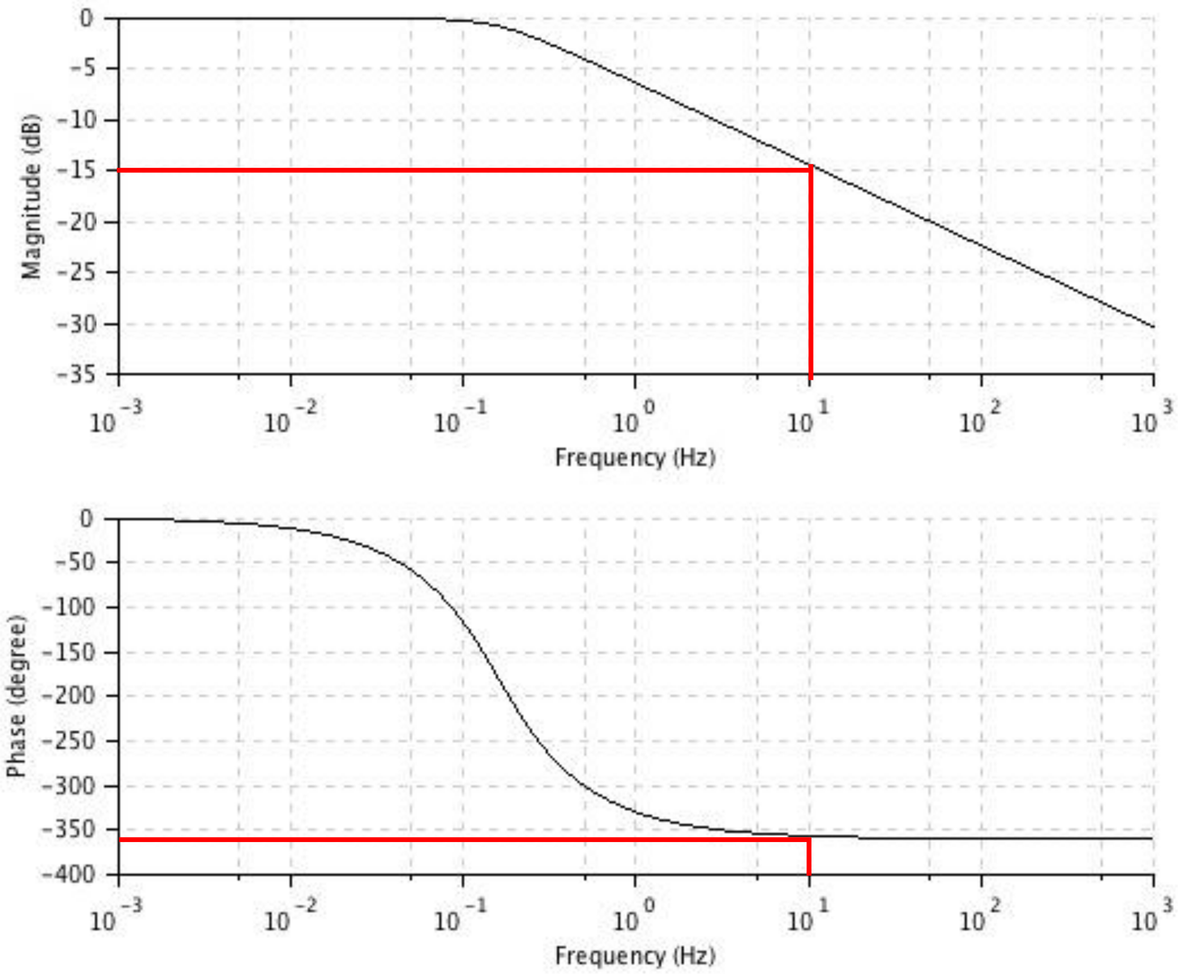
EECS/CSE 70A Network Analysis I

Homework #6
Solution Key

Problem 1 (30pts.)

