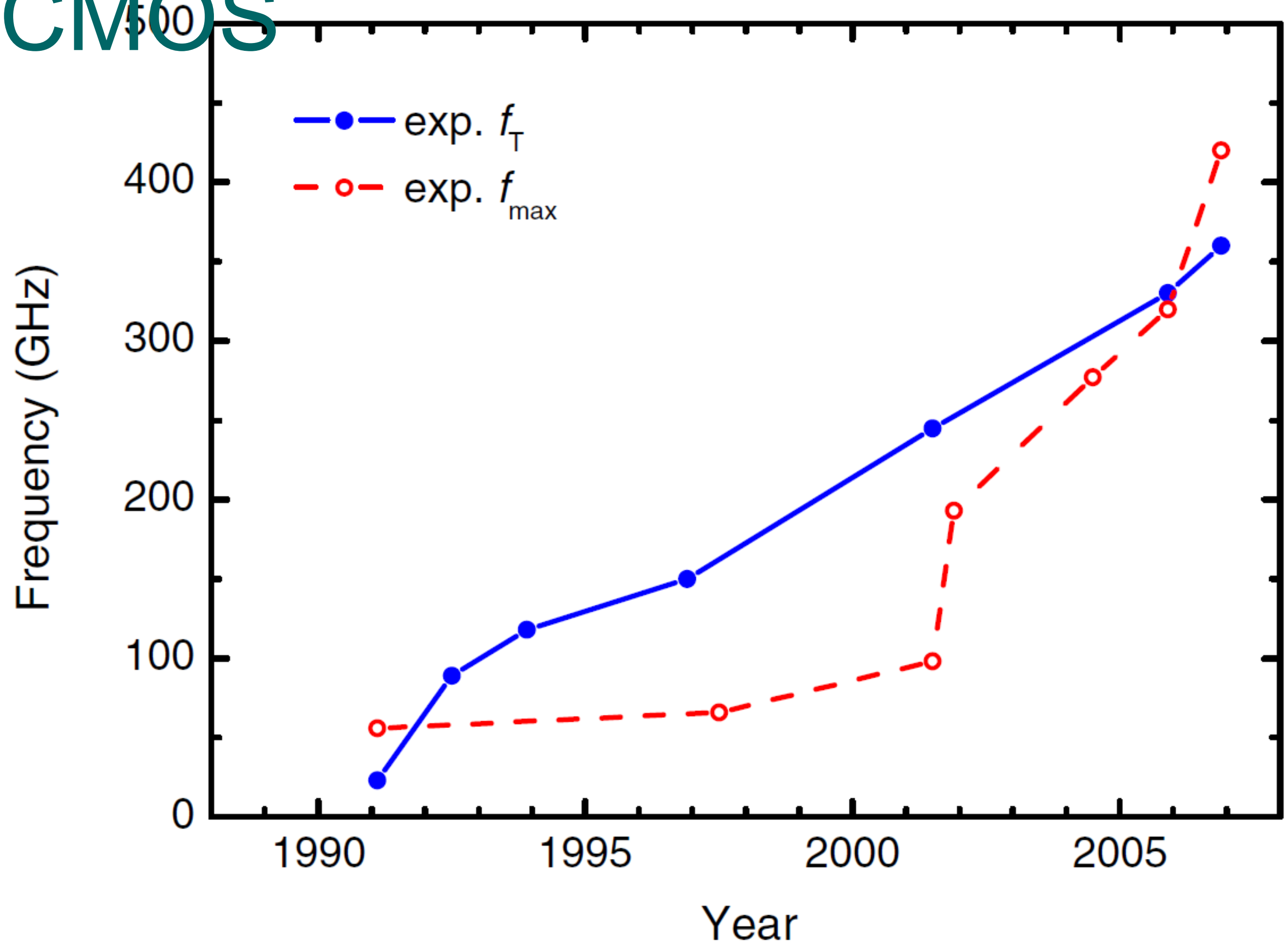
