EECS277C Winter 2015 **Midterm #1** Name: Solutions

2/26/2015 3:30 to 4:50 pm ID no.:

Professor Peter Burke

1	2	3	4	Total
/25	25	/25	/25	/100

Helpful constants for you:

$c = 3\ 10^8$ m/s
$e = 1.6\ 10^{-19}$ coulombs
$h = 6.63\ 10^{-34}$ J-s
$m = 9.1\ 10^{-31}$ kg
$k_B = 1.38\ 10^{-23}$ J/K
$h/e^2 = 25$ kW

1. [25 pts.] Resistance quantization

2) A) [16 pts.] Which of the following systems shows resistance quantization?

Quantized resistance?	Yes	No
Ballistic, wide, long wire	X	
Ballistic, narrow, long wire	X	
Ballistic, wide, short wire	X	
Ballistic narrow, short wire	X	
Diffusive wide long wire		X
Diffusive narrow long wire		X
Diffusive wide short wire		X
Diffusive narrow short wire		X

B) [9 pts.] For those systems that do show resistance quantization, what is the resistance if the wire is wide enough to support one mode? (Be sure to take into account the fact that electrons have spin.) Express your answer in ohms!

$\frac{h}{2e^2} \approx 12\ k\Omega$

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6) Dimensionality

a) In the demonstration tunnel junction in class we had two Al electrodes separated by a thin oxide tunnel barrier. The Al electrodes were about 1 cm x 1 mm x 100 nm. Are the electrodes 1d,2d, or 3d in the quantum mechanical sense?

$\boxed{3d}$

b) How does the Fermi energy depend on particle density n in 3d systems? (Linear, quadratic, square root, exponential, independent, etc?) How does the density of states depend on energy in 3d systems?

$n = \frac{N}{L^3} = \int_0^{E_F} D(E)\,dE \sim E_F^{3/2}$

$\Rightarrow \boxed{E_F \propto n^{2/3}}$

$\boxed{D(E) \propto \sqrt{E}}$

c) Same for 2d systems

$n = \frac{N}{L^2} = \int_0^{E_F} D(E)\,dE \sim E_F$

$\boxed{E_F \propto n}$

$\boxed{D(E) \sim \text{constant}}$

d) Same for 1d systems

$\boxed{E_F \propto n^2}$

$\boxed{D(E) \sim \frac{1}{\sqrt{E}}}$

$n = \frac{N}{L} = \int_0^{E_F} D(E)\,dE \sim E_F^{+1/2}$

3) Quantum dots

Calculate the energy levels in a quantum dot (in eV as a function of quantum numbers n_x n_y n_z) in the form of a cube, 5 nm on a side. Assume zero potential energy in the quantum dot, and an infinitely high potential bounding the dot. Assume $m^*=0.045m_e$ in the dot material. Is this a 0d system at room temperature? Why?

$$E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$

To be 0D need $\Delta E < kT$

$$\Delta E \simeq \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$$

$m = 9.1 \times 10^{-31}$ kg $\times$ 0.045

$L = 5$ nm

$\Rightarrow \Delta E =$ ~~2×10^{-18} J = 13 eV~~

5×10^{-20} J $\simeq 0.3$ eV

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$kT = 26$ meV $= 4 \times 10^{-21}$ J @ 300K

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$\Rightarrow$ YES 0D

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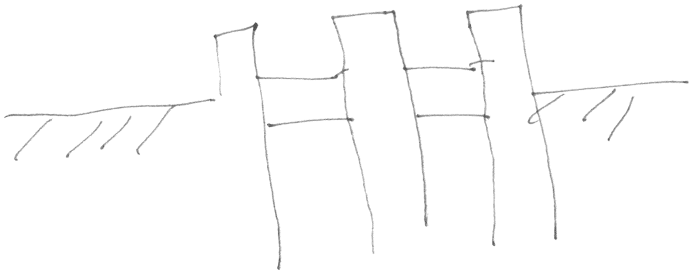
4) Coulomb blockade.

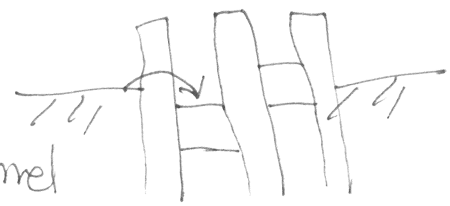
Consider the two-island circuit with two gates. Draw the band diagram for small source-drain voltage at V_{g1}=, V_{g2}=0. Is current flowing under this condition? Next, draw the band diagram at several different times and the associated gate voltages (time dependent) in order to allow current to flow from source to drain at small source-drain bias voltage. How much current flows and how does this depend on the waveform of the gate voltages and the source drain bias voltage?



No current flows. 5

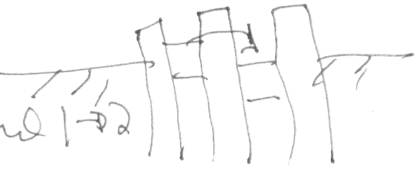
To get current to flow, we need to do things in this order

1) Increase V_{g1}, one electron will tunnel onto island!



2) Reset V_{g1} to 0

3) Increase V_{g2} electron will tunnel island 1→2



4) Reset V_{g2} to 0 electron will tunnel of island 2 to drain.

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$I = Fe$ F = frequency of clock

Independent of S-D voltage... 10