Q1	Q2	Q3	Q4	Q5	Total
/20	/25	/25	/10	/20	/100

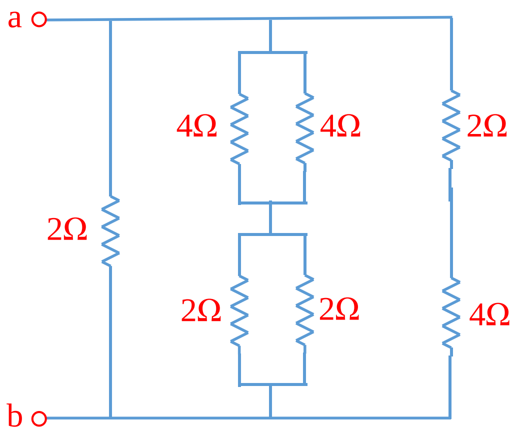
EECS / CSE 70A Midterm Exam #1
SOLUTION KEY

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Print your name on all pages.

PROBLEM 1: (20 points)

(a) Solve for the equivalent resistance, R_{eq}, across terminals a-b.

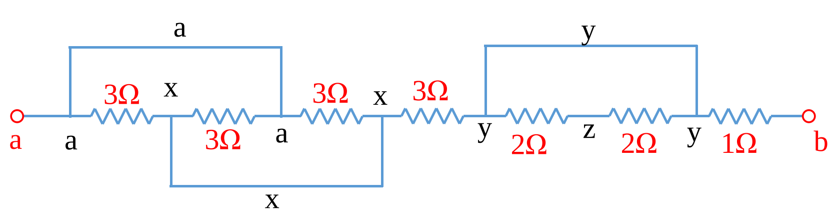


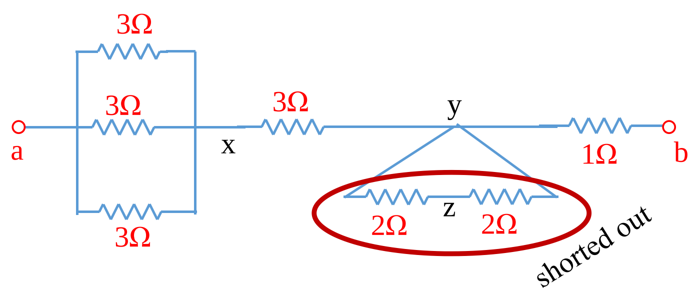
$$R_{eq} = 2\Omega \,||\, \left[(4\Omega \,||\, 4\Omega) + (2\Omega \,||\, 2\Omega) \right] \,||\, (2\Omega + 4\Omega) =$$

$$= 2\Omega \,||\, \left[2\Omega + 1\Omega \right] \,||\, 6\Omega = 2\Omega \,||\, \underbrace{3\Omega \,||\, 6\Omega}_{2\Omega} = 2\Omega \,||\, 2\Omega =$$

$$= 1\Omega$$

(b) Solve for the equivalent resistance, R_{eq}, across terminals a-b.



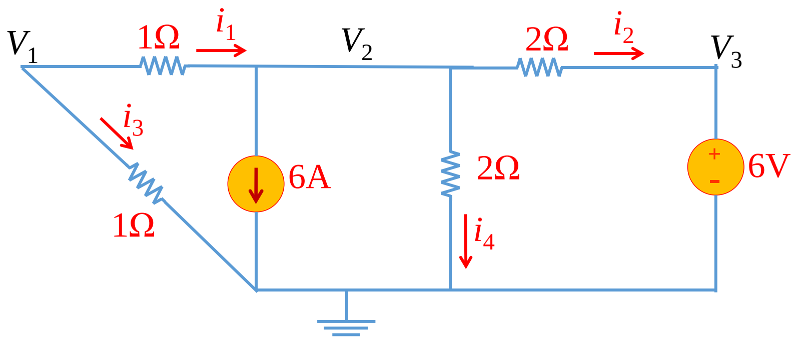


$$R_{eq} = (3\Omega \,||\, 3\Omega \,||\, 3\Omega) + 3\Omega + 1\Omega =$$

$$= 1\Omega + 3\Omega + 1\Omega = 5\Omega$$

PROBLEM 2: (25 points)

Use nodal analysis, and solve for the node voltages and the labeled currents.