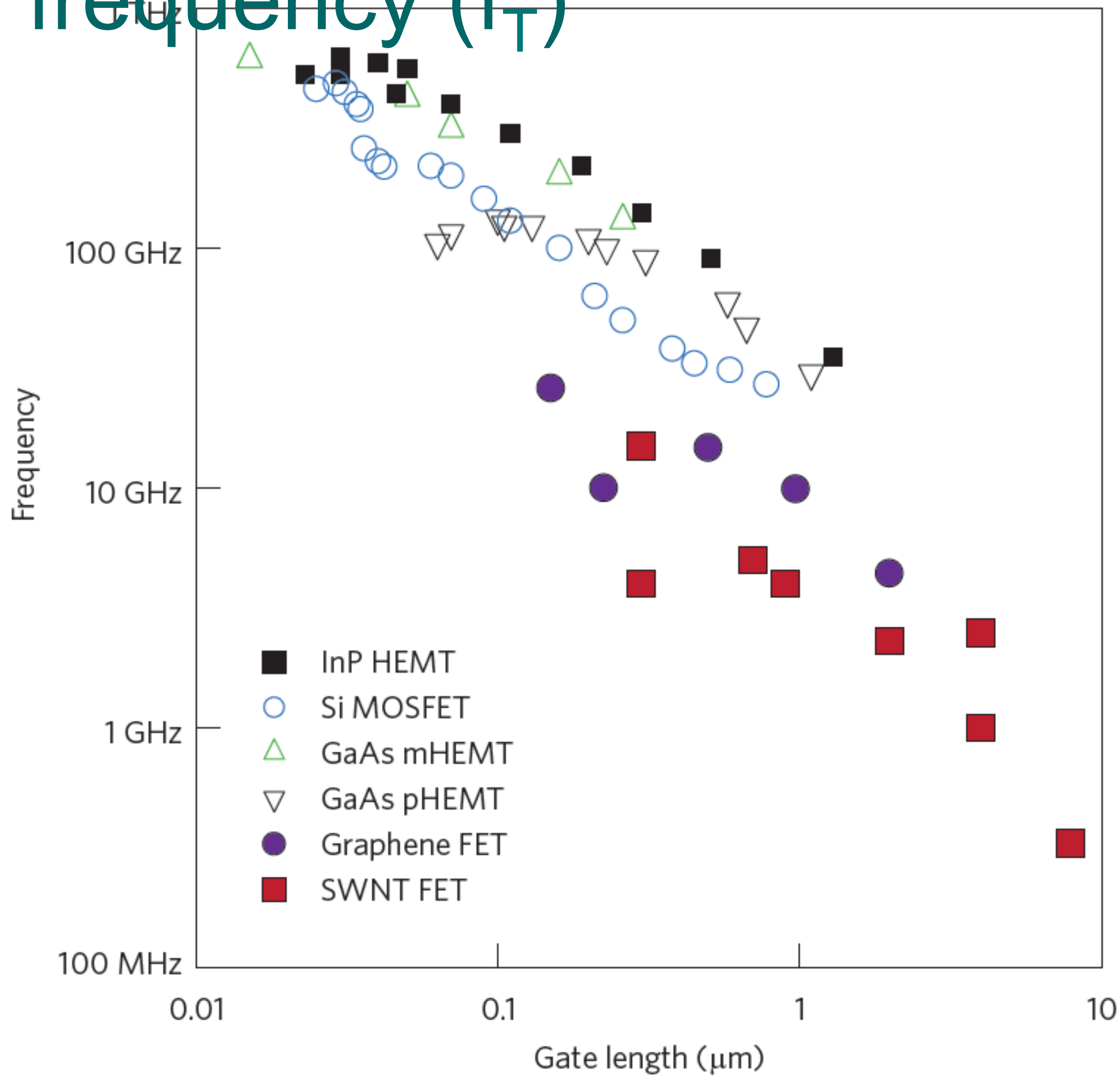


