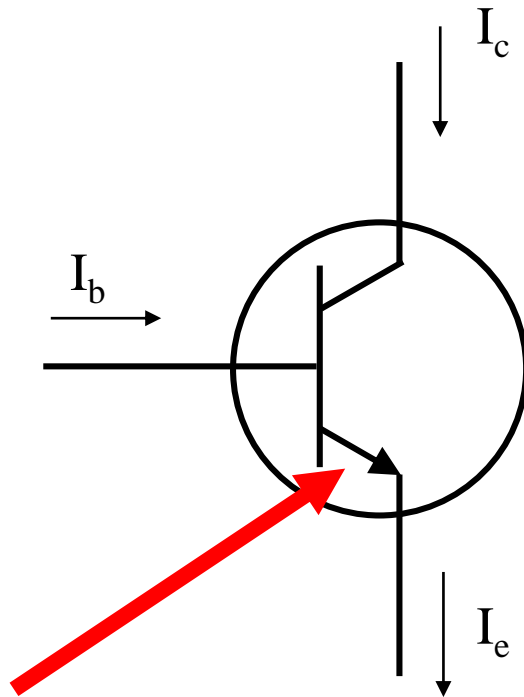


# Lecture 5: HBT AC properties

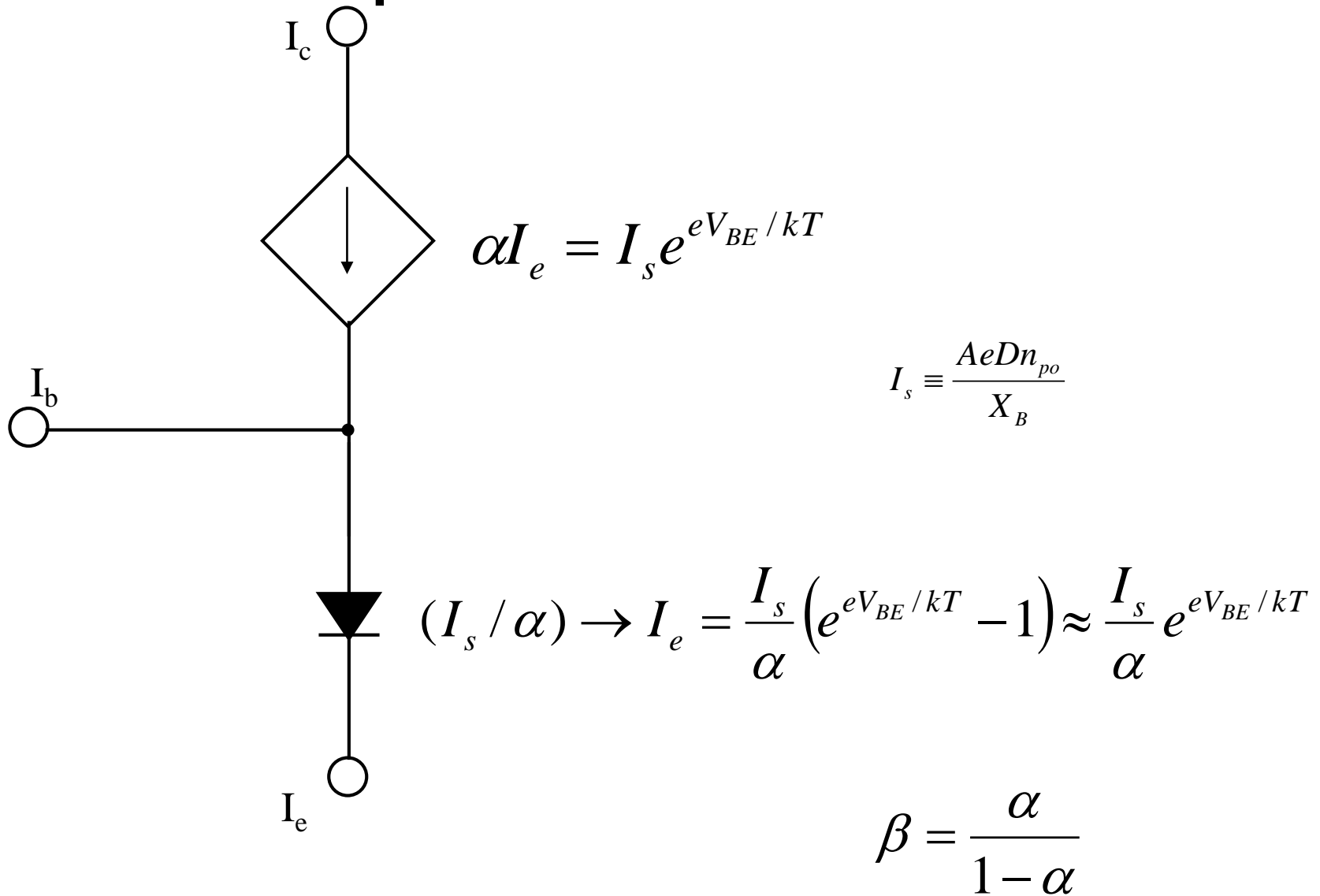
# “Normal active” bias



Like a diode.

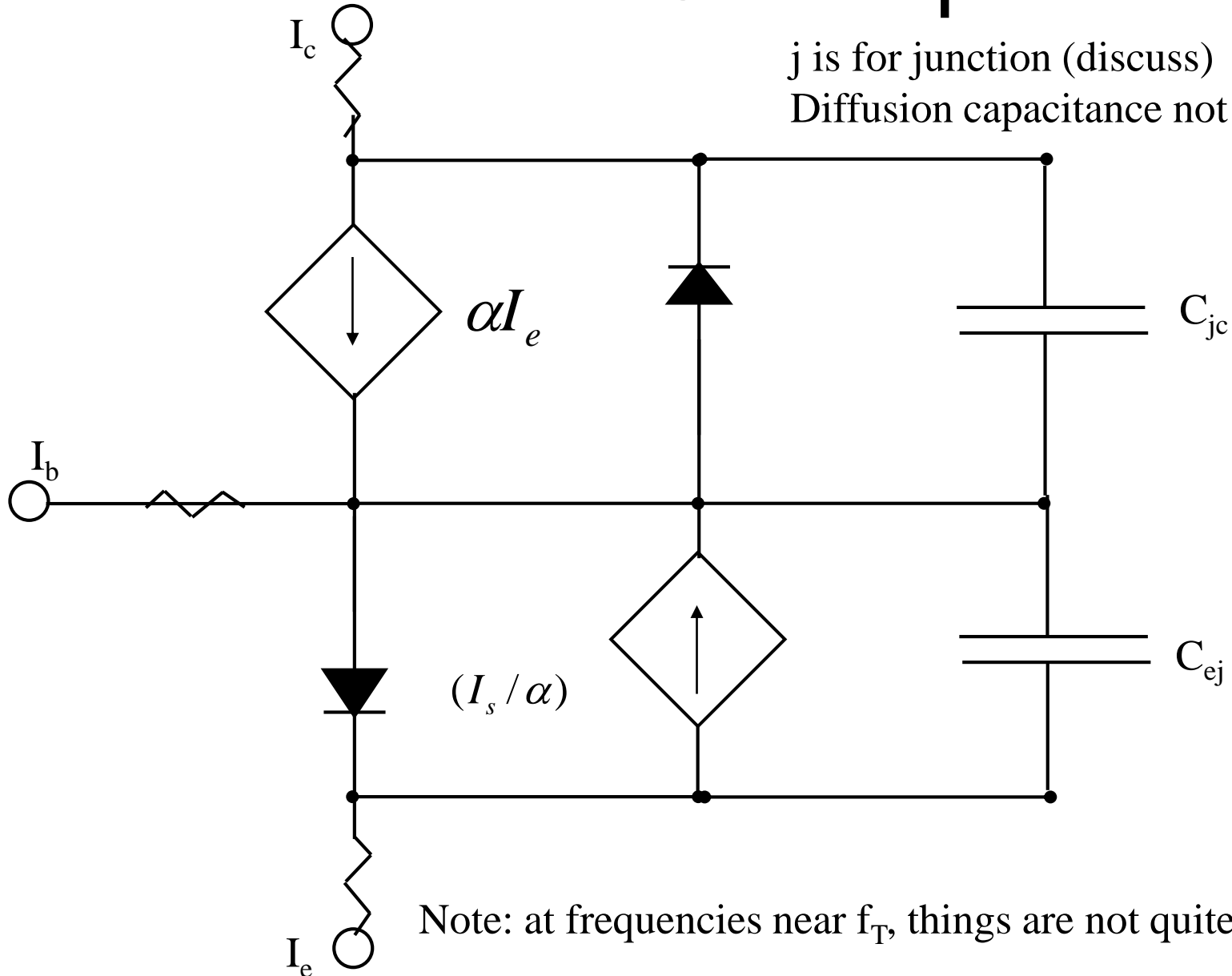
- E-B forward bias  
( $V_b > V_e$ )
- C-B reverse bias  
( $V_c > V_b$ )
- $I_{ce} = 100 I_{be} = \beta I_{be}$

# Equivalent circuit



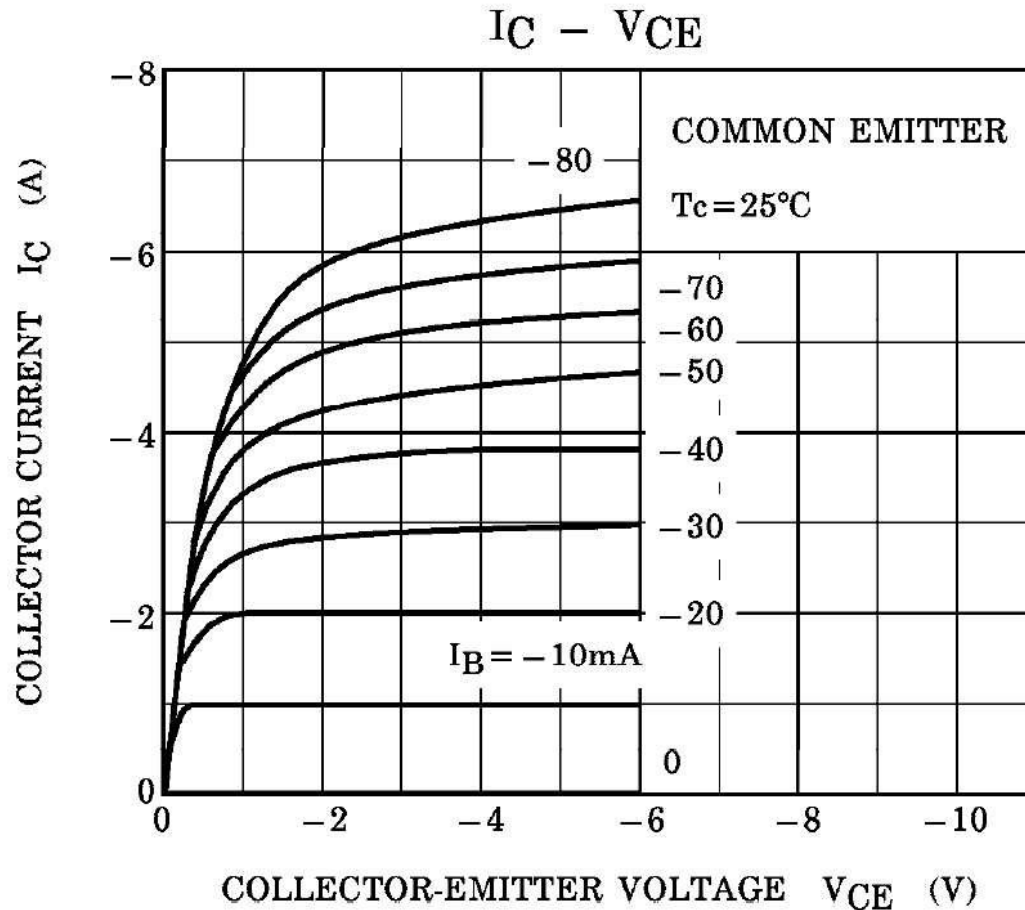
# E-M model AC with parasitics

j is for junction (discuss)  
Diffusion capacitance not shown.



Note: at frequencies near  $f_T$ , things are not quite this simple

# Global dc properties

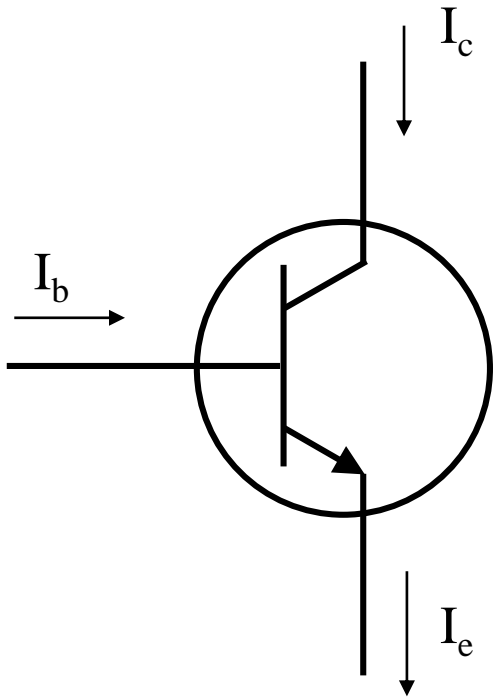


Note  
Early effect.

It is assumed you know this, so it is rare to see on data sheets!

