EECS/CSE 70A Network Analysis I

Homework #5
Solution Key

Problem 1 (20pts)

Part (a): $u = (A + jB)(C + jD) = AC + jAD + jBC - BD$

$$\mathrm{Re}\{u\} = AC - BD$$
$$\mathrm{Im}\{u\} = AD + BC$$

$$r = |u| = \sqrt{u(u^*)} = \sqrt{(\mathrm{Re}\{u\})^2 + (\mathrm{Im}\{u\})^2}$$
$$= \sqrt{(AC - BD)^2 + (AD + BC)^2}$$
$$\phi = \tan^{-1}\frac{\mathrm{Im}\{u\}}{\mathrm{Re}\{u\}} = \tan^{-1}\frac{AD + BC}{AC - BD}$$
$$u = \sqrt{(AC - BD)^2 + (AD + BC)^2}\, e^{j\tan^{-1}\frac{AD+BC}{AC-BD}}$$

$$u = x + jy =$$
$$= (AC - BD) + j(AD + BC)$$

$$\mathrm{Re}\{ue^{j\omega t}\} = \sqrt{(AC - BD)^2 + (AD + BC)^2}\cos\left[\omega t + \tan^{-1}\frac{AD + BC}{AC - BD}\right]$$

Problem 1 cont'd

Part (a) 2nd way:

$$A+jB=\sqrt{A^2+B^2}\,e^{j\tan^{-1}\frac{B}{A}},\quad C+jD=\sqrt{C^2+D^2}\,e^{j\tan^{-1}\frac{D}{C}}$$

$$u=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\,e^{j\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)}$$

$$\mathrm{Re}\{u\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\cos\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)$$

$$\mathrm{Im}\{u\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\sin\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)$$

$$r=|u|=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}$$

$$\phi=\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}$$

$$\mathrm{Re}\left\{ue^{j\omega t}\right\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\cos\left[\omega t+\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right]$$

Problem 1 cont'd

Part (b): $u = \frac{A + jB}{C + jD} = \frac{(A + jB)(C + jD)^*}{(C + jD)(C + jD)^*} = \frac{(A + jB)(C - jD)}{C^2 + D^2} = \frac{AC - jAD + jBC + BD}{C^2 + D^2}$

$$\mathrm{Re}\{u\} = \frac{AC + BD}{C^2 + D^2}$$

$$\mathrm{Im}\{u\} = \frac{-AD + BC}{C^2 + D^2}$$

$$u = x + jy = \\ = \left(\frac{AC + BD}{C^2 + D^2}\right) + j\left(\frac{-AD + BC}{C^2 + D^2}\right)$$

$$r = |u| = \frac{\sqrt{(AC + BD)^2 + (-AD + BC)^2}}{C^2 + D^2}$$

$$\phi = \tan^{-1}\frac{\mathrm{Im}\{u\}}{\mathrm{Re}\{u\}} = \tan^{-1}\frac{-AD + BC}{AC + BD}$$

$$u = \frac{\sqrt{(AC + BD)^2 + (-AD + BC)^2}}{C^2 + D^2} e^{j\tan^{-1}\frac{-AD + BC}{AC + BD}}$$

$$\mathrm{Re}\left\{ue^{j\omega t}\right\} = \frac{\sqrt{(AC + BD)^2 + (-AD + BC)^2}}{C^2 + D^2}\cos\left[\omega t + \tan^{-1}\frac{-AD + BC}{AC + BD}\right]$$

Problem 1 cont'd

Part (b) 2nd way:

$$A + jB = \sqrt{A^2 + B^2}\, e^{j\tan^{-1}\frac{B}{A}}, \quad C + jD = \sqrt{C^2 + D^2}\, e^{j\tan^{-1}\frac{D}{C}}$$

$$u = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}\, e^{j\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)}$$

$$\operatorname{Re}\{u\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \cos\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)$$

$$\operatorname{Im}\{u\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \sin\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)$$

$$r = |u| = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}$$

$$\phi = \tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}$$

$$\operatorname{Re}\left\{u e^{j\omega t}\right\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \cos\left[\omega t + \tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right]$$