The Bode plots show the magnitude and phase of the transfer function of a circuit. The input voltage $v_{in}(t) = 1\text{ mV}\cos([2\pi\cdot 10\text{Hz}]t)$. Find the output voltage $v_{out}(t)$.



$$v_{\text{in}} \underset{\text{to phasor}}{\Rightarrow} V_{\text{in}} = 1\,\text{mV}$$

$$V_{\text{out}} = H\left(\omega = 2\pi\cdot 10\text{Hz}\right)V_{\text{in}}$$

From the graphs :

$$\left|H\left(\omega = 2\pi\cdot 10\text{Hz}\right)\right|_{\text{dB}} = -15\text{dB},$$

$$\left|H\left(\omega = 2\pi\cdot 10\text{Hz}\right)\right| = 10^{-15/20} \approx 0.18$$

$$\arg\left[H\left(\omega = 2\pi\cdot 10\text{Hz}\right)\right] = -360° = 0° \equiv 0\text{ rad}$$

$$H\left(\omega = 2\pi\cdot 10\text{Hz}\right) \approx 0.18e^{j0} = 0.18$$

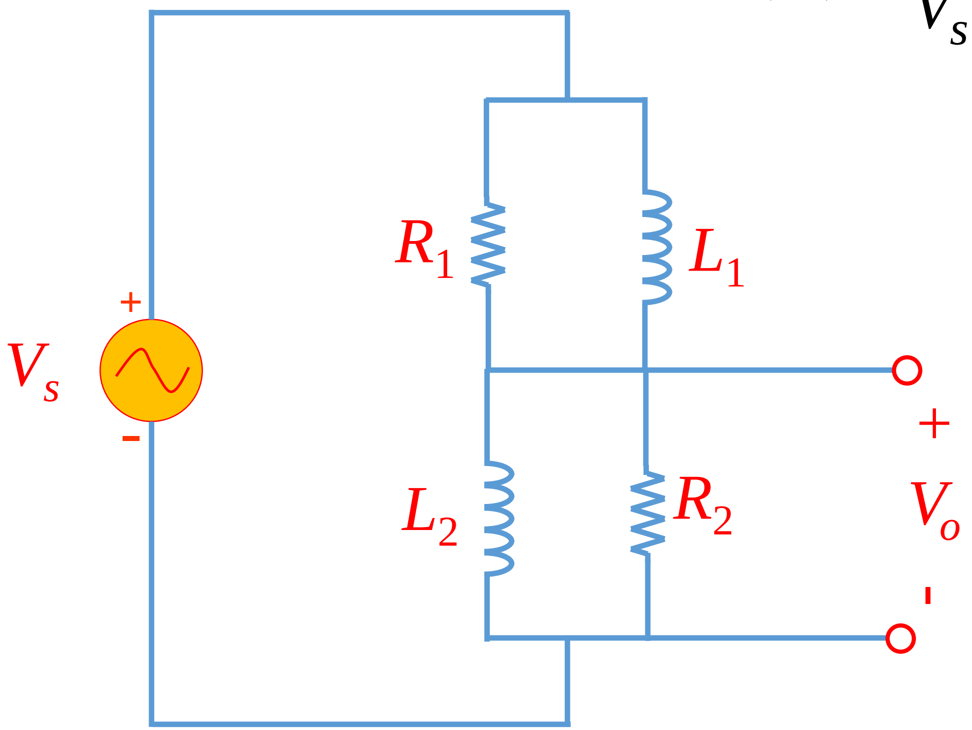
$$V_{\text{out}} = 0.18\text{ mV} \Rightarrow v_{\text{out}}(t) = \text{Re}\left\{V_{out}e^{j(2\pi\cdot 10)t}\right\}$$

$$v_{\text{out}}(t) = 0.18\cos\left(\left[\omega = 2\pi\cdot 10\text{Hz}\right]t\right)\text{ mV}$$

Problem 2 (35pts.)

