| Q1 | Q2 | Q3 | Q4 | Q5 | Total |
|---|---|---|---|---|---|
| /20 | /20 | /20 | /20 | /20 | /100 |

# EECS / CSE 70A Midterm Exam #1 SOLUTION KEY

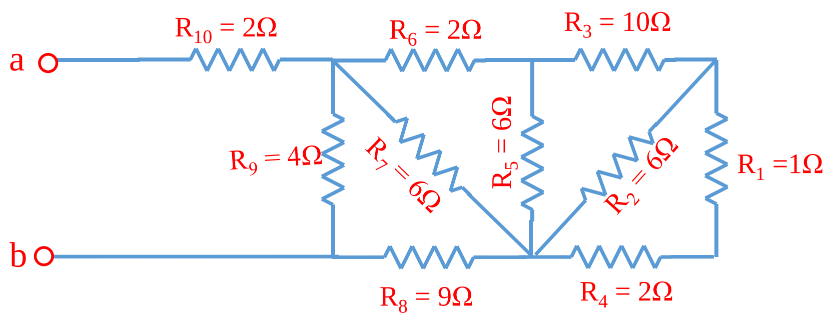
# DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

# Print your name on all pages.

# Write your solutions in clear steps with concise explanations.

**PROBLEM 1: (20 points)**

(a) Solve for the equivalent resistance, $R_{eq}$, across terminals a-b.




| $R_{eq}$ | 5Ω |
|---|---|

$R_{eq} = \{([([([(R_1 + R_4) \parallel R_2] + R_3) \parallel R_5] + R_6) \parallel R_7] + R_8) \parallel R_9\} + R_{10}$

$= \{([([([(1\Omega + 2\ \Omega) \parallel 6\ \Omega] + 10\ \Omega) \parallel 6\ \Omega] + 2\ \Omega) \parallel 6\ \Omega] + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

$= \{([([([3\Omega \parallel 6\ \Omega] + 10\ \Omega) \parallel 6\ \Omega] + 2\ \Omega) \parallel 6\ \Omega] + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

$= \{([([(2\Omega + 10\ \Omega) \parallel 6\ \Omega] + 2\ \Omega) \parallel 6\ \Omega] + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

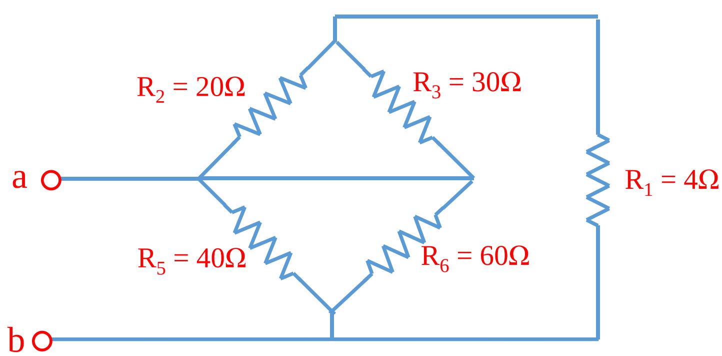
$= \{([([12\Omega \parallel 6\ \Omega] + 2\ \Omega) \parallel 6\ \Omega] + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

$= \{([(4\Omega + 2\ \Omega) \parallel 6\ \Omega] + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

$= \{(3\ \Omega + 9\ \Omega) \parallel 4\ \Omega\} + 2\ \Omega$

$= 5\ \Omega$

(b) Solve for the equivalent resistance, $R_{eq}$, across terminals a-b.




| $R_{eq}$ | 9.6Ω |
|---|---|

$R_{eq} = \{[(R_2 \parallel R_3) + R_1] \parallel (R_5 \parallel R_6)\}$

$= \{[(20\Omega \parallel 30\Omega) + 4\Omega] \parallel (40\Omega \parallel 60\Omega)\}$

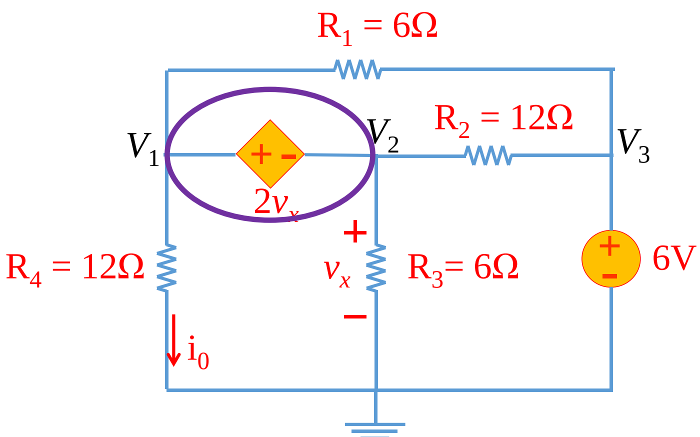
$= \{[12\Omega + 4\Omega] \parallel 24\Omega\}$

