EECS/CSE 70A Spring 2016 **Final Exam**

June 7th, 2016, 10:30 am to 12:30 pm

Professor Peter Burke

Name: Peter the Anteater

ID no.:

Q1	Q2	Q3	Q4	Q5	Q6	Total
/20	/15	/15	/20	/15	/15	/100

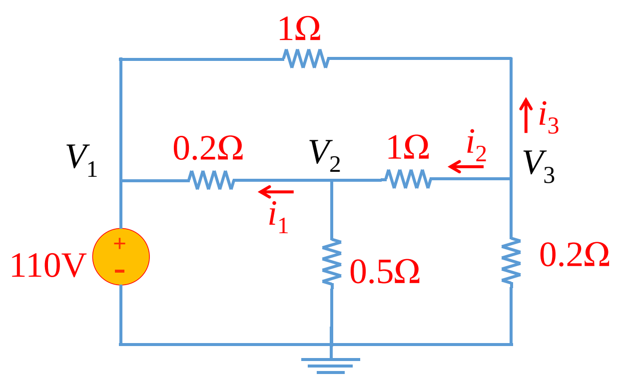
EECS / CSE 70A Final Exam
SOLUTION KEY

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Print your name on all pages.

Write your solutions in clear steps with concise explanations.

PROBLEM 1: (20 points)



Use nodal analysis and find all node voltages and the currents i_1, i_2, i_3.

Set by the voltage source: $V_1 = 110\text{V}$

KCL at node 2: $\frac{V_2 - V_1}{0.2\Omega} + \frac{V_2 - 0\text{V}}{0.5\Omega} + \frac{V_2 - V_3}{1\Omega} = 0$

$5V_2 - 5V_1 + 2V_2 + V_2 - V_3 = 0$

$8V_2 - V_3 = 5V_1 = 550$

KCL at node 3: $\frac{V_3 - V_1}{1\Omega} + \frac{V_3 - V_2}{1\Omega} + \frac{V_3 - 0\text{V}}{0.2\Omega} = 0$

$V_3 - V_1 + V_3 - V_2 + 5V_3 = 0$

$-V_2 + 7V_3 = V_1 = 110$

$8V_2 - V_3 = 5V_1 = 550$
$-V_2 + 7V_3 = V_1 = 110$

$8V_2 - V_3 = 5V_1 = 550$
$8(-V_2 + 7V_3) = 8V_1 = 880$

$7(8V_2 - V_3) = 35V_1 = 3850$
$-V_2 + 7V_3 = V_1 = 110$

$8V_2 - V_3 = 5V_1 = 550$
$-8V_2 + 56V_3 = 8V_1 = 880$

$56V_2 - 7V_3 = 35V_1 = 3850$
$-V_2 + 7V_3 = V_1 = 110$

$55V_3 = 13V_1 = 1430$

$55V_2 = 36V_1 = 3960$

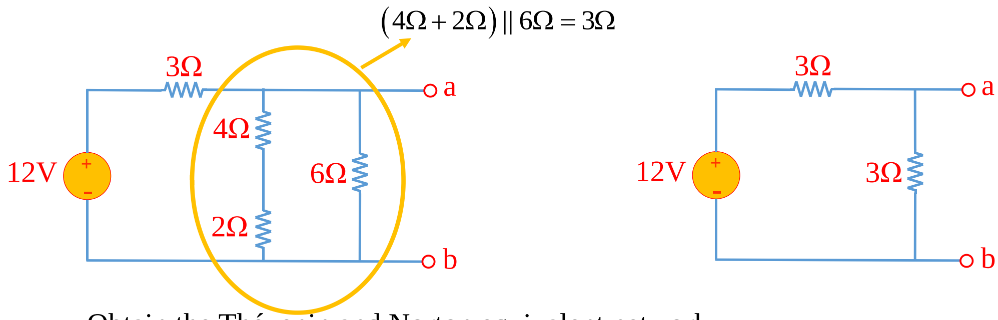
$V_3 = \frac{13V_1}{55} = \frac{1430}{55} = 26\text{V}$ $V_2 = \frac{36V_1}{55} = \frac{3960}{55} = 72\text{V}$

$i_1 = \frac{V_2 - V_1}{0.2\Omega} = \frac{72 - 110}{0.2} = -190\text{A}$

$i_2 = \frac{V_3 - V_2}{1\Omega} = 26 - 72 = -46\text{A}$

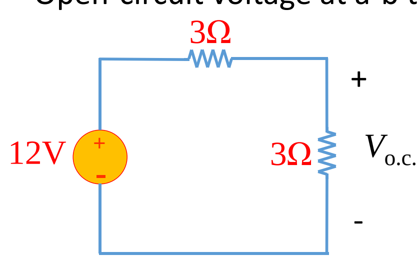
$i_2 = \frac{V_3 - V_1}{1\Omega} = 26 - 110 = -84\text{A}$

PROBLEM 2: (15 points)



Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b.

