

Q1	Q2	Q3	Q4	Q5	Total
/15	/20	/25	/20	/20	/100

## **EECS / CSE 70A Final Exam**

**DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.**

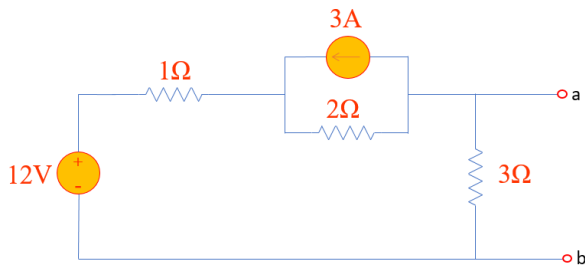
**Print your name on all pages.**

**Write your solutions in clear steps with concise explanations.**

**PROBLEM 1: (15 points)**

**In the following circuit:**

- (a) Find equivalent Thevenin voltage source**
- (b) Find equivalent Norton Current source**
- (c) Find equivalent Thevenin and Norton Resistor and give the equivalent circuits.**



$V_{th}$	3V
$I_N$	2A
$R_{th}$	1.5Ω

Super-position Theorem: (effect of two independent sources in a linear circuit is sum of effects of each source while the other one is zero).

$$(a) V_{th1} = 12V \times \frac{3\Omega}{1\Omega + 2\Omega + 3\Omega} = 6V \quad V_{th2} = -3A \times \frac{2\Omega}{1\Omega + 3\Omega + 2\Omega} \times 3\Omega = -3V$$

$$V_{th} = V_{th1} + V_{th2} = 3V$$

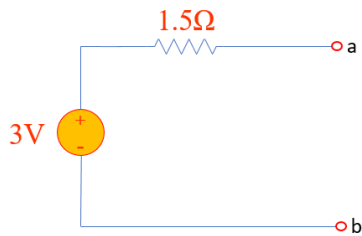
$$(b) I_{N1} = \frac{12V}{1\Omega + 2\Omega} = 4A \quad I_{N2} = -3A \times \frac{2\Omega}{1\Omega + 2\Omega} = -2A$$

$$I_N = I_{N1} + I_{N2} = 2A$$

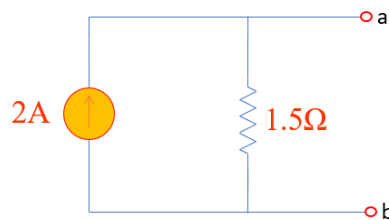
$$(c) R_{th} = \frac{V_{th}}{I_N} = \frac{3V}{2A} = 1.5\Omega$$

$V_{th}$  and  $I_N$  can be found directly without using superposition theorem.

Also  $R_{th}$  can be directly found by setting sources to zero and finding the equivalent resistance observed from a-b.



**Thevenin**



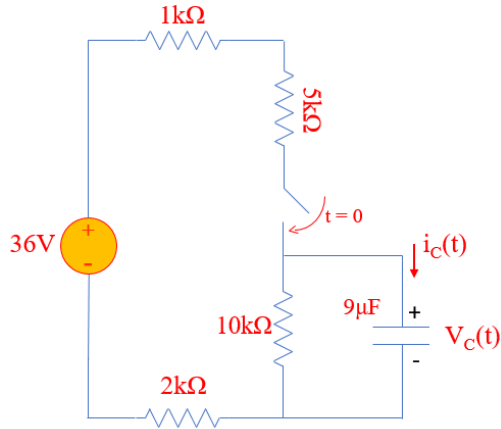
**Norton**

**PROBLEM 2: (20 points)**

The switch in the circuit in the figure below was open for a long time and is closed at  $t = 0$ .

(a) Find the expression of the  $V_C(t)$  for  $t > 0$ .

(b) Find the expression of the current of the capacitor,  $i_C(t)$  for  $t > 0$ .



$V_C(t)$	$20(1 - e^{-\frac{t}{\tau}})$ V
$i_C(t)$	$4.5e^{-\frac{t}{40ms}}$ mA

(a) For  $t < 0$ :  $V_C(t) = 0V$

For  $t > 0$ :

$$V_C(t=0^+) = V_C(t=0^-) = 0V$$

$$V_C(t=\infty) = \frac{10k\Omega}{1k\Omega + 5k\Omega + 10k\Omega + 2k\Omega} \times 36V = 20V$$

$$R \times C = \{10K\Omega \parallel (1K\Omega + 5K\Omega + 2K\Omega)\} \times 9\mu F = \frac{40}{9} K\Omega \times 9\mu F = 40ms$$