Problem 2 (20pts)

(a) Convert the phasor $V = 5 + j3$ to time domain expression $v(t)$.

$$|V| = |5 + j3| = \sqrt{5^2 + 3^2} = \sqrt{34} \approx 5.83$$

$$\tan^{-1}\frac{3}{5} \approx 0.54\text{rad} \approx 31°$$

$$V = 5.83e^{j0.54}$$

$$v(t) = \text{Re}\left\{Ve^{j\omega t}\right\} =$$
$$= 5.83\cos(\omega t + 0.54)$$

(b) Convert the phasor $I = 15 - j8$ to time domain expression $i(t)$.

$$|I| = |15 - j8| = \sqrt{15^2 + (-8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

$$\tan^{-1}\frac{-8}{15} \approx -0.49\text{rad} \approx -28°$$

$$I = 17e^{-j0.49}$$

$$i(t) = \text{Re}\left\{Ie^{j\omega t}\right\} =$$
$$= 17\cos(\omega t - 0.49)$$

Problem 2 cont'd

(c) Convert $v(t) = 12\sin\left(\omega t - \frac{\pi}{6}\right)$ to the phasor domain both in Cartesian and polar forms $V = x + jy = re^{i\phi}$

$$v(t) = 12\cos\left(\omega t - \frac{\pi}{6} - \frac{\pi}{2}\right) = 12\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$V = 12e^{-j\frac{2\pi}{3}} = 12\underbrace{\cos\left(-\frac{2\pi}{3}\right)}_{-0.5} + j12\underbrace{\sin\left(-\frac{2\pi}{3}\right)}_{-\sqrt{3}/2=-0.866} \approx -6 - j10.4$$

(d) Convert $i(t) = 4\cos\left(\omega t + \frac{\pi}{4}\right)$ to the phasor domain both in Cartesian and polar forms $I = x + jy = re^{i\phi}$

$$I = 4e^{j\frac{\pi}{4}} = 4\underbrace{\cos\left(\frac{\pi}{4}\right)}_{\sqrt{2}/2} + j4\underbrace{\sin\left(\frac{\pi}{4}\right)}_{\sqrt{2}/2=0.707} \approx 2.8284 - j2.8384$$

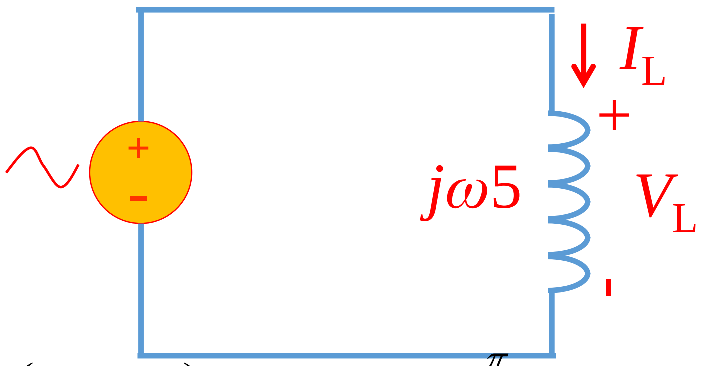
Problem 3 (30pts.)

Part (a): Find the current $i_L(t)$ at the frequency 80Hz.

$$v(t) = 4\cos\left(\omega t - \frac{\pi}{3}\right)$$

Convert to phasor domain

$$V = 4e^{-j\frac{\pi}{3}}$$



$$v(t) = 4\cos\left(\omega t - \frac{\pi}{3}\right) \Leftrightarrow V = 4e^{-j\frac{\pi}{3}}$$

$$V_L = V$$

$$V_L = I_L(j\omega L) \qquad \omega = 2\pi f = 2\pi 80 \text{ rad/s} = 160\pi \text{ rad/s}$$

$$I_L = \frac{V}{j\omega L} = \frac{4e^{-j\frac{\pi}{3}}}{j800\pi} = \frac{1}{200\pi} e^{j\left(-\frac{\pi}{3} - \frac{\pi}{2}\right)} = \frac{1}{200\pi} e^{-j\frac{5\pi}{6}} \text{ A}$$

$$i_L(t) = \text{Re}\left\{I_L e^{j\omega t}\right\} = \text{Re}\left\{\frac{1}{200\pi} e^{-j\frac{5\pi}{6}} e^{j160\pi t}\right\} =$$

$$= \frac{1}{200\pi}\cos\left(160\pi t - \frac{5\pi}{6}\right) \text{A} =$$

$$= \frac{1}{628.3}\cos(502.7t - 150°) \text{A} =$$

$$= 0.0016\cos(502.7t - 150°) \text{A}$$

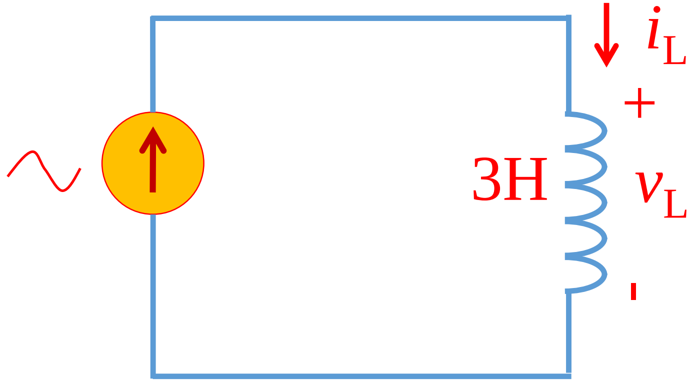
Other answer can be found with:

$$\sin(x) = \cos(x - \pi/2), \; -\sin(x) = \sin(x \pm \pi), \; -\cos(x) = \cos(x \pm \pi)$$

Problem 3 (30pts.)

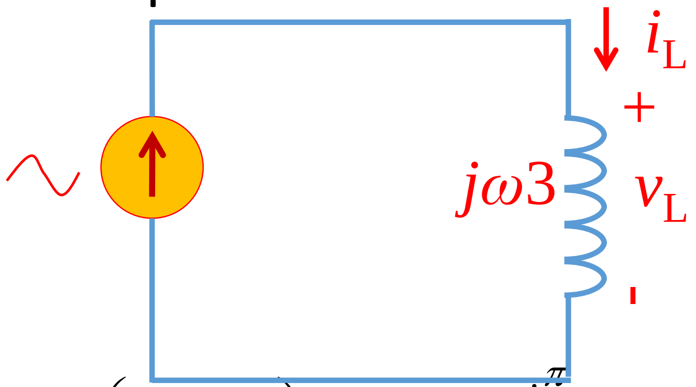
Part (b): Find the voltage $v_L(t)$ at the frequency 30Hz.

$$i(t) = 7\cos\left(\omega t + \frac{\pi}{8}\right)$$



Convert to phasor domain

$$I = 7e^{j\frac{\pi}{8}}$$



$$i(t) = 7\cos\left(\omega t + \frac{\pi}{8}\right) \Leftrightarrow I = 7e^{j\frac{\pi}{8}}$$

$$I_L = I$$

$$V_L = I_L\left(j\omega L\right) \qquad \omega = 2\pi f = 2\pi 30 \text{ rad/s} = 60\pi \text{ rad/s}$$

$$= 7e^{j\frac{\pi}{8}}\left(j180\pi\right) = 1260\pi e^{j\left(\frac{\pi}{8}+\frac{\pi}{2}\right)} = 1260\pi e^{j\frac{5\pi}{8}} \text{ V}$$