Open-circuit voltage at a-b terminals:

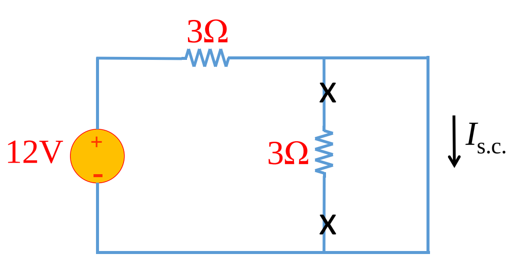


By voltage division:

$$V_{Th} = V_{o.c.} = 3\Omega \frac{12V}{3\Omega + 3\Omega}$$

$$V_{Th} = 6V$$

Short-circuit current through a-b terminals:

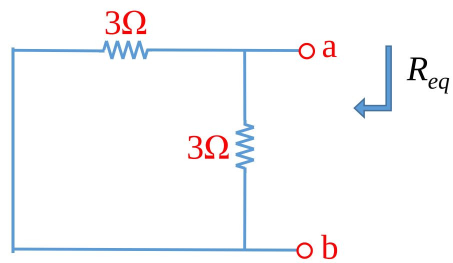


By Ohm's Law:

$$I_{No} = I_{s.c.} = \frac{12V}{3\Omega}$$

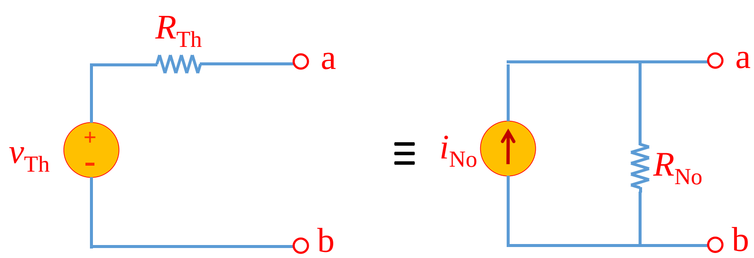
$$I_{No} = 4A$$

Equivalent resistance found by killing independent sources:



$$R_{eq} = 3\Omega \,||\, 3\Omega$$

$$R_{\text{Th}} = R_{\text{No}} = R_{eq} = \frac{3}{2}\Omega$$

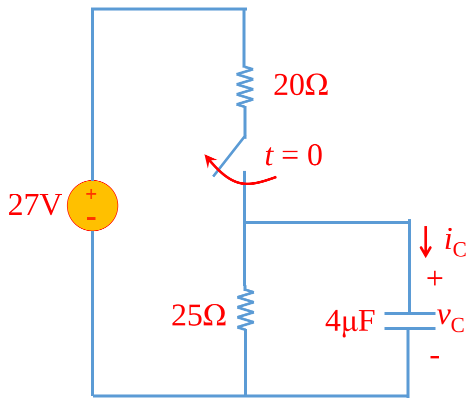


$$v_{\text{Th}} = 6\text{V}$$

$$i_{\text{No}} = 4\text{A}$$

$$R_{\text{No}} = R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{No}}} = \frac{3}{2}\Omega$$

PROBLEM 3: (15 points)



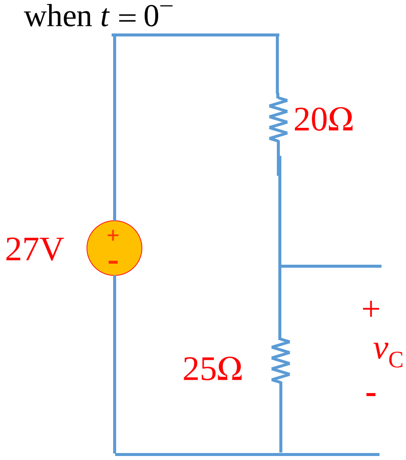
(a) Find the voltage across the capacitor $v_C(t)$ for $t > 0$.

(b) Find the current through the capacitor $i_C(t)$ for $t > 0$.

