May 25th, 2017, 11:00 am to 12:10 pm ID no.:__________________________

Professor Peter Burke

Q1	Q2	Q3	Q4	Total
/20	/30	/30	/20	/100

EECS / CSE 70A Midterm Exam #2
SOLUTION KEY

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

Print your name on all pages.

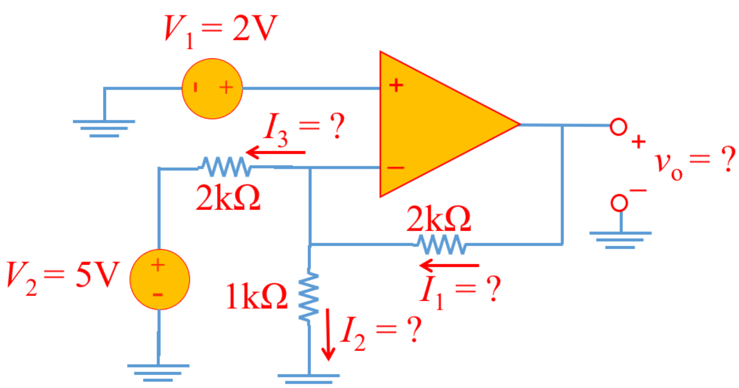
Write your solutions in clear steps with concise explanations.

PROBLEM 1: (20 points)

Assuming ideal op-amp:

(a) **Calculate currents I_1, I_2 and I_3.**

(b) **Find the output voltage v_o**



I_1	0.5mA
I_2	2mA
I_3	-1.5mA
v_o	3V

SOLUTION:

Opamp is ideal so voltage at its positive and negative inputs are equal: $V_+ = V_- = 2V$ and there is no current flowing in to the opamp input pins.

$$I_3 = \frac{V_- - V_2}{2k\Omega} = -1.5mA$$

$$I_2 = \frac{V_-}{1k\Omega} = 2mA$$

KCL at V_- : $I_1 = I_2 + I_3 = 0.5mA$

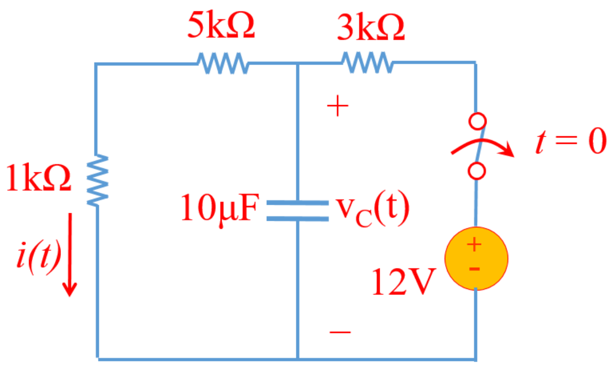
KVL: vo = $V_- + 2k\Omega * I_1 = 2 + 1 = 3V$

PROBLEM 2: (30 points)

The switch in the circuit in the figure below has been closed for a long time (from t = -∞ till t = 0) and is opened at t = 0.

(a) Find the voltage across the capacitor, $v_C(t)$ for t > 0.

(b) Find the current passing through the 1kΩ resistor for t > 0.



$v_C(t)$	$v_C(t) = 8e^{\frac{-t}{0.06}}$ (V)
$i(t)$	$i(t) = 1.33e^{\frac{-t}{0.06}}$ (mA)

SOLUTION:

Capacitor is open-circuit at DC, which is the state the circuit is in for t < 0 and t = ∞.

For t < 0 : $v_C(t) = \frac{5k\Omega+1k\Omega}{5k\Omega+1k\Omega+3k\Omega}\times 12V = 8V$ **and** $i(t) = \frac{12V}{5k\Omega+1k\Omega+3k\Omega} = 1.33mA$

For t = ∞ : notice the switch is open so the voltage source and 3kΩ resistors are out of the circuit. Capacitor is also open-circuit so no current passes through it → $v_C(t=\infty) = 0$ **and** $i(t=\infty) = 0$**.**

After the switch opens, capacitor sees 1kΩ and 5kΩ series resistances → circuit time constant = $\tau = RC = 6k\Omega\times 10uF = 6\times 10^{-2}s$

Solving for first order differential equation, we know for t > 0 :

$v_C(t) = v_C(t=\infty)+[v_C(t=0^+) - v_C(t=\infty)]e^{\frac{-t}{\tau}}$ **and** $i(t) = i(t=\infty)+[i(t=0^+) - i(t=\infty)]e^{\frac{-t}{\tau}}$

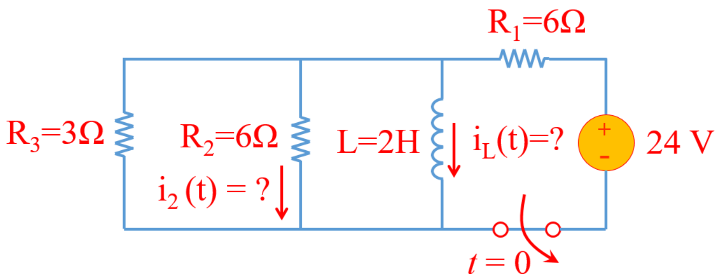
→ $v_C(t) = 8e^{\frac{-t}{0.06}}$ **(V) and** $i(t) = 1.33e^{\frac{-t}{0.06}}$ (mA)

PROBLEM 3: (30 points)

The switch in the circuit in the figure below was closed for a long time and is opened at t = 0.