Fig. 1. Evolution of the record cutoff frequency f_T and the record maximum frequency of oscillation f_{max} of RF Si MOSFETs versus time.

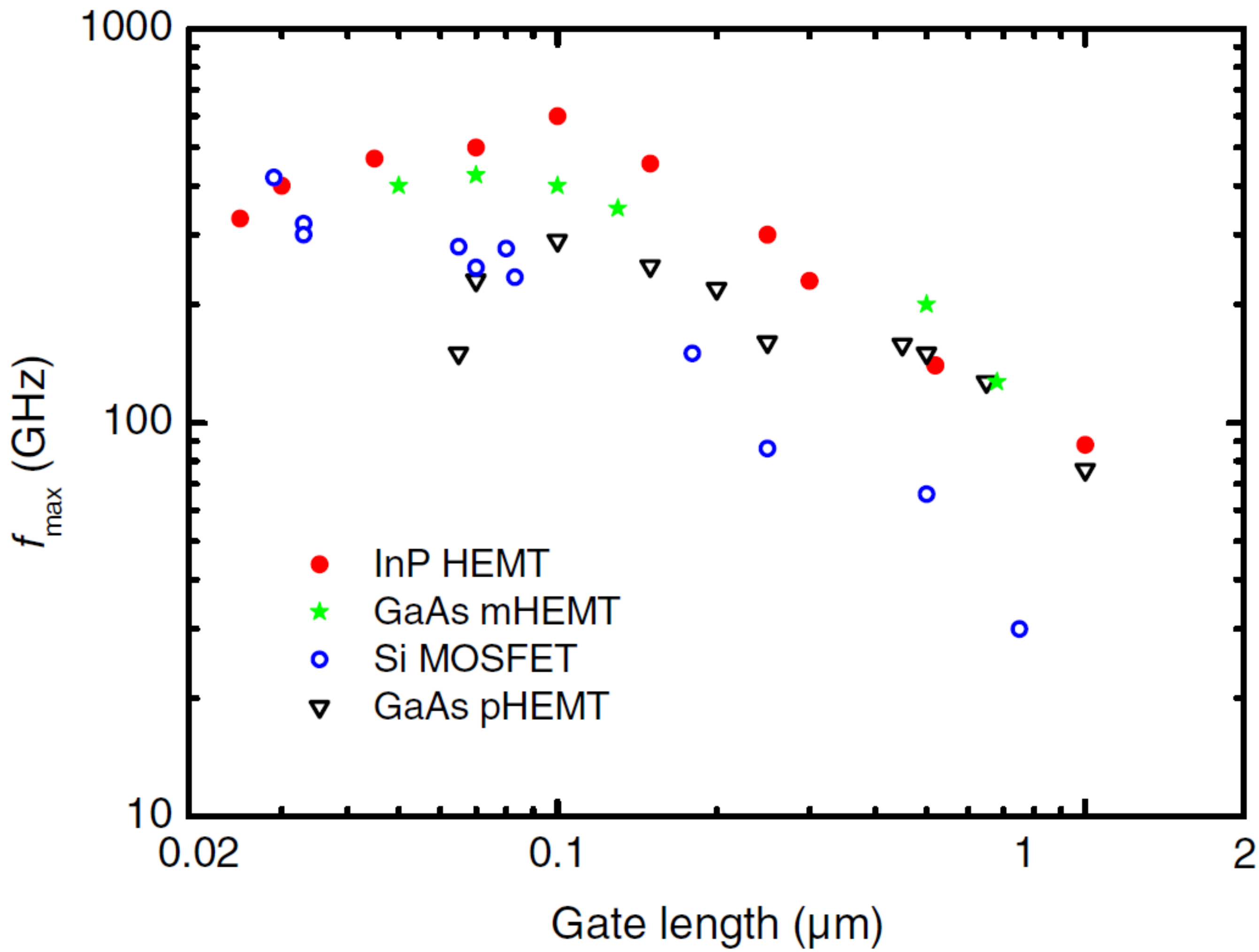
F. Schwierz and J. J. Liou, "RF Transistors: Recent Developments and Roadmap toward Terahertz Applications", *Solid-State Electronics,* ***51, 1079-1091, (2007).***

Cutoff frequency (f_T)