(a)

Initial state:

when $t = 0^-$



$$i_C = C\frac{dv_C}{dt} \quad \text{at } t = 0^- \quad \frac{dv_C}{dt} = 0$$

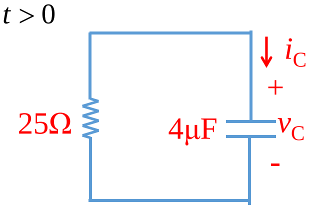
$$i_C\left(0^-\right) = 0\text{A}$$

$$v_C\left(0^-\right) = 25\Omega\frac{27\text{V}}{20\Omega + 25\Omega} = 15\text{V}$$

The voltage through the capacitor is continuous $v_C\left(0^+\right) = v_C\left(0^-\right)$

Transient response:

when $t > 0$



Stored energy in the capacitor discharges through dissipation at the resistor. The voltage decays exponentially with the time constant

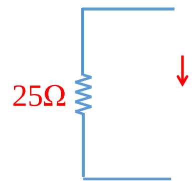
$$\tau = RC = (25\Omega)\left(4\cdot 10^{-6}\text{F}\right) = 10^{-4}\text{s} = 0.1\text{ms}$$

Steady-state response:

as $t \to \infty$

$$\frac{dv_C}{dt} = 0 \quad i_C(\infty) = 0\text{A}$$

The current through the resistor tends to zero, therefore, with respect to Ohm's law, the voltage also tends to zero



$$i_C(\infty) = 0\text{A}$$

$$v_C(\infty) = 0\text{V}$$

Plugging $v_C\left(0^+\right)$, $v_C(\infty)$, and τ in the following formula

Complete response = Steady-state response + Transient response

$$v_C(t) = v_C(\infty) + \left[v_C\left(0^+\right) - v_C(\infty)\right]e^{-\frac{t}{\tau}}, \ t > 0$$

$$v_C(t) = 15e^{-10000t} \text{ V}, \ t > 0$$

(b) $$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left\{v_C(\infty) + \left[v_C\left(0^+\right) - v_C(\infty)\right]e^{-\frac{t}{\tau}}\right\}$$

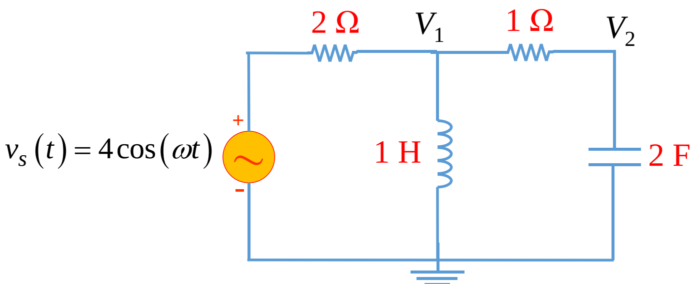
$$= C\left(-\frac{1}{\tau}\right)\left[i_L\left(0^+\right) - i_L(\infty)\right]e^{-\frac{t}{\tau}}, \ t > 0$$

If you use the following relation, you have to show the steps $C\left(-\frac{1}{\tau}\right) = -C\frac{1}{RC} = -\frac{1}{R}$

$$i_C(t) = \left(4\cdot 10^{-6}\text{F}\right)\left(-\frac{1}{10^{-4}\text{s}}\right)\left(15e^{-10000t}\text{V}\right)$$

$$= -\frac{15}{25}e^{-10000t} = -\frac{3}{5}e^{-10000t} \text{ A}, \ t > 0$$

PROBLEM 4: (20 points)

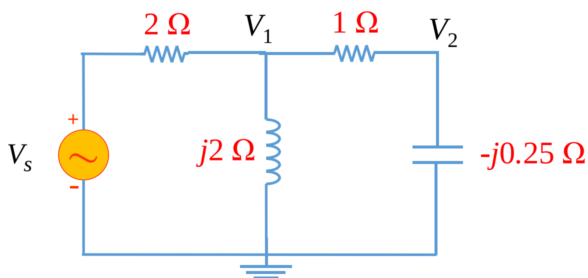


Use nodal analysis to find the voltage phasors V_1 and V_2 and their corresponding time-domain expressions $v_1(t)$ and $v_2(t)$ at $\omega = 2$ rad/s.

