EECS277C Winter 2015 **Midterm #1** Name: Solutions

1/29/2015 3:30 to 4:50 pm
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Professor Peter Burke
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ID no.:
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1	2	3	4	5	6	Total
/20	20	/10	/10	/20	/20	/100

Helpful constants for you:

$c = 3\ 10^{8}$ m/s
$e = 1.6\ 10^{-19}$ coulombs
$h = 6.63\ 10^{-34}$ J-s
$m = 9.1\ 10^{-31}$ kg
$k_B = 1.38\ 10^{-23}$ J/K
$h/e^2 = 25$ kΩ

1. [20 pts.] Calculate the density of states in a 1 dimensional world.

1 dimension

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{2}{\pi} dk$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{2}{\pi}\sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}} = \frac{1}{\hbar\pi}\sqrt{\frac{2m}{E}} dE}$$

$= \frac{\sqrt{m}}{\sqrt{E}} \frac{1}{\hbar} \frac{\sqrt{2}}{\pi}$

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2. [20 pts.] Calculate average energy in terms of the Fermi energy of an electron in a 2d world.

In 2D, DOS constant. We don't need to have it memorized, just that it is constant. Call it K.

By definition of E_F, $N = \int_0^{E_F} K\,dE = K E_F$.

$$\langle E \rangle = \frac{E_{TOTAL}}{N} = \frac{\sum E_i}{N} = \frac{\int_0^{E_F} E K\,dE}{N}$$

$$= \frac{1}{2} E_F^2 K \frac{1}{N} = \boxed{\frac{1}{2} E_F = \langle E \rangle}$$

3. [10 pts.] Approximately what is the Fermi energy for electrons in metals such as gold, aluminum, copper, etc?

$\sim$ 1-10 eV.

Specifically (not required on test):

Au 5.51 eV

Al 11.63 eV

Cu 8.12 eV

4. [10 pts.] Approximately what is the Fermi wavelength for electrons in metals such as gold, aluminum, copper, etc?

$\sim$ Angstroms

Specifically (not required for test)

Al 3.6 Å

Cu 4.6 Å

Au 5.2 Å

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5. [20 pts.] Consider a 1 nm x 1 nm x 1 nm metal nanoparticle. Find the energy level spacing between states at the Fermi energy. Hint: Calculate the DOS. Recall that the energy level spacing at the Fermi energy is not the same as the spacing between the lowest 2 energy levels.

$N(E) \equiv \frac{\# \text{ states}}{\text{energy}} \Rightarrow \text{spacing between states} = \left.\frac{1}{N(E)}\right|_{E_F}$

$\rho = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \equiv \frac{1}{L^3} N(E)$

$\Rightarrow \delta = \left.\frac{1}{N(E)}\right|_{E_F} = \left.\frac{1}{L^3 \rho(E)}\right|_{E_F} = \frac{1}{L^3} 2\pi^2 \left(\frac{\hbar^2}{2m}\right)^{3/2} \frac{1}{\sqrt{E_F}}$

Take $E_F = 1eV = 1.6 \times 10^{-19} J$

$\Rightarrow \delta = \frac{1}{(10^{-9} m)^3} 2\pi^2 \left[\left[\frac{6.6 \times 10^{-34}}{2\pi}\right]^2 \frac{1}{2 \cdot 9.1\, 10^{-31} Kg}\right]^{3/2} \left[1.6 \times 10^{-19} J\right]^{-\frac{1}{2}}$

$= 10^{+9+9+9}\, 10^{-34-34-34}\, 10^{\underset{(46.5)}{31 \times \frac{3}{2}}}\, 10^{\underset{(9.5)}{+19 \times \frac{1}{2}}}\, 2^{1-3-\frac{3}{2}}\, \pi^{2-3}\, 9.1^{-3/2}\, 1.6^{-1/2}\, J$

$= 10^{-19}\, 2^{-3.5}\, \pi^{-1}\, 9.1^{-3/2}\, 1.6^{-1/2} J = 10^{-19} (0.088)(0.318)(0.036)(0.79) J$

$= 800\, 10^{-6}\, 10^{-19} J = 80 \times 10^{-24} J = 0.5\, meV$

6. [20 pts.] Consider two hypothetical metal nanowires of diameter 1 nm each, in an X configuration separated by a tunnel barrier of thickness 10 Angstroms. At room temperature, would you expect to see Coulomb blockade behavior in the current flowing from one wire to the other? Why or why not?

$C = \frac{\epsilon A}{d} = \frac{8.85 \times 10^{-12} \frac{F}{m}\ 10\ 10^{-9} m\ 10^{-9} m}{10\ 10^{-10} m} = 8.85 \times 10^{-20} F$

$\frac{e^2}{C} \approx 1.8 eV$

Maybe

We need

1. $\underset{1.8eV}{\frac{e^2}{C}} > \underset{26meV}{kT}$

2. $R_T \gg R_q$?

3. $Z(\omega) > R_T$?

−5 pts if missing these