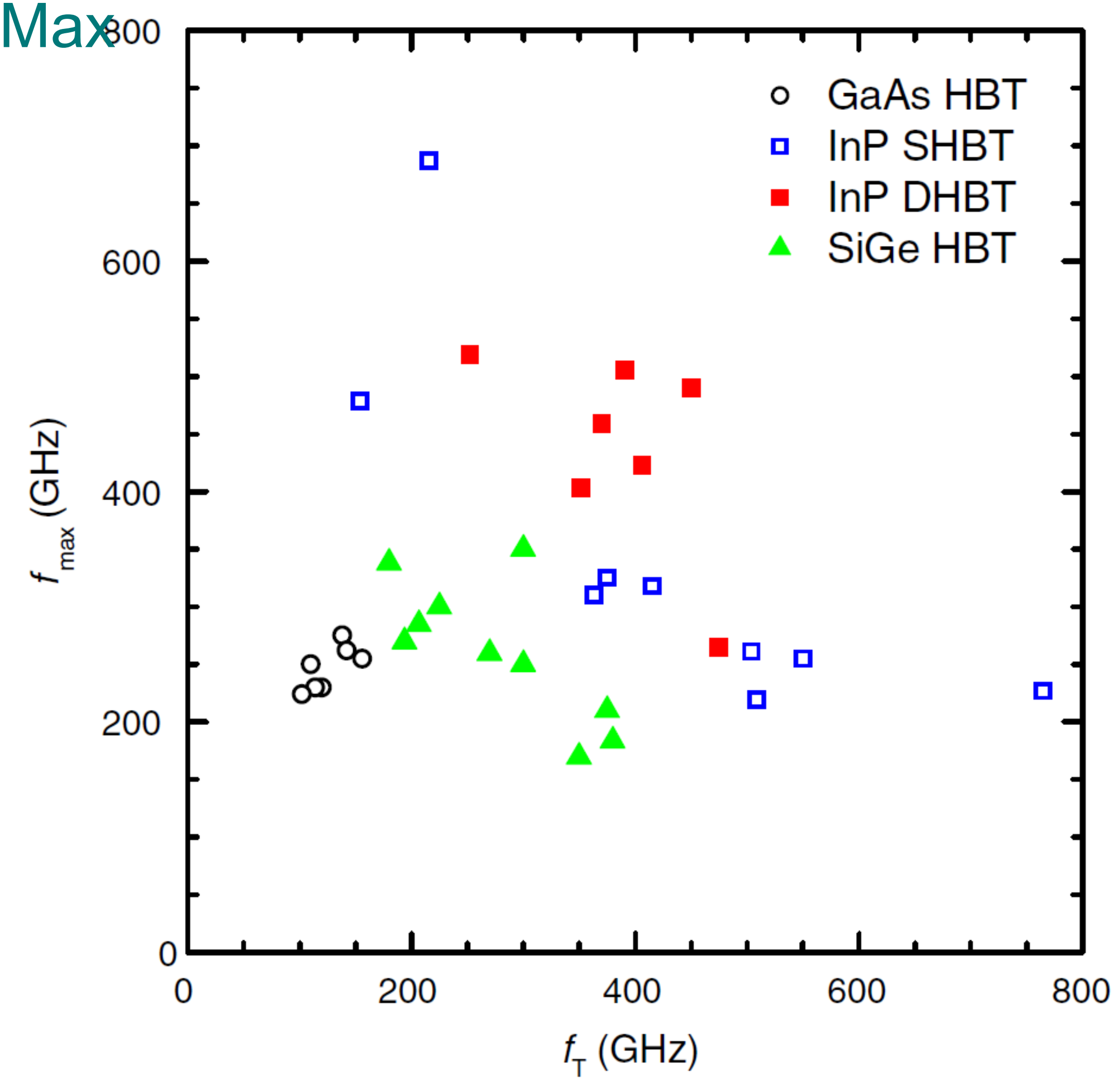
C. Rutherglen, D. Jain and P. Burke, "Nanotube Electronics for Radiofrequency Applications", *Nature Nanotechnology*, ***4*, 811-819, (2009).**

III-V f_{Max}



F. Schwierz and J. J. Liou, "RF Transistors: Recent Developments and Roadmap toward Terahertz Applications", *Solid-State Electronics*, ***51, 1079-1091, (2007).***

f_T vs f_{Max}



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