Q1	Q2	Q3	Q4	Total
/25	/25	/25	/25	/100

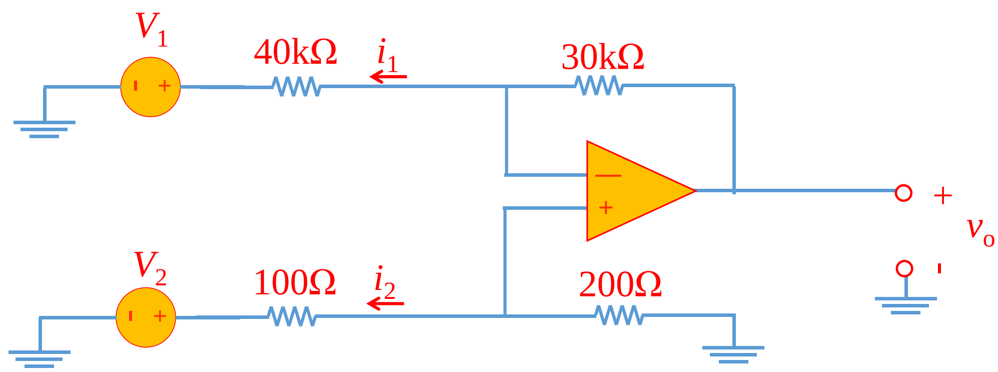
EECS / CSE 70A Midterm Exam #2
SOLUTION KEY

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Print your name on all pages.

Write your solutions in clear steps with concise explanations.

PROBLEM 1: (25 points)



(a) Find the output voltage v_o in terms of V_1 and V_2.

(b) Calculate v_o , i_1, and i_2 when V_1 = 8V and V_2 = 12V.

Solution:

(a) Ideal op-amp has $V_- = V_+$ and there's no current into or out of the -/+ terminals.

KCL at - terminal of the op-amp $$\frac{V_- - V_1}{40\text{k}\Omega} + \frac{V_- - v_o}{30\text{k}\Omega} = 0 \Rightarrow v_o = \frac{7V_- - 3V_1}{4}$$

KCL at + terminal of the op-amp $$\frac{V_+ - V_2}{100\Omega} + \frac{V_+ - 0\text{V}}{200\Omega} = 0 \Rightarrow V_+ = \frac{2}{3}V_2$$

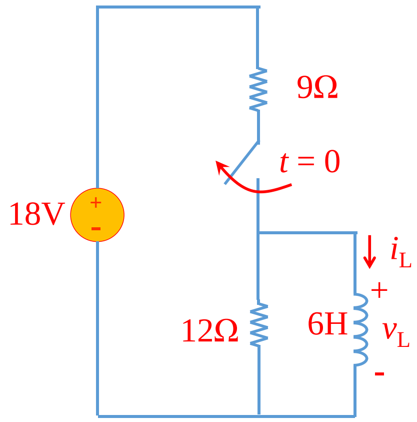
(you can also apply voltage division, since no current flows through + terminal)

Using $V_- = V_+$ $$v_o = \frac{\frac{14}{3}V_2 - 3V_1}{4} = \frac{7}{6}V_2 - \frac{3}{4}V_1$$

(b) Substitute V_1 and V_2 in the equation above: $$v_o = \frac{7}{6}12 - \frac{3}{4}8 = 8\text{V}$$

$$V_- = V_+ = \frac{2}{3}V_2 = 8\text{V}$$ $$i_1 = \frac{V_- - V_1}{40\text{k}\Omega} = 0\text{A} \qquad i_2 = \frac{V_+ - V_2}{100\Omega} = -0.04\text{A}$$

PROBLEM 2: (25 points)



(a) Find the current through the inductor $i_L(t)$ for $t > 0$.

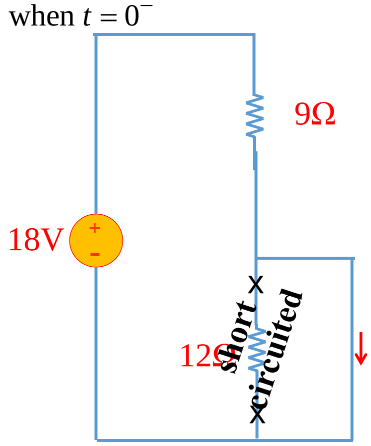
(b) Find the voltage across the inductor $v_L(t)$ for $t > 0$.