<http://www.toshiba.com/taec/components/Datasheet/2SA1244DS.pdf>

# ac properties: notation



$I_E$  dc

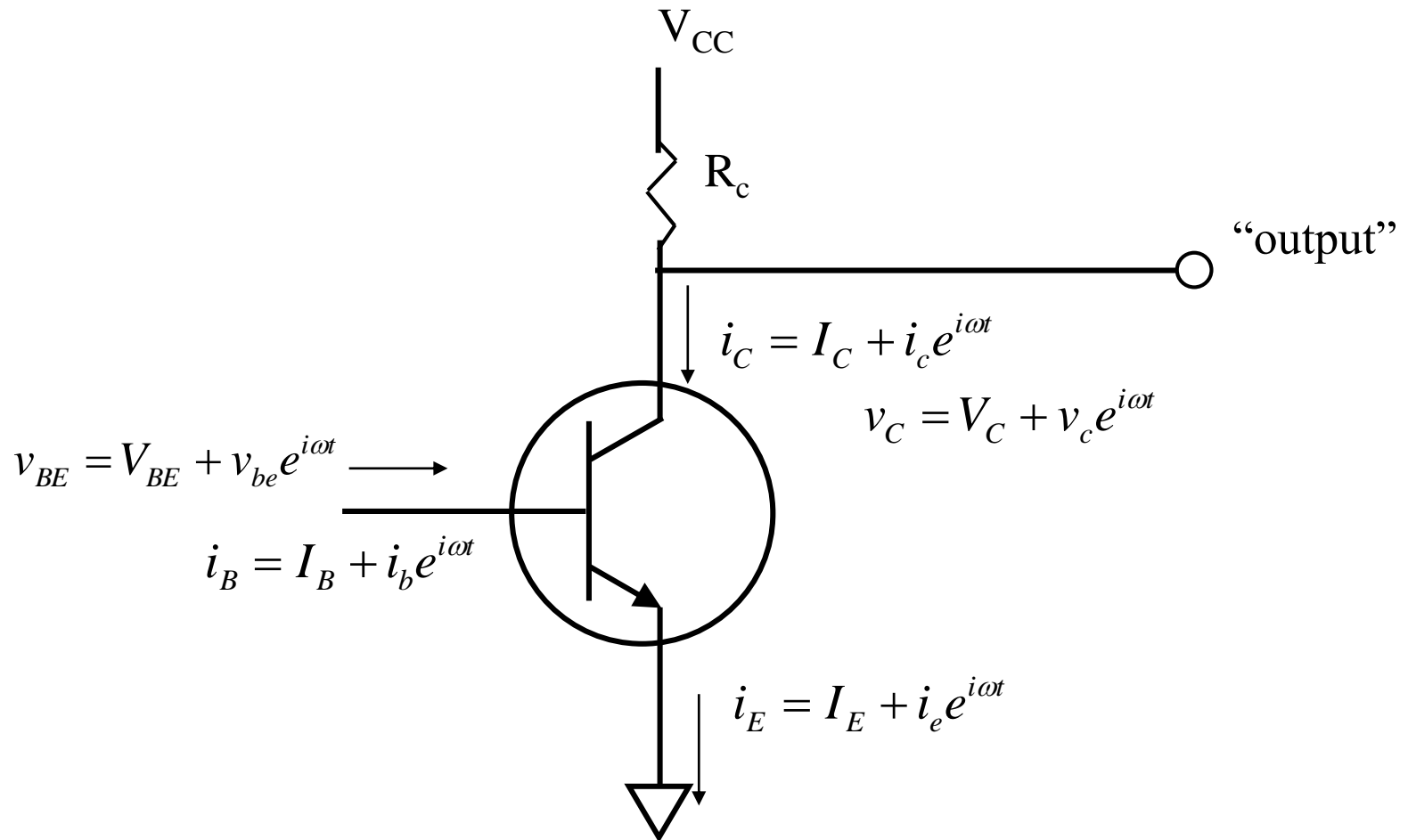
$i_e$  ac

$$i_E \text{ total} = I_E + i_e$$

We will use equivalent circuit #1 (implicitly).

# ac properties

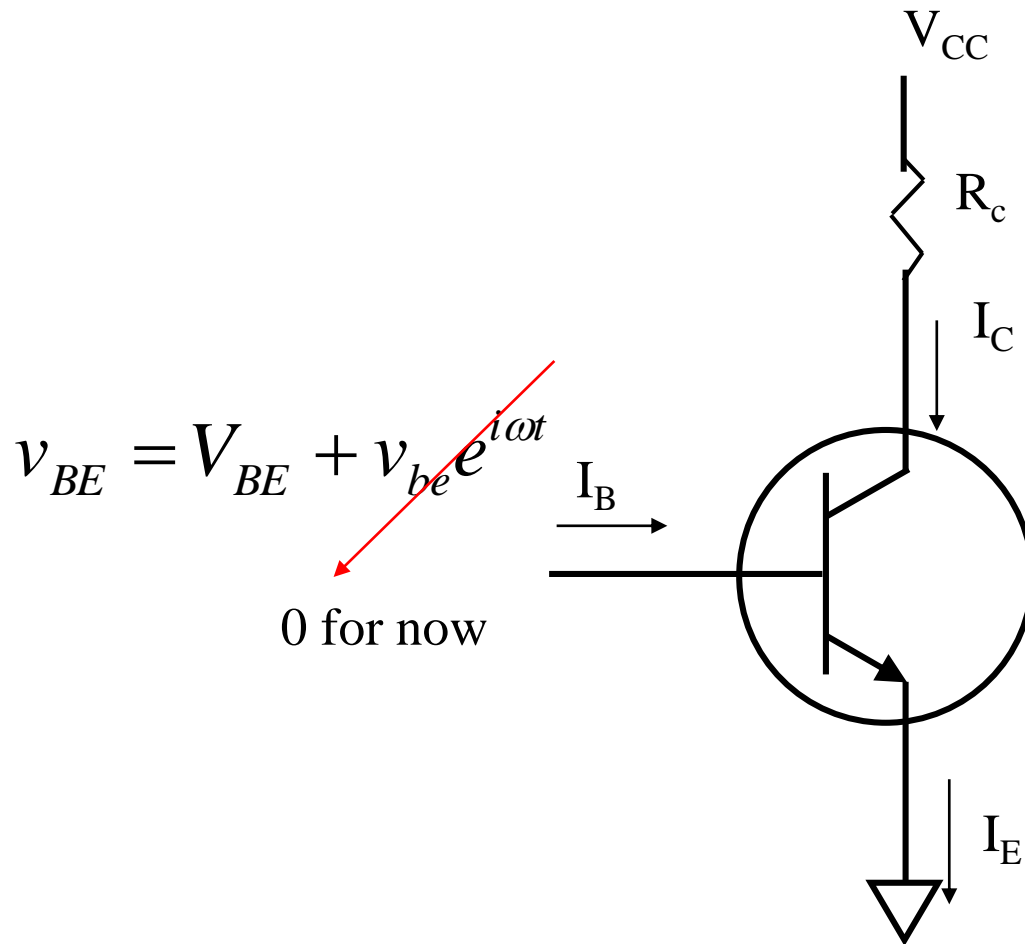
common-emitter configuration



Note: three terminal device has three-terminal equivalent ac circuit.

# dc analysis

common-emitter configuration



$$I_C = I_S e^{eV_{BE}/kT}$$

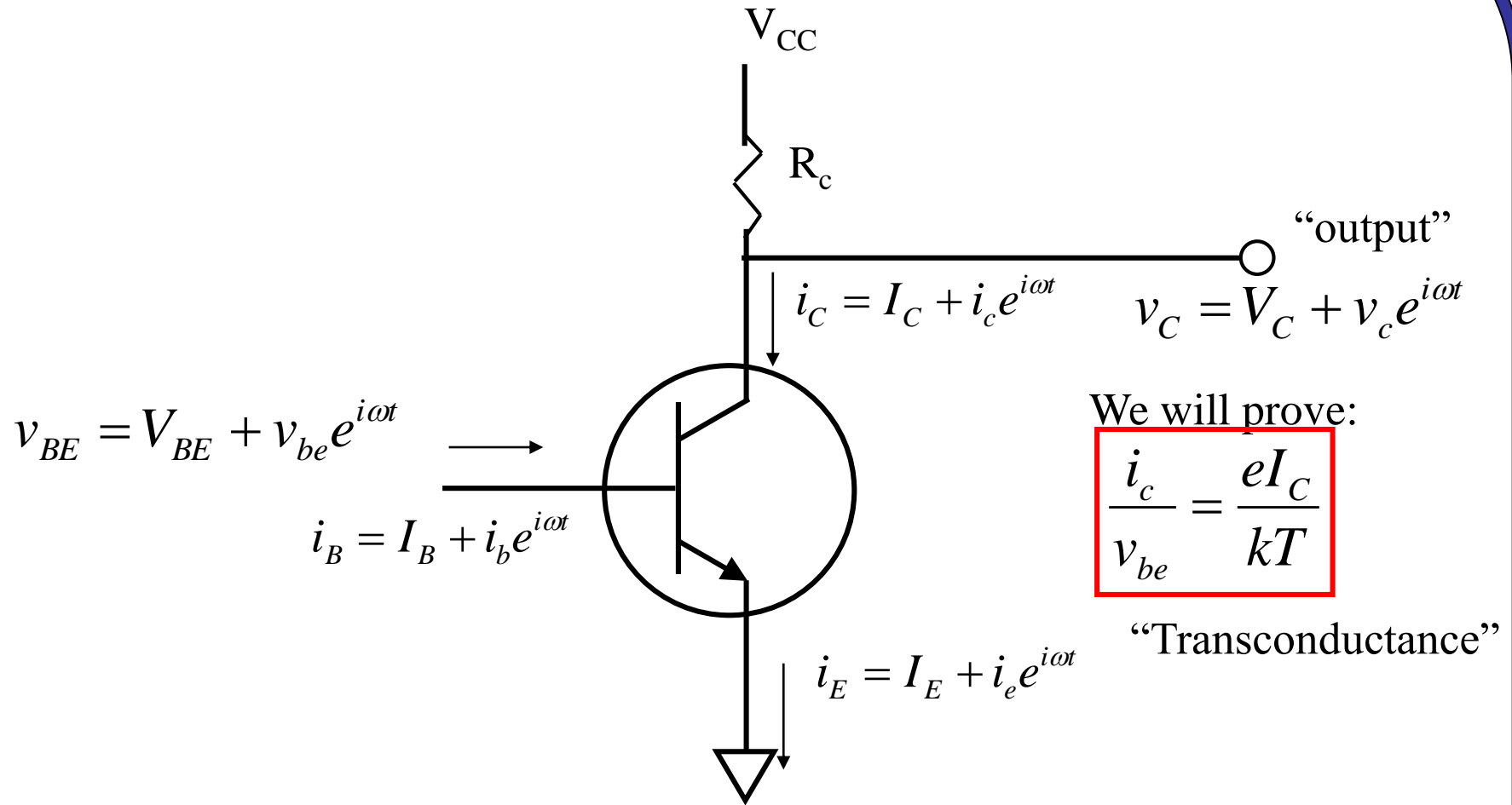
$$I_E = I_C / \alpha$$

$$I_B = I_C / \beta$$

$$V_C = V_{CE} = V_{CC} - I_C R_C$$

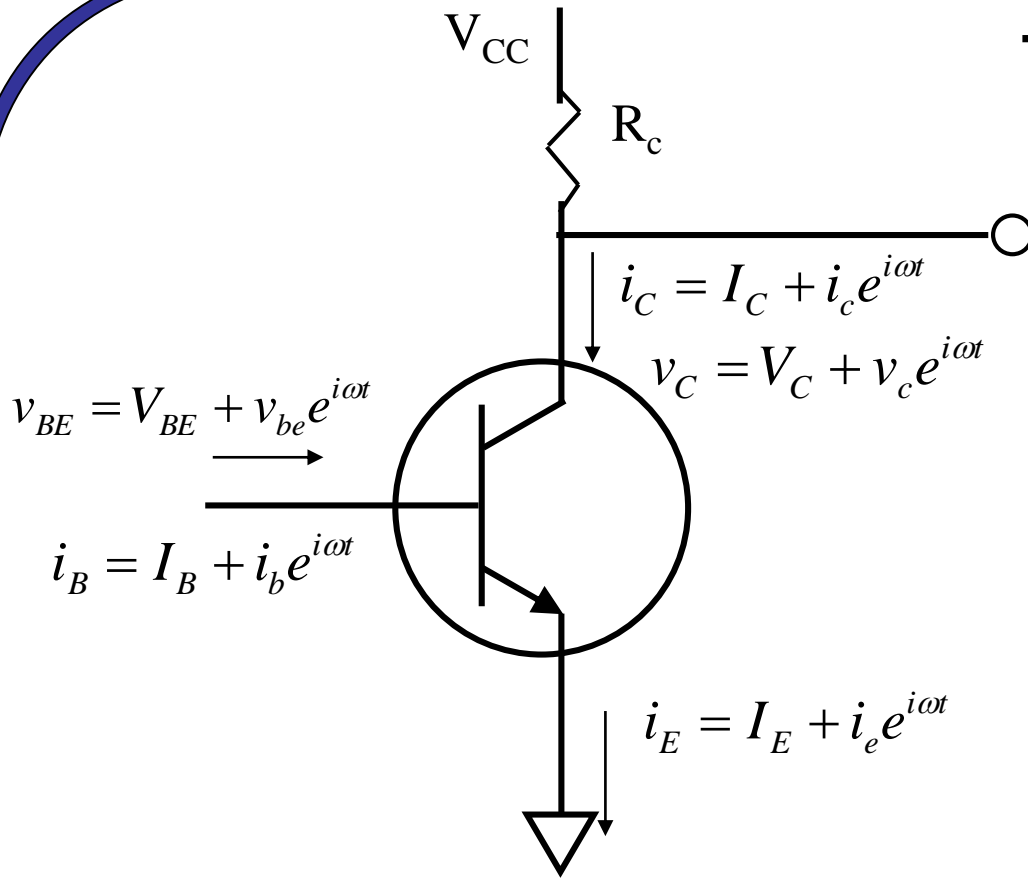


# ac analysis



Note: three terminal device has three-terminal equivalent ac circuit.

# Transconductance



$$v_{BE} = V_{BE} + v_{be} e^{i\omega t}$$

$$i_C = I_S e^{e v_{BE} / kT}$$

$$= I_S e^{e(V_{BE} + v_{be} e^{i\omega t}) / kT}$$

$$= I_S e^{e V_{BE} / kT} e^{e v_{be} e^{i\omega t} / kT}$$

$$= I_C e^{e v_{be} e^{i\omega t} / kT}$$

$$e^x \approx 1 + x \text{ for small } x$$

$$i_C \approx I_C \left( 1 + \frac{e v_{be}}{kT} e^{i\omega t} \right)$$

Bu

$$i_c = I_C + i_c e^{i\omega t}$$

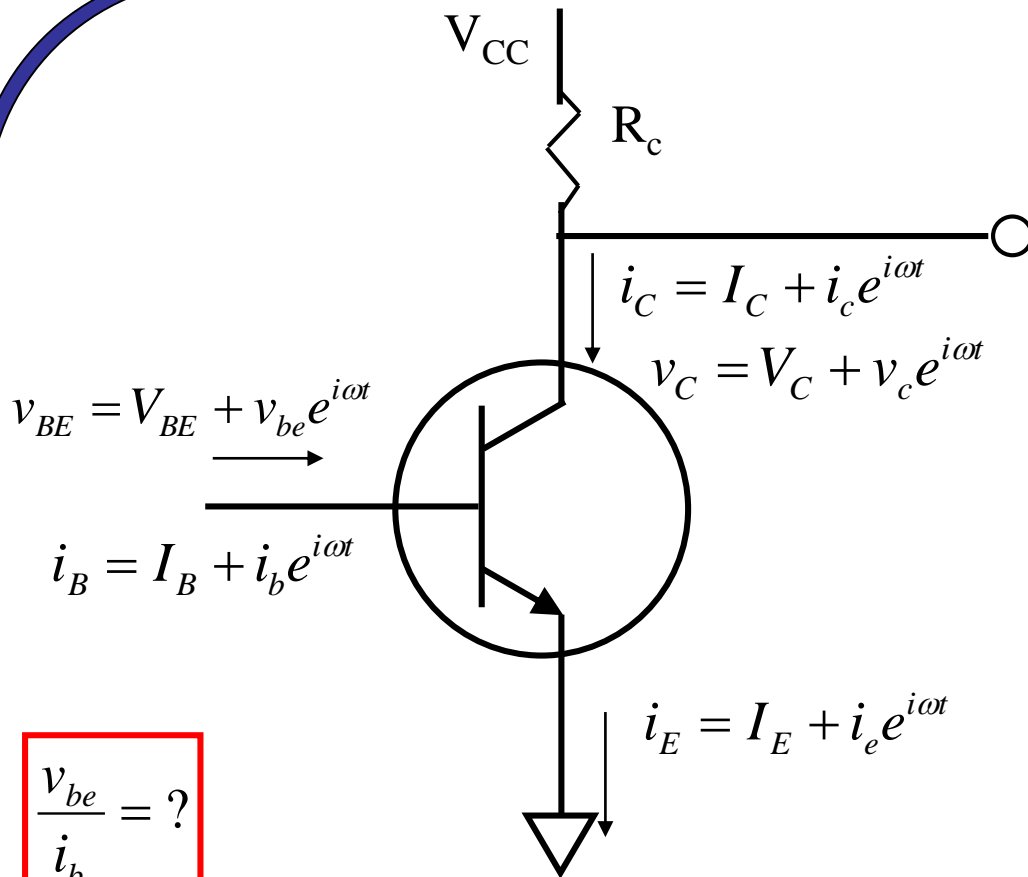
So

$$i_c = I_C \frac{e v_{be}}{kT} \Rightarrow \frac{i_c}{v_{be}} = \frac{e I_C}{kT}$$

Typical number is 40 mA/V.