Node 1 KCL: $\frac{V_1 - 0\text{V}}{1\Omega} + \frac{V_1 - V_2}{1\Omega} = 0$

Node 2 KCL: $\frac{V_2 - V_1}{1\Omega} + 6\text{A} + \frac{V_2 - 0\text{V}}{2\Omega} + \frac{V_2 - V_3}{2\Omega} = 0$

Node 3 set by voltage source: $V_3 = 6\text{V}$

-Rearrange the equations and substitute V_3.

$$\begin{aligned} 2V_1 - V_2 &= 0 \\ -2V_1 + 4V_2 &= -6 \end{aligned} \quad \text{and we reach} \quad \begin{aligned} V_1 &= -1\text{V} \\ V_2 &= -2\text{V} \end{aligned}$$

Currents:

$$i_1 = \frac{V_1 - V_2}{1\Omega} = \frac{-1-(-2)}{1} = 1\text{A},$$

$$i_2 = \frac{V_2 - V_3}{2\Omega} = \frac{-2-6}{2} = -4\text{A},$$

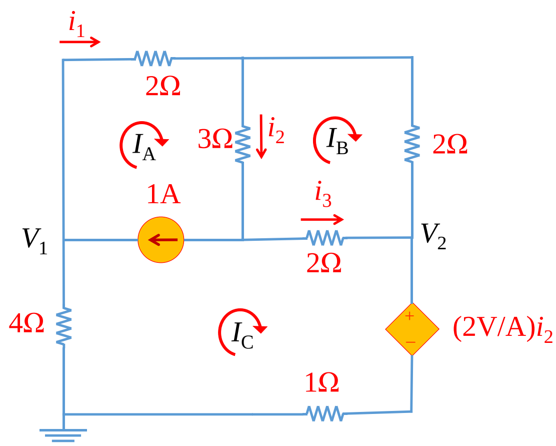
$$i_3 = \frac{V_1}{1\Omega} = \frac{-1}{1} = -1\text{A},$$

$$i_4 = \frac{V_2}{2\Omega} = \frac{-2}{2} = -1\text{A}.$$

$$\boxed{\begin{aligned} V_1 &= -1\text{V} \\ V_2 &= -2\text{V} \\ V_3 &= 6\text{V} \end{aligned} \qquad \begin{aligned} i_1 &= 1\text{A} \\ i_2 &= -4\text{A} \\ i_3 &= -1\text{A} \\ i_4 &= -1\text{A} \end{aligned}}$$

PROBLEM 3: (25 points)

Use mesh analysis, and solve for the mesh currents and the labeled voltages.



Due to the 1A current source, KVL in meshes A and C cannot be written in terms of mesh currents. We need to use a supermesh.

Supermesh A&C, KVL:

$$2\Omega\cdot I_A + 3\Omega\cdot(I_A - I_B) + 2\Omega\cdot(I_C - I_B) + (2\text{V/A})\cdot i_2 + 1\Omega\cdot I_C + 4\Omega\cdot I_C = 0 \quad \text{where } i_2 = I_A - I_B$$

Current source on common branch of meshes A and C: $I_A - I_C = 1\text{A}$

Mesh B, KVL: $2\Omega\cdot I_B + 2\Omega\cdot(I_B - I_C) + 3\Omega\cdot(I_B - I_A) = 0$