$$v_L(t) = \mathrm{Re}\left\{V_L e^{j\omega t}\right\} = \mathrm{Re}\left\{1260 p e^{j\frac{5\pi}{8}} e^{j60\pi t}\right\} =$$

$$= 1260\pi\cos\left(60\pi t + \frac{5\pi}{8}\right)\text{V} =$$

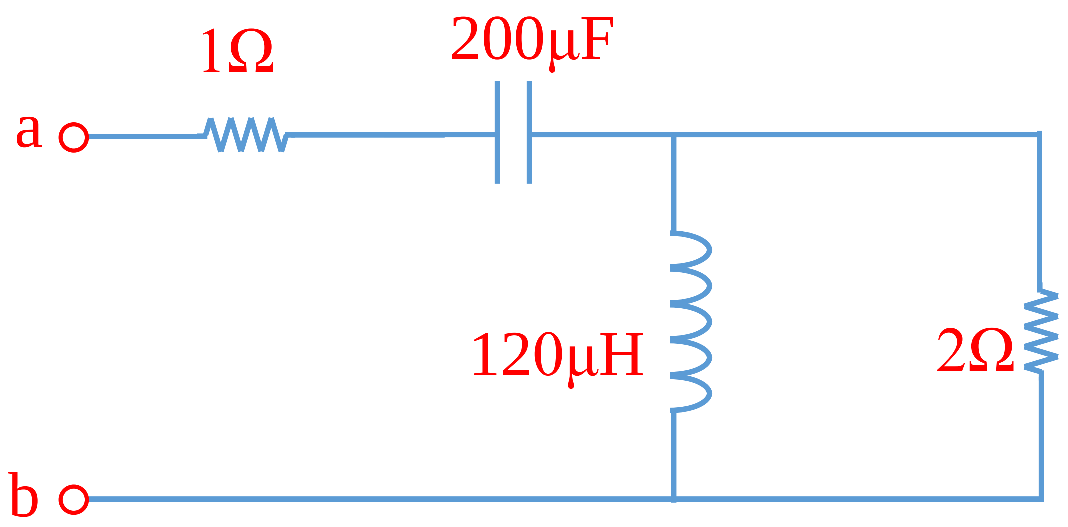
$$= 3958\cos\left(188.5t - 112.5°\right)\text{V}$$

Other answer can be found with:

$$\sin(x) = \cos(x - \pi/2), \quad -\sin(x) = \sin(x \pm \pi), \quad -\cos(x) = \cos(x \pm \pi)$$

Problem 4 (30pts.)

Part (a): Find the impedance seen from terminals a-b as a function of the angular frequency ω.



$$Z_{eq} = 1\Omega + \frac{1}{j\omega 200\mu\text{F}} + \left(j\omega 120\mu\text{H} \,||\, 2\Omega \right)$$

$$= 1 + \frac{1}{j\omega 200\times 10^{-6}} + \frac{2 j\omega 120\times 10^{-6}}{2 + j\omega 120\times 10^{-6}}$$

$$= 1 - j\frac{1}{\omega 200\times 10^{-6}} + \frac{2 j\omega 120\times 10^{-6}\left(2 - j\omega 120\times 10^{-6}\right)}{4 + \left(\omega 120\times 10^{-6}\right)^2}$$

$$= 1 - j\frac{1}{\omega 200\times 10^{-6}} + \frac{\omega 2\left(120\times 10^{-6}\right)^2 + j\omega 480\times 10^{-6}}{4 + \left(\omega 120\times 10^{-6}\right)^2}$$

Part (b): Evaluate the impedance at 750Hz

$$\omega = 2\pi 750 \text{ rad/s} = 1500\pi \text{ rad/s}$$

$$Z_{eq} = 1.1481 + j0.5374 \ \Omega$$

Part (c): Evaluate the impedance at 3kHz

$$\omega = 2\pi 3000 \text{ rad/s} = 6000\pi \text{ rad/s}$$

$$Z_{eq} = 2.1225 + j0.7272 \ \Omega$$