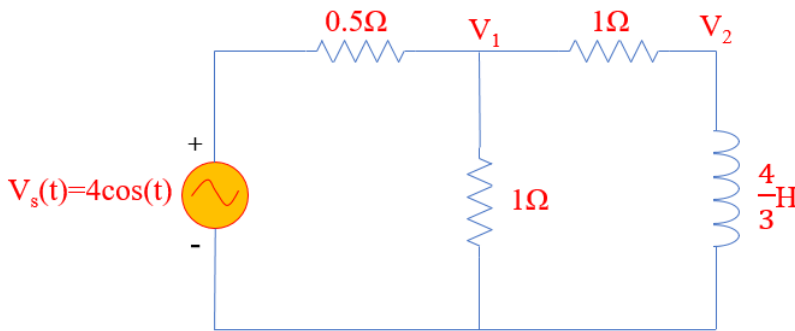
$$V_C(t > 0) = V_C(t=\infty) + \{V_C(t=0^+) - V_C(t=\infty)\} e^{-\frac{t}{\tau}}$$

$$V_C(t > 0) = 20(1 - e^{-\frac{t}{40ms}}) V$$

$$(b) i_C(t) = C \frac{dV_C(t)}{dt} = 4.5e^{-\frac{t}{40ms}} \text{ mA}$$

**PROBLEM 3: (25 points)**

Find  $V_1(t)$  and  $V_2(t)$  and  $V_1$  and  $V_2$ . ( $V_1$  is phasor of  $V_1(t)$ ).  $\tan(8^\circ)=1/7$



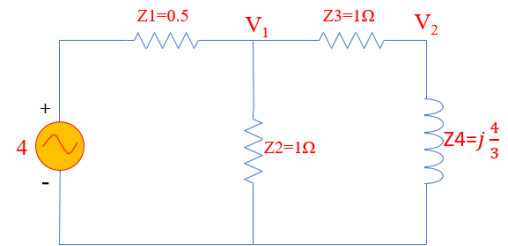
$V_1(t)$	$\frac{5\sqrt{2}}{3} \cos(t + 8 \times \frac{\pi}{180})V$
$V_2(t)$	$\frac{4\sqrt{2}}{3} \cos(t + \frac{\pi}{4})V$
$V_1$	$\frac{5\sqrt{2}}{3} e^{j8^\circ} V$
$V_2$	$\frac{4\sqrt{2}}{3} e^{j45^\circ} V$

**Voltage Division:**

$$V_1 = V_s \times \frac{Z_2 || (Z_3 + Z_4)}{Z_2 || (Z_3 + Z_4) + Z_1} = 4 \times \frac{1 + \frac{4}{3}j}{2 + \frac{4}{3}j} \div \frac{1 + \frac{4}{3}j}{2 + \frac{4}{3}j + 0.5}$$

$$= 4 \times \frac{1 + \frac{4}{3}j}{2 + 2j} = \frac{1}{3} \times (7 + j) = \frac{5\sqrt{2}}{3} e^{j8^\circ}$$

$$\rightarrow V_1(t) = \frac{5\sqrt{2}}{3} \cos(t + 8 \times \frac{\pi}{180})$$



**Voltage Division 2:**

$$V_2 = V_1 \times \frac{Z_4}{Z_3 + Z_4} = V_1 \times \frac{\frac{4}{3}j}{1 + \frac{4}{3}j}$$

So:

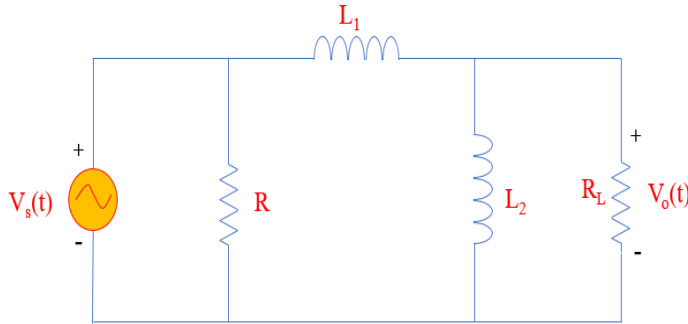
$$V_2 = 4 \times \frac{1 + \frac{4}{3}j}{2 + 2j} \times \frac{\frac{4}{3}j}{1 + \frac{4}{3}j} = \frac{4}{3} (1 + j) = \frac{4\sqrt{2}}{3} e^{j45^\circ} \rightarrow V_2(t) = \frac{4\sqrt{2}}{3} \cos(t + \frac{\pi}{4})$$

**PROBLEM 4: (20 points)**

In the circuit below, the Transfer Function is defined as:  $H(\omega) = \frac{V_o(\omega)}{V_s(\omega)}$

a) Find the transfer function  $H(\omega)$  in terms of  $R$ ,  $R_L$ ,  $L_1$  and  $L_2$ .

b) Find  $\lim_{\omega \rightarrow 0} |H(\omega)|$  and  $\lim_{\omega \rightarrow \infty} |H(\omega)|$ .



