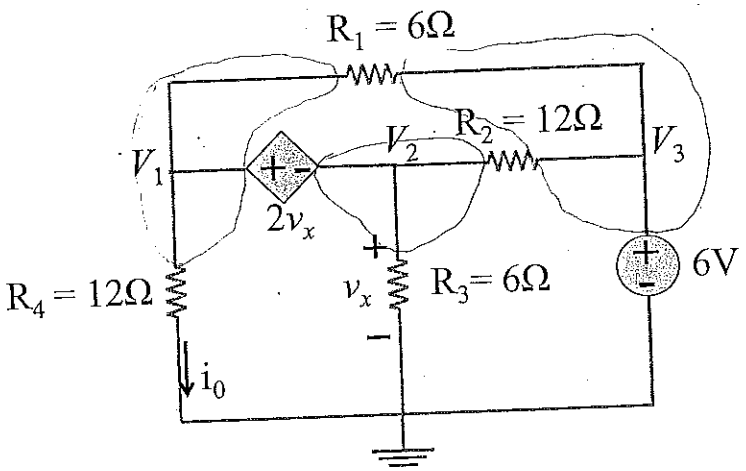
PROBLEM 2: (20 points)

Use nodal analysis, and solve for the node voltages and the current i_0.

Use KCL



V_1	1V
V_2	3V
V_3	6V
i_0	1/12 A

-17

KCL @ node 1: $\frac{V_1-0V}{12\Omega} + \frac{V_1-V_2}{2V_x} + \frac{V_1-V_3}{6\Omega} = 0$ (1) ✗

⊥ @ node 2: $\frac{V_2-V_1}{2V_x} + \frac{V_2}{6\Omega} + \frac{V_2-V_3}{12\Omega} = 0$ (2) ✗

⊥ @ node 3: $V_3 = 6V$ (3) ✓2

$V_x = \frac{V_2}{6\Omega}$ ✗

$* i_0 = \frac{V_1}{12\Omega} = \frac{1V}{12\Omega}$

$V_x = \frac{3V}{6\Omega} = \frac{1}{2}A \rightarrow 2V_x = 1A$

Rearrange equations:

$\frac{V_1}{12\Omega} + \frac{V_1}{2V_x} + \frac{V_1}{6\Omega} - \frac{V_2}{2V_x} - \frac{V_3}{6\Omega} = 0$

$\frac{V_1}{12\Omega} + \frac{V_1}{1\Omega} - \frac{V_2}{1\Omega} + \frac{V_1}{6\Omega} - \frac{V_3}{6\Omega} = 0$

$\frac{V_2}{2V_x} + \frac{V_2}{6\Omega} + \frac{V_2}{12\Omega} - \frac{V_3}{12\Omega} = 0$

$\frac{V_2}{2V_x} + \frac{3V_2}{12\Omega} - \frac{V_3}{12\Omega} = 0$

$\frac{V_2}{2(\frac{V_2}{6\Omega})} + \frac{3V_2}{12\Omega} = \frac{1}{2}A$

$\frac{6\Omega}{V_2} + \frac{3V_2}{12\Omega} = 1/2A$

$\frac{V_2}{2V_2} + \frac{3V_2}{12\Omega} = \frac{1}{2}A$

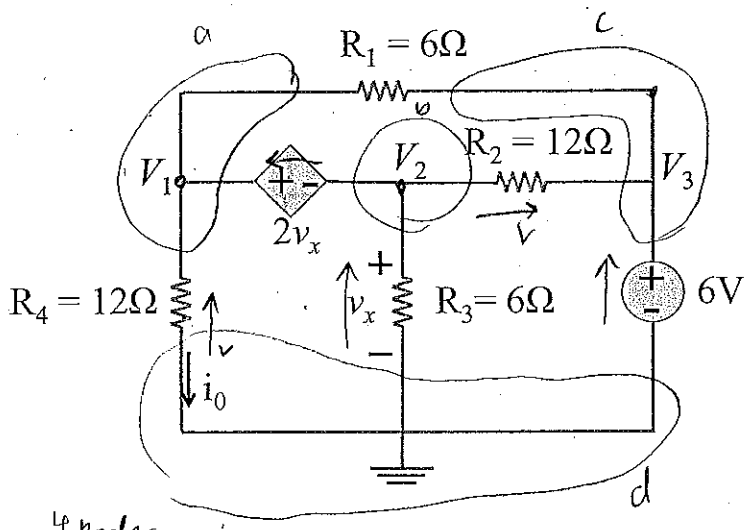
$\frac{3V_2}{12\Omega} = (\frac{1}{2}A - \frac{6\Omega}{V_2})12\Omega$

$\frac{6\Omega V_2}{2V_2} + \frac{3V_2}{12\Omega} = \frac{1}{2}A$

$3V_2 = 6V - \frac{72\Omega}{V_2}$

Below average score

...nodal analysis, and solve for the node voltages and the current i_0.