$g_m$

# Input impedance



$$i_B = \frac{i_c}{\beta} = \frac{I_C + i_c e^{i\omega t}}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta} I_C \frac{e v_{be}}{kT} e^{i\omega t}$$

But

$$i_B = I_B + i_b e^{i\omega t}$$

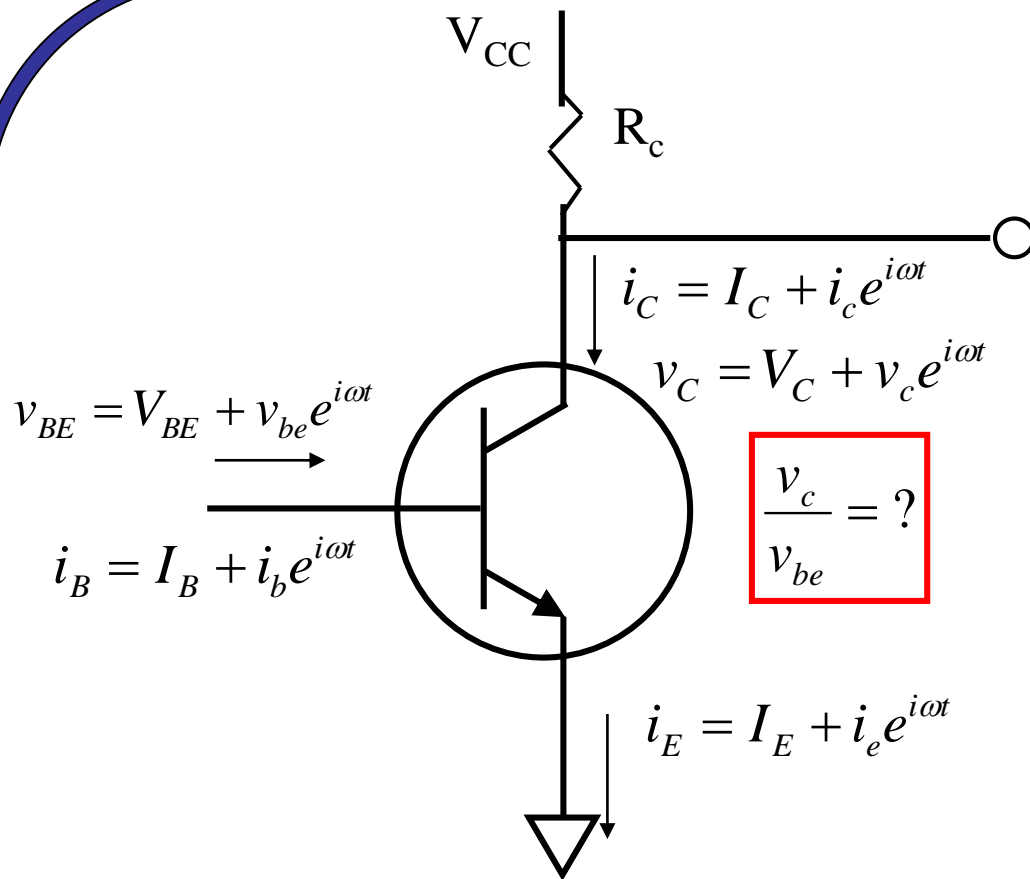
So

$$i_b = \frac{1}{\beta} I_C \frac{e v_{be}}{kT} = \frac{g_m}{\beta} v_{be}$$

So

$$\frac{v_{be}}{i_b} = \frac{g_m}{\beta}$$

What is typical input impedance?



## Gain

$$\begin{aligned}
 v_C &= V_{CC} - i_C R_C \\
 &= V_{CC} - (I_C + i_c e^{i\omega t}) R_C \\
 &= (V_{CC} - I_C R_C) + i_c R_C e^{i\omega t} \\
 &= V_C + i_c R_C e^{i\omega t}
 \end{aligned}$$

Bu

$$v_C = V_C + v_c e^{i\omega t}$$

So

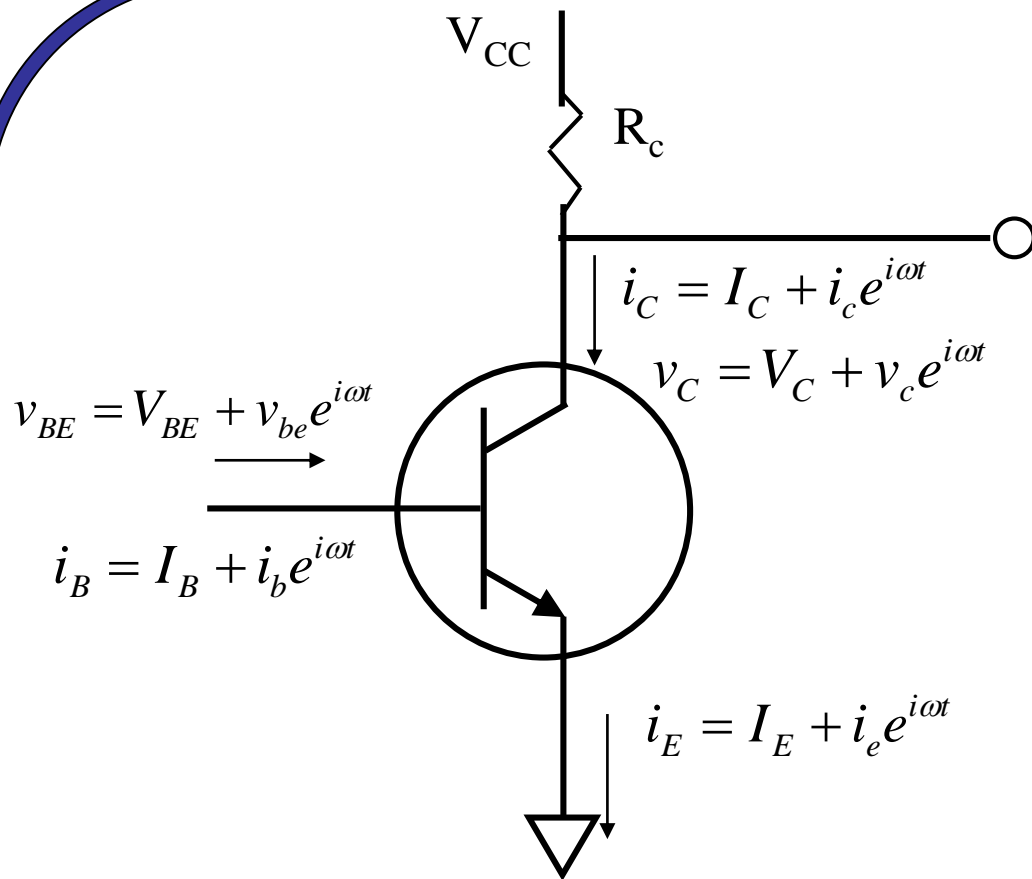
:

$$v_C = -i_c R_C = -g_m v_{be} R_C$$

So

$$\frac{v_c}{v_{be}} = g_m R_C$$

What is typical gain?



# Summary

$$i_b = \frac{g_m}{\beta} v_{be} \quad \text{input imp.}$$

$$i_c = \frac{eI_C}{kT} v_{be} \quad \text{transcond.}$$

In *matrix* form:

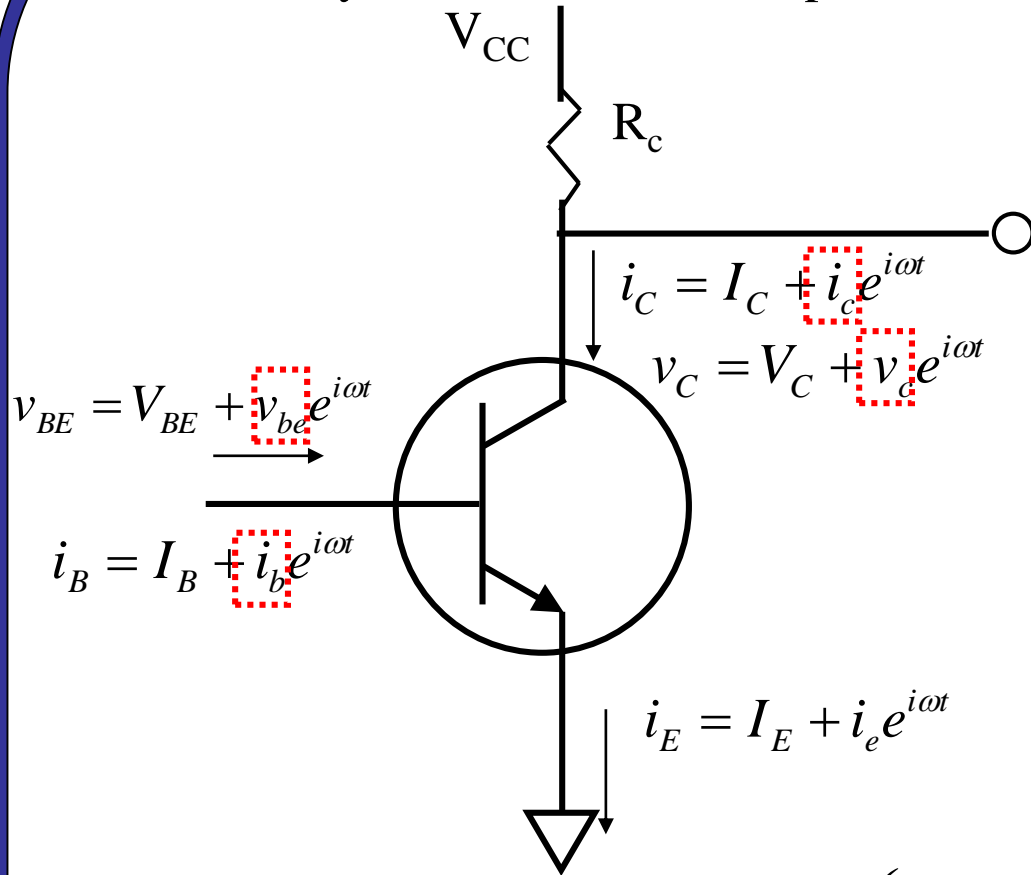
$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$



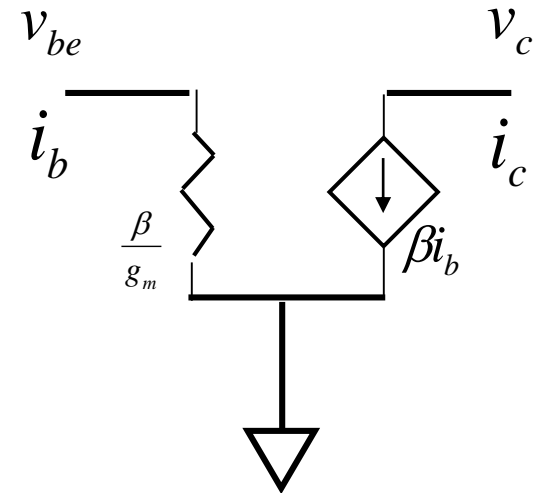
Admittance (*Y*) *matrix*

# AC equivalent circuit:

If we are only interested in ac components, life can be simplified:



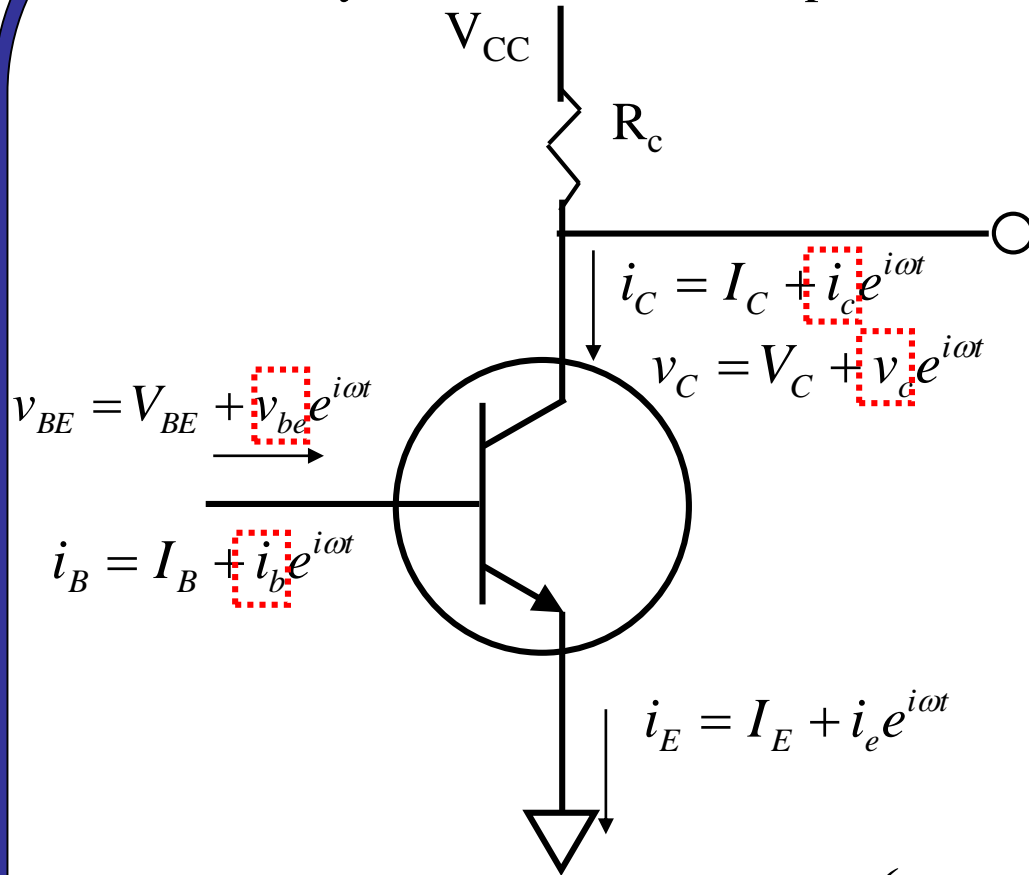
Hybrid  $\pi$  model:



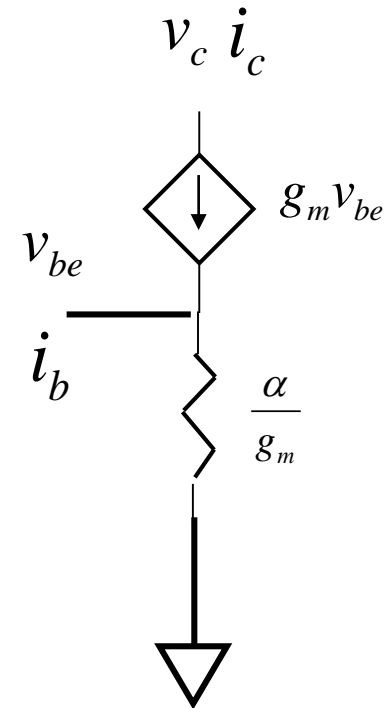
$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

# T-model

If we are only interested in ac components, life can be simplified:



T model:



$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

# Rules for ac analysis

- From complete circuit, calculate dc currents and voltages
- For ac analysis only:
  - dc voltage source -> short circuit
  - dc current source -> open circuit
- Replace transistor with  $\pi$  or T-model
- Now solve (simplified) ac circuit

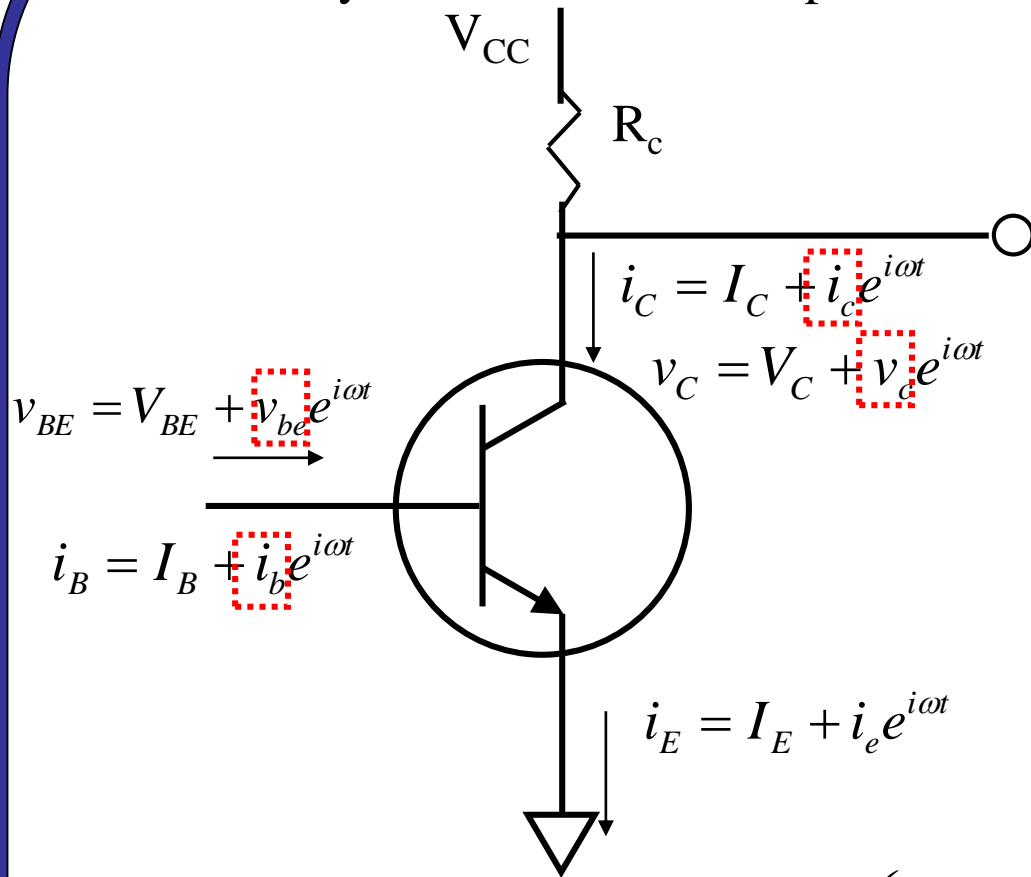


# Next

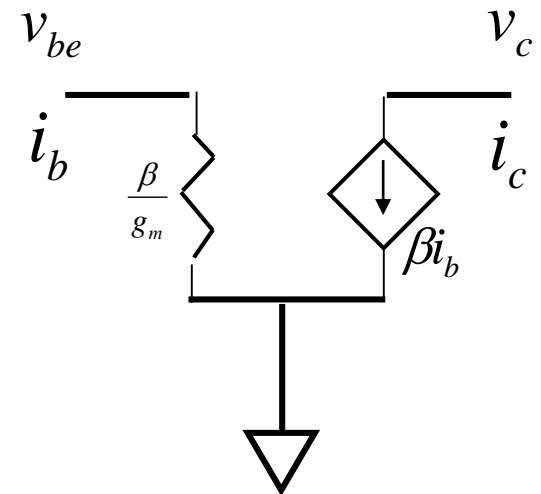
- Generalized y-parameters
- not just common emitters
- Capacitances
- y-parameters from doping profile
- Definition of  $f_T$

# AC equivalent circuit:

If we are only interested in ac components, life can be simplified:



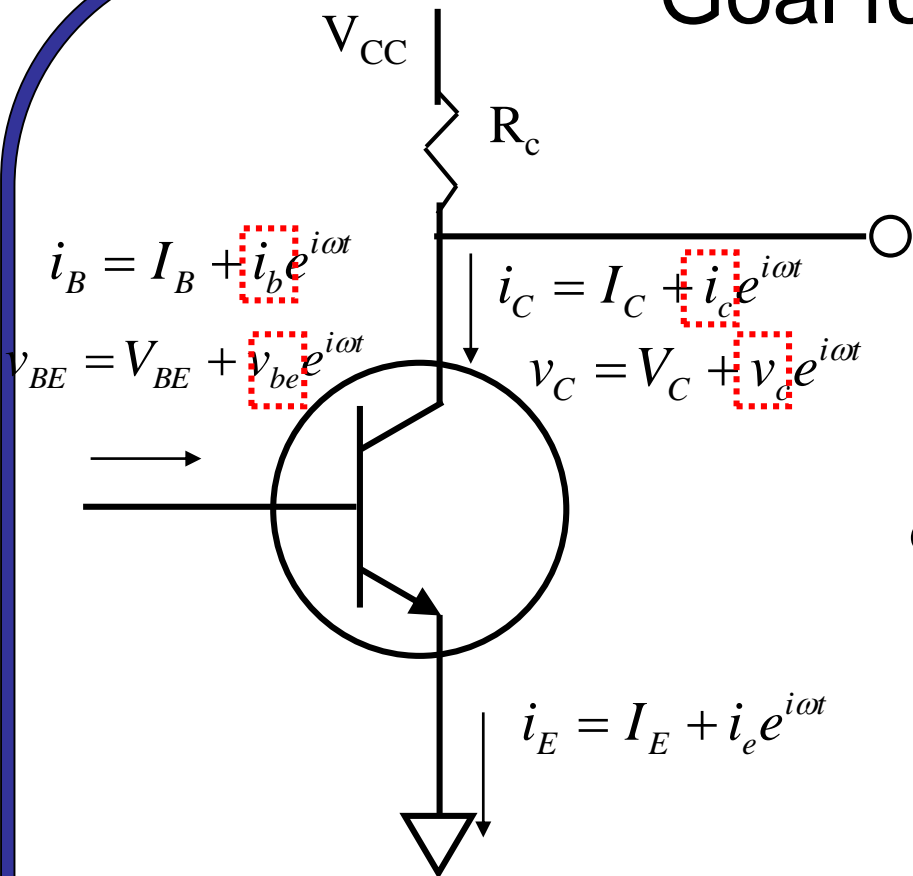
Hybrid  $\pi$  model:



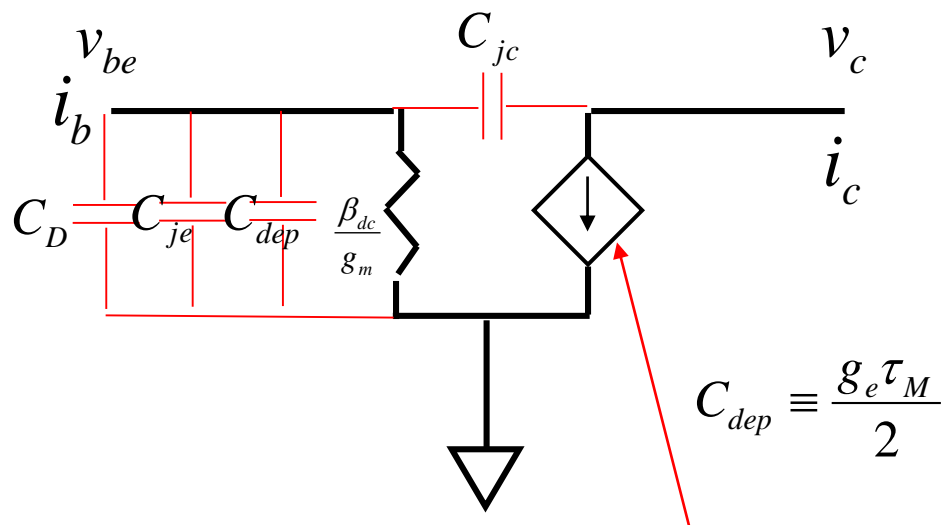
$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{g_m}{\beta} & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

Discuss easy interpretation of  $\pi$  model.

# Goal for today:



Hybrid  $\pi$  model:  
 Red is new for ac:



$$\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i \frac{\omega}{3\omega_0}\right] e^{\omega\tau_M/2} v_{be}$$

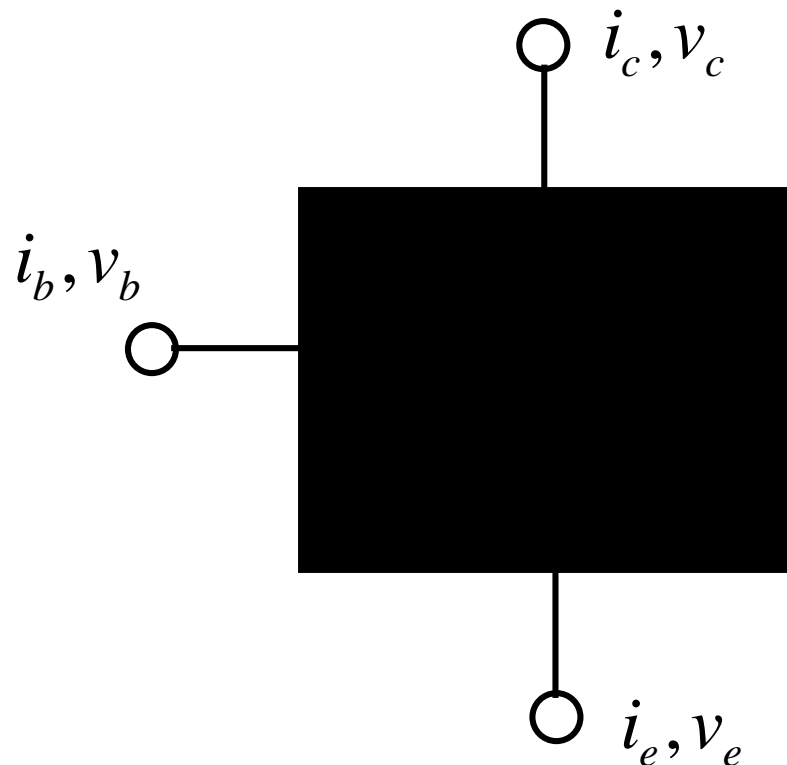
# General admittance matrix

Last lecture, we had emitter grounded.  
Called common emitter configuration:

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} g_m & 0 \\ \beta & 0 \\ \frac{eI_C}{kT} & 0 \end{pmatrix} \begin{pmatrix} v_{be} \\ v_c \end{pmatrix}$$

In general:

$$\begin{pmatrix} i_e \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} v_e \\ v_b \\ v_c \end{pmatrix}$$



Y-matrix has 9 elements, but once you know 4 you know them all because:

$$i_e = i_b + i_c$$

and:  $v_{cb} + v_{be} = v_{ce}$

See book about details procedure to get 9 parameters from only 4.

# Three configurations:

Common emitter configuration ( $v_e=0$ ):

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} y_{bb} & y_{bc} \\ y_{cb} & y_{cc} \end{pmatrix} \begin{pmatrix} v_b \\ v_c \end{pmatrix} = [y]_e \begin{pmatrix} v_b \\ v_c \end{pmatrix}$$

Common base configuration ( $v_b=0$ ):

$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} y_{ee} & y_{ec} \\ y_{ce} & y_{cc} \end{pmatrix} \begin{pmatrix} v_e \\ v_c \end{pmatrix} = [y]_b \begin{pmatrix} v_e \\ v_c \end{pmatrix}$$



Easiest to  
calculate from  
doping profile.

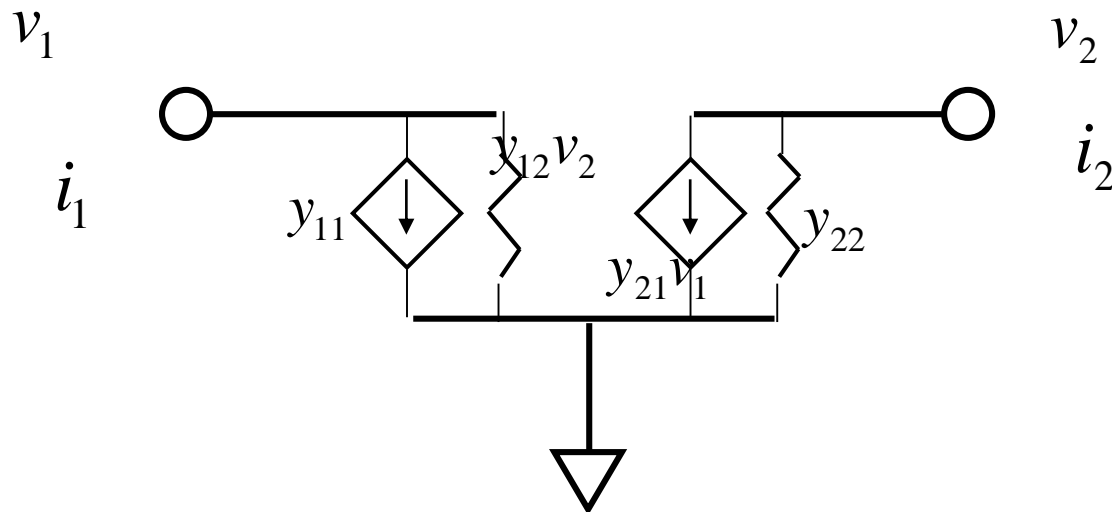
Common collector configuration ( $v_c=0$ ):

$$\begin{pmatrix} i_b \\ i_e \end{pmatrix} = \begin{pmatrix} y_{bb} & y_{be} \\ y_{eb} & y_{ee} \end{pmatrix} \begin{pmatrix} v_b \\ v_e \end{pmatrix} = [y]_c \begin{pmatrix} v_b \\ v_e \end{pmatrix}$$

# Generalized $\pi$ model:

Regardless of which configuration you use, the following  $\pi$  model applies:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$



Common emitter: 1=base, 2=collector

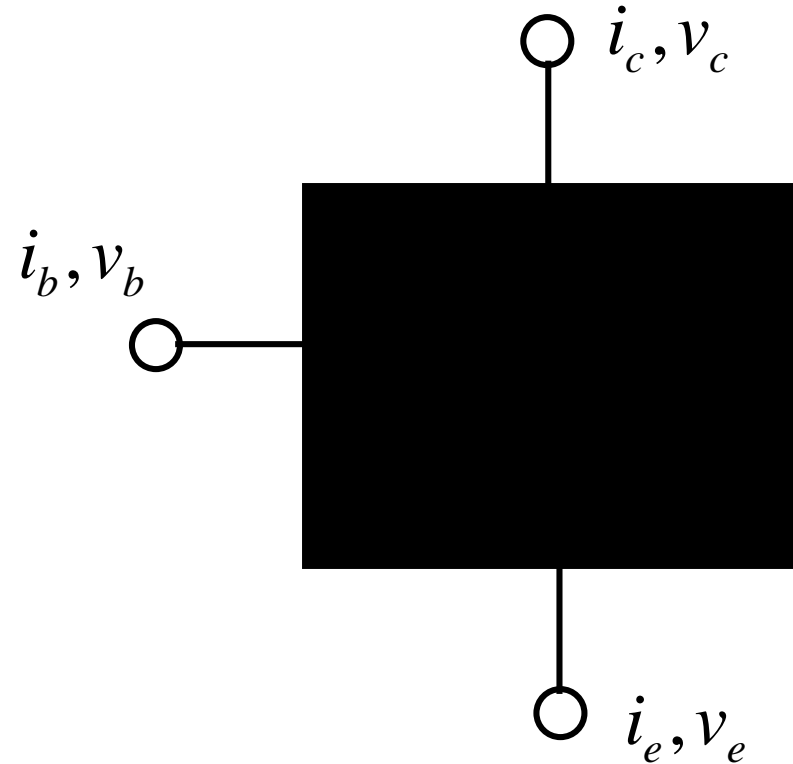
Common base: 1=emitter, 2=collector

Common collector: 1=base, 2=emitter

You might be used to  
 $V=IR$

# General impedance matrix

$$\begin{pmatrix} v_e \\ v_b \\ v_c \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} i_e \\ i_b \\ i_c \end{pmatrix}$$



Y-matrix has 9 elements, but once you know 4 you know them all because:



## h matrix:

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

Common emitter: 1=base, 2=collector

Common base: 1=emitter, 2=collector

Common collector: 1=base, 2= emitter

Note: In general, matrix elements depend on dc currents, dc voltages, and frequency. Spec. sheet (or model) will provide the matrix elements as a table vs. frequency, usually for only one bias current.

## Common emitter h matrix:

$$\begin{pmatrix} v_b \\ i_c \end{pmatrix} = \begin{pmatrix} h_{11e} & h_{12e} \\ h_{21e} & h_{22e} \end{pmatrix} \begin{pmatrix} i_b \\ v_c \end{pmatrix}$$

- Early effect:

Collector voltage changes current gain ( $\beta$ ).

- $\beta$  depends on frequency *and* collector voltage.
- How do we define frequency at which  $\beta = 1$ ?
- At  $v_c=0$ . This *is*  $h_{21e}$

$$i_c = h_{21e}i_b + h_{22e}v_c \rightarrow h_{21e}i_b$$

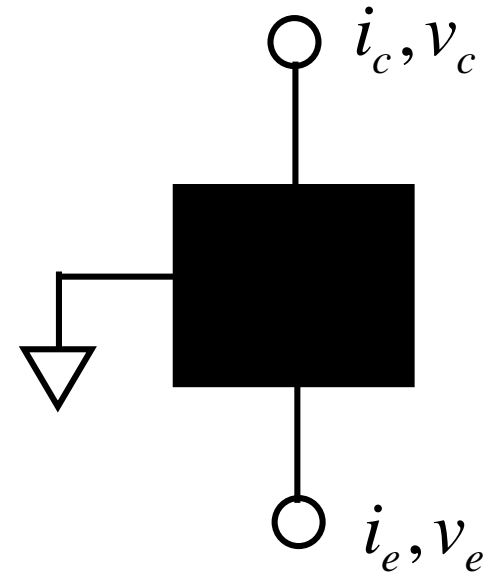
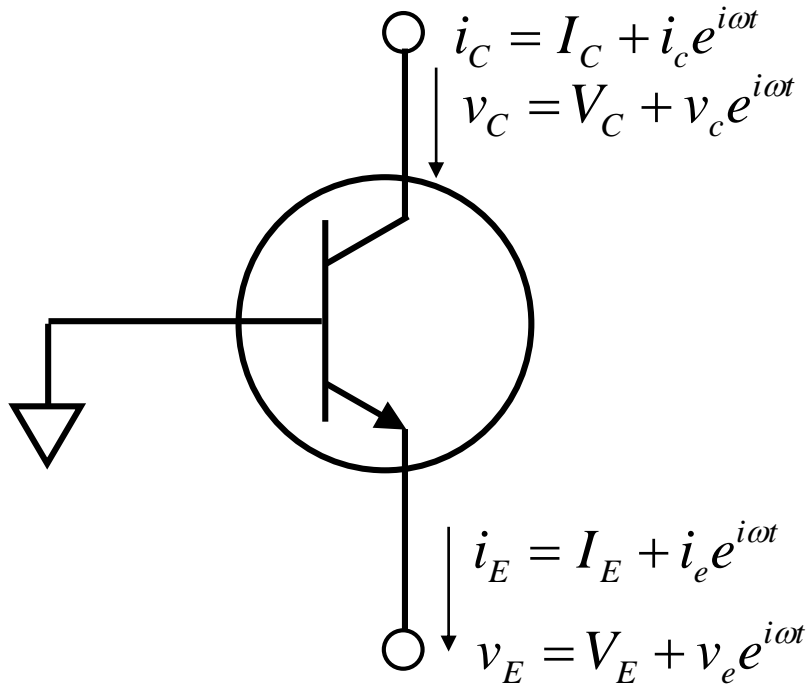
- We define  $f_T$  such that:

$$|h_{21e}|(f_T) = 1$$

# Next:

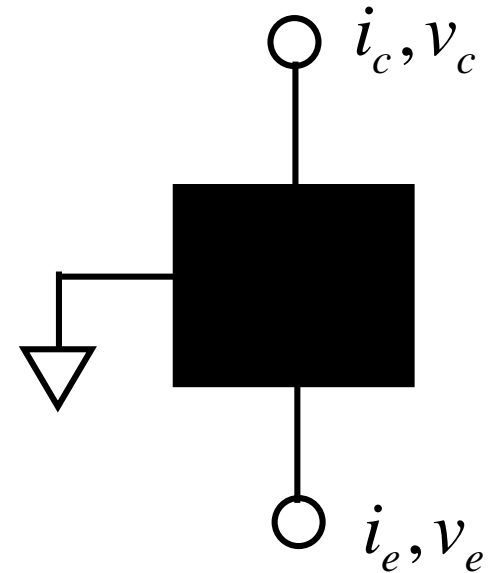
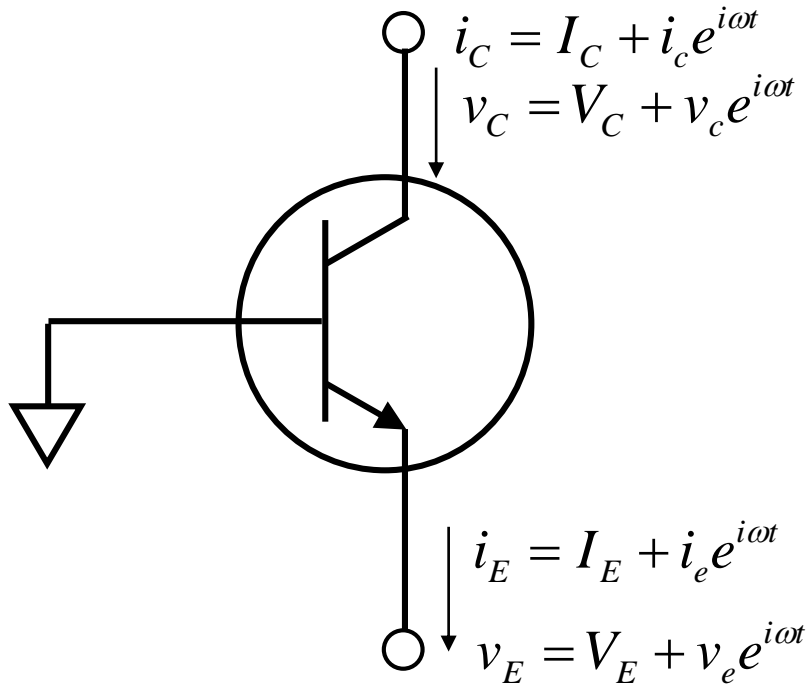
- How do we relate doping profile and geometry to Y-matrix?
- We will calculate common-base Y-matrix because it is easiest
- You can get any other Y-matrix from this.

# Common base



$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} y_{ee} & y_{ec} \\ y_{ce} & y_{cc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix} = \begin{pmatrix} \left. \frac{\partial i_E}{\partial v_{BE}} \right|_{V_{CB}} & \left. \frac{\partial i_E}{\partial v_{CB}} \right|_{V_{BE}} \\ \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{V_{CB}} & \left. \frac{\partial i_C}{\partial v_{CB}} \right|_{V_{BE}} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix}$$

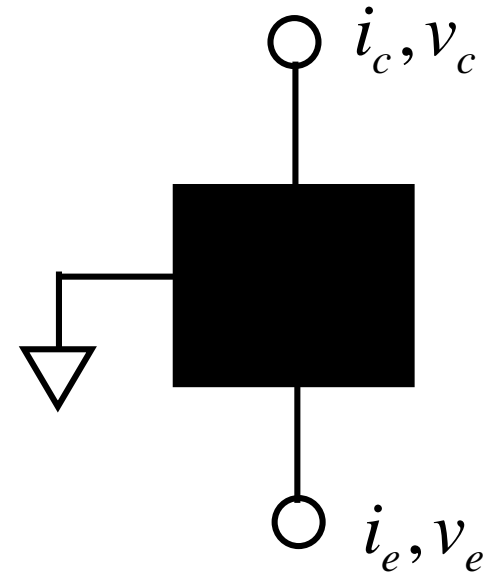
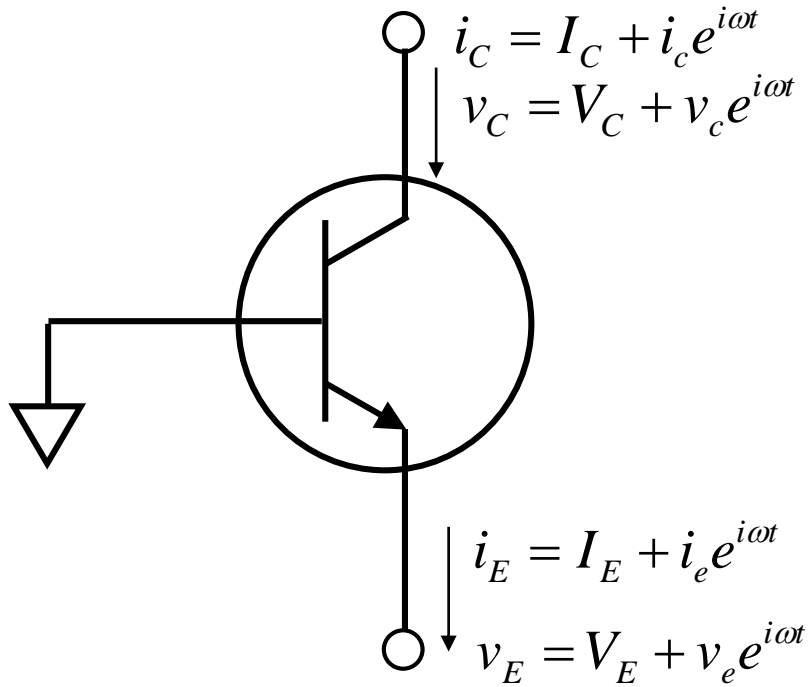
$y_{ec}$



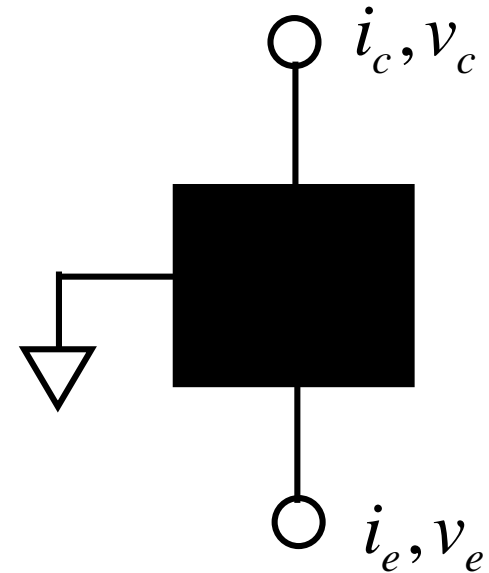
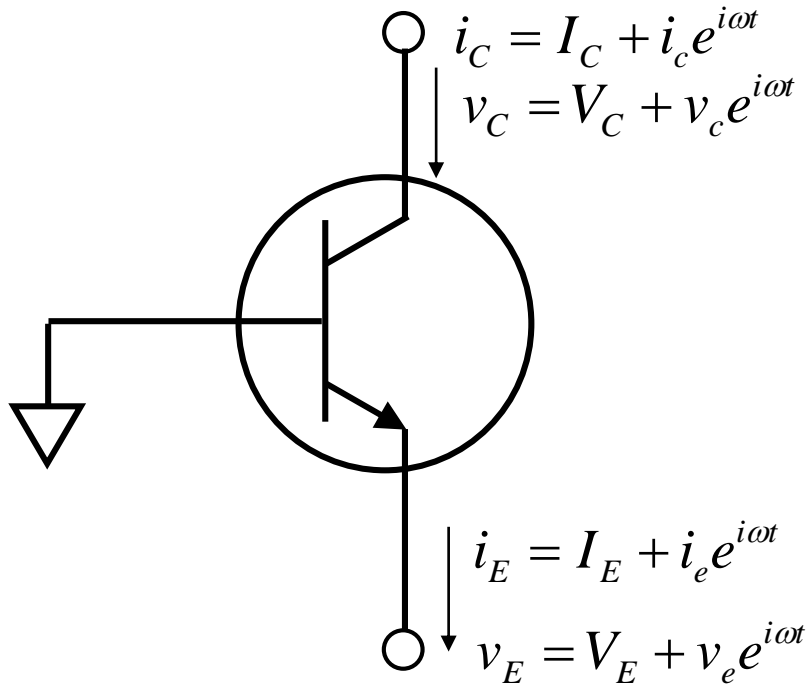
Neglecting Early effect,  $y_{ec} = \left. \frac{\partial i_E}{\partial v_{CB}} \right|_{V_{BE}} = 0$

OK for GaAs HBTs, but not Si.

So far:



$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} y_{ee} & y_{ec} \\ y_{ce} & y_{cc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix} = \begin{pmatrix} ? & 0 \\ ? & ? \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix}$$

$y_{cc}$ 

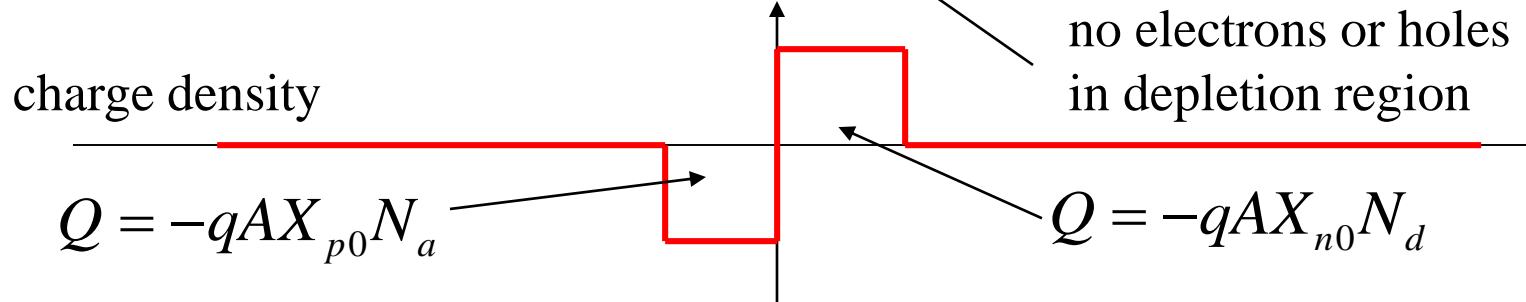
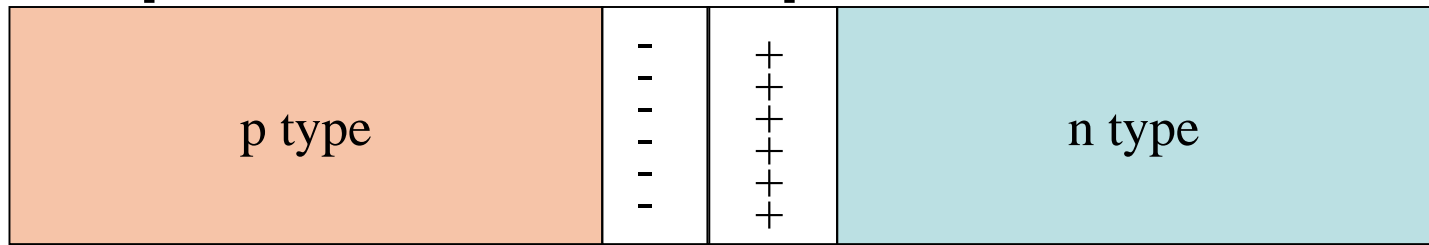
Neglecting Early effect,  
this is just capacitance of a  
p-n junction!

(Space charge region shrinks/grows  
with applied voltage.)

$$y_{cc} = \left. \frac{\partial i_c}{\partial v_{CB}} \right|_{V_{BE}} = \text{capacitor}$$

Recall lecture 4 slide 20:

# p n diode capacitance



$$Q = -qAX_{p0}N_a = -qAN_a \sqrt{\frac{2\varepsilon(\phi_{bi} - V)}{q} \frac{1}{N_a(1 + N_a/N_d)}}$$

$$C = \frac{dQ}{dV} = \frac{A}{2} \sqrt{\frac{2q\varepsilon}{(\phi_{bi} - V)} \frac{N_d N_a}{(N_d + N_a)}}$$

In Liu notation:  
jc stands for  
*junction capacitance*

$$C_{jc} = \frac{\varepsilon A_C}{X_{dep}} \quad \text{where:} \quad X_{dep} = \sqrt{\frac{2\varepsilon}{qN_C} (\phi_{CB} + V_{CB})}$$



$y_{cc}$ 

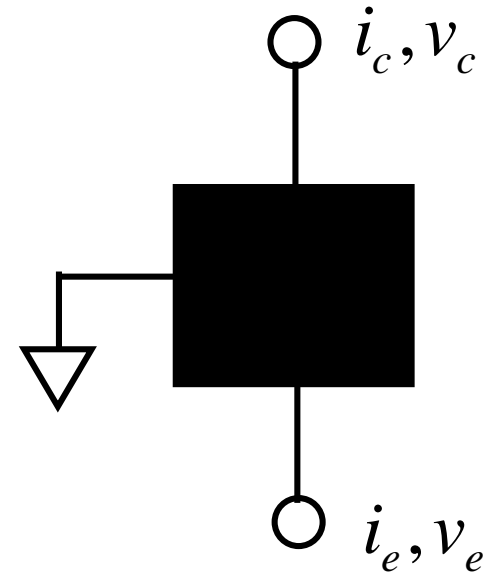
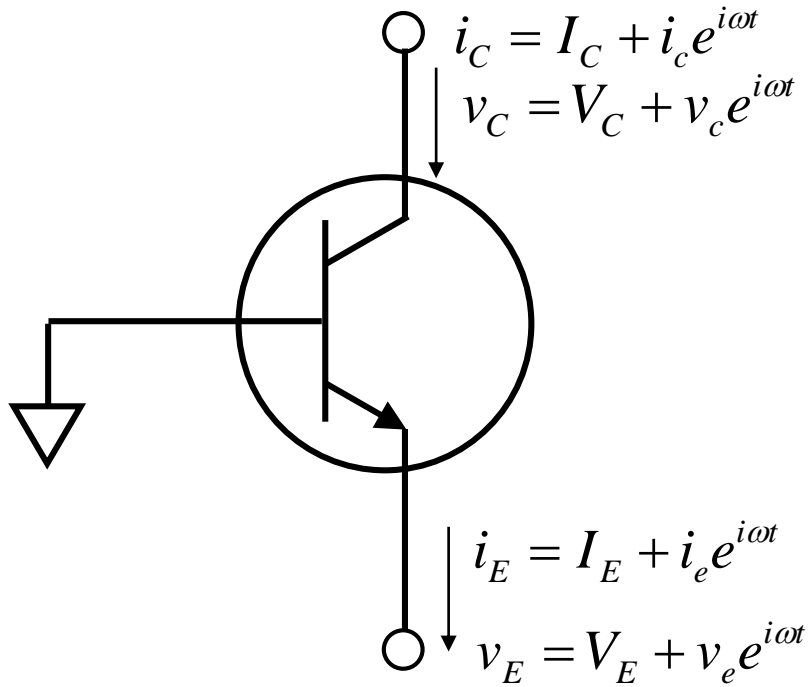
$$y_{cc} = \left. \frac{\partial i_c}{\partial v_{CB}} \right|_{V_{BE}} = \textit{capacitor}$$

$$Q = CV$$

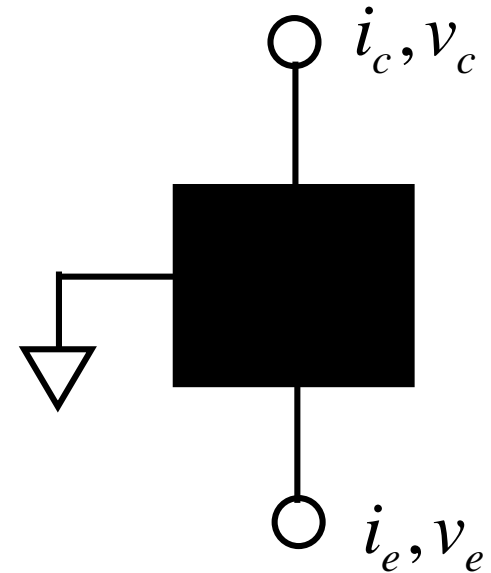
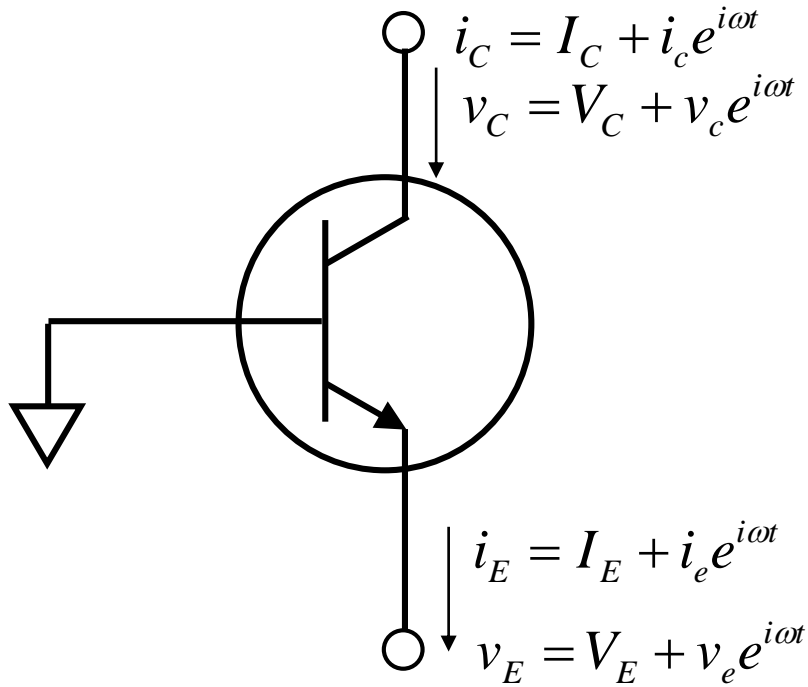
$$I = \frac{dQ}{dt} = C \frac{dV}{dt} = C(i\omega)V$$

$$\Rightarrow y_{cc} = i\omega C_{jc}$$

So far:



$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} y_{ee} & y_{ec} \\ y_{ce} & y_{cc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix} = \begin{pmatrix} ? & 0 \\ ? & i\omega C_{jc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix}$$

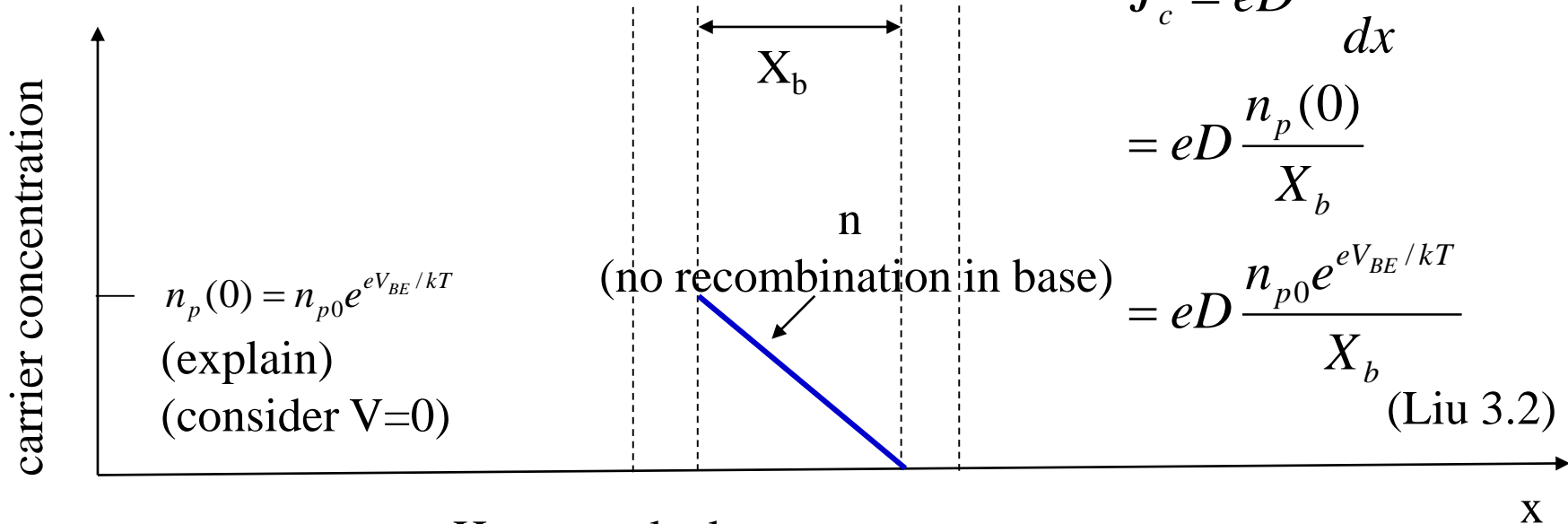
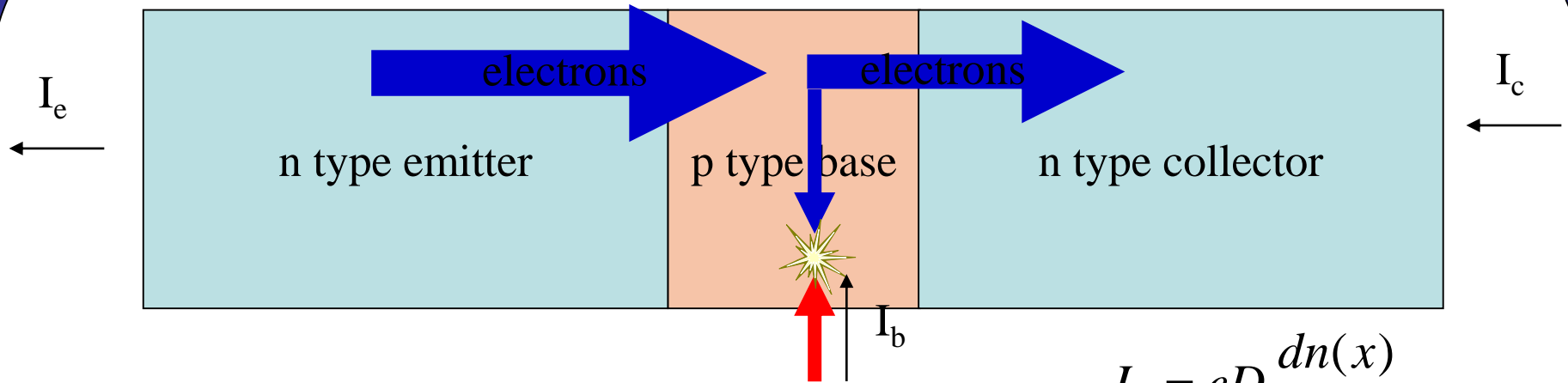
$y_{ee}$ 

$$y_{ee} = \left. \frac{\partial i_E}{\partial v_{BE}} \right|_{V_{CB}} = \text{capacitor} + \text{diodelike} = i\omega C_{jc} + \text{diodelike}$$

$$C_{je} = \frac{\epsilon A_E}{X_{dep,E}} \quad X_{dep,E} = \sqrt{\frac{2\epsilon}{qN_E} (\phi_{BE} + V_{BE})}$$

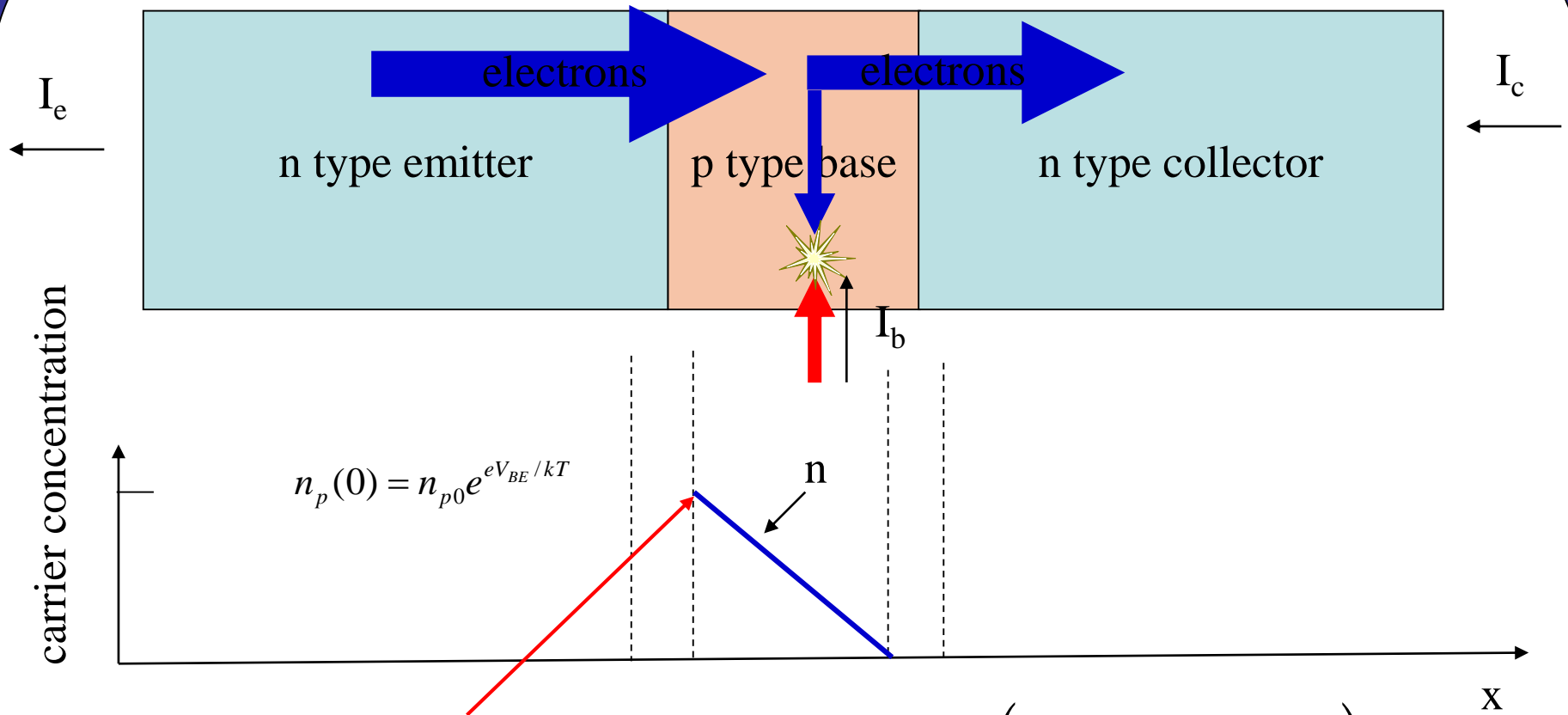
Recall lecture 5 slides 14,19 (and HW3):

# “Normal active” schematic



How to calculate:  
 Don't use line for  $n(x)$  but tanh.  
 $dn/dx$  em. –  $dn/dx$  coll = base curr.

# Time dependence:

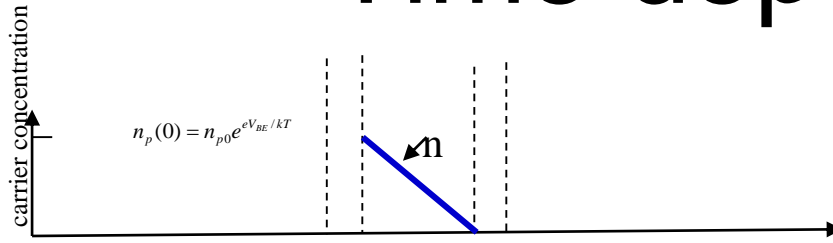


$$n_p(x=0, t) = n_{p0} e^{e(V_{BE} + v_{be} e^{i\omega t})/kT}$$

$$n_p(x = X_B, t) \approx 0$$

Must solve for  $n(x, t)$ !  
Current is  $dn/dx$ .

# Time dependence:



$$n_p(x=0, t) = n_{p0} e^{e(V_{BE} + v_{be} e^{i\omega t})/kT} = n_{p0} e^{eV_{BE}/kT} e^{ev_{be} e^{i\omega t}/kT}$$

$$\approx n_{p0} e^{eV_{BE}/kT} \left( 1 + \frac{ev_{be}}{kT} e^{i\omega t} \right) = n_{p0} e^{eV_{BE}/kT} + n_{p0} e^{eV_{BE}/kT} \frac{ev_{be}}{kT} e^{i\omega t}$$

$$\rightarrow n_{dc}(0) + \tilde{n}(0) e^{i\omega t} \text{ If solution is of form : } n(x, t) = n_{dc}(x) + \tilde{n}(x) e^{i\omega t}$$

From lecture 2, slide 22:

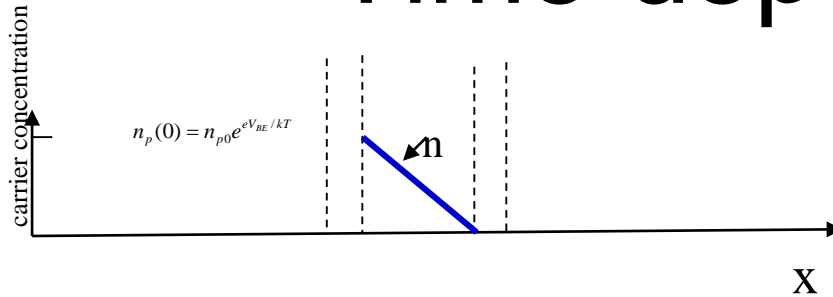
$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} - \frac{n}{\tau} \Rightarrow \frac{\partial (n_{dc}(x) + \tilde{n}(x) e^{i\omega t})}{\partial t} = D \frac{\partial^2}{\partial x^2} (n_{dc}(x) + \tilde{n}(x) e^{i\omega t}) - \frac{1}{\tau} (n_{dc}(x) + \tilde{n}(x) e^{i\omega t})$$

(discuss)

$$\Rightarrow 0 = D \frac{\partial^2}{\partial x^2} n_{dc}(x) - \frac{1}{\tau} n_{dc}(x) \quad \Rightarrow n_{dc}(x) = \frac{n_{dc}(0)}{\sinh(X_B / L_n)} \sinh\left(\frac{X_B - x}{L_n}\right)$$

$$\Rightarrow i\omega \tilde{n}(x) = D \frac{\partial^2}{\partial x^2} \tilde{n}(x) - \frac{1}{\tau} \tilde{n}(x) \quad \Rightarrow \text{see next page!}$$

# Time dependence:



$$\Rightarrow 0 = D \frac{\partial^2}{\partial x^2} n_{dc}(x) - \frac{1}{\tau} n_{dc}(x) \quad \Rightarrow n_{dc}(x) = \frac{n_{dc}(0)}{\sinh(X_B / L_n)} \sinh\left(\frac{X_B - x}{L_n}\right)$$

$$\Rightarrow i\omega \tilde{n}(x) = D \frac{\partial^2}{\partial x^2} \tilde{n}(x) - \frac{1}{\tau} \tilde{n}(x)$$

$$\tilde{n}(x) = n_{dc}(0) \frac{e v_{eb}}{kT} \frac{\sinh(\xi_{ac}(X_B - x))}{\sinh(\xi_{ac} X_B)}$$

$$\xi_{ac} \equiv \sqrt{\frac{1 + i\omega\tau}{D\tau}}$$

From lecture 2:

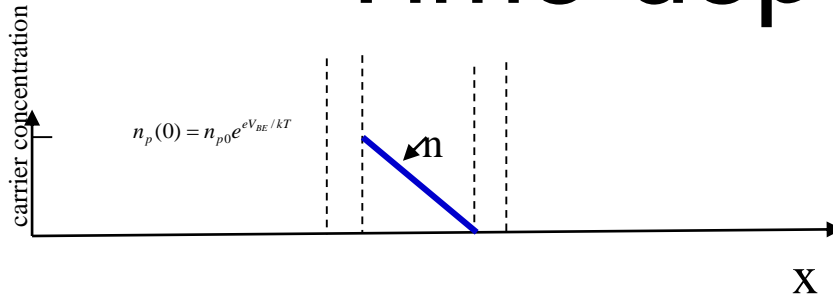
$$I = eAD \frac{dn(x,t)}{dx} = eAD \frac{d}{dx} (n_{dc}(x) + \tilde{n}(x)e^{i\omega t}) = eAD \frac{dn_{dc}(x)}{dx} + eAD \frac{d\tilde{n}(x)}{dx} e^{i\omega t}$$

We only want ac part at  $x=0$  (discuss):

$$I_{ac} = I_{dc}(0) \frac{e v_{eb}}{kT} \xi_{ac} \coth(\xi_{ac} X_B) e^{i\omega t} \Rightarrow$$

$$y_{ee} \equiv \frac{I_{ac}}{v_{eb}} = I_{dc}(0) \frac{e}{kT} \xi_{ac} \coth(\xi_{ac} X_B)$$

# Time dependence:



$$y_{ee} \equiv \frac{I_{ac}}{v_{eb}} = I_{dc}(0) \frac{e}{kT} \xi_{ac} \coth(\xi_{ac} X_B)$$

$$\omega_0 \equiv \frac{1}{\tau_{transit}} = \frac{2D}{X_B^2}$$

For small frequencies,

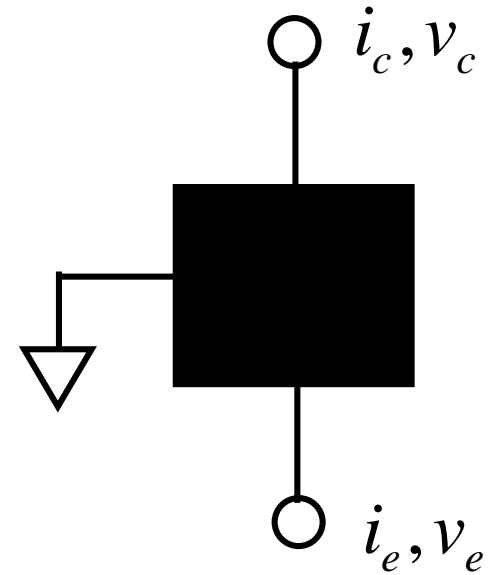
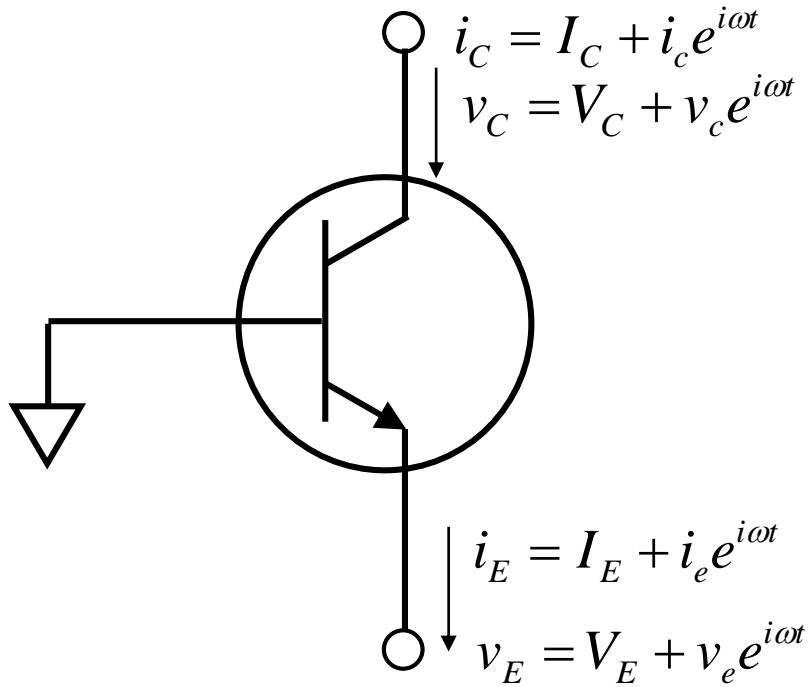
$$I_{dc}(0) \frac{e}{kT} \xi_{ac} \coth(\xi_{ac} X_B) \approx g_e + i\omega C_D \qquad g_e \equiv I_{dc}(0) \frac{e}{kT}$$

The “diffusion” capacitance is the extra (time dependent) charge stored as excess electrons in the base:

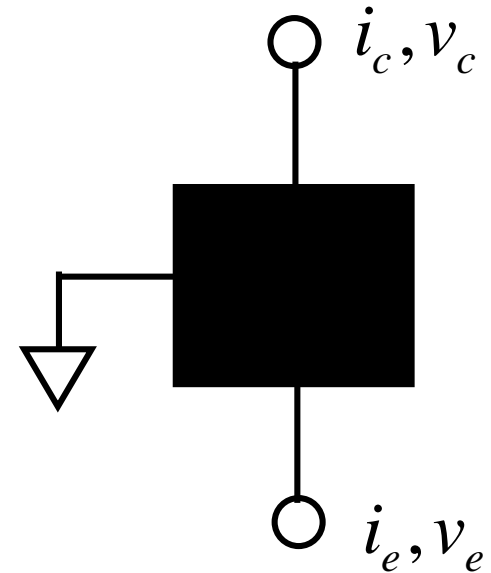
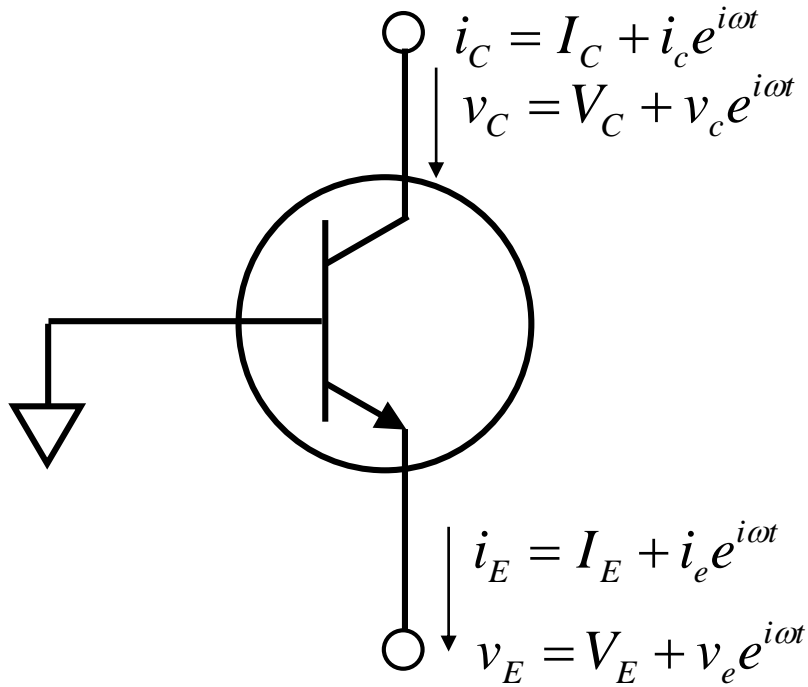
$$C_D \equiv g_e \frac{X_B^2}{3D}$$



So far:



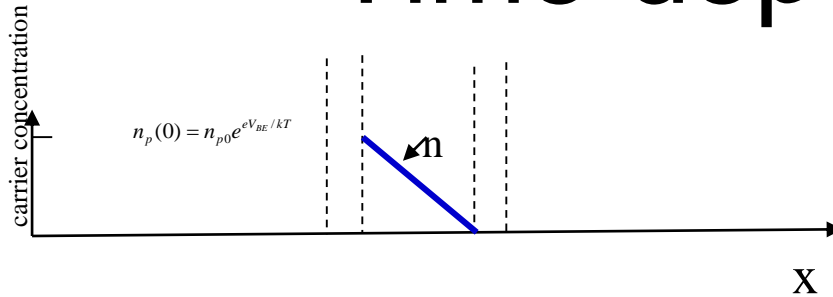
$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} g_e + i\omega C_D + i\omega C_{je} & 0 \\ ? & i\omega C_{jc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix}$$

$y_{ce}$ 

$$y_{ce} = \left. \frac{\partial i_c}{\partial v_{BE}} \right|_{V_{CB}} = ?$$

We need solution of time-dependent diffusion equation again.  
 $i_c$  equal to  $dn/dx$  at *collector* side.

# Time dependence:



$$\Rightarrow 0 = D \frac{\partial^2}{\partial x^2} n_{dc}(x) - \frac{1}{\tau} n_{dc}(x)$$

$$\Rightarrow i\omega \tilde{n}(x) = D \frac{\partial^2}{\partial x^2} \tilde{n}(x) - \frac{1}{\tau} \tilde{n}(x)$$

$$\tilde{n}(x) = n_{dc}(0) \frac{e v_{eb}}{kT} \frac{\sinh(\xi_{ac}(X_B - x))}{\sinh(\xi_{ac} X_B)}$$

$$\xi_{ac} \equiv \sqrt{\frac{1 + i\omega\tau}{D\tau}}$$

From lecture 2:

$$I = eAD \frac{dn(x,t)}{dx} = eAD \frac{d}{dx} (n_{dc}(x) + \tilde{n}(x)e^{i\omega t}) = eAD \frac{dn_{dc}(x)}{dx} + eAD \frac{d\tilde{n}(x)}{dx} e^{i\omega t}$$

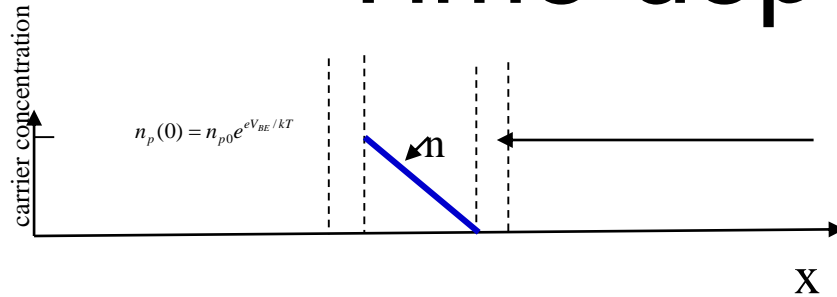
We only want ac part at  $x=X_B$  (discuss):

$$I_{ac} = v_{eb} \left( 1 - \frac{X_B^2}{2L_n^2} \right) g_e \xi_{ac} \csc h(\xi_{ac} X_B) \Rightarrow$$

$$y_{ce} \equiv \frac{I_{ac}}{v_{eb}} \left( 1 - \frac{X_B^2}{2L_n^2} \right) g_e \xi_{ac} \csc h(\xi_{ac} X_B)$$

not quite...

# Time dependence:



It takes an electron some time to get through depletion region here.

(Question for class:

What about the other depletion region?)

$$y_{ce} \equiv \frac{I_{ac}}{v_{eb}} = \left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \xi_{ac} \csc h(\xi_{ac} X_B)$$

$$\rightarrow \left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \xi_{ac} \csc h(\xi_{ac} X_B) e^{-i\omega\tau_M/2}$$

$$\tau_M \equiv \frac{X_{dep}}{v_{sat}}$$

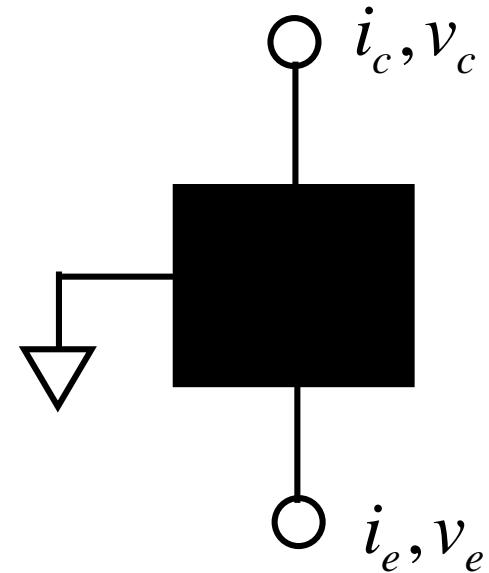
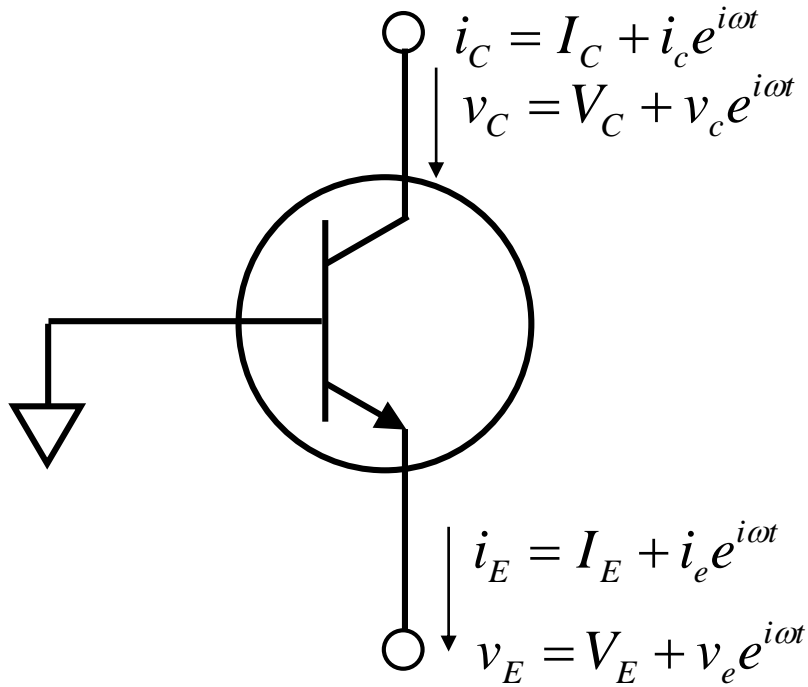
This is how long it takes an electron to get through depletion region

(There is no scattering in the depletion region in this approximation)

At low frequencies:

$$y_{ce} \approx -\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i \frac{\omega}{3\omega_0}\right] e^{-i\omega\tau_M/2}$$

So far:



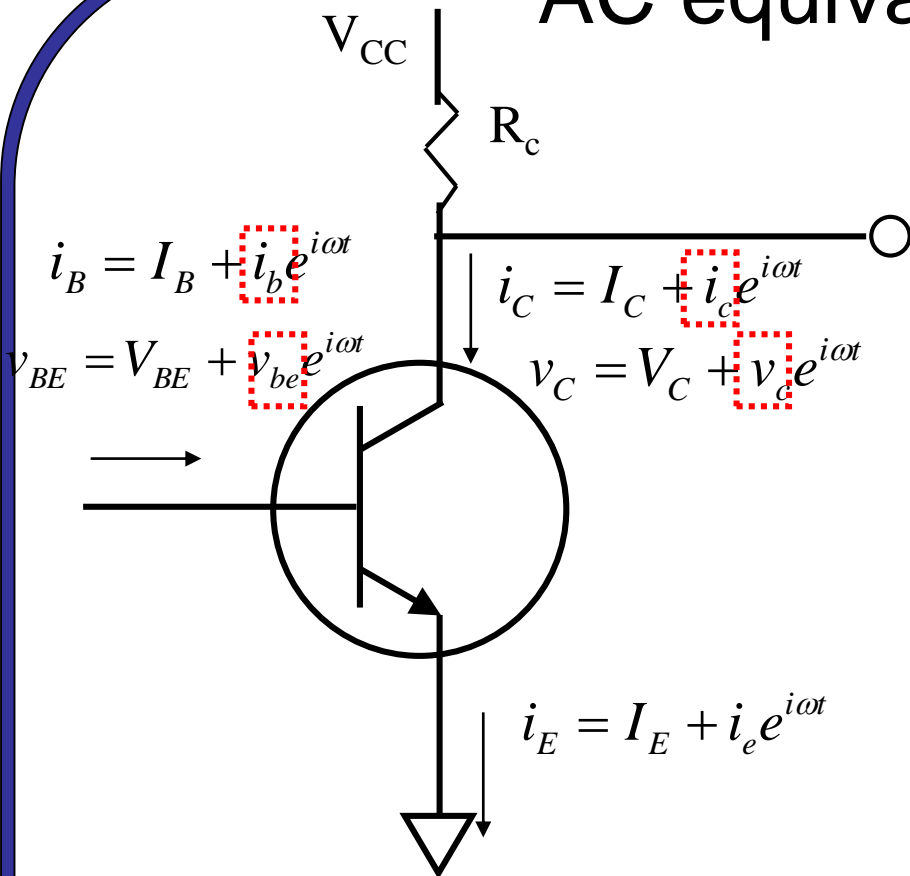
$$\begin{pmatrix} i_e \\ i_c \end{pmatrix} = \begin{pmatrix} g_e + i\omega C_D + i\omega C_{je} & 0 \\ y_{ce} \approx -\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i \frac{\omega}{3\omega_0}\right] e^{-i\omega\tau_M/2} & i\omega C_{jc} \end{pmatrix} \begin{pmatrix} v_{be} \\ v_{cb} \end{pmatrix}$$

You will calculate for typical parameters in HW.

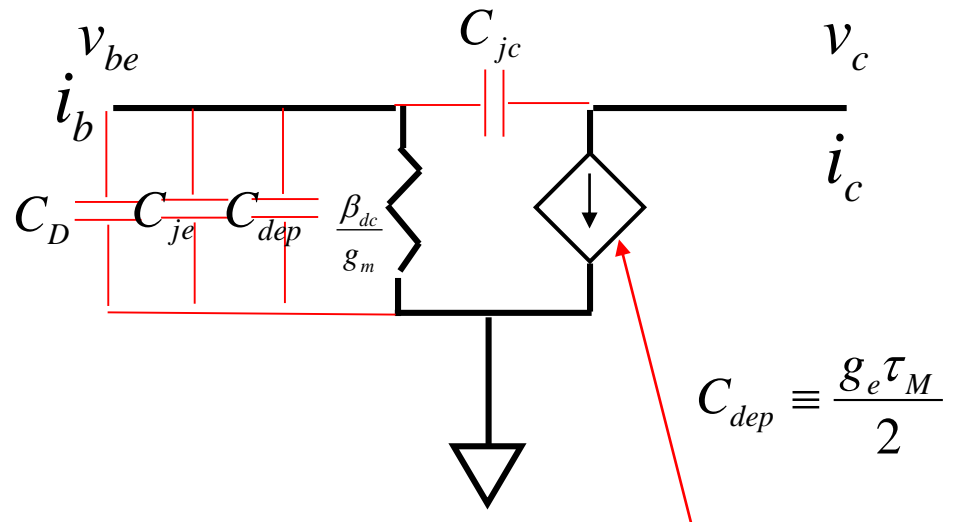
# Next steps

- Translate back to common-emitter model
- Generate equivalent circuit.

# AC equivalent circuit:



Hybrid  $\pi$  model:  
Red is new for ac:



$$\left(1 - \frac{X_B^2}{2L_n^2}\right) g_e \left[1 - i \frac{\omega}{3\omega_0}\right] e^{\omega\tau_M/2} v_{be}$$

- Circuit model good only for low frequencies
- At high frequencies computer must be used!
- That concludes our derivation of intrinsic HBT behavior.
- Next will include parasitics, and discuss  $f_T$ ,  $f_{max}$