Find the transfer function $H(\omega)$ in terms of R_1, R_2, L_1 and L_2. And simplify $|H(\omega)|$ at $\omega = 0$ and as $\omega \longrightarrow \infty$.

$$H(\omega) = \frac{V_o}{V_s}$$



By voltage division

$$V_0 = V_s \frac{R_2 \parallel j\omega L_2}{(R_1 \parallel j\omega L_1) + (R_2 \parallel j\omega L_2)}$$

$$H(\omega) = \frac{R_2 \parallel j\omega L_2}{(R_1 \parallel j\omega L_1) + (R_2 \parallel j\omega L_2)}$$

$$= \frac{\dfrac{j\omega L_2 R_2}{R_2 + j\omega L_2}}{\dfrac{j\omega L_1 R_1}{R_1 + j\omega L_1} + \dfrac{j\omega L_2 R_2}{R_2 + j\omega L_2}} = \frac{\dfrac{L_2 R_2}{R_2 + j\omega L_2}}{\dfrac{L_1 R_1}{R_1 + j\omega L_1} + \dfrac{L_2 R_2}{R_2 + j\omega L_2}}$$

Problem 2 (35pts.) cont'd

$$\left|H(\omega=0)\right| = \left| \frac{\dfrac{L_2R_2}{R_2 + j0L_2}}{\dfrac{L_1R_1}{R_1 + j0L_1} + \dfrac{L_2R_2}{R_2 + j0L_2}} \right| = \left| \frac{\dfrac{L_2R_2}{R_2}}{\dfrac{L_1R_1}{R_1} + \dfrac{L_2R_2}{R_2}} \right| = \boxed{\frac{L_2}{L_1 + L_2}}$$

Let us make an algebraic manipulation as

$$H(\omega) = \frac{\dfrac{\frac{1}{j\omega}L_2R_2}{\frac{1}{j\omega}(R_2 + j\omega L_2)}}{\dfrac{\frac{1}{j\omega}L_1R_1}{\frac{1}{j\omega}(R_1 + j\omega L_1)} + \dfrac{\frac{1}{j\omega}L_2R_2}{\frac{1}{j\omega}(R_2 + j\omega L_2)}} = \frac{\dfrac{L_2R_2}{\frac{R_2}{j\omega} + L_2}}{\dfrac{L_1R_1}{\frac{R_1}{j\omega} + L_1} + \dfrac{L_2R_2}{\frac{R_2}{j\omega} + L_2}}$$

$$\lim_{\omega\to\infty}\left|H(\omega)\right| = \left| \frac{\dfrac{L_2R_2}{0 + L_2}}{\dfrac{L_1R_1}{0 + L_1} + \dfrac{L_2R_2}{0 + L_2}} \right| = \boxed{\frac{R_2}{R_1 + R_2}}$$

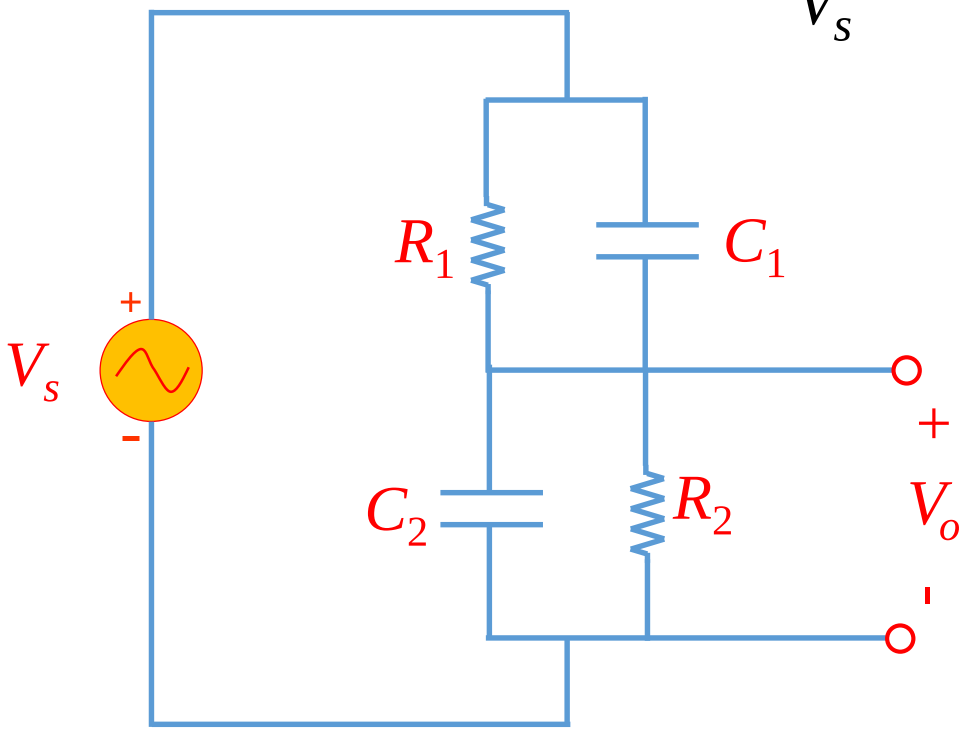
The answers are expected since the inductors approach to: (i) short circuit , i.e. lowest impedance path, as the frequency decreases; and (ii) open circuit as frequency increases.