Solution:

(a)

Initial state:

when $t = 0^-$



$$v_L = L\frac{di_L}{dt}$$

at $t = 0^-$ $\frac{di_L}{dt} = 0$

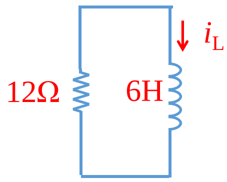
$$v_L(0^-) = 0\text{V}$$

$$i_L(0^-) = \frac{18\text{V}}{9\Omega} = 2\text{A}$$

The current across the inductor is continuous $i_L(0^+) = i_L(0^-)$

Transient response:

when $t > 0$



Stored energy in the inductor discharges through dissipation at the resistor. The current decays exponentially with the time constant

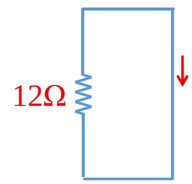
$$\tau = \frac{L}{R} = \frac{6\text{H}}{12\Omega} = 0.5\text{s}$$

Steady-state response:

as $t \to \infty$

$$\frac{di_L}{dt} = 0 \quad v_L(\infty) = 0\text{V}$$

The voltage across the resistor tends to zero, therefore, with respect to Ohm's law, the current also tends to zero



$$v_L(\infty) = 0\text{V}$$

$$i_L(\infty) = 0\text{A}$$

Plugging $i_L(0^+)$, $i_L(\infty)$, and τ in the following formula

Complete response = Steady-state response + Transient response

$$i_L(t) = i_L(\infty) + \left[i_L(0^+) - i_L(\infty)\right]e^{-\frac{t}{\tau}}, \ t > 0$$

$$\boxed{i_L(t) = 2e^{-2t} \text{ A}, \ t > 0}$$

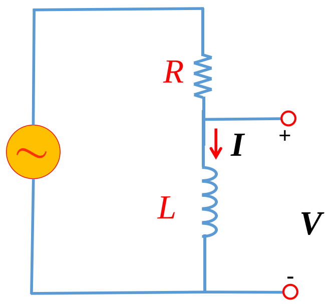
(b)
$$v_L(t) = L\frac{di_L(t)}{dt} = L\frac{d}{dt}\left\{i_L(\infty) + \left[i_L(0^+) - i_L(\infty)\right]e^{-\frac{t}{\tau}}\right\}$$

$$= L\left(-\frac{1}{\tau}\right)\left[i_L(0^+) - i_L(\infty)\right]e^{-\frac{t}{\tau}}, \ t > 0$$

$$\boxed{v_L(t) = (6\text{H})\left(-\frac{1}{0.5\text{s}}\right)\left(2e^{-2t}\text{A}\right) = -24e^{-2t} \text{ V}, \ t > 0}$$

PROBLEM 3: (25 points)

$$v_s(t) = 3\cos\left(\omega t - \frac{\pi}{6}\right)$$



Find the following in terms of L, R, and ω:

(a) the voltage phasor $\boldsymbol{V}$ that corresponds to the voltage across the inductor

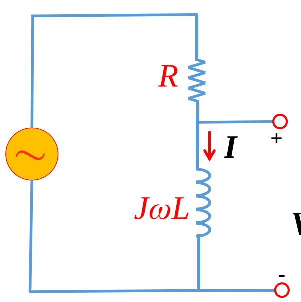
(b) the current phasor $\boldsymbol{I}$ that corresponds to the current through the inductor

Answers in either polar/exponential form or rectangular/Cartesian form will be accepted.

Solution:

(a)

$$V_s = 3e^{-j\frac{\pi}{6}}$$



$$v_s(t) = 3\cos\left(\omega t - \frac{\pi}{6}\right) \Rightarrow V_s = 3e^{-j\frac{\pi}{6}}$$

By voltage division

$$V = \frac{j\omega L}{R + j\omega L} V_s$$

In exponential form, using $\begin{cases} j\omega L = \omega L e^{j\frac{\pi}{2}} \\ R + j\omega L = \sqrt{R^2 + (\omega L)^2}\, e^{j\tan^{-1}\frac{\omega L}{R}} \end{cases}$

$$V = \frac{3\omega L}{\sqrt{R^2 + (\omega L)^2}} e^{j\left(-\frac{\pi}{6} + \frac{\pi}{2} - \tan^{-1}\frac{\omega L}{R}\right)} = \boxed{\frac{3\omega L}{\sqrt{R^2 + (\omega L)^2}} e^{j\left(\frac{\pi}{3} - \tan^{-1}\frac{\omega L}{R}\right)}}$$

In Cartesian form

$$V = \frac{j\omega L}{R + j\omega L} V_s = \frac{j\omega L(R - j\omega L)3\left(\cos\left(\frac{\pi}{6}\right) - j\sin\left(\frac{\pi}{6}\right)\right)}{R^2 + (\omega L)^2}$$

$$= \frac{j\omega L(R - j\omega L)3\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)}{R^2 + (\omega L)^2} = \boxed{\frac{3\omega L}{R^2 + (\omega L)^2}\left[\left(\frac{1}{2}R + \frac{\sqrt{3}}{2}\omega L\right) + j\left(\frac{\sqrt{3}}{2}R - \frac{1}{2}\omega L\right)\right]}$$

(b) $V = (j\omega L) I$

Starting from the exponential form V

$$I = \frac{V}{j\omega L} = \frac{\dfrac{3\omega L}{\sqrt{R^2 + (\omega L)^2}} e^{j\left(\frac{\pi}{3} - \tan^{-1}\frac{\omega L}{R}\right)}}{\omega L e^{j\frac{\pi}{2}}} =$$

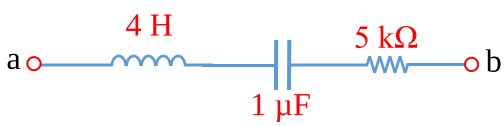
$$= \frac{3}{\sqrt{R^2 + (\omega L)^2}} e^{j\left(-\frac{\pi}{6} - \tan^{-1}\frac{\omega L}{R}\right)}$$

Starting from the Cartesian form V

$$I = \frac{V}{j\omega L} = \frac{\dfrac{3\omega L}{R^2 + (\omega L)^2}\left[\left(\frac{1}{2}R + \frac{\sqrt{3}}{2}\omega L\right) + j\left(\frac{\sqrt{3}}{2}R - \frac{1}{2}\omega L\right)\right]}{j\omega L} =$$

$$= \frac{3}{R^2 + (\omega L)^2}\left[\left(\frac{\sqrt{3}}{2}R - \frac{1}{2}\omega L\right) - j\left(\frac{1}{2}R + \frac{\sqrt{3}}{2}\omega L\right)\right]$$

PROBLEM 4: (25 points)



(a) Find $Z_{\text{eq}}(\omega)$, the equivalent impedance between terminals a-b as a function of the angular frequency ω.

(b) Give $\text{Re}\{Z_{\text{eq}}(\omega)\}$.

(c) Give $\text{Im}\{Z_{\text{eq}}(\omega)\}$.

(d) Find the value of ω at which $\text{Im}\{Z_{\text{eq}}(\omega)\} = 0$.

Solution:

(a)

$$Z_{eq} = j\omega(4\text{H}) + \frac{1}{j\omega\left(10^{-6}\text{F}\right)} + 5000\Omega =$$

$$\frac{1}{j} = \frac{1\times(j)^*}{j\times(j)^*} = \frac{1\times(-j)}{j\times(-j)} = \frac{-j}{1} = -j$$

$$= j4\omega - j\frac{1}{10^{-6}\omega} + 5000 =$$

$$= 5000 + j\left(4\omega - \frac{10^6}{\omega}\right)$$

(b)

$$\text{Re}\left\{Z_{eq}\right\} = \text{Re}\left\{5000 + j\left(4\omega - \frac{10^6}{\omega}\right)\right\} = \boxed{5000\Omega}$$

(c)

$$\text{Im}\left\{Z_{eq}\right\} = \text{Im}\left\{5000 + j\left(4\omega - \frac{10^6}{\omega}\right)\right\} = \boxed{4\omega - \frac{10^6}{\omega} = 4\left(\frac{\omega}{\text{rad}\cdot\text{s}^{-1}}\right) - \frac{10^6}{\left(\frac{\omega}{\text{rad}\cdot\text{s}^{-1}}\right)}\ \Omega}$$

(d)

$$\text{Im}\left\{Z_{eq}\right\} = 0$$

$$4\omega - \frac{10^6}{\omega} = 0$$

$$\omega^2 = \frac{10^6}{4},\quad \omega = \boxed{\sqrt{\frac{10^6}{4}} = \frac{10^3}{2} = 500\text{rad/s}}$$

Exam cheat sheet

This will be provided with the exam.

radians :	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	-1
tan	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{1}}$	DNE	0

where $\sqrt{\cdot}$ always denotes the positive square root, and DNE means does not exist.

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