Source voltage phasor $V_s = 4e^{j0} = 4$

$$j\omega L = j(2\text{rad/s})\cdot 1\text{H} = j2\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(2\text{rad/s})\cdot 2\text{F}} = -j\frac{1}{4}\Omega$$



KCL at node 1:

$$\frac{V_1 - V_s}{2\Omega} + \frac{V_1 - 0\text{V}}{j2\Omega} + \frac{V_1 - V_2}{1\Omega} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{j2} + 1\right) - V_2 = 2$$

$$V_1\left(\frac{3}{2} - j\frac{1}{2}\right) - V_2 = 2$$

KCL at node 2:

$$\frac{V_2 - V_1}{1\Omega} + \frac{V_2 - 0\text{V}}{-j\frac{1}{4}\Omega} = 0$$

$$-V_1 + V_2\left(1 + \frac{1}{-j\frac{1}{4}}\right) = 0$$

$$-V_1 + V_2(1 + j4) = 0$$

$$\begin{pmatrix} \left(\frac{3}{2} - j\frac{1}{2}\right) & -1 \\ -1 & (1 + j4) \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Using Cramer's Rule:

$$V_1 = \frac{\begin{vmatrix} 2 & -1 \\ 0 & (1+j4) \end{vmatrix}}{\begin{vmatrix} \left(\frac{3}{2} - j\frac{1}{2}\right) & -1 \\ -1 & (1+j4) \end{vmatrix}} = \frac{2+j8}{\frac{5}{2}+j\frac{11}{2}} = \frac{4+j16}{5+j11}\text{V}$$

$$V_1 = \frac{\sqrt{4^2+16^2}}{\sqrt{5^2+11^2}} e^{j\left(\tan^{-1}4 - \tan^{-1}\frac{11}{5}\right)}$$

or

$$V_1 = \frac{(4+j16)(5-j11)}{(5+j11)(5-j11)} = \frac{20-j44+j80+176}{5^2+11^2} =$$

$$= \frac{196+j36}{5^2+11^2} = \frac{\sqrt{196^2+36^2}}{5^2+11^2} e^{j\tan^{-1}\frac{36}{196}}$$

$$V_2 = \frac{\begin{vmatrix} \left(\frac{3}{2} - j\frac{1}{2}\right) & 2 \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} \left(\frac{3}{2} - j\frac{1}{2}\right) & -1 \\ -1 & (1+j4) \end{vmatrix}} = \frac{2}{\frac{5}{2}+j\frac{11}{2}} = \frac{4}{5+j11}\text{V}$$

$$V_2 = \frac{4}{\sqrt{5^2+11^2}} e^{-j\tan^{-1}\frac{11}{5}}$$

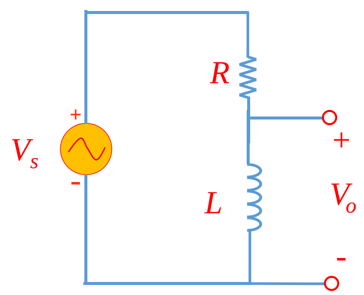
Going back to time domain

$$v_1(t) = \text{Re}\left\{e^{j2t}V_1\right\} = \frac{\sqrt{4^2+16^2}}{\sqrt{5^2+11^2}}\cos\left(2t + \tan^{-1}4 - \tan^{-1}\frac{11}{5}\right)$$

$$= \frac{\sqrt{196^2+36^2}}{5^2+11^2}\cos\left(2t + \tan^{-1}\frac{36}{196}\right)$$

$$v_2(t) = \text{Re}\left\{e^{j2t}V_2\right\} = \frac{4}{\sqrt{5^2+11^2}}\cos\left(2t - \tan^{-1}\frac{11}{5}\right)$$

PROBLEM 5: (15 points)



$$H(\omega) = \frac{V_o}{V_s}$$

Find the transfer function $H(\omega)$ in terms of R and L. Find the limit of $|H(\omega)|$ as $\omega \to 0$ and as $\omega \to \infty$.

$$\lim_{\omega \to 0} |H(\omega)| = \left| \frac{j0L}{R + j0L} \right| = 0$$

$$V_o = V_s \frac{j\omega L}{R + j\omega L}$$

$$H(\omega) = \frac{j\omega L}{R + j\omega L}$$

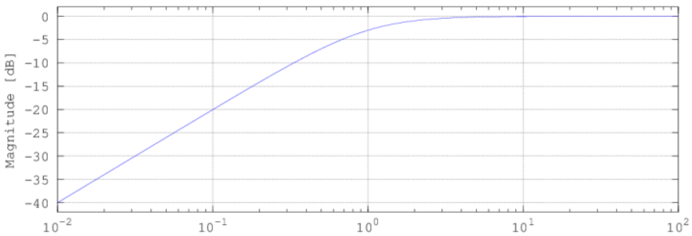
$$\lim_{\omega \to \infty} |H(\omega)| = \lim_{\omega \to \infty} \left| \frac{\frac{1}{j\omega} j\omega L}{\frac{1}{j\omega}(R + j\omega L)} \right| = \lim_{\omega \to \infty} \left| \frac{L}{\frac{R}{j\omega} + L} \right| = \left| \frac{L}{0 + L} \right| = 1$$