$= 16\Omega \parallel 24\Omega$

$= 9.6\Omega$

**PROBLEM 2: (20 points)**

Use nodal analysis, and solve for the node voltages and the current $i_0$.




| | |
|---|---|
| $V_1$ | 4.5V |
| $V_2$ | 1.5V |
| $V_3$ | 6V |
| $i_0$ | 0.375A |

Due to the VSCV, KCL in nodes 1 and 2 can not be written in terms of node voltages. We need to use a supernode:

KCL at supernode: $\frac{V_1 - 0\,\text{V}}{R_4} + \frac{V_1 - V_3}{R_1} + \frac{V_2 - 0\,\text{V}}{R_3} + \frac{V_2 - V_3}{R_2} = 0$ (1)

Node 3 set by voltage source: $V_3 = 6\,\text{V}$ (2)

Substitute (2) and (3) in (1):

$$\frac{V_1 - 0\,\text{V}}{12\Omega} + \frac{V_1 - 6\,\text{V}}{6\Omega} + \frac{V_2 - 0\,\text{V}}{6\Omega} + \frac{V_2 - 6\,\text{V}}{12\Omega} = 0 \rightarrow V_1 + V_2 = 6 \quad (3)$$

Voltage source controlled by voltage: $V_1 - V_2 = 2v_x$ , also $V_2 = v_x$ , as a result $V_1 = 3V_2$ (4)

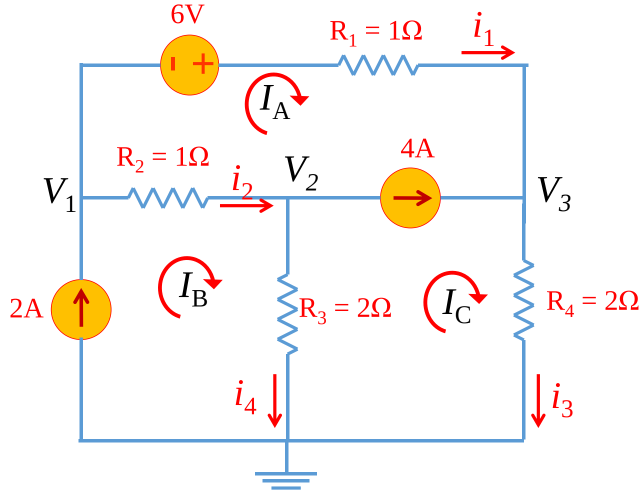
Substitute (4) in (3):

$4V_2 = 6\,\text{V}$, so $V_2 = 1.5\,\text{V}$ and $V_1 = 4.5\,\text{V}$

Also the current $i_0 = \frac{V_1 - 0\,\text{V}}{R_4} = \frac{4.5\,\text{V} - 0\,\text{V}}{12\Omega} = 0.375\,\text{A}$

**PROBLEM 3: (20 points)**

Use mesh analysis, and solve for the mesh currents and the labeled voltages.




| | |
|---|---|
| $I_A$ | -0.67 A |
| $I_B$ | 2 A |
| $I_C$ | 3.3 A |
| $i_1$ | -0.67 A |
| $i_2$ | 2.67 A |
| $i_3$ | 3.3 A |
| $i_4$ | -1.3 A |
| $V_1$ | 0 V |
| $V_2$ | -2.6 V |
| $V_3$ | 6.6 V |

The current of mesh B, $I_B$ set by the current source: $I_B = 2\,\text{A}$

Due to the 4A current source, KVL in meshes A and C can not be written in terms of mesh currents. We need to use a supermesh:

$$-6\,\text{V} + R_1 \cdot I_A + R_4 \cdot I_C + R_3 \cdot (I_C - I_B) + R_2 \cdot (I_A - I_B) = 0$$

Substitute the resistors value and the $I_B$ :

$$-6\,\text{V} + 1\Omega \cdot I_A + 2\Omega \cdot I_C + 2\Omega \cdot (I_C - 2\,\text{A}) + 1\Omega \cdot (I_A - 2\,\text{A}) = 0$$

So $2I_A + 4I_C = 12$ (1)

Also, based on the 4A current source: $I_C - I_A = 4$ (2)