(a) Find the expression of the $i_L(t)$ for t > 0.

(b) Find the expression of the current passing R_2, $i_2(t)$ for t > 0.



$i_L(t)$	$4e^{\frac{-t}{1}}$ (A)
$i_2(t)$	$-\frac{4}{3}e^{\frac{-t}{1}}$ (A)

SOLUTION:

inductor is short-circuit at DC, which is the state the circuit is in for t < 0 and t = ∞.

For t < 0 : $i_L(t) = \frac{24V}{6\Omega} = 4A$ **and** $i_2(t < 0) = 0$

For t = ∞ : notice the switch is open so the voltage source and 6kΩ resistor are out of the circuit. Inductor is also short-circuit and since there is no independent source → $i_L(t = \infty) = 0.$

From KCL we see after the switch is open $i_2(t) = -i_L(t)/3$ **and the current passing R_3** $i_1(t) = -2i_L(t)/3.$

The equivalent resistance seen from the inductor is $R_2 \| R_1 = 2\Omega$ → circuit time constant = τ = L/R = 2H/2Ω = 1*s*

Solving for first order differential equation, we know for t > 0 :

$i_L(t) = i_L(t{=}\infty) + [i_L(t{=}0^+) - i_L(t{=}\infty)]e^{\frac{-t}{\tau}}$ **and** $i_L(t) = -3i_2(t)$

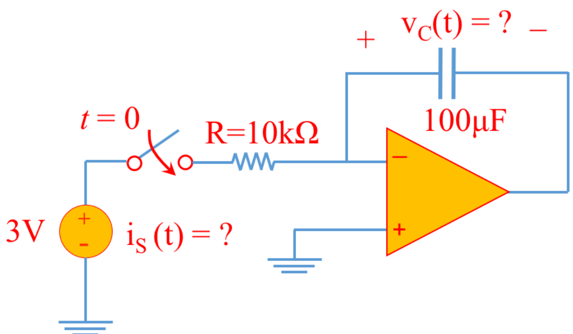
→ $i_L(t) = 0 + [4 - 0]e^{\frac{-t}{1}}$ **(mA)** $= 4e^{\frac{-t}{1}}$ **(mA) and** $i_2(t) = -\frac{4}{3}e^{\frac{-t}{1}}$ **(A)**

PROBLEM 4: (20 points)

The switch in the circuit in the figure below was open for a long time (from t = -∞ till t = 0) and is closed at t = 0. Assume the capacitor is completely discharged before t = 0. Opamp is ideal.

(a) Find the current supplied by the voltage source, $i_S(t)$ for t > 0.

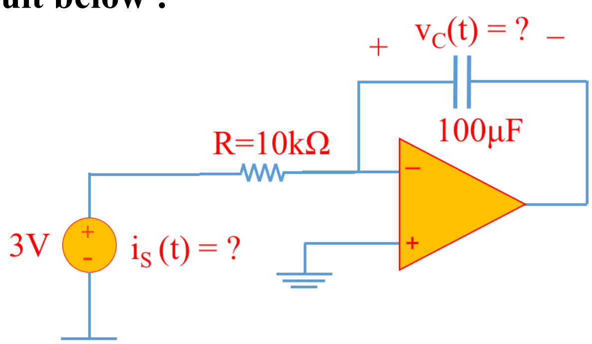
(b) Find the voltage across the capacitor, $v_C(t)$ at t = 2s.



$i_S(t)$	**0.3mA**
$v_C(t)$	**6V**

SOLUTION:

for t < 0 capacitor is fully discharged so $v_C(t < 0) = 0$. After the switch closes, we will have the circuit below :



Opamp is ideal → negative and positive opamp input pins have the same voltage: $V_- = V_+ = 0V$ → $i_S(t) = 3V / 10k\Omega = 0.3mA$.

Since no current enters the opamp input pins, $i_S(t)$ has nowhere to go but through the capacitor: $i_C(t) = i_S(t) = 0.3mA$.

For capacitor we know: $i_C(t) = C\frac{dv_C(t)}{dt}$ or $v_C(t) = \int \frac{1}{C} i_C(t)dt$ →

To find the voltage at t = 2 we do the integration fro t = 0 to t = 2:

$$v_C(t) = \int_0^2 10^4 \times 0.3mA\, dt = \int_0^2 3\, dt = 6V$$