- Rearrange equations

$$\left.\begin{array}{l} 7I_A - 7I_B + 7I_C = 0 \\ I_A - I_C = 1 \\ -3I_A + 7I_B - 2I_C = 0 \end{array}\right\} \begin{array}{l} I_A - I_B + I_C = 0 \\ I_A - I_C = 1 \\ -3I_A + 7I_B - 2I_C = 0 \end{array} \quad \text{substitute } I_C = I_A - 1 \text{ in the other two equations}$$

We reach : $\left.\begin{array}{l} I_A - I_B + I_A - 1 = 0 \\ -3I_A + 7I_B - 2(I_A - 1) = 0 \end{array}\right\} \left.\begin{array}{l} 2I_A - I_B = 1 \\ -5I_A + 7I_B = -2 \end{array}\right\} \left.\begin{array}{l} 14I_A - 7I_B = 7 \\ -5I_A + 7I_B = -2 \end{array}\right\} I_A = \frac{5}{9}\text{A},\ I_B = \frac{1}{9}\text{A},\ I_C = -\frac{4}{9}\text{A}$

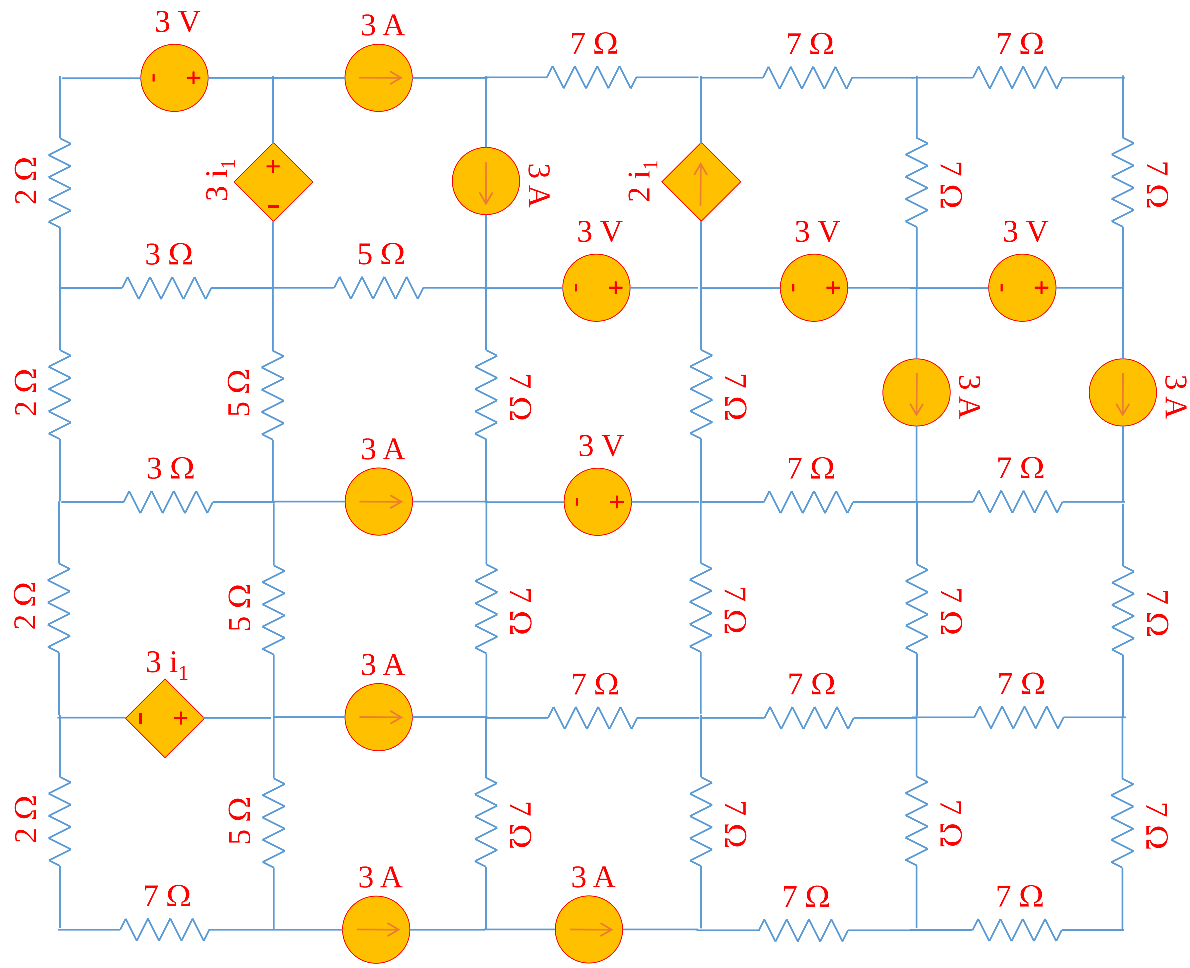
The currents $i_1 = I_A = \frac{5}{9}\text{A},\ \ i_2 = I_A - I_B = \frac{4}{9}\text{A},\ \ i_3 = I_C - I_B = -\frac{5}{9}\text{A}.$

