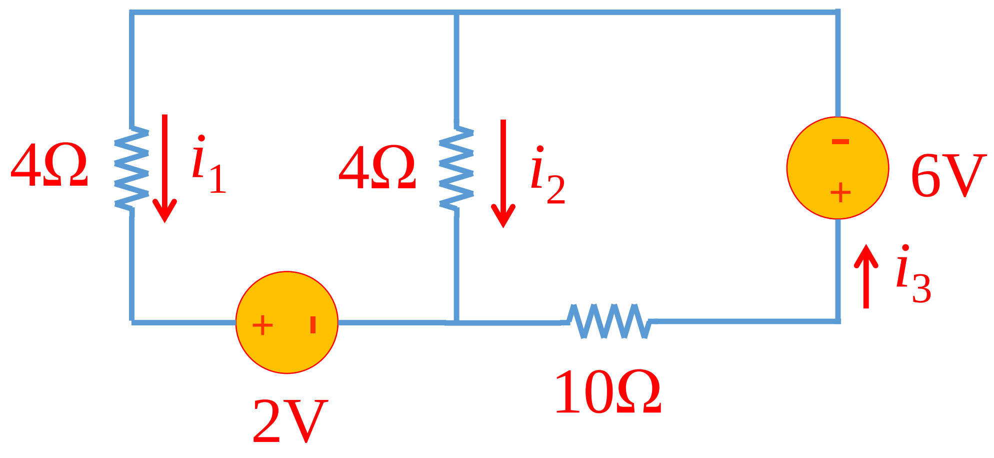
EECS/CSE 70A Network Analysis I

Homework #3

Solution Key

Problem 1: (KCL, KVL, Ohm's Law) Find currents i_1, i_2, i_3. (10pts.)

Problem 1 Solution



KVL in the left loop: $4i_1 + 2 = 4i_2 \quad (1)$

KVL in the right loop: $4i_2 + 10i_3 + 6 = 0 \quad (2)$

KCL in the top node: $i_3 = i_1 + i_2 \quad (3)$

Substitute (3) in (2): $4i_2 + 10(i_1 + i_2) + 6 = 0$

$$i_2 = \frac{-6 - 10i_1}{14} = \frac{-3 - 5i_1}{7} \quad (4)$$

Substitute (4) in (1): $4i_1 + 2 = 4 \times \frac{-3 - 5i_1}{7}$

$$28i_1 + 14 = -12 - 20i_1$$

$$i_1 = -\frac{26}{48}\text{A} = \boxed{-\frac{13}{24}\text{A}}$$

Using (1): $i_2 = i_1 + \frac{1}{2} = -\frac{13}{24} + \frac{1}{2} = \boxed{-\frac{1}{24}\text{A}}$

Using (3): $i_3 = i_1 + i_2 = -\frac{13}{24} - \frac{1}{24} = \boxed{-\frac{14}{24}\text{A}}$

Problem 2: Use nodal analysis and find all node voltages and the currents i_1, i_2, i_3. (10pts.)

Problem 2 Solution

Nodal Analysis by Inspection

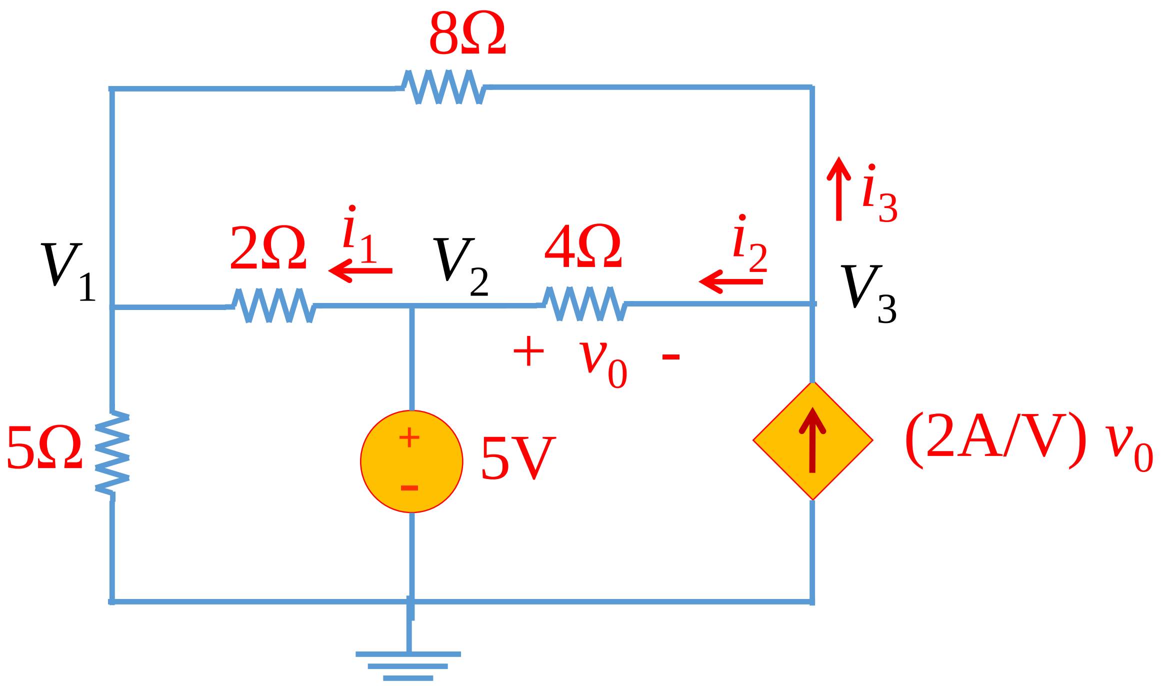
KCL at node 1: $\frac{V_1}{5} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{8} = 0$

node 2 is set by the source: $V_2 = 5\text{V}$

KCL at node 3: $\frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{8} - 2v_0 = 0$

$$\frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{8} - 2(V_2 - V_3) = 0$$

where $v_0 = V_2 - V_3$



By using node 2 voltage and rearranging the KCL equations

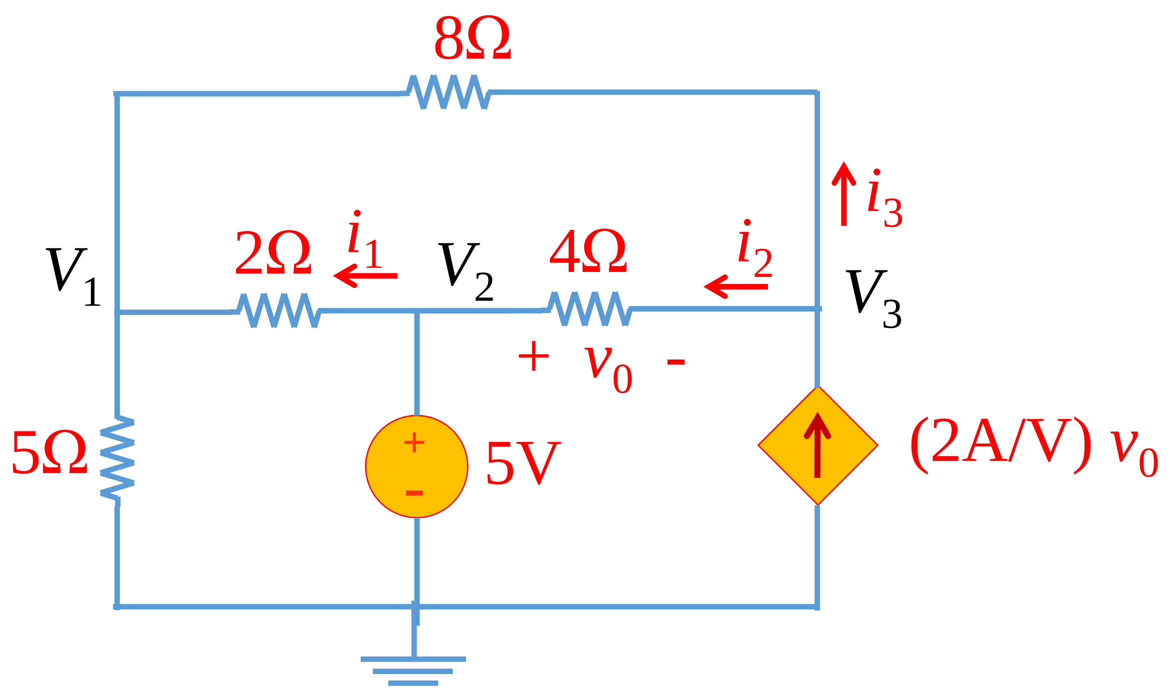
$$\left.\begin{aligned} 33V_1 - 5V_3 &= 100 \\ -V_1 + 19V_3 &= 90 \end{aligned}\right\} \quad \begin{aligned} 33V_1 - 5V_3 &= 100 \\ -33V_1 + 627V_3 &= 2970 \end{aligned}$$

Problem 2: Use nodal analysis and find all node voltages and the currents i_1, i_2, i_3. (10pts.)

Problem 2 Solution cont'd

$$V_3 = \frac{3070}{622}\text{V} \approx \boxed{4.94\text{V}}$$

$$V_1 = \frac{100 + 5\times\frac{3070}{622}}{33}\text{V} = \frac{77850}{20625}\text{V} \approx \boxed{3.78\text{V}}$$



Labeled currents are

$$i_1 = \frac{V_2 - V_1}{2} = \frac{5 - 3.78}{2} = 0.61\text{A}$$

$$i_2 = \frac{V_3 - V_2}{4} = \frac{4.94 - 5}{4} = -0.015\text{A}$$

$$i_3 = \frac{V_3 - V_1}{8} = \frac{4.94 - 3.78}{8} = 0.145\text{A}$$

CORRECTION MADE HERE

Problem 3: Use nodal analysis and find all node voltages and the currents i_1, i_2, i_3 , i_4. (10pts.)

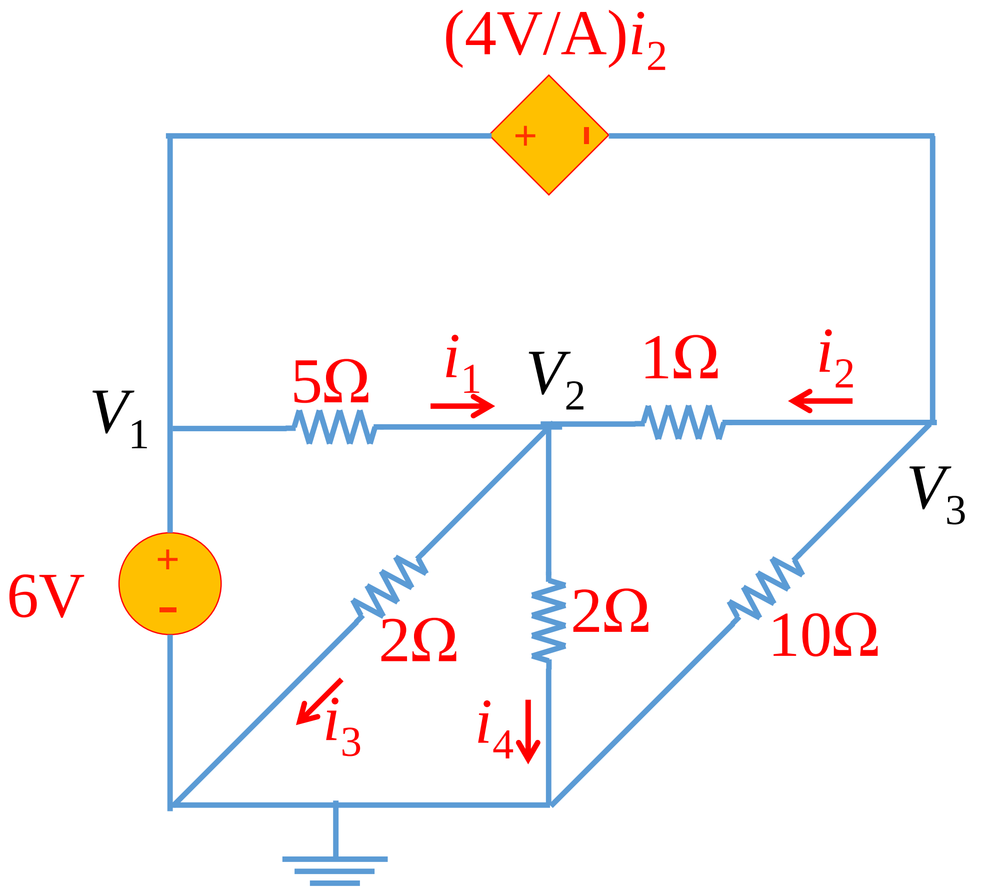
<u>Problem 3 Solution</u> Nodal Analysis by Inspection

node 1 voltage set by the source: $V_1 = 6\text{V}$

KCL at node 2: $\frac{V_2 - V_1}{5} + \frac{V_2}{2} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$

Due to the CCVC at the top branch we cannot write KCL at node 3. But we can apply KVL through the top branch $V_1 = V_3 + 4i_2$

$$= V_3 + 4\left(\frac{V_3 - V_2}{1}\right)$$



Problem 3: Use nodal analysis and find all node voltages and the currents i_1, i_2, i_3 , i_4. (10pts.)

Problem 3 Solution cont'd

Let us substitute node 1 voltage and rewrite the two equations

$$\left.\begin{array}{l} 11V_2 - 5V_3 = 6 \\ -4V_2 + 5V_3 = 6 \end{array}\right\}$$

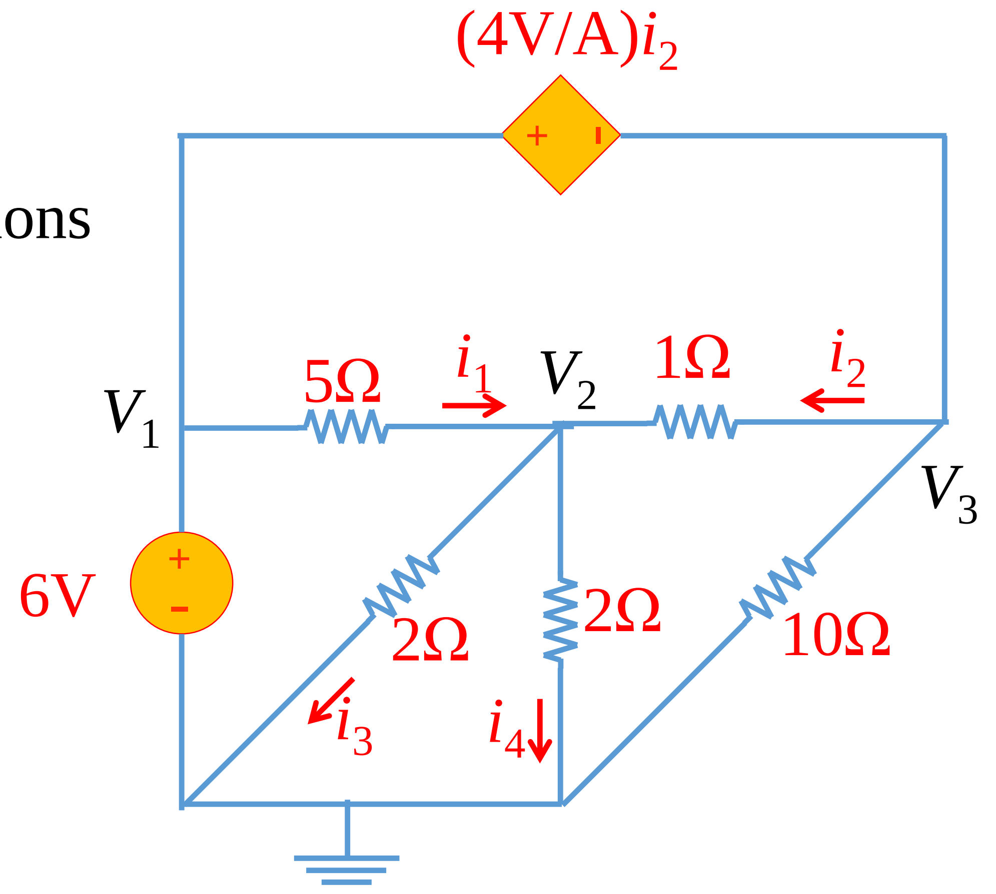
$$V_2 = \frac{12}{7}\text{V} = 1.71\text{V}$$

$$V_3 = \frac{11V_2 - 6}{5} = \frac{18}{7}\text{V} = 2.57\text{V}$$

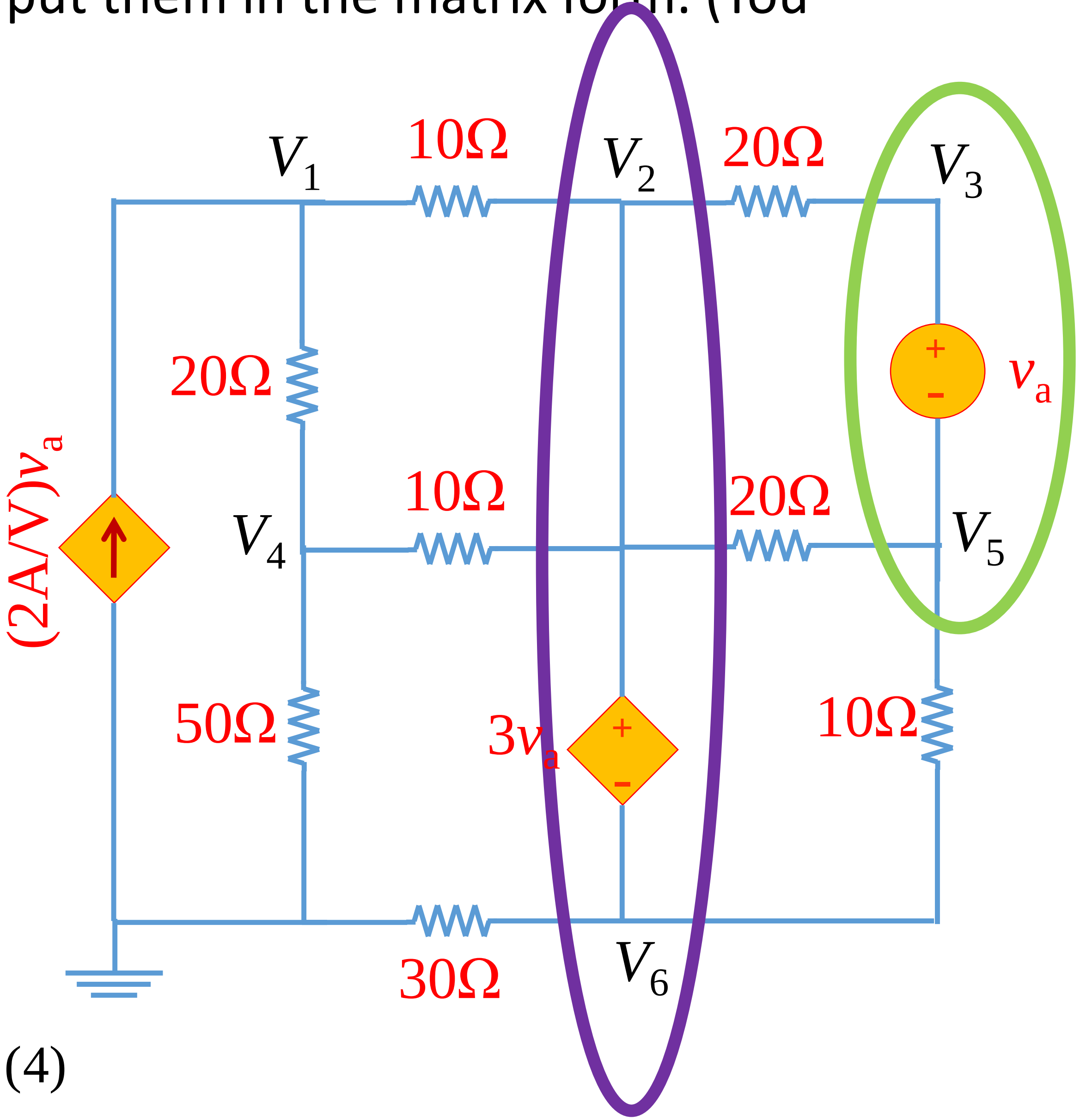
$$i_1 = \frac{V_1 - V_2}{5} = \frac{6 - \frac{12}{7}}{5} = \frac{6}{7}\text{A} = 0.86\text{A}$$

$$i_2 = \frac{V_3 - V_2}{1} = \frac{18}{7} - \frac{12}{7} = \frac{6}{7}\text{A}$$

$$i_3 = i_4 = \frac{V_2 - 0}{2} = \frac{6}{7}\text{A}$$



Problem 4: Write all node voltage equations and put them in the matrix form. (You do not need to solve.) (10pts.)



Problem 4 Solution

(Note that v_a is a given parameter, not a variable of nodal analysis)

We apply supernode when there is a voltage source in the branch connected to a node

KCL at node 1: $-2v_a + \frac{V_1 - V_2}{10} + \frac{V_1 - V_4}{20} = 0$ (1)

KCL at supernode 2&6:

$$\frac{V_2 - V_1}{10} + \frac{V_2 - V_3}{20} + \frac{V_2 - V_4}{10} + \frac{V_2 - V_5}{20} + \frac{V_6}{30} + \frac{V_6 - V_5}{10} = 0 \quad (2)$$

KVL connecting nodes 2&6: $V_2 - V_6 = 3v_a$ (3)

KCL at supernode 3&5: $\frac{V_3 - V_2}{20} + \frac{V_5 - V_2}{20} + \frac{V_5 - V_6}{10} = 0$ (4)

KVL connecting nodes 3&5: $V_3 - V_5 = v_a$ (5)

Problem 4: Write all node voltage equations and put them in the matrix form. (You do not need to solve.) (10pts.)

Problem 4 Solution cont'd

KCL at node 4: $\frac{V_4 - V_1}{20} + \frac{V_4}{50} + \frac{V_4 - V_2}{10} = 0 \quad (6)$

We have 6 unknown node voltages and 6 equations, now let us rewrite the equations

$$3V_1 - 2V_2 - V_4 = 40v_a \quad (1)$$

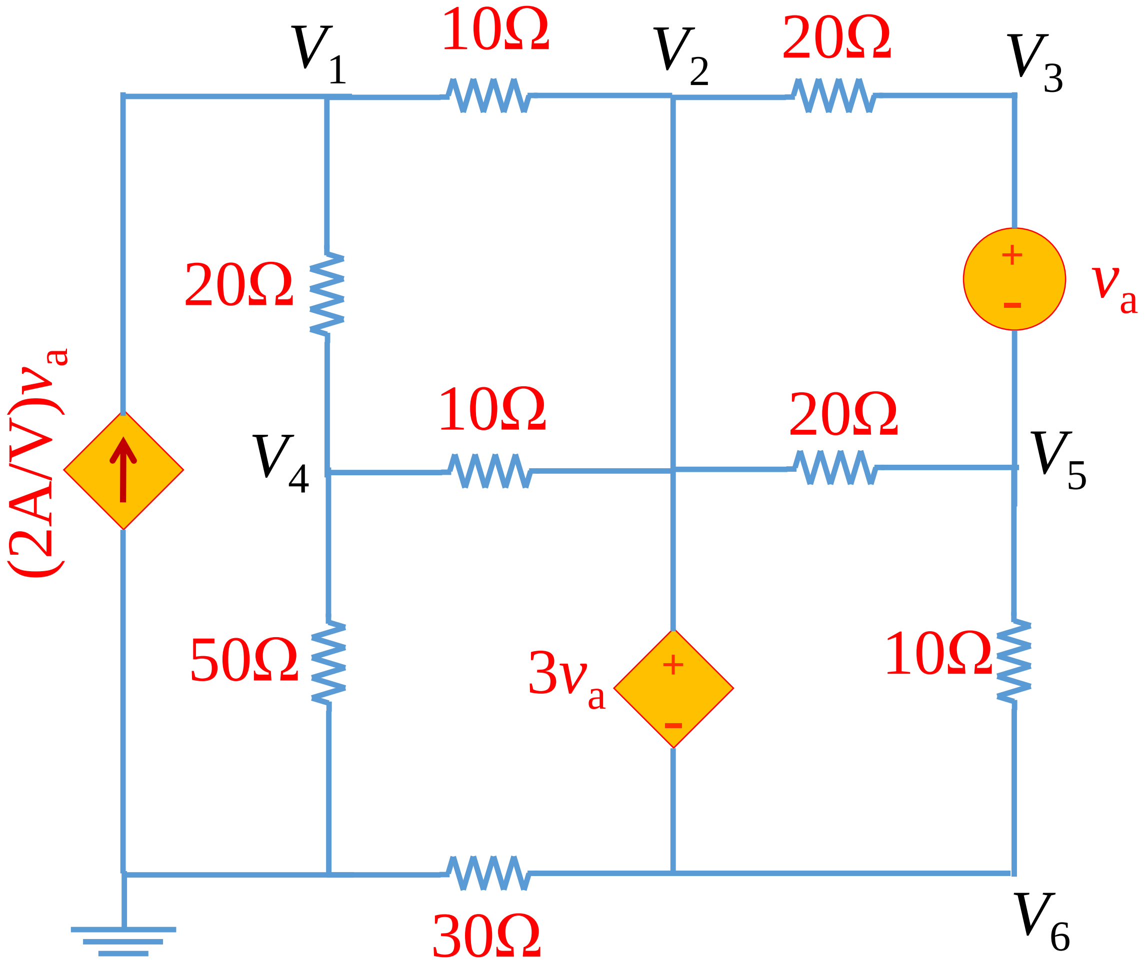
$$-6V_1 + 18V_2 - 3V_3 - 6V_4 - 9V_5 + 8V_6 = 0 \quad (2)$$

$$V_2 - V_6 = 3v_a \quad (3)$$

$$-2V_2 + V_3 + 3V_5 - 2V_6 = 0 \quad (4)$$

$$V_3 - V_5 = v_a \quad (5)$$

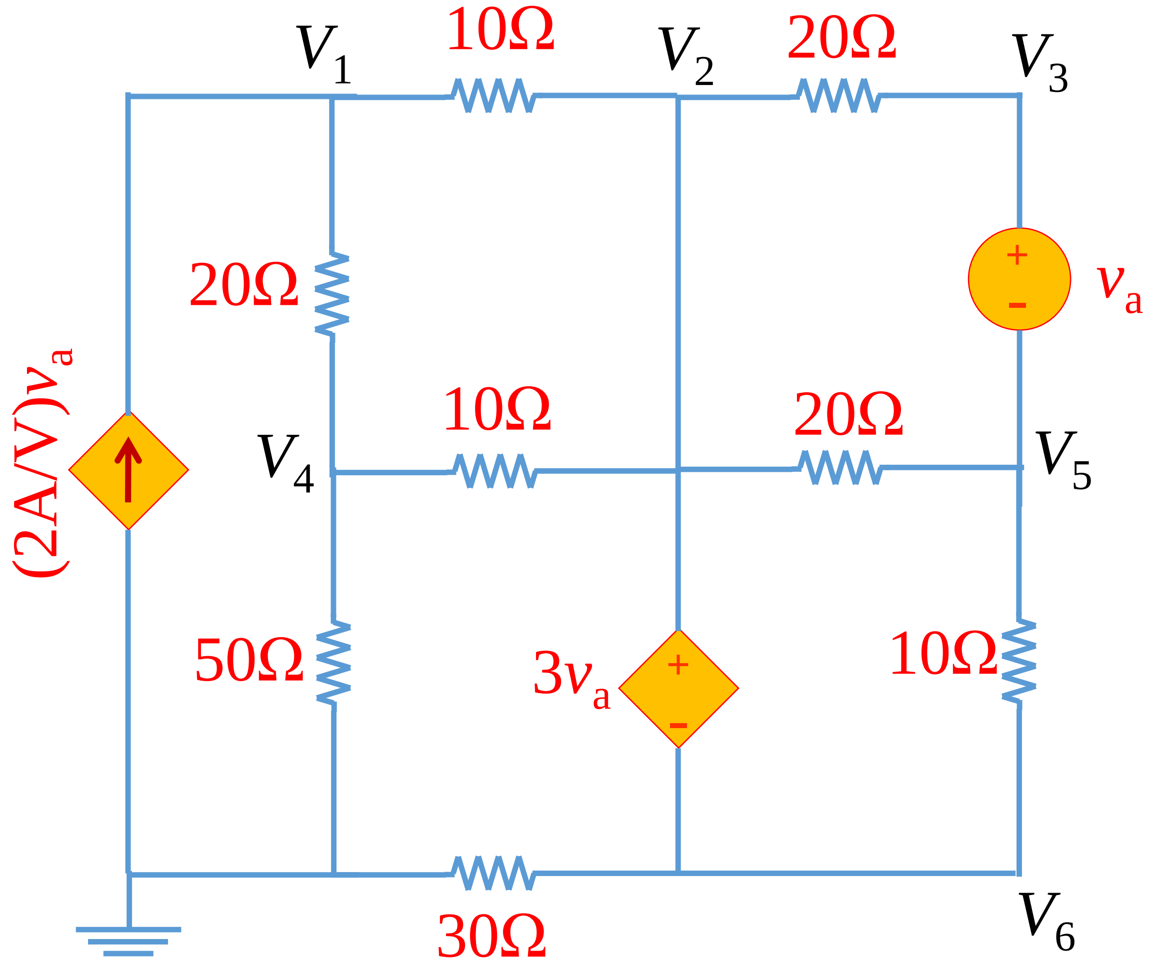
$$-5V_1 - 10V_2 + 17V_4 = 0 \quad (6)$$



Problem 4: Write all node voltage equations and put them in the matrix form. (You do not need to solve.) (10pts.)

Problem 4 Solution cont'd

$$\begin{pmatrix} 3 & -2 & 0 & -1 & 0 & 0 \\ -6 & 18 & -3 & -6 & -9 & 8 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & -2 & 1 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ -5 & -10 & 0 & 17 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} = \begin{pmatrix} 40v_a \\ 0 \\ 3v_a \\ 0 \\ v_a \\ 0 \end{pmatrix}$$



Problem 5: Use mesh analysis to find all the labeled currents and node voltages. (10pts.)

Problem 5 Solution

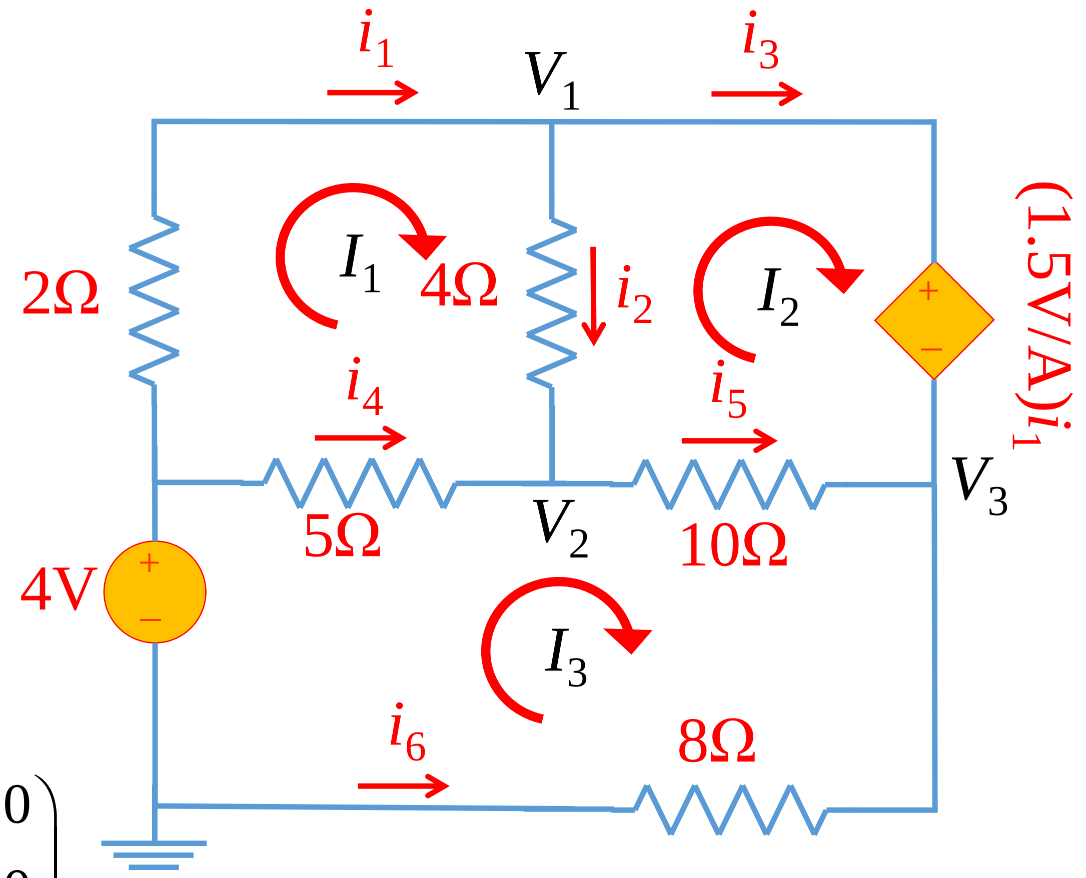
Note the annotated loop currents

KVL at loop 1: $4(I_1 - I_2) + 5(I_1 - I_3) + 2I_1 = 0$ (1)

KVL at loop 2: $1.5I_1 + 10(I_2 - I_3) + 4(I_2 - I_1) = 0$ (2)

where we used $i_1 = I_1$

KVL at loop 3: $-4 + 5(I_3 - I_1) + 10(I_3 - I_2) + 8I_3 = 0$ (3)



Rewriting all equations

$11I_1 - 4I_2 - 5I_3 = 0$ (1)

$-5I_1 + 28I_2 - 20I_3 = 0$ (2)

$-5I_1 - 10I_2 + 23I_3 = 4$ (3)

$$\begin{pmatrix} 11 & -4 & -5 \\ -5 & 28 & -20 \\ -5 & -10 & 23 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

Problem 5: Use mesh analysis to find all the labeled currents and node voltages. (10pts.)

Problem 5 Solution cont'd

Using Cramer's rule or matrix operations:

$$I_1 = 0.2863\text{A},\ \ I_2 = 0.3188\text{A},\ \ I_3 = 0.3748\text{A}.$$

CORRECTION MADE HERE

$i_1 = I_1 = 0.2863\text{A}$

$i_2 = I_1 - I_2 = -0.0325\text{A}$

$i_3 = I_2 = 0.3188\text{A}$

$i_4 = I_3 - I_1 = 0.0885\text{A}$

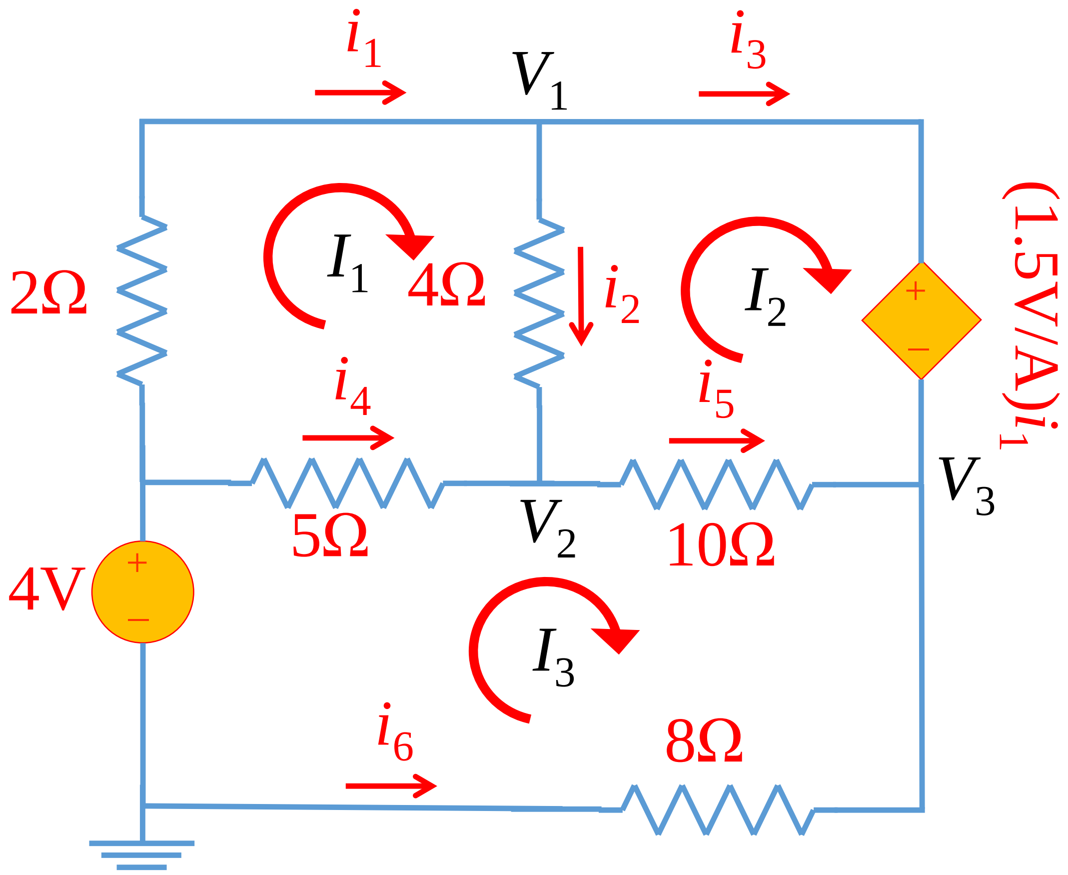
$i_5 = I_3 - I_2 = 0.056\text{A}$

$i_6 = -I_3 = -0.3748\text{A}$

$V_1 = -I_1 \times 2\Omega + 4\text{V} = 3.4275\text{V}$

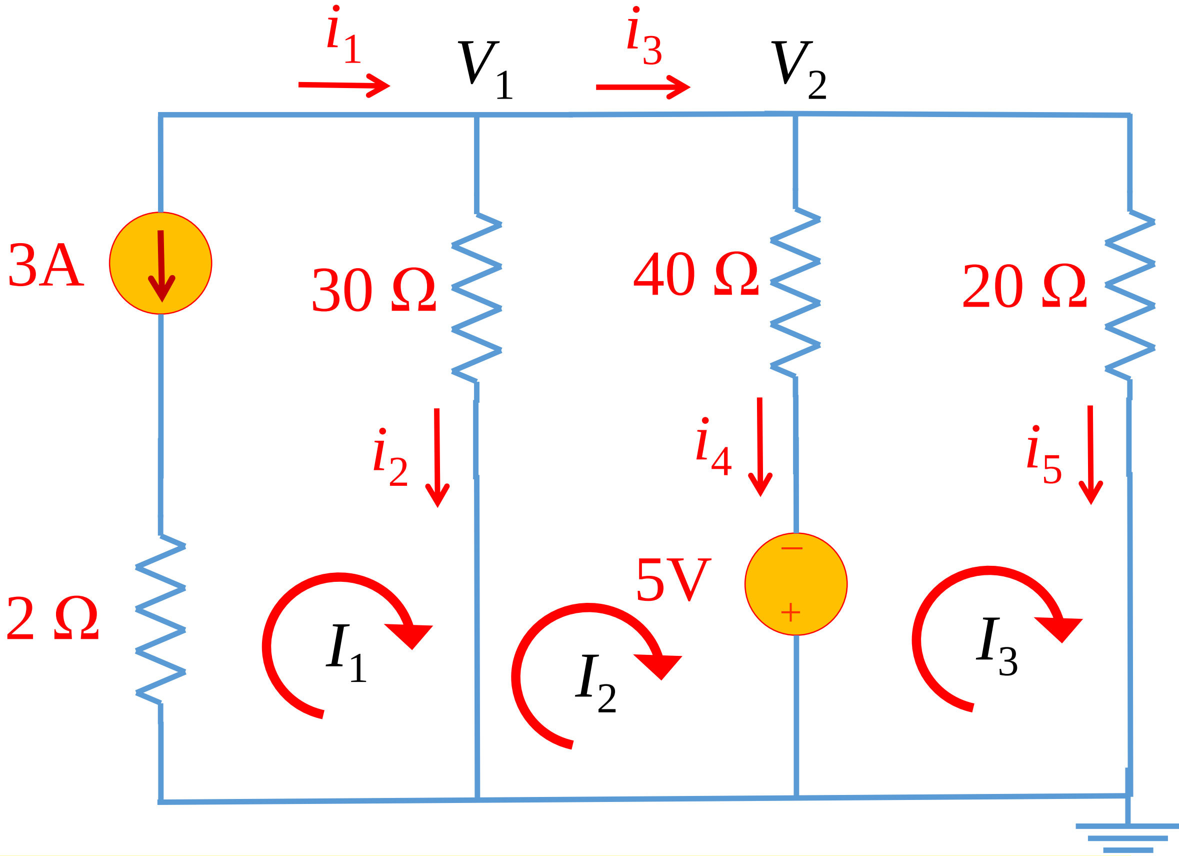
$V_2 = (I_1 - I_3) \times 5\Omega + 4\text{V} = 3.5576\text{V}$

$V_3 = I_3 \times 8\Omega = 2.998\text{V}$



Problem 6: Use mesh analysis to find all the labeled currents and node voltages. (10pts.)

Problem 6 Solution



Loop 1 current is set by the current source: $I_1 = -3\text{A}$

KVL at loop 2: $30(I_2 - I_1) + 40(I_2 - I_3) - 5 = 0$

KVL at loop 3: $5 + 40(I_3 - I_2) + 20I_3 = 0$

Rewriting KVL equations by substituting I_1

$$\left.\begin{aligned} 14I_2 - 8I_3 &= -17 \\ -8I_2 + 12I_3 &= -1 \end{aligned}\right\} \begin{aligned} 42I_2 - 24I_3 &= -51 \\ -16I_2 + 24I_3 &= -2 \end{aligned}$$

$$I_2 = -\frac{53}{26}\text{A} = -2.039\text{A},\ I_3 = \frac{-1+8I_2}{12} = \frac{-450}{312}\text{A} = -1.442\text{A}$$

Labeled currents and voltages are

$i_1 = I_1 = -3\text{A}$

$i_2 = I_1 - I_2 = -0.961\text{A}$

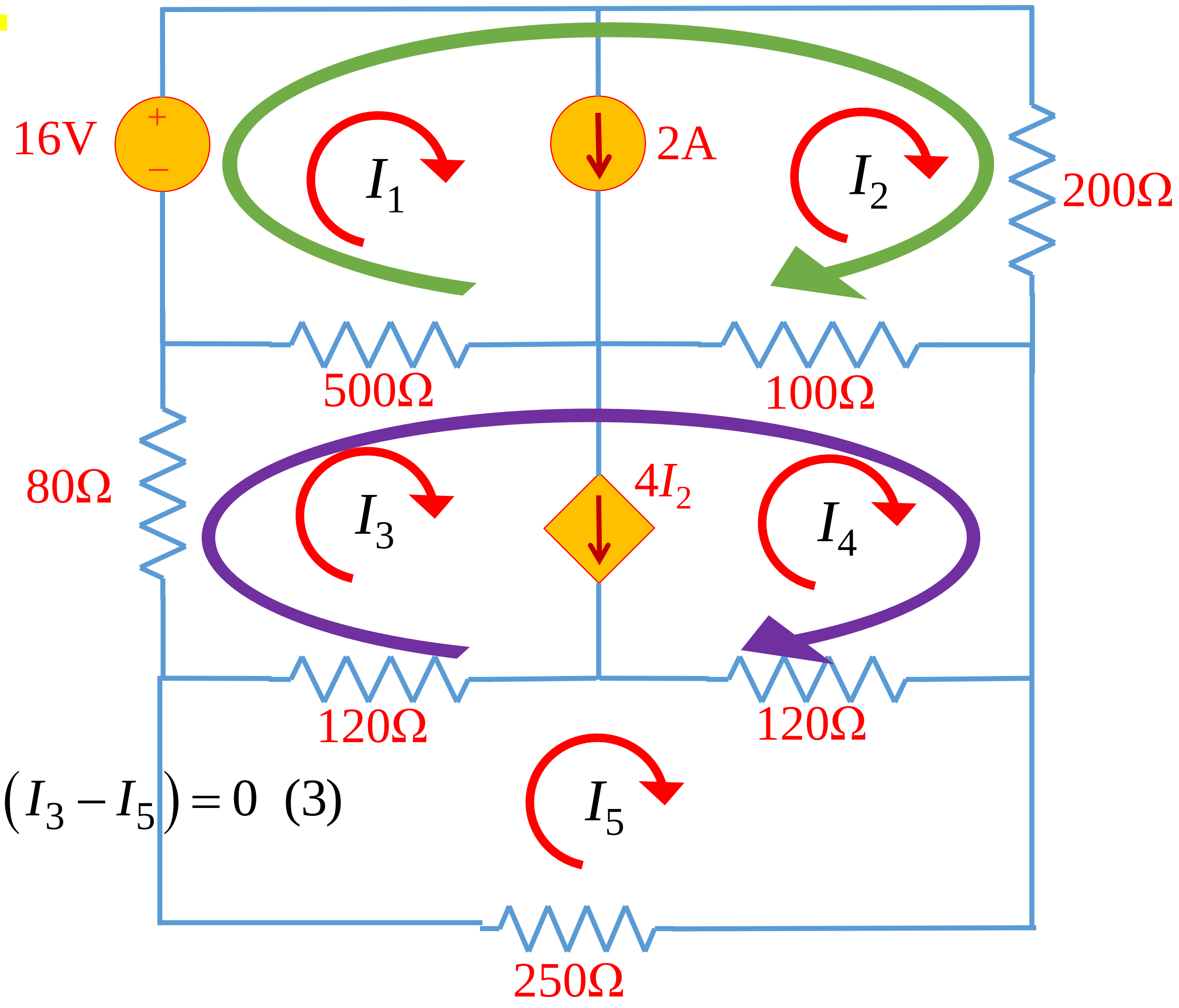
$i_3 = I_2 = -2.039\text{A}$

$i_4 = I_2 - I_3 = -0.597\text{A}$

$i_5 = I_3 = -1.442\text{A}$

$V_1 = V_2 = i_2 \times 30\Omega = i_4 \times 40\Omega - 5\text{V} = i_5 \times 20\Omega = -28.8\text{V}$

Problem 7: Write all the mesh current equations and put them in the matrix form. You don't have to solve. (10pts.)



Problem 7 Solution

We need to use superloops by-passing the current sources, and use the current sources to obtain equations

KVL at superloop 1&2:

$$-16+200I_2+100(I_2-I_4)+500(I_1-I_3)=0 \quad (1)$$

Current source inside the super loop 1&2:

$$I_1-I_2=2 \quad (2)$$

KVL at superloop 3&4:

$$80I_3+500(I_3-I_1)+100(I_4-I_2)+120(I_4-I_5)+120(I_3-I_5)=0 \quad (3)$$

Current source inside the super loop 3&4:

$$I_3-I_4=4I_2 \quad (4)$$

KVL at loop 5:

$$120(I_5-I_3)+120(I_5-I_4)+250I_4=0 \quad (5)$$

Problem 7: Write all the mesh current equations and put them in the matrix form. You don't have to solve. (10pts.)

Problem 7 Solution cont'd

Rewrite all equations

$$500I_1 + 300I_2 - 500I_3 - 100I_4 = 16 \quad (1)$$

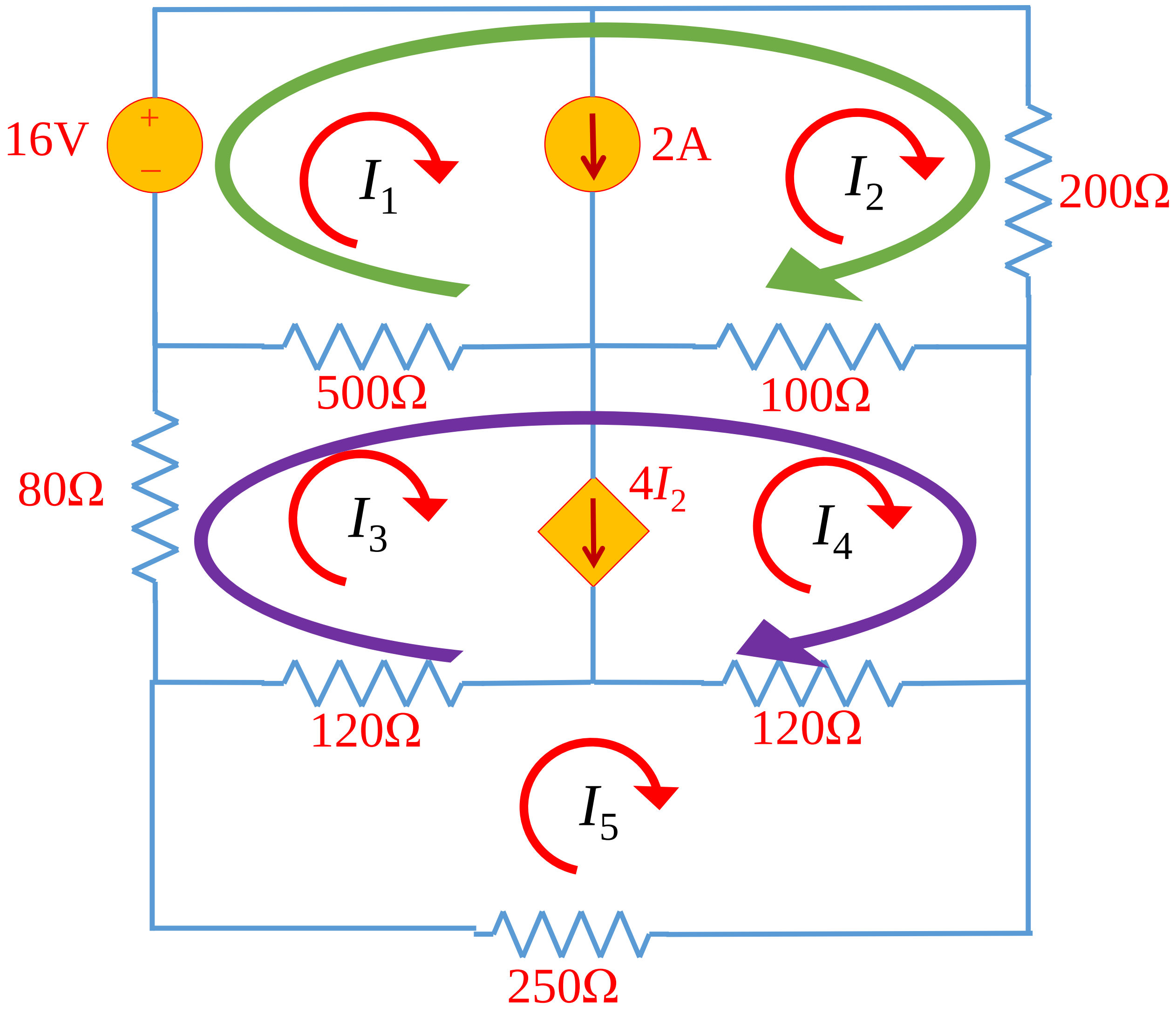
$$I_1 - I_2 = 2 \quad (2)$$

$$-500I_1 - 100I_2 + 700I_3 + 220I_4 - 240I_5 = 0 \quad (3)$$

$$-4I_2 + I_3 - I_4 = 0 \quad (4)$$

$$-120I_3 - 120I_4 + 490I_5 = 0 \quad (5)$$

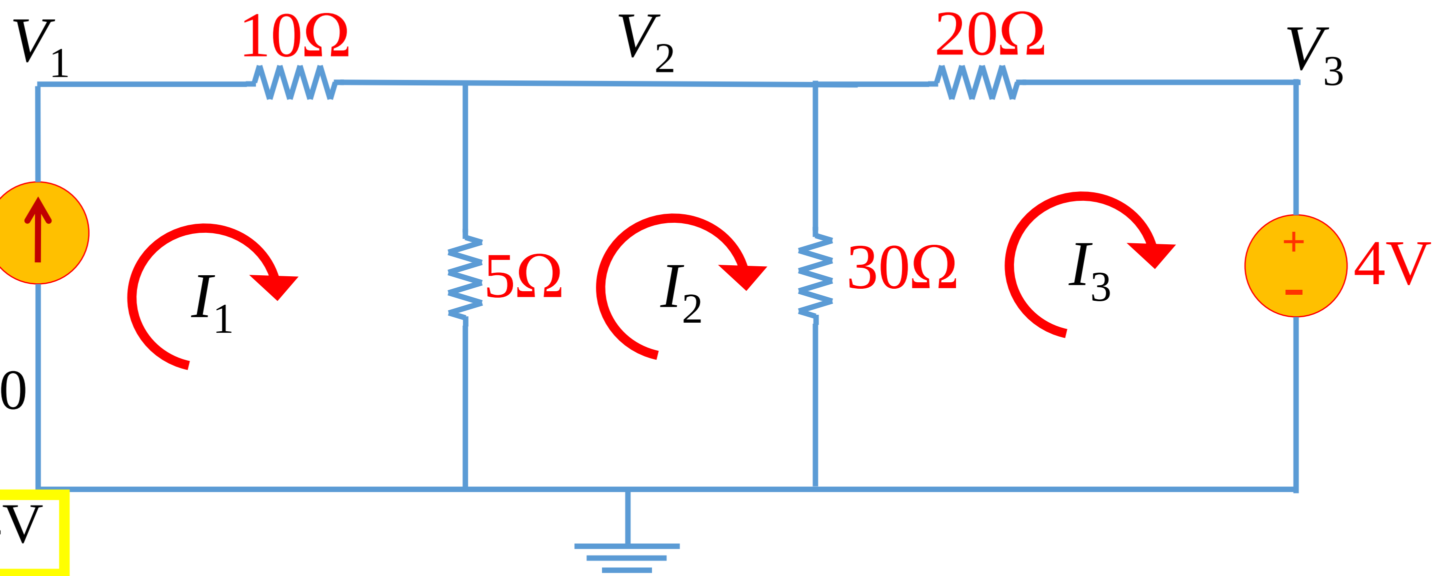
$$\begin{pmatrix} 500 & 300 & -500 & -100 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ -500 & -100 & 700 & 220 & -240 \\ 0 & -4 & 1 & -1 & 0 \\ 0 & 0 & -120 & -120 & 490 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



Problem 8: Use both nodal and mesh analyses to solve for all the node voltages and loop currents. (10pts.)

Problem 8 Solution

Nodal Analysis



KCL at node 1: $-3+\frac{V_1-V_2}{10}=0$

KCL at node 2: $\frac{V_2-V_1}{10}+\frac{V_2}{30}+\frac{V_2}{5}+\frac{V_2-V_3}{20}=0$

Node 3 voltage is set by the source: $V_3=4\text{V}$

Rewrite the equation substituting node 3 voltage

$$\left.\begin{array}{r} V_1-V_2=30 \\ -6V_1+23V_2=12 \end{array}\right\} \begin{array}{c} -6V_1+23(V_1-30)=12 \\ 17V_1=702 \end{array}$$

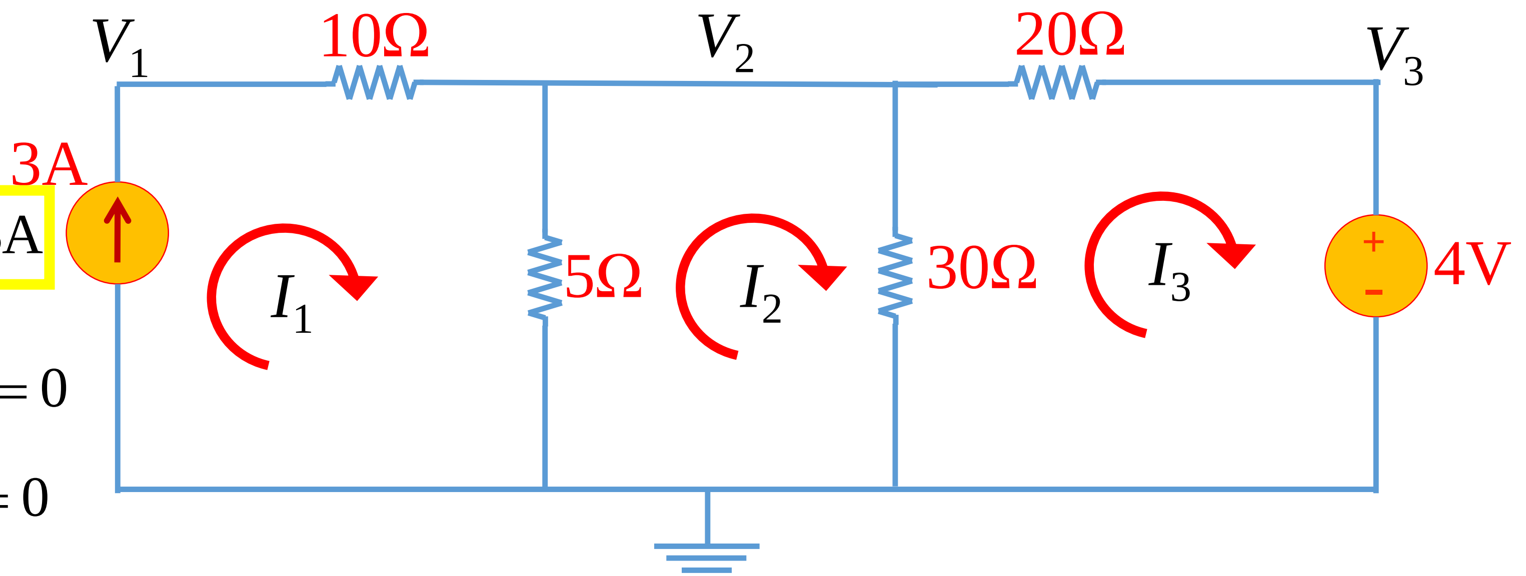
$$V_1=\frac{702}{17}\text{V}=41.3\text{V}$$

$$V_2=V_1-30=11.3\text{V}$$

Problem 8: Use both nodal and mesh analyses to solve for all the node voltages and loop currents. (10pts.)

Problem 8 Solution cont'd

Mesh Analysis



Loop 1 current set by the source: $I_1 = 3\text{A}$

KVL at loop 2: $5(I_2 - I_1) + 30(I_2 - I_3) = 0$

KVL at loop 3: $30(I_3 - I_2) + 20I_3 + 4 = 0$

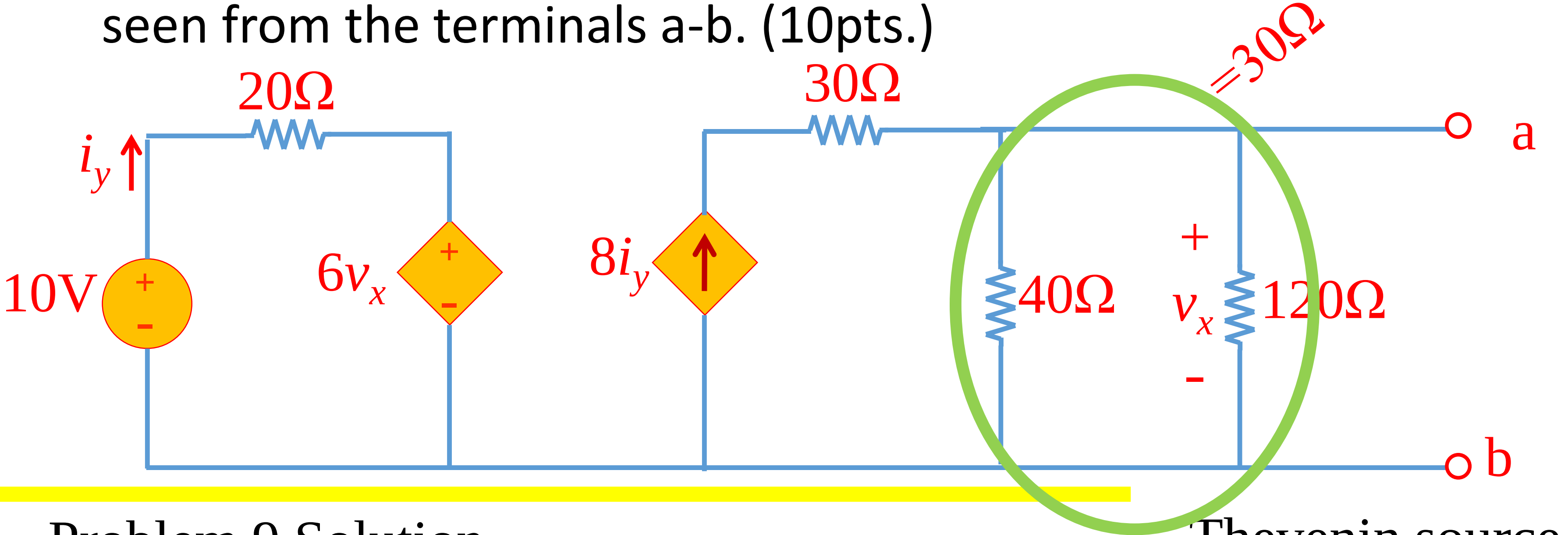
Rewrite the equation substituting loop 1 current

$$\left.\begin{aligned} 35I_2 - 30I_3 &= 15 \\ -30I_2 + 50I_3 &= -4 \end{aligned}\right\} \begin{aligned} 175I_2 - 150I_3 &= 75 \\ -90I_2 + 150I_3 &= -12 \end{aligned}$$

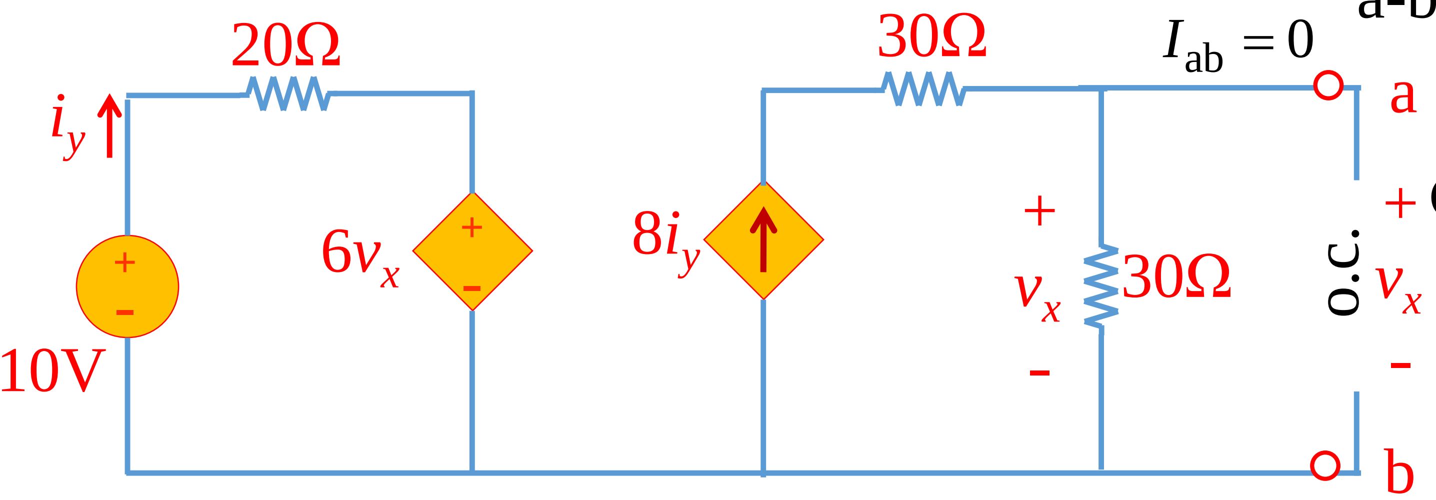
$$I_2 = \frac{63}{85}\text{A} = 0.741\text{A}$$

$$I_3 = \frac{35I_2 - 15}{30} = 0.365\text{A}$$

Problem 9: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



Problem 9 Solution



Thevenin source voltage is the voltage across a-b terminals, when a-b is left as open circuit

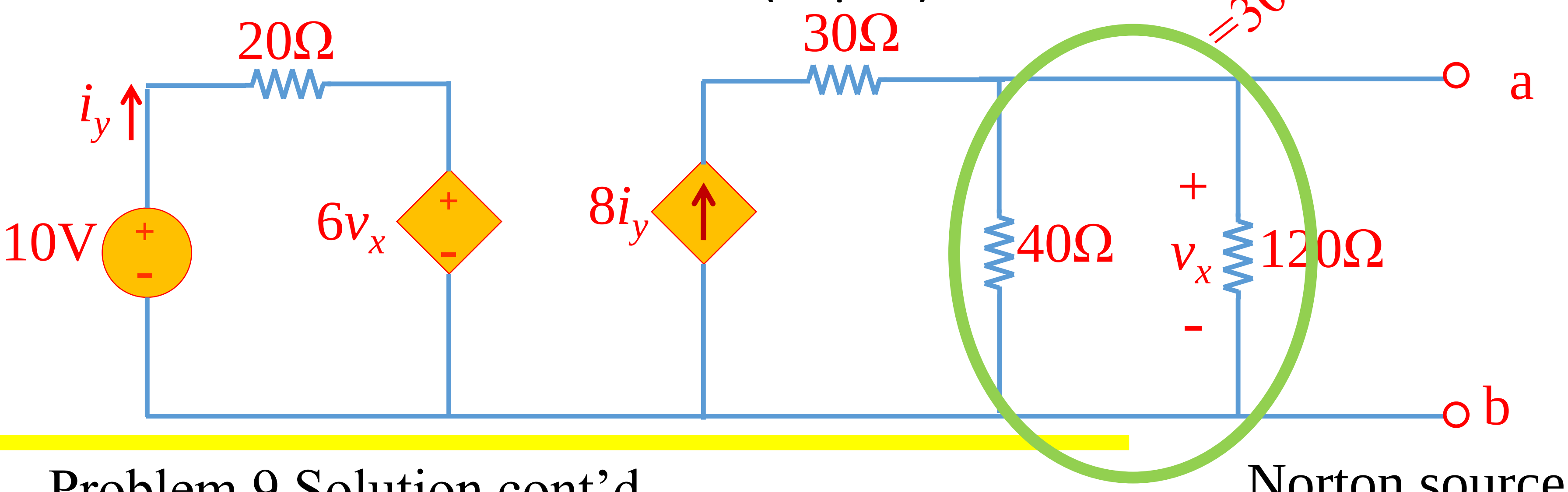
KVL in the left loop: $20i_y + 6v_x = 10$

+ Ohm's Law across 30Ω: $v_x = 8i_y \times 30 = 240i_y$

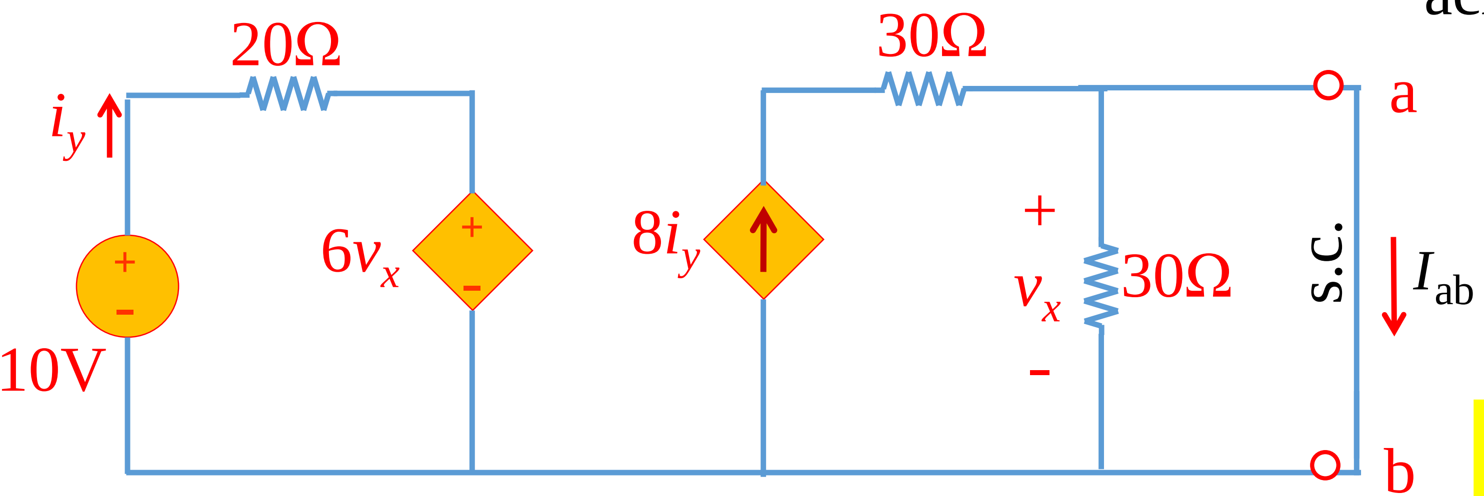
$$20\frac{v_x}{240} + 6v_x = v_x\frac{73}{12} = 10 \qquad v_{ab,oc} = v_x = \frac{120}{73}\text{V}$$

$$v_{Th} = \frac{120}{73}\text{V}$$

Problem 9: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



Problem 9 Solution cont'd



Norton source current is the current through across a-b terminals, when a-b is shorted.

$$v_x = 0$$

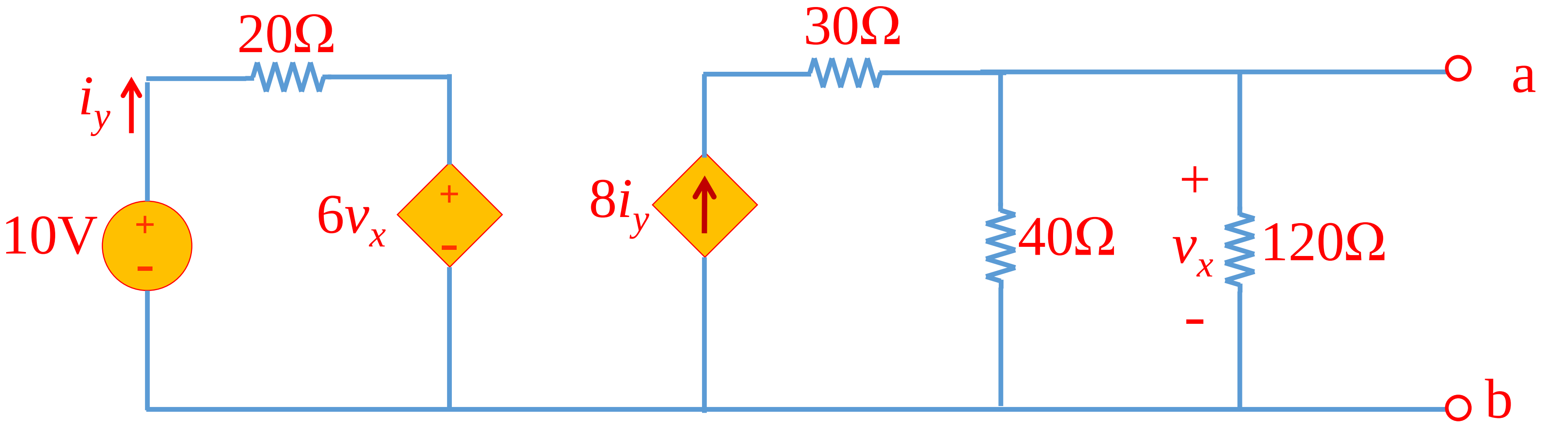
KVL in the left loop: $20i_y = 10$

$$i_y = 0.5\text{A}$$

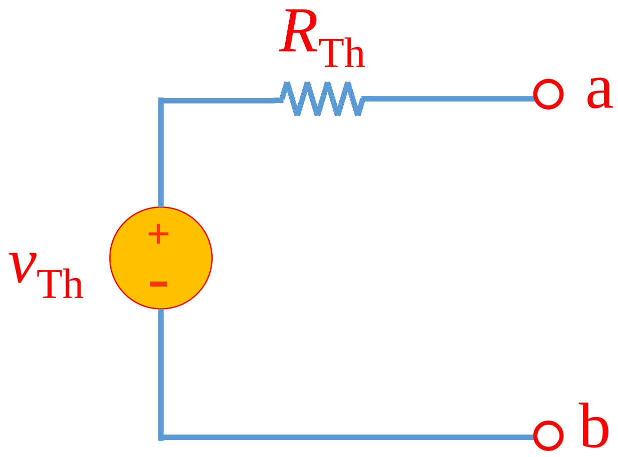
All the current from CCCS will flow through the short circuit $I_{ab,sc} = 8i_y = 4\text{A}$

$$i_{\text{No}} = 4\text{A}$$

Problem 9: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



Problem 9 Solution cont'd



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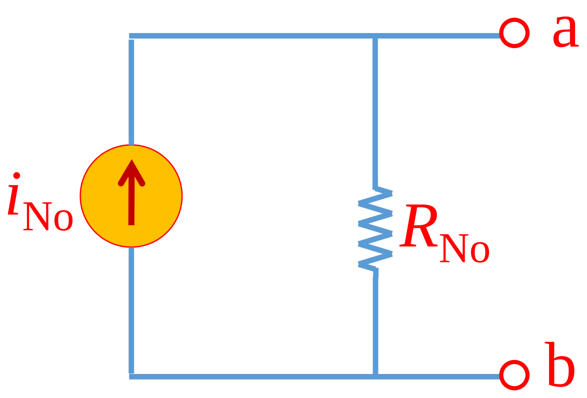
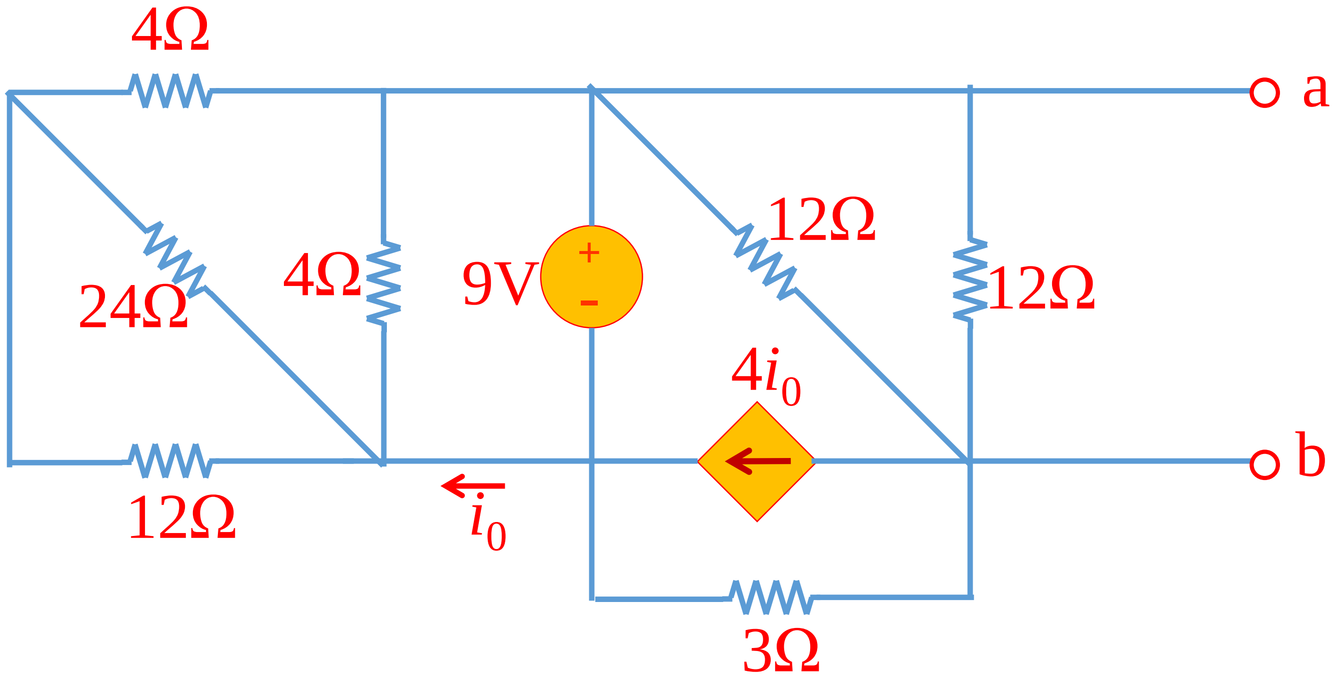


$$v_{Th} = \frac{120}{73}\text{V}$$

$$i_{No} = 4\text{A}$$

$$R_{No} = R_{Th} = \frac{v_{Th}}{i_{No}} = \frac{30}{73}\Omega$$

Problem 10: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



Problem 10 Solution

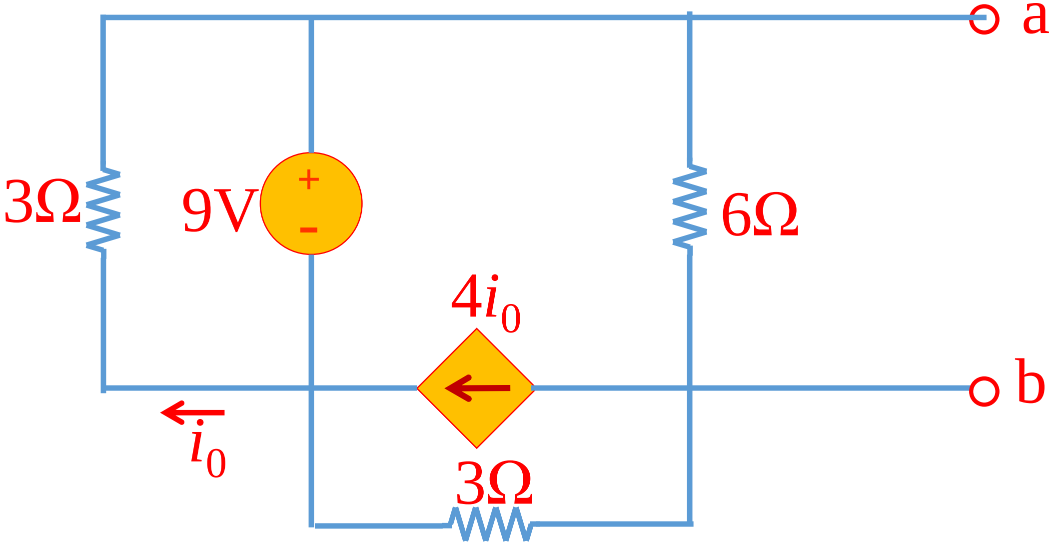
The left-most resistances simplify as

$$\left[(12\Omega \,||\, 24\Omega) + 4\Omega\right] || \, 4\Omega = \left[8\Omega + 4\Omega\right] || \, 4\Omega =$$

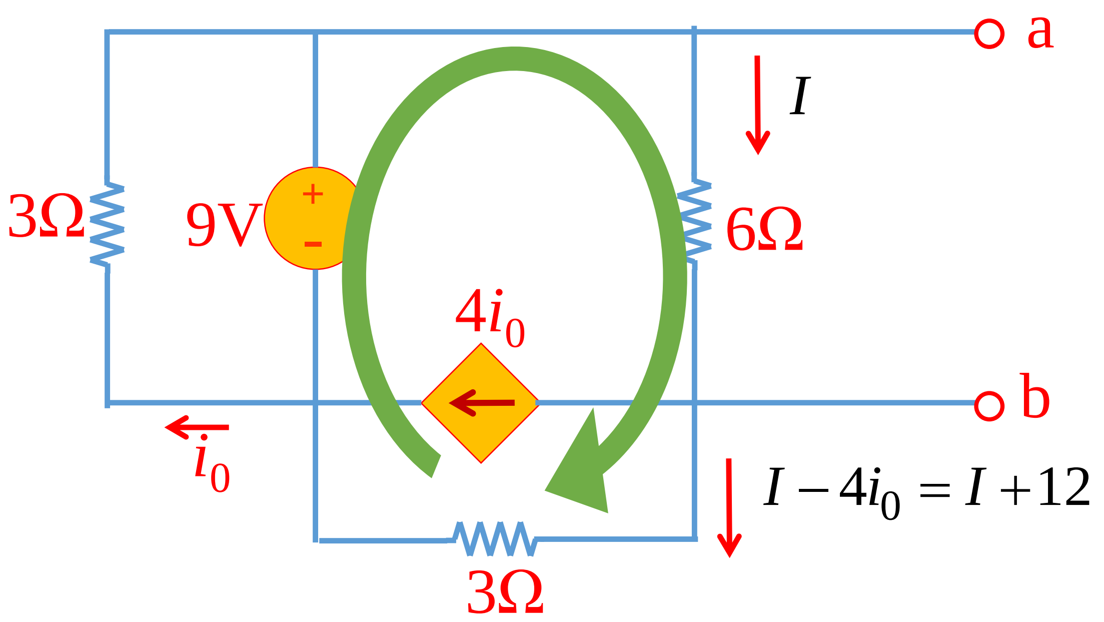
$$= 12\Omega \,||\, 4\Omega = 3\Omega$$

The 12Ω's on the right are also in parallel

The circuit is simplified as



Problem 10: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



Problem 10 Solution cont'd

Let us find the open circuit voltage across a-b

KVL at left-most loop: $-3i_0=9$ $\quad i_0=-3\text{A}$

CCCS current: $4i_0=-12\text{A}$

The current flowing through the bottom 3Ω resistor is $I-4i_0=I+12$

KVL in the looped marked by the green arrow:

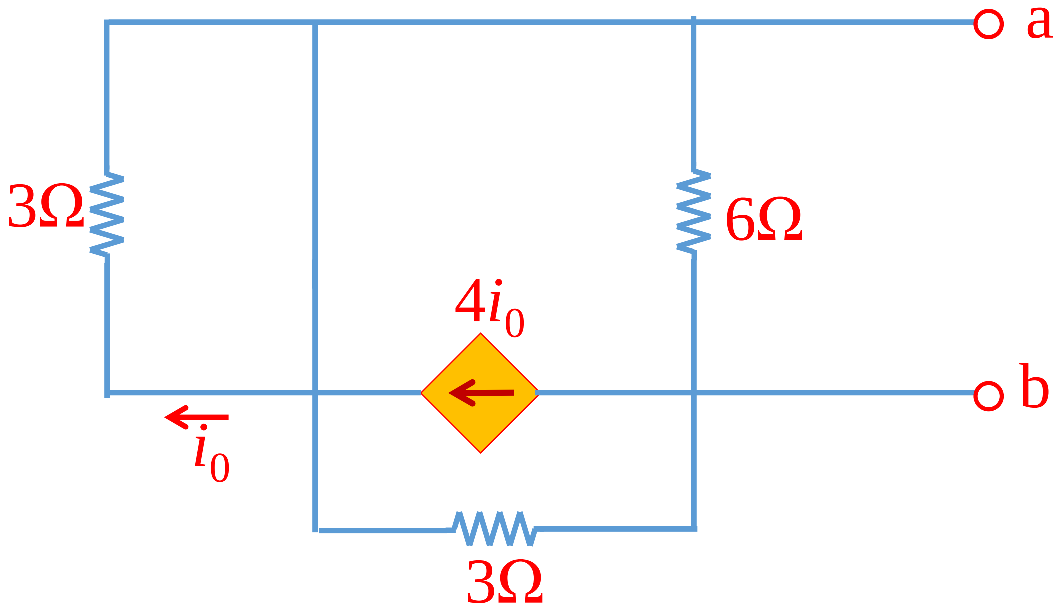
$$-9+6I+3(I+12)=0$$

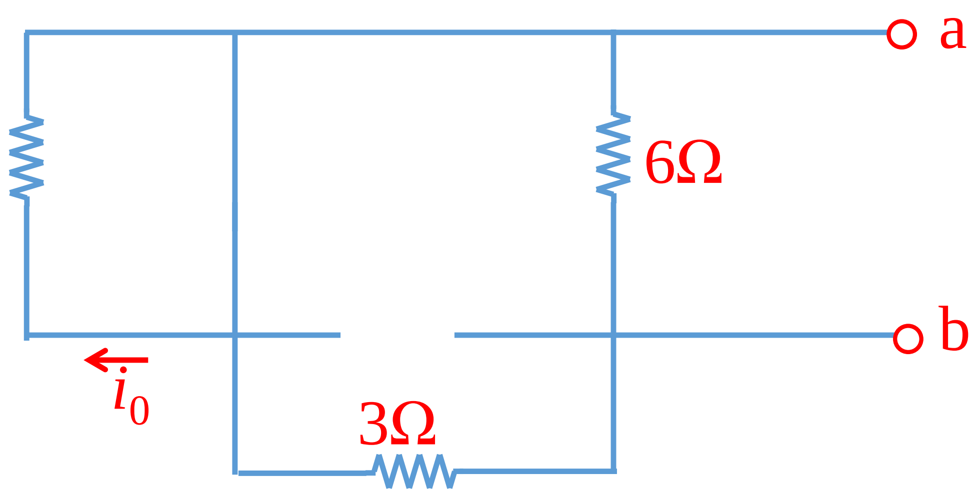
$$9I=-27$$

$$I=-3\text{A}$$

The open circuit voltage across a-b is $v_{\text{Th}}=I\times6\Omega=-3\text{A}\times6\Omega=-18\text{V}$

Problem 10: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)





Problem 10 Solution cont'd

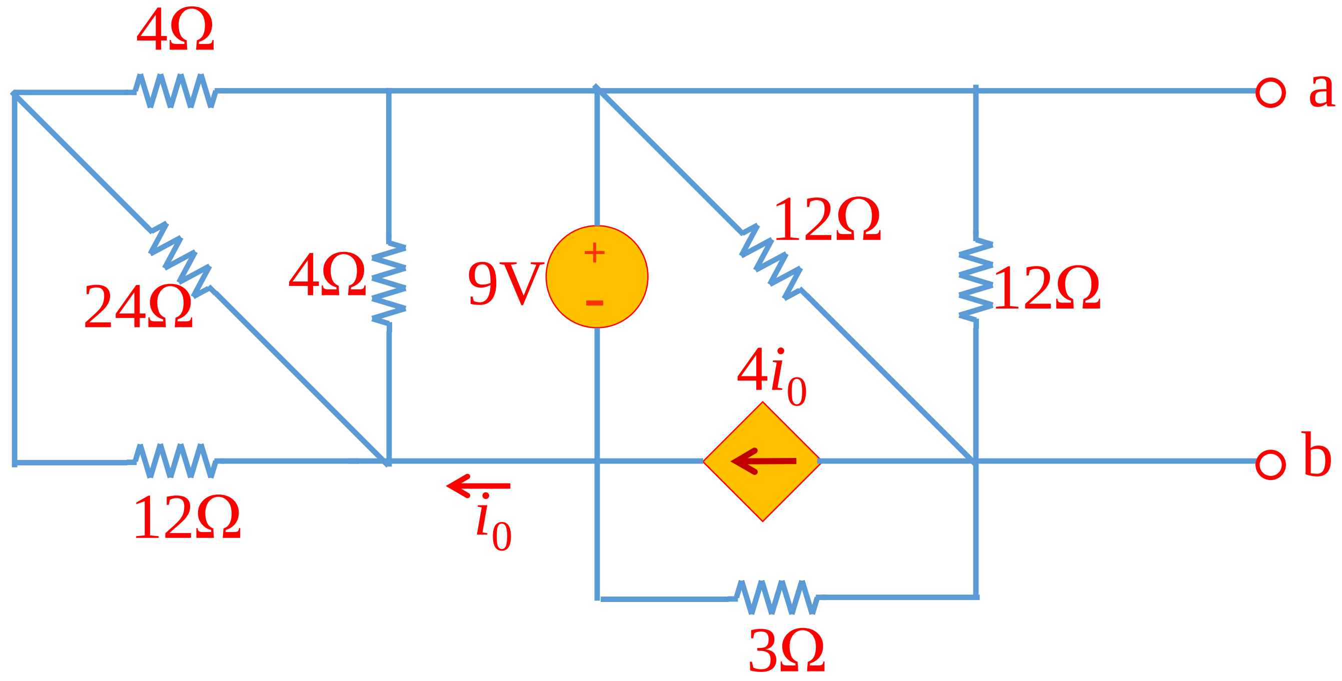
Let us kill the independent source and find the equivalent resistance across a-b which provides us with the Thevenin resistance

When 9V source is killed, it is shorted. Then the voltage across the 3Ω resistor (vertical one) is zero, so $i_0 = 0$

The current flowing through CCCS is also zero so this branch is "open"

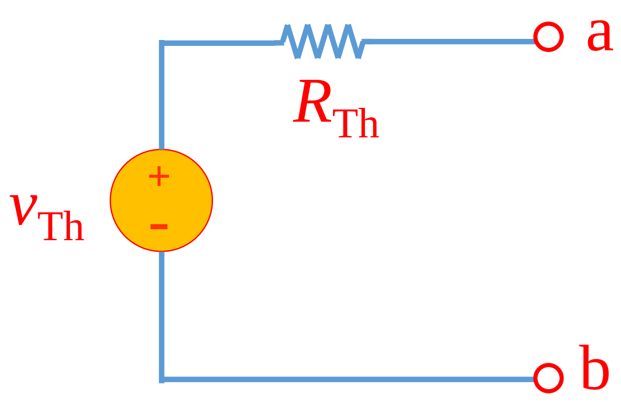
$$R_{eq,ab} = 6\Omega \,||\, 3\Omega = 2\Omega$$

Problem 10: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)



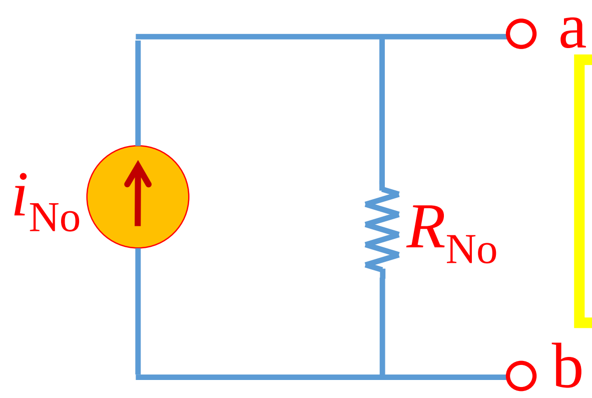
Problem 10 Solution cont'd

We found Thevenin equivalent



$$v_{Th} = -18\text{V}$$
$$R_{Th} = 2\Omega$$

Norton equivalent follows as



$$R_{No} = R_{Th} = 2\Omega$$
$$i_{No} = \frac{v_{Th}}{R_{Th}} = -9\text{A}$$