Problem 3 (35pts.)

Find the transfer function $H(\omega)$ in terms of R_1, R_2, C_1 and C_2. And simplify $|H(\omega)|$ at $\omega = 0$ and as $\omega \to \infty$.

$$H(\omega) = \frac{V_o}{V_s}$$

Similarly as in Problem 2



$$H(\omega) = \frac{R_2 \parallel \dfrac{1}{j\omega C_2}}{\left(R_1 \parallel \dfrac{1}{j\omega C_1}\right) + \left(R_2 \parallel \dfrac{1}{j\omega C_2}\right)}$$

$$= \frac{\dfrac{\dfrac{R_2}{j\omega C_2}}{R_2 + \dfrac{1}{j\omega C_2}}}{\dfrac{\dfrac{R_2}{j\omega C_2}}{R_2 + \dfrac{1}{j\omega C_2}} + \dfrac{\dfrac{R_1}{j\omega C_1}}{R_1 + \dfrac{1}{j\omega C_1}}} = \frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{R_1}{1 + j\omega C_1 R_1} + \dfrac{R_2}{1 + j\omega C_2 R_2}}$$

Problem 3 (35pts.) cont'd

$$\left|H(\omega=0)\right| = \left|\frac{\dfrac{R_2}{1+j0C_2R_2}}{\dfrac{R_1}{1+j0C_1R_1}+\dfrac{R_2}{1+j0C_2R_2}}\right| = \boxed{\frac{R_2}{R_1+R_2}}$$

Let us make an algebraic manipulation as

$$H(\omega) = \frac{\dfrac{\frac{1}{j\omega}R_2}{\frac{1}{j\omega}(1+j\omega C_2R_2)}}{\dfrac{\frac{1}{j\omega}R_1}{\frac{1}{j\omega}(1+j\omega C_1R_1)}+\dfrac{\frac{1}{j\omega}R_2}{\frac{1}{j\omega}(1+j\omega C_2R_2)}} = \frac{\dfrac{R_2}{\frac{1}{j\omega}+C_2R_2}}{\dfrac{R_1}{\frac{1}{j\omega}+C_1R_1}+\dfrac{R_2}{\frac{1}{j\omega}+C_2R_2}}$$

$$\lim_{\omega\to\infty}\left|H(\omega)\right| = \left|\frac{\dfrac{R_2}{0+C_2R_2}}{\dfrac{R_1}{0+C_1R_1}+\dfrac{R_2}{0+C_2R_2}}\right| = \boxed{\frac{\dfrac{1}{C_2}}{\dfrac{1}{C_1}+\dfrac{1}{C_2}}}$$

The answers are expected since the capacitors approach to: (i) open circuit as frequency decreases; and (ii) short circuit, i.e. lowest impedance path, as the frequency increases.