PROBLEM 6: (15 points)

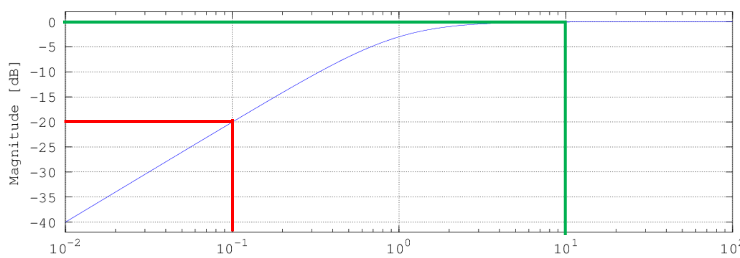
The Bode plots show the magnitude of the transfer function of a circuit. The phase of the transfer function does not change with respect to frequency, it is constant and equal to 0 rad.

The input voltage is $v_{\text{in}}(t) = \cos([2\pi \cdot 0.1\text{Hz}]t) + \cos([2\pi \cdot 10\text{Hz}]t)$

Find the output voltage $v_{\text{out}}(t)$.



Frequency (Hz)



Frequency (Hz)

Via superposition we can find the output voltage at two frequencies separately and superpose them.

$$v_{\text{in}}(t) = v_{\text{in},1}(t) + v_{\text{in},2}(t)$$

$$v_{\text{in},1}(t) = \cos([2\pi \cdot 0.1\text{Hz}]t)$$

$$v_{\text{in},2}(t) = \cos([2\pi \cdot 10\text{Hz}]t)$$

At 0.1 Hz, $v_{\text{in},1}(t) = \cos([2\pi \cdot 0.1\text{Hz}]t) \Leftrightarrow V_{\text{in},1} = 1$

$$V_{\text{out},1} = H(\omega = 2\pi \cdot 0.1\text{Hz})V_{\text{in},1}$$

At 10 Hz, $v_{\text{in},2}(t) = \cos([2\pi \cdot 10\text{Hz}]t) \Leftrightarrow V_{\text{in},2} = 1$

$$V_{\text{out},2} = H(\omega = 2\pi \cdot 10\text{Hz})V_{\text{in},2}$$

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$$H\left(\omega = 2\pi \cdot 0.1\text{Hz}\right) = \left|H\left(\omega = 2\pi \cdot 0.1\text{Hz}\right)\right| e^{j0}$$
$$\left|H\left(\omega = 2\pi \cdot 0.1\text{Hz}\right)\right|_{\text{dB}} = -20\text{dB}$$
$$\left|H\left(\omega = 2\pi \cdot 0.1\text{Hz}\right)\right| = 10^{-20/20} = 0.1$$
$$H\left(\omega = 2\pi \cdot 0.1\text{Hz}\right) = 0.1$$

$$H\left(\omega = 2\pi \cdot 10\text{Hz}\right) = \left|H\left(\omega = 2\pi \cdot 10\text{Hz}\right)\right| e^{j0}$$
$$\left|H\left(\omega = 2\pi \cdot 10\text{Hz}\right)\right|_{\text{dB}} = -20\text{dB}$$
$$\left|H\left(\omega = 2\pi \cdot 10\text{Hz}\right)\right| = 10^{0/20} = 1$$
$$H\left(\omega = 2\pi \cdot 10\text{Hz}\right) = 1$$

Using the transfer function values at the frequencies 0.1Hz and 10Hz

$V_{\text{out},1} = 0.1$ and $V_{\text{out},2} = 1$

$$v_{\text{out},1}(t) = \text{Re}\left\{e^{j2\pi 0.1t} V_{\text{out},1}\right\} = 0.1\cos\left(\left[2\pi \cdot 0.1\text{Hz}\right]t\right) \quad \text{and}$$
$$v_{\text{out},2}(t) = \text{Re}\left\{e^{j2\pi 10t} V_{\text{out},2}\right\} = \cos\left(\left[2\pi \cdot 10\text{Hz}\right]t\right)$$
$$v_{\text{out}}(t) = v_{\text{out},1}(t) + v_{\text{out},2}(t)$$
$$= 0.1\cos\left(\left[2\pi \cdot 0.1\text{Hz}\right]t\right) + \cos\left(\left[2\pi \cdot 10\text{Hz}\right]t\right) \text{ V}$$

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Exam cheat sheet

This will be provided with the exam.

radians :	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	-1
tan	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{1}}$	DNE	0

where $\sqrt{\cdot}$ always denotes the positive square root, and DNE means does not exist.