By solving (1) and (2), we reach: $I_A = \frac{-2}{3}\text{A} = -0.67\,\text{A}$ and $I_C = \frac{10}{3}\text{A} = 3.3\,\text{A}$

Also,

$i_1 = I_A = -0.67\,\text{A}$, $\quad i_2 = I_B - I_A = 2.67\,\text{A}$

$i_3 = I_C = 3.3\,\text{A}$, $\quad i_4 = I_B - I_C = -1.3\,\text{A}$

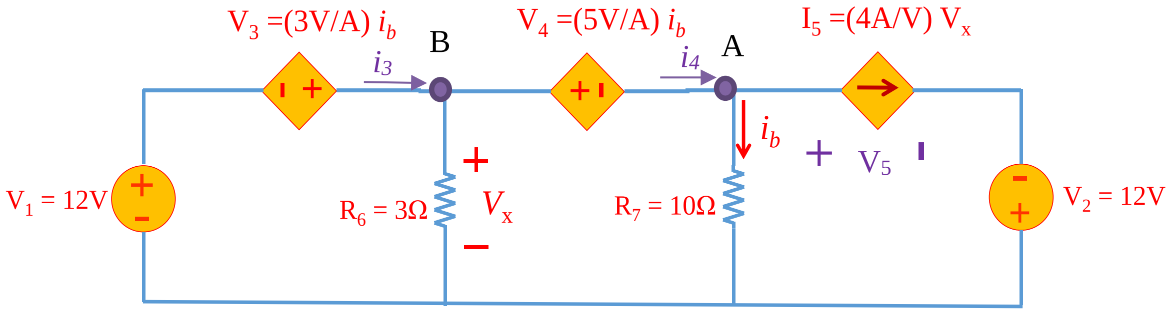
$$V_2 = R_3 \cdot i_4 = 2\Omega \cdot (-1.3\,\text{A}) = -2.6\,\text{V}$$

$$V_3 = R_4 \cdot i_3 = 2\Omega \cdot (3.3\,\text{A}) = 6.6\,\text{V}$$

$$V_1 - V_2 = R_2 \cdot i_2 = 1\Omega \cdot (2.67\,\text{A}) \rightarrow V_1 = 0\,\text{V}$$

**PROBLEM 4: (20 points)**

Find the absorbed or supplied power by each dependent source and indicate if it is source or sink.




KVL in the left mesh: $-V_1 - V_3 + V_x = 0$, So $V_x - 3i_b = 12$ (1)

KVL in the middle mesh: $-V_x + V_4 + R_7 \cdot i_b = 0$, So $-V_x + 15i_b = 0$ (2)

By solving (1) and (2), we reach $i_b = 1\text{A}$ and $V_x = 15\text{V}$

To find the powered absorbed/supplied by elements 3, 4 and 5, we need to calculate $i_3, V_3, i_4, V_4, I_5$ and $V_5$. To calculate the currents, we need to write the KCL at nodes A and B.

KCL at node A: $i_4 = i_b + I_5 = 1\text{A} + (4\text{A/V}) \cdot 15\text{V} = 61\text{A}$

KCL at node B: $i_3 = \dfrac{V_x}{R_6} + i_4 = \dfrac{15\text{V}}{3\Omega} + 61\text{A} = 66\text{A}$

To calculate $V_5$, we write the KVL in the right mesh: $V_5 - V_2 - R_7 \cdot i_b = 0$, so $V_5 = 12\text{V} + 10\Omega \cdot 1\text{A} = 22\text{V}$

Absorbed power by element 3: $P_3 = -i_3 \cdot V_3 = -66\text{A} \cdot (3\text{V/A})1\text{A} = -198\text{W}$, so element 3 is power source.

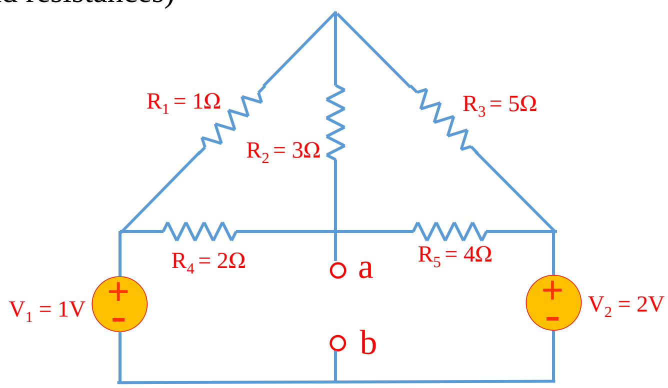
Absorbed power by element 4: $P_4 = i_4 \cdot V_4 = 61\text{A} \cdot (5\text{V/A})1\text{A} = 305\text{W}$, so element 4 is power sink.