$H(\omega)$	$\frac{L_2 R_L}{(L_1 + L_2) R_L + j\omega L_1 L_2}$
$\lim_{\omega \rightarrow 0}  H(\omega) $	$\frac{L_2}{(L_1 + L_2)}$
$\lim_{\omega \rightarrow \infty}  H(\omega) $	0

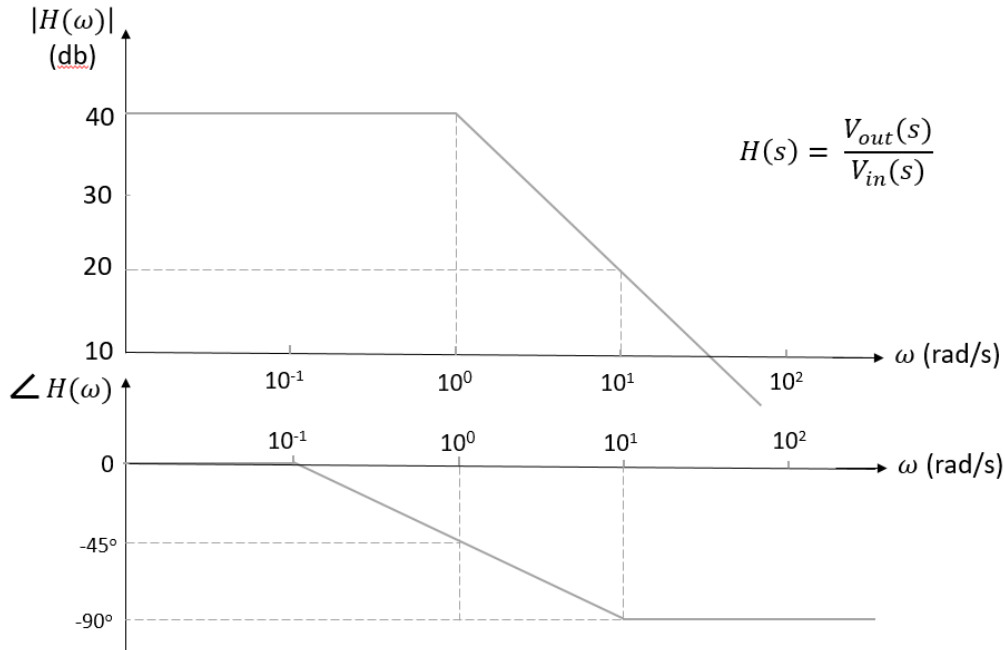
$$(a) H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{R_L \| j\omega L_2}{\{R_L \| j\omega L_2\} + j\omega L_1} = \frac{j\omega L_2 R_L}{j\omega(L_1 + L_2)R_L - \omega^2 L_1 L_2} = \frac{L_2 R_L}{(L_1 + L_2)R_L + j\omega L_1 L_2}$$

$$(b) \lim_{\omega \rightarrow 0} |H(\omega)| = \lim_{\omega \rightarrow 0} \left| \frac{L_2 R_L}{(L_1 + L_2)R_L + j\omega L_1 L_2} \right| = \frac{L_2}{(L_1 + L_2)}$$

$$\lim_{\omega \rightarrow \infty} \left| \frac{L_2 R_L}{(L_1 + L_2)R_L + j\omega L_1 L_2} \right| = 0$$

**PROBLEM 5: (20 points)**

The bode plots in the following represent the magnitude and phase for Transfer Function of an amplifier. If the input voltage is  $V_{in}(t) = \cos(t) + 10\sin(10t)$ , find the output voltage  $V_{out}(t)$ .



$$V_{in1}(t) = \cos t, (|H(\omega)|)_{\omega=1} = 40\text{db} = 10^{\frac{40}{20}} = 100, \text{Phase}(H(\omega))_{\omega=1} = -45^\circ \rightarrow V_{out1}(t) = 100\cos(t - \frac{\pi}{4})$$

$$V_{in2}(t) = 10 \sin 10t, (|H(\omega)|)_{\omega=10} = 20\text{db} = 10^{\frac{20}{20}} = 10, \text{Phase}(H(\omega))_{\omega=10} = -90^\circ \rightarrow V_{out2}(t) = 100\sin(10t - \frac{\pi}{2})$$

$$V_{in}(t) = V_{in1}(t) + V_{in2}(t) \rightarrow V_{out}(t) = V_{out1}(t) + V_{out2}(t)$$

$$V_{out}(t) = 100\cos(t - \frac{\pi}{4}) + 100\sin(10t - \frac{\pi}{2})$$

$V_o(t)$	$V_{out}(t) = 100\cos(t - \frac{\pi}{4}) + 100\sin(10t - \frac{\pi}{2})$
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