The voltages $V_1 = -I_C\cdot 4\Omega = \frac{16}{9}\text{V},\ V_2 = (2\text{V/A})\cdot i_2 + 1\Omega\cdot I_C = \frac{4}{9}\text{V}$

	$i_1 = \frac{5}{9}\text{A}$
$I_A = \frac{5}{9}\text{A}$	$i_2 = \frac{4}{9}\text{A}$
$I_B = \frac{1}{9}\text{A}$	$i_3 = -\frac{5}{9}\text{A}$
$I_C = -\frac{4}{9}\text{A}$	$V_1 = \frac{16}{9}\text{V}$
	$V_2 = \frac{4}{9}\text{V}$

PROBLEM 4: (10 points)

Write down the number of meshes and nodes.

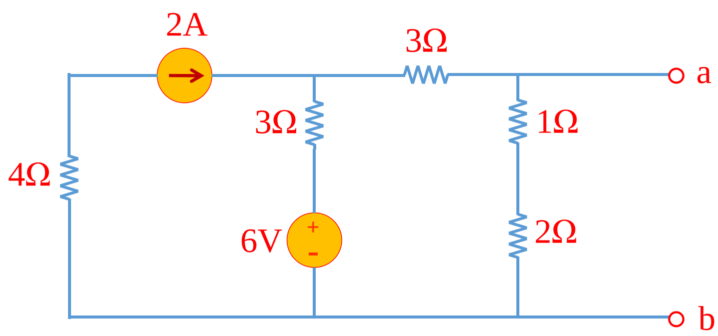


Number of nodes: 30

Number of meshes: 20

PROBLEM 5: (20 points)

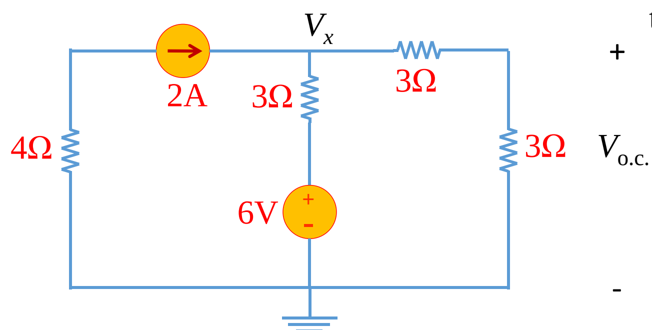
Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (Please draw the equivalent network representations and annotate the source voltages or currents and resistances)



One needs to solve two of the following, since the third can be found by the previous two parameters:

$V_{\text{Th}}, I_{\text{No}}, R_{\text{Th}} = R_{\text{No}}$ where $V_{\text{Th}} = I_{\text{No}} R_{\text{Th}}$

Open-circuit voltage at a-b terminals:



Let us write KCL at the top node in terms of V_x (the voltage thereof):

$$-2\text{A} + \frac{V_x - 6\text{V}}{3\Omega} + \frac{V_x}{6\Omega} = 0$$

$$3V_x = 12 + 12 = 24\text{V}$$

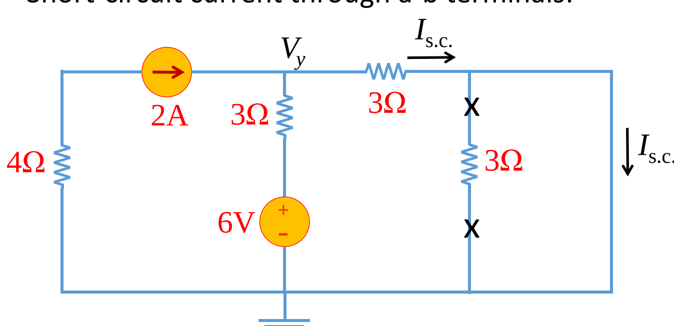
$$V_x = 8\text{V}$$

By voltage division:

$$V_{\text{Th}} = V_{\text{o.c.}} = 3\Omega \frac{V_x}{6\Omega}$$

$$V_{\text{Th}} = 4\text{V}$$

Short-circuit current through a-b terminals:



Let us write KCL at the top node in terms of V_y (the voltage thereof):

$$-2\text{A} + \frac{V_y - 6\text{V}}{3\Omega} + \frac{V_y}{3\Omega} = 0$$

$$2V_y = 6 + 6 = 12\text{V}$$

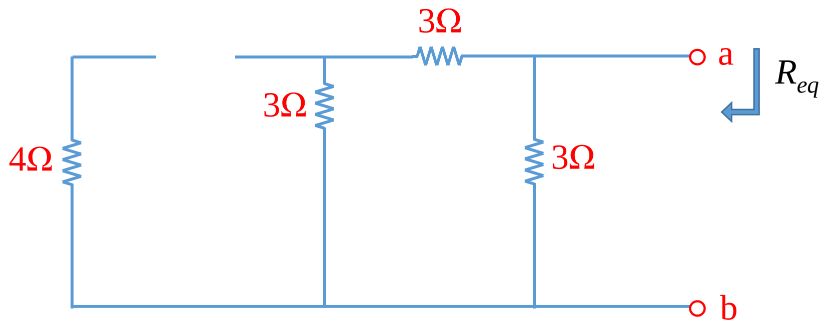
$$V_y = 6\text{V}$$

By Ohm's Law:

$$I_{\text{No}} = I_{\text{s.c.}} = \frac{V_y}{3\Omega}$$

$$I_{\text{No}} = 2\text{A}$$

Equivalent resistance found by killing independent sources:

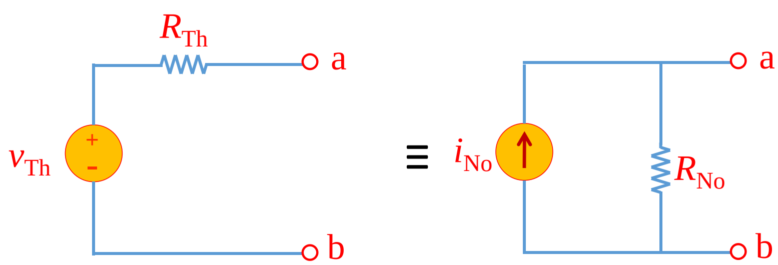


$$R_{eq} = 3\Omega \,||\, (3\Omega + 3\Omega)$$

$$R_{\mathrm{Th}} = R_{\mathrm{No}} = R_{eq} = 3\Omega \,||\, 6\Omega$$

$$R_{\mathrm{Th}} = 2\Omega$$

Finally we have



$$v_{\mathrm{Th}} = 4\mathrm{V}$$

$$i_{\mathrm{No}} = 2\mathrm{A}$$

$$R_{\mathrm{No}} = R_{\mathrm{Th}} = \frac{v_{\mathrm{Th}}}{i_{\mathrm{No}}} = 2\Omega$$