V_1	6V
V_2	$3-14V_x$
V_3	$-2V_x+30V$
i_0	5Amp

4 nodes

$V=IR$

$V_{in} = V_{out}$

$I_{in} = I_{out}$

node a: $\frac{V_1-V_3}{R_1} = 2V_x + \frac{(V_1-6V)}{R_4} \Rightarrow \frac{V_1-V_3}{6\Omega} = 2V_x + \frac{(V_1-6V)}{12\Omega}$

node b: $\frac{V_2-V_3}{R_2} + 2V_x = V_x \Rightarrow \frac{V_2-V_3}{12\Omega} + 2V_x = V_x \Rightarrow \frac{V_2-V_3}{6\Omega} = -V_x \Rightarrow V_2-V_3 = -12V_x$

$V_2 = V_3 - 12V_x$

node c: $\frac{V_3-V_1}{R_1} = \frac{V_3-V_2}{R_2} + 6 \Rightarrow \frac{V_3-V_1}{6} = \frac{V_3-V_2}{12} + 6 \Rightarrow V_3-V_1 = \frac{V_3-V_2}{2} + 36$

$V_3-V_1 = \frac{V_3}{2} - \frac{V_2}{2} + 36$

$2V_3-2V_1 = V_3-V_2+72$

$V_1-V_3 = 2V_x + \frac{(V_1-6V)}{2}$

$V_1-V_3 = 2V_x + \frac{V_1}{2} + \frac{6V}{2}$

$\frac{V_1-6V}{R_4} = i_0$

$V_3-2(2V_3+4V_x+6) = -V_3+12V_x+72$

$V_3-4V_3+8V_x+12 = -V_3+12V_x+72$

$\frac{V_1}{2}-V_3 = 2V_x+3$

$\frac{66-6}{12} = \frac{60}{12} = 5Amp$

$-2V_3 = 4V_x-60$

$V_1 = 2V_3+4V_x+6$

$V_3 = -2V_x+30$

$V_1 = -4V_x+60+4V_x+6$

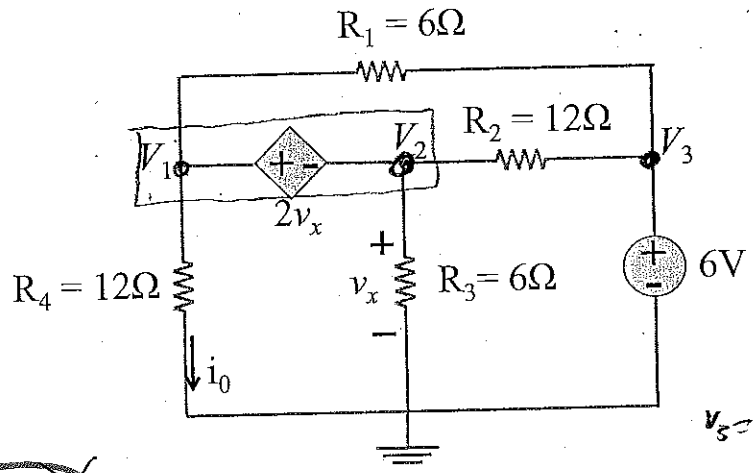
$V_1 = 66$

$V_1 = -2V_x+3-12V_x$

Below average score

≈ 20

Use nodal analysis, and solve for the node voltages and the current i_0.



V_1	4V
V_2	10V
V_3	6V
i_0	1/3 A

✓ +2

$V_3 = 6$

KCL N3! $\frac{V_3 - V_2}{12} + \frac{V_3 - V_1}{6} = 0$ Plug $V_3 = 6$ $\Rightarrow \frac{6 - V_2}{12} + \frac{6 - V_1}{6} = 0 \Rightarrow \frac{1}{2} - \frac{V_2}{12} + 1 - \frac{V_1}{6} = 0$

$\Rightarrow \frac{V_2}{12} + \frac{V_1}{6} = 1.5$

$\Rightarrow V_2 + 2V_1 = 18$

+2

KCL SN: $\frac{V_1 - V_3}{6} + \frac{V_1}{12} + \frac{V_2 - V_3}{12} + \frac{V_2}{6} = 0$

Plug $V_3 = 6$ $\Rightarrow \frac{V_1 - 6}{6} + \frac{V_1}{12} + \frac{V_2 - 6}{12} + \frac{V_2}{6} = 0$

$\Rightarrow \frac{V_1}{6} - 1 + \frac{V_1}{12} + \frac{V_2}{12} - \frac{1}{2} + \frac{V_2}{6} = 0$

$\Rightarrow \frac{2V_1}{12} + \frac{V_1}{12} + \frac{V_2}{12} + \frac{2V_2}{12} = 1.5$

$\Rightarrow \frac{3V_1}{12} + \frac{3V_2}{12} = 1.5$

$\Rightarrow 3V_1 + 3V_2 = 18$ (+4)

$\begin{cases} 3V_1 + 3V_2 = 18 \\ -2V_1 + V_2 = 8 \\ -8V_1 - 3V_2 = -54 \end{cases}$

$-9V_1 = -36$

$V_1 = 4V$, $V_2 = 10V$

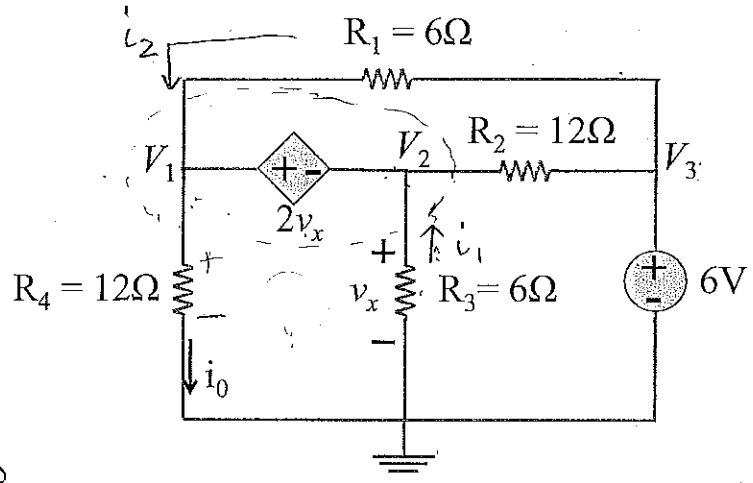
$V = IR$ +2

$I_0 = \frac{V_1}{R_4} = \frac{4V}{12\Omega} = \frac{1}{3}A$ +1

Average score ≈ 55

PROBLEM 2: (20 points)

Use nodal analysis, and solve for the node voltages and the current i_0.



V_1	4 V
V_2	$\frac{4}{3}$ V
V_3	6 V
i_0	$\frac{1}{3}$ A

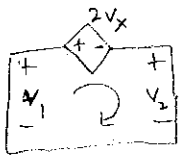
KCL @ Supernode +2

$i_0 = i_1 + i_2$

$\frac{V_1}{2} = \frac{V_x - V_2}{6} + \frac{V_3 - V_1}{6} = 0$ ← multiply by 12

$V_3 = 6$ V +2

KVL Supernode



$-V_1 + 2v_x + V_2 = 0$ +2

$-(12 i_0) + 2v_x + v_x = 0$ +2

$-V_1 + 2v_x + v_x = 0$

$3v_x = V_1$

$v_x = \frac{V_1}{3}$

$V_1 = 2v_x - 2V_2 + 2V_3 - 2V_1$

$V_1 = 2\left(\frac{V_1}{3}\right) - 2\left(\frac{V_1}{3}\right) + 2(6) - 2V_1$

$V_1 = \frac{2V_1}{3} - \frac{2V_1}{3} + 12 - 2V_1$

$V_1 = 12 - 2V_1$

$3V_1 = 12$

$V_1 = 4$

$-V_1 + 2\left(\frac{V_1}{3}\right) + V_2 = 0$

$-V_1 + \frac{2V_1}{3} + V_2 = 0$

$-\frac{V_1}{3} + V_2 = 0$

$V_2 = \frac{V_1}{3}$

$i = 2v_x - 2V_2 + 2V_3 - 2V_1$

$= 2\left(\frac{4}{3}\right) - 2V_2 + 12 - 8$

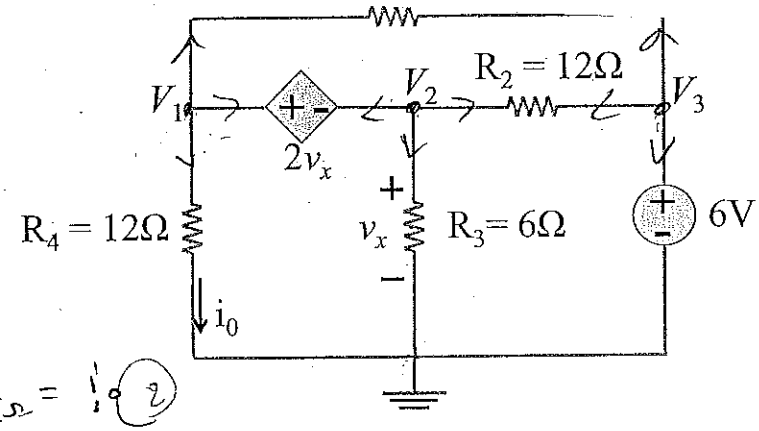
$= \frac{8}{3} - 2V_2 + 4$

$0 = \frac{8}{3} - 2V_2$

$-\frac{8}{3} = -2V_2$

$-\frac{8}{3} \times \frac{1}{-2} =$

Average score ≈ 55



V_1	
V_2	
V_3	
i_0	

where $\frac{V_1}{12\Omega} = i_0$ (2)

(9)

(a) V_1: $i_0 + \frac{V_1 - V_3}{6\Omega} = 0$

(b) V_2: $\frac{V_2}{6\Omega} + \frac{V_2 - V_3}{12\Omega} = 0$

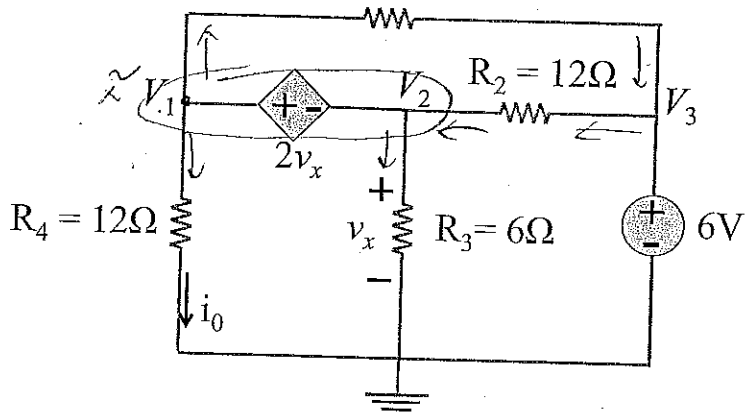
(c) V_3: $\frac{V_3 - V_2}{12} + \frac{V_3 - V_1}{6\Omega} = 0$

19

$V_2 = \dots$

Below average score

$\approx$ 30/100



V_1	$\frac{24}{7}$ V
V_2	$\frac{18}{7}$ V
V_3	6 V
i_0	$\frac{2}{7}$ A

12

node 1 and 2: $\frac{V_1}{12} + \frac{V_1 - V_3}{6} + \frac{V_2}{6} = \frac{V_3 - V_2}{12}$ ✓

$V_1 + 2V_1 - 2V_3 + 2V_2 = V_3 - V_2$

$3V_1 + 3V_2 - 3V_3 = 0$

$V_1 + V_2 - V_3 = 0$ ✓

node 3: $V_3 = \boxed{6\text{ V}}$ $\times 2$

$V_2 + 2V_x = V_1 \times 2$

$V_x = \frac{V_2}{6}$

$V_2 + \frac{V_2}{3} = V_1$

$\frac{4V_2}{3} = V_1$

$V_1 + V_2 - V_3 = 0$

$\frac{4}{3}V_2 + V_2 = 6$

$\frac{7}{3}V_2 = 6$

$V_2 = \frac{3}{7} \times 6\text{ V} = \boxed{\frac{18}{7}\text{ V}}$

$V_1 = \frac{4}{3}V_2 = \frac{4}{3} \times \frac{\overset{6}{\cancel{18}}}{7} = \boxed{\frac{24}{7}\text{ V}}$

$i_0 = \frac{V_1}{12} = \frac{24}{7} \times \frac{1}{12}\text{ A} = \boxed{\frac{2}{7}\text{ A}}$ $\times 2$

well above average score.

≈ 70/100