2 dimensions

$$N_k dk = ?$$

Volume of circular shell

$=2\pi k dk/4$

4 is for upper right quadrant

Number of states in area=
area x States/area

States/area = $1 / (\pi/L)^2$:

$$N_k dk = \left(2\pi k dk / 4\right) \cdot \left(\frac{1}{(\pi / L)^2} \right) \cdot 2 = L^2 \frac{k dk}{\pi}$$

$$\rho_k dk \equiv \frac{N_k dk}{\text{area}} = \frac{k dk}{\pi}$$

2 dimensions

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{kdk}{\pi}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{m}{\pi\hbar^2}dE}$$

1) $E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n_x^2 + n_y^2 + n_z^2$

$\Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 \left[(1^2+1^2+1^2) - (2^2+1^2+1^2)\right]$

$= \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 3$

$\hbar = 6.6\times10^{-34}\ J\text{-}s \qquad k = 1.38\times10^{-23}\ \frac{J}{K}$

$= 4.1\times10^{-15}\ eV\text{-}s$

$m = 9.1\times10^{-31}\ kg$

$\hbar = 10^{-34}\ J\text{-}s$

$= 6.5\times10^{-16}\ eV\text{-}s$

(1m) $\Delta E = \frac{(10^{-34}\ J\text{-}s)^2}{2\ 9.1\times10^{-31}\ kg}\left(\frac{\pi}{1m}\right)^2 3 =$

$= 1.6\times10^{-37}\ J = 10^{-18}\ eV$

(1μm) $\Delta E = 10^{-6}\ eV = 1\mu eV$

(1nm) $\Delta E = 10^{-0}\ eV = 1\ eV$

(1Å) $\Delta E =$ [redacted] $10\ eV$

$kT = 26\ meV$ @ $T = 300K$

4) $kT \sim \frac{e^2}{C}$

$\Rightarrow C \sim \frac{e^2}{kT} \simeq 1.6\,10^{-19}\,F$

$\sim 10^{-18}\,F$

5) $C = \frac{\epsilon A}{d} = \frac{5.53 \times 10^{-2}\,F/m \cdot 10 \times 10^{-6}\,m^2}{10 \times 10^{-9}\,m}$

$= 5.9 \times 10^{-17}\,F$

$\frac{e^2}{C} = \frac{1.6 \times 10^{-19}}{5.9 \times 10^{-17}}\,V = 1.8\,meV$

$kT = 26\,meV$

Need $14\times \downarrow C \Rightarrow T = 3.7\times \downarrow$ K, w

We are getting close...