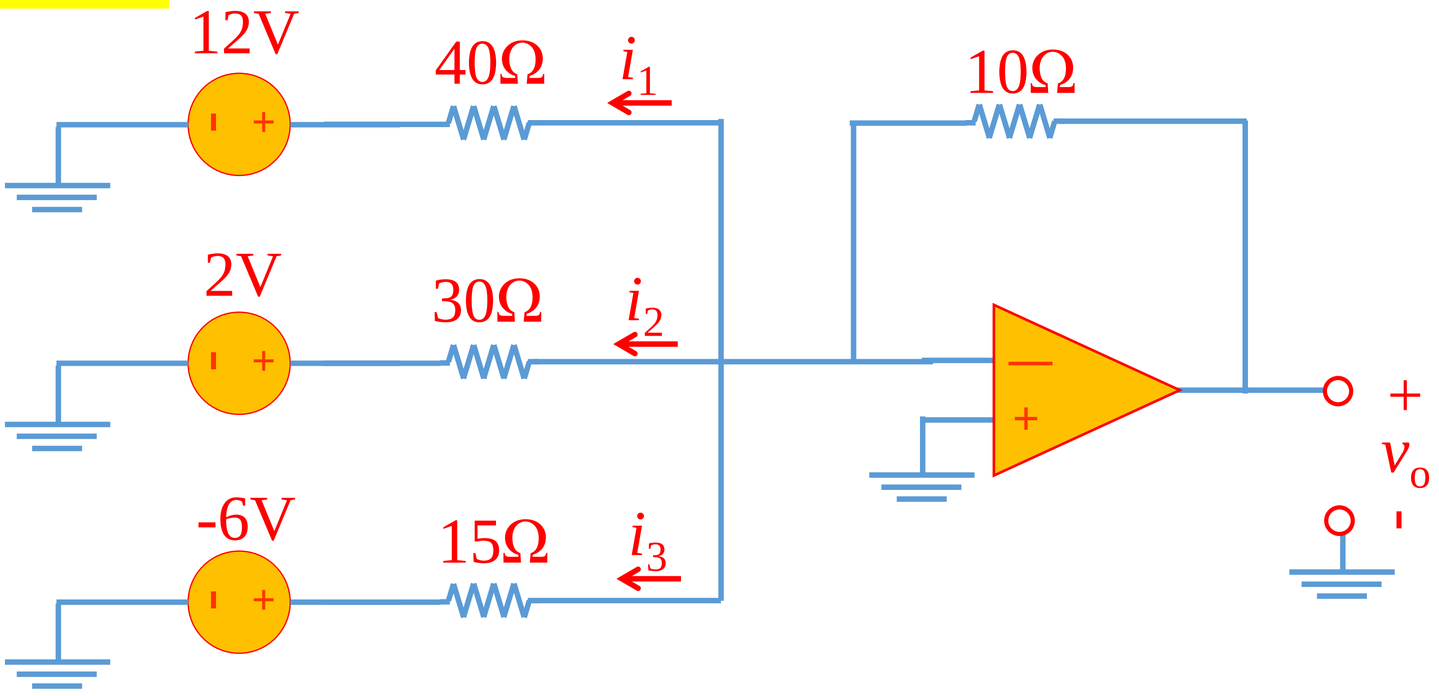
EECS/CSE 70A Network Analysis I

Homework #4
Solution Key

Problem 1: (Ideal Opamp) Find currents i_1, i_2, i_3 and the output voltage v_o (30pts.)



Problem 1 Solution

In ideal opamp, no current flows into -/+ terminals, and the voltages at those terminals are equal

$$i_1 = \frac{V_- - 12\text{V}}{40\Omega}, \quad i_2 = \frac{V_- - 2\text{V}}{30\Omega}, \quad i_3 = \frac{V_- - (-6\text{V})}{15\Omega}$$

KCL at - terminal

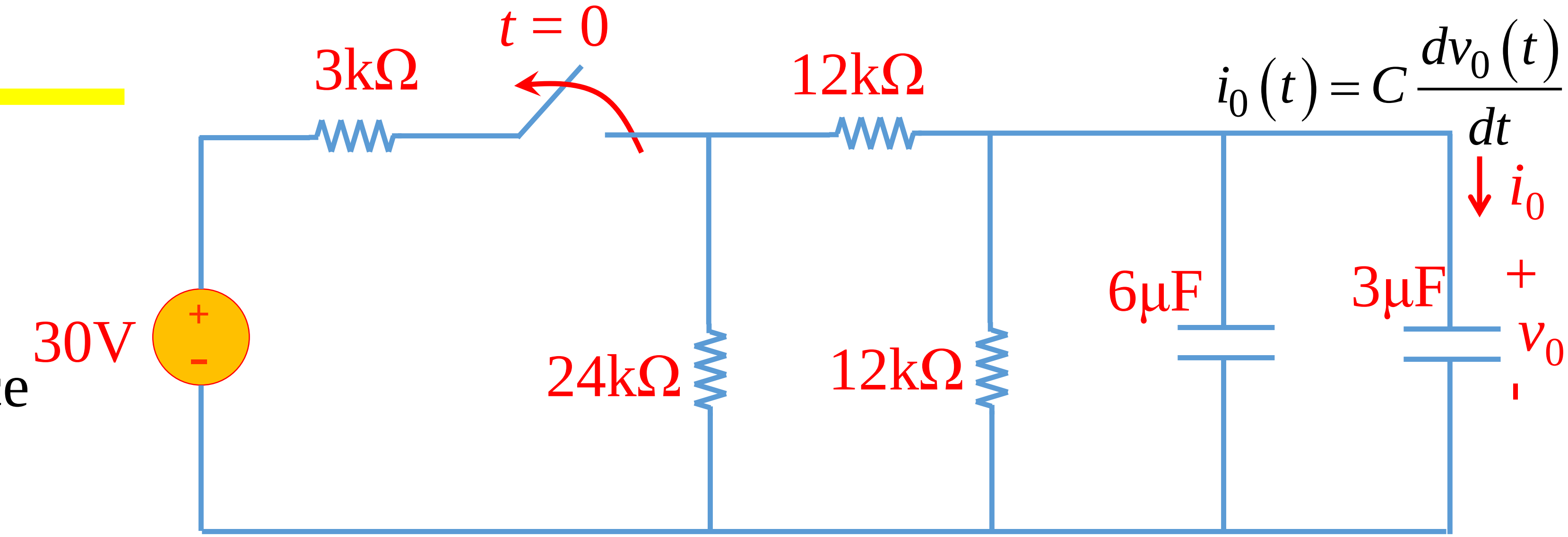
$$\frac{V_- - 12\text{V}}{40\Omega} + \frac{V_- - 2\text{V}}{30\Omega} + \frac{V_- - (-6\text{V})}{15\Omega} + \frac{V_- - v_0}{10\Omega} = 0$$

$$27V_- + 4 = 12v_0$$

Using $V_- = V_+ = 0\text{V}$

$$v_0 = \frac{1}{3}\text{V} \qquad i_1 = \frac{-12\text{V}}{40\Omega} = -0.3\text{A}, \quad i_2 = \frac{-2\text{V}}{30\Omega} = -0.066\text{A}, \quad i_3 = \frac{6\text{V}}{15\Omega} = 0.4\text{A}$$

Problem 2: (RC circuit) Write the expression for the voltage v_0 for t>0. Please clearly show the time constant calculation, initial and steady state voltage across the 3μF capacitor (35pts.)

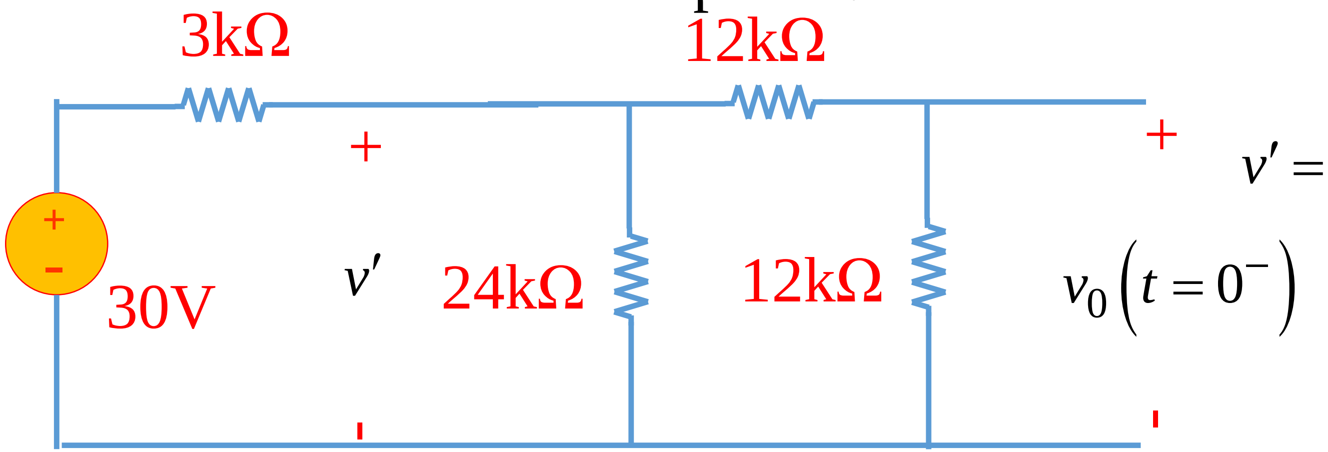


$$i_0(t) = C\frac{dv_0(t)}{dt}$$

Problem 2 Solution

The capacitors are in parallel, the voltage across them is equal, we can use the equivalent capacitance of 9μF in the following steps

$@t = 0^-$ The capacitors act as open circuit since the circuit was in steady state before the switch is opened. All the time derivatives are zero $\frac{d}{dt}(\cdot) \to 0$ $\quad i_0(t=0^-) = 0$



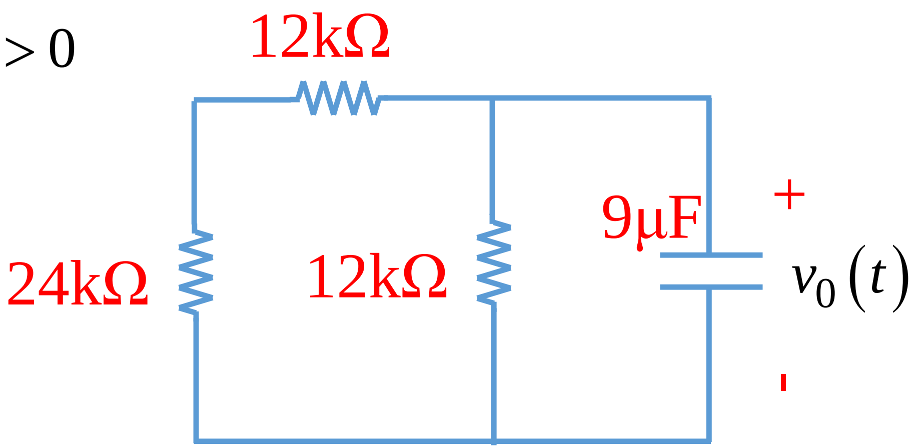
$$v' = 30\text{V}\frac{(12\text{k}\Omega + 12\text{k}\Omega) \,||\, 24\text{k}\Omega}{\left[(12\text{k}\Omega + 12\text{k}\Omega) \,||\, 24\text{k}\Omega\right] + 3\text{k}\Omega} = 30\frac{12}{15} = 24\text{V}$$

$$v_0(t=0^-) = \frac{v'}{2} = 12\text{V}$$

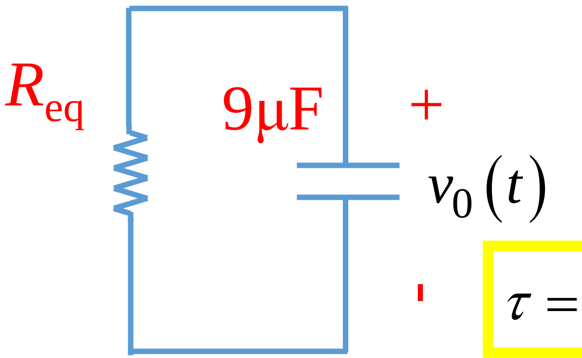
Problem 2: (RC circuit) Write the expression for the voltage v_0 for t>0. Please clearly show the time constant calculation, initial and steady state voltage across the 3μF capacitor (35pts.)

Problem 2 Solution (cont'd)

$t > 0$



$$R_{eq} = (12\text{k}\Omega + 24\text{k}\Omega) \| 12\text{k}\Omega = 9\text{k}\Omega$$



The capacitors discharges in source free setup with the time constant

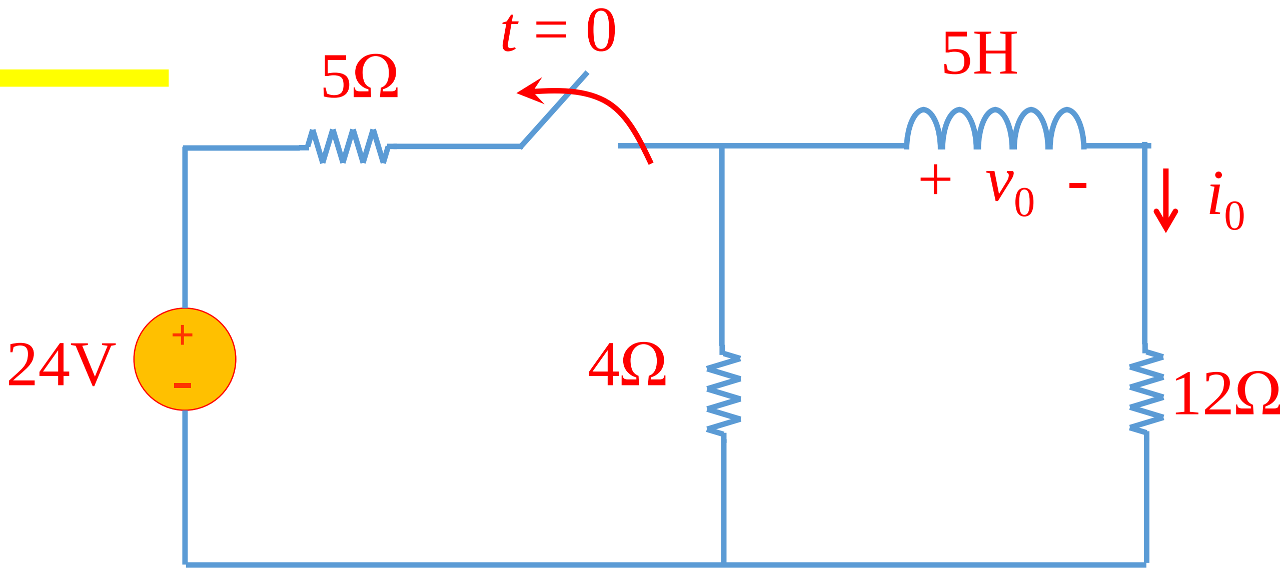
$$\tau = R_{eq}C = (9\times10^3\Omega)(9\times10^{-6}\text{F}) = 81\text{ms}$$

$t \to \infty$ The current through capacitor vanishes, and the voltage decays towards zero $v_0(\infty) = 0\text{V}$

$$v_0(t) = v_0(\infty) + [v_0(0) - v_0(\infty)]e^{-t/\tau}$$

$$= 12e^{-t/(81\times10^{-3})}\text{V} = 12e^{-12.35t}\text{V} \quad t > 0$$

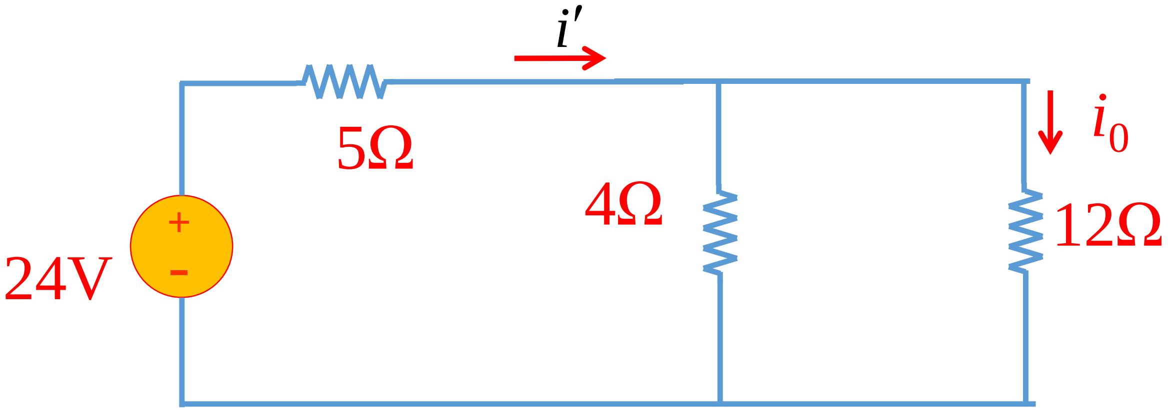
Problem 3: (RL circuit) Write the expressions for the current i_0 and the voltage v_0 for t>0. Please clearly show the time constant calculation, initial and steady state current through the inductor. (35pts.)



Problem 3 Solution

$@t=0^-$ The inductor acts as short circuit since all the time derivatives are zero $\frac{d}{dt}(\cdot)\to 0$

$$v_0(t)=L\frac{di_0(t)}{dt} \qquad v_0(t=0^-)=0\text{V}$$



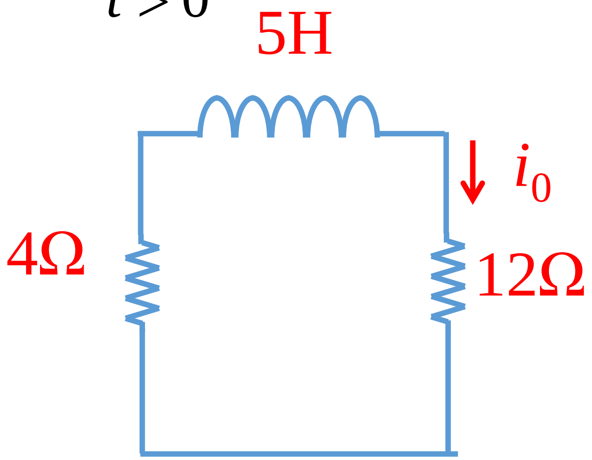
$$i'=\frac{24\text{V}}{(12\Omega\,||\,4\Omega)+5\Omega}=\frac{24}{8}=3\text{A}$$

$$i_0(t=0^-)=i'\frac{(12\Omega\,||\,4\Omega)}{12\Omega}=i'\frac{3}{12}=\frac{3}{4}\text{A}$$

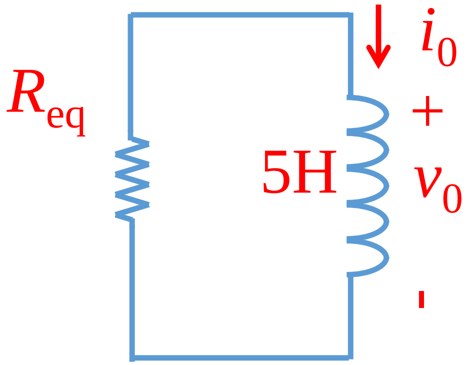
Problem 3: (RL circuit) Write the expressions for the current i_0 and the voltage v_0 for $t>0$. Please clearly show the time constant calculation, initial and steady state current through the inductor. (35pts.)

Problem 3 Solution (cont’d)

$t > 0$



$$R_{eq} = 4\Omega + 12\Omega = 16\Omega$$



The stored energy in the inductor discharges through the resistor in absence of a source.
The time constant is

$$\tau = \frac{L}{R_{eq}} = \frac{5\text{H}}{16\Omega} = 0.3125\text{s}$$

$t \to \infty$ The voltage across the inductor vanishes, and the current decays towards zero $i_0(\infty) = 0\text{A}$

$$i_0(t) = i_0(\infty) + \left[i_0(0) - i_0(\infty)\right]e^{-t/\tau}$$
$$= \frac{3}{4}e^{-t/0.3125}\text{A} = 0.75e^{-3.2t}\text{A} \quad t > 0$$

$$v_0(t) = L\frac{di_0(t)}{dt} = -\frac{1}{\tau}Li_0(t)$$
$$= -12e^{-t/0.3125}\text{A} = -12e^{-3.2t}\text{A} \quad t > 0$$