EECS 277C Nanotechnology HW #2

1. Find the relationship between the Fermi energy and the average energy of electrons in a box.
2. Same for the average wavelength.
3. Find the Fermi wavelength of electrons in a typical metal, e.g, Cu.

1) $\langle E \rangle = \frac{1}{N} \int_0^{\infty} F(E)\, E\, N(E)\, dE$

F(E) is appr. step function.

$\cong \frac{1}{N} \int_0^{E_F} E\, N(E)\, dE$

$N(E) = \frac{L^3}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$

$\langle E \rangle = \frac{1}{N} \frac{L^3}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{E_F} E^{3/2}\, dE$

$= \frac{1}{N} \frac{L^3}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{2}{5} E_F^{5/2}$ (**)

Recall $E_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{L^3}\right)^{2/3}$ (*)

Combining (*), (**) yields

$\langle E \rangle = \frac{3}{5} E_F$

2) $\langle \lambda \rangle = \frac{1}{N} \int_0^\infty \lambda \, F(E) N(E) \, dE$

$$\cong \frac{1}{N} \int_0^{E_F} \lambda \, N(E) \, dE \qquad E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{\lambda}\right)^2$$

$$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, 2\pi \int_0^{E_F} \frac{1}{\sqrt{E}} N(E)\, dE \qquad \lambda = \sqrt{\frac{\hbar^2}{2mE}}\, 2\pi$$

N(E) from before

$$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, 2\pi \, \frac{L^3}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{E_F} \frac{1}{\sqrt{E}} \sqrt{E}\, dE$$

$$= \frac{1}{N}\sqrt{\frac{\hbar^2}{2m}}\, \cancel{2\pi}\, \frac{L^3}{\cancel{2}\pi^{\cancel{2}}}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F$$

$$= \frac{L^3}{N}\frac{1}{\pi}\left(\frac{2m}{\hbar^2}\right) E_F$$

$$= \frac{\frac{1}{\cancel{\pi}} \frac{2m}{\hbar^2} E_F}{E_F^{3/2} \frac{1}{3\pi^{\cancel{2}}} \left(\frac{2m}{\hbar^2}\right)^{3/2}} = 3\pi E_F^{-1/2}\left(\frac{2m}{\hbar^2}\right)^{-1/2} = \langle \lambda \rangle$$

If we define λ_F as $\sqrt{\frac{2mE_F}{\hbar^2}}$ we get

$$\langle \lambda \rangle = \lambda_F \times \frac{3}{2}$$

3) $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{L^3}\right)^{2/3}$ (*)

For copper, electron density = 1/atom

Using atom/m³ for copper, we find

$$n \equiv \frac{N}{L^3} = 8.5\times10^{28}\ \frac{\text{electrons}}{m^3}$$

Using (*) $\Rightarrow E_F \cong 7\,eV$