Absorbed power by element 5: $P_5 = I_5 \cdot V_5 = (4\text{A/V})15\text{V} \cdot 22\text{V} = 1320\text{W}$, so element 5 is power sink.

| element | Power | Type (sink/source) |
|---|---|---|
| 3 | -198W | source |
| 4 | 305W | sink |
| 5 | 1320W | sink |

**PROBLEM 5: (20 points)**

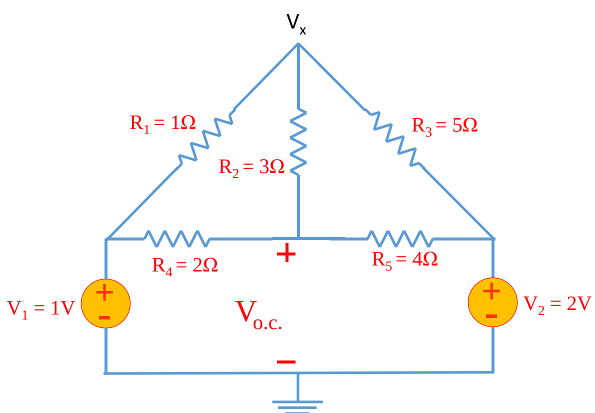
Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (Please draw the equivalent network representations and annotate the source voltages or currents and resistances)




| | |
|---|---|
| $v_{Th}$ | 1.29 V |
| $R_{Th}$ | 0.98Ω |
| $i_{No}$ | 1.3A |
| $R_{No}$ | 0.98Ω |

One needs to solve two of the following, since the third can be found by the previous two parameters: $v_{Th}$, $i_{No}$, $R_{Th}=R_{No}$ where $v_{Th} = i_{No}R_{Th}$

Open-circuit voltage at a-b terminals:




let us write KCL at the node "a" in terms of $V_x$

$$\frac{V_{o.c.}-V_x}{R_2}+\frac{V_{o.c.}-V_1}{R_4}+\frac{V_{o.c.}-V_2}{R_5}=0$$

So $\frac{V_{o.c.}-V_x}{3\Omega}+\frac{V_{o.c.}-1\text{V}}{2\Omega}+\frac{V_{o.c.}-2\text{V}}{4\Omega}=0$

As a result the first equation would be:

$$13V_{o.c.}-4V_x=12 \qquad (1)$$

Also, KCL at the top node:

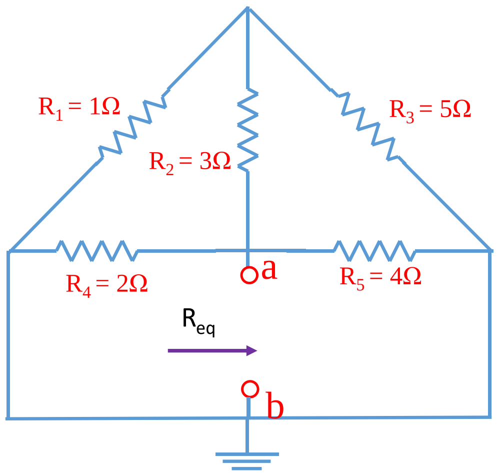
$$\frac{V_x-V_{o.c.}}{R_2}+\frac{V_x-V_1}{R_1}+\frac{V_x-V_2}{R_3}=0$$

So $\frac{V_x-V_{o.c.}}{3\Omega}+\frac{V_x-1\text{V}}{1\Omega}+\frac{V_x-2\text{V}}{5\Omega}=0$ , as a result the second equation would be:

$$-5V_{o.c.}+23V_x=21 \qquad (2)$$

By solving (1) and (2), we reach $V_x$=1.193V and $V_{o.c.}$ = 1.29 V. So $v_{Th} = V_{o.c.}$=1.29 V

To calculate the $R_{Th}$, we kill the independent voltage sources and find the equivalent resistance from a-b terminal:




$R_{Th} = R_{eq} = [(R_1||R_3)+R_2]||(R_4||R_5)$

$= [(1\Omega||5\Omega)+3\Omega]||(2\Omega||4\Omega)$

$= [\frac{5}{6}\Omega + 3\Omega]||\left(\frac{8}{6}\Omega\right) = \frac{92}{93}\Omega = 0.98\Omega$

$R_{No} = R_{Th} = \frac{92}{93}\Omega = 0.98\Omega$

$i_{No} = v_{Th}/R_{Th} = 1.3A$

Finally we have: