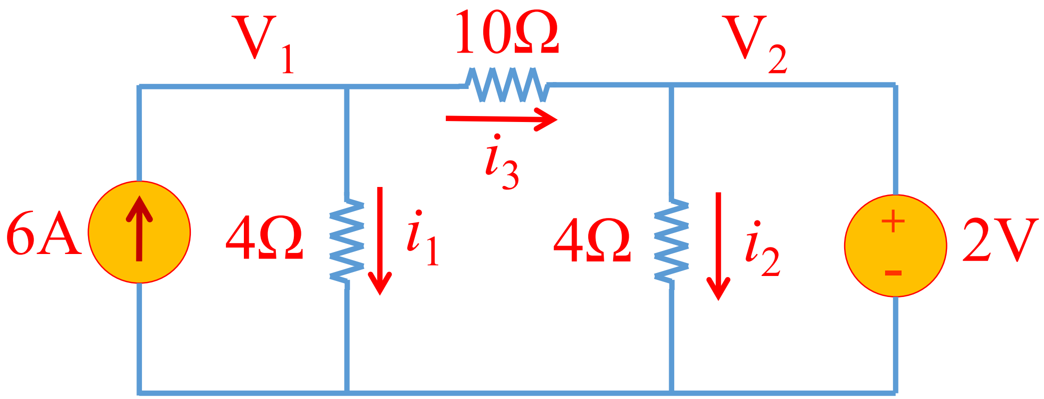
EECS/CSE 70A Network Analysis I

Homework #3

Solution Key

Problem 1: (KCL, KVL, Ohm's Law) Find currents i_1, i_2, i_3. (10pts.)



Solution:

KCL @ node 1: 6 A = $i_1 + i_3$

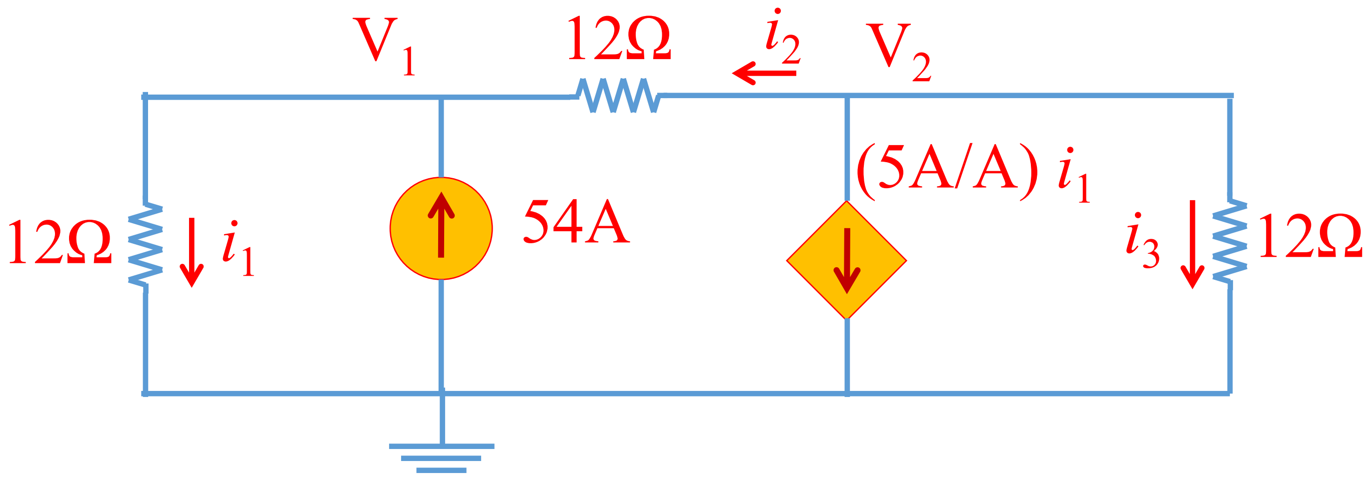
$i_1 = V_1/4\ \Omega$, $i_3 = (V_1 - V_2)/10\ \Omega = (V_1\text{-}2)/10\ \Omega$

→ 6 A = $V_1/4 + (V_1\text{-}2)/10\ \Omega$ → $V_1 = 17.71\text{V}$

→ $i_1 = 4.42\text{A}$, $i_3 = 1.57\text{A}$

$I_2 = 2\text{V}/4\ \Omega = 0.5\text{A}$

Problem 2: Use nodal analysis and find i_3. (10pts.)



Solution:

$i_1 = V_1/12\ \Omega$, $i_3 = V_2/12\ \Omega$, $i_2 = (V_2 - V_1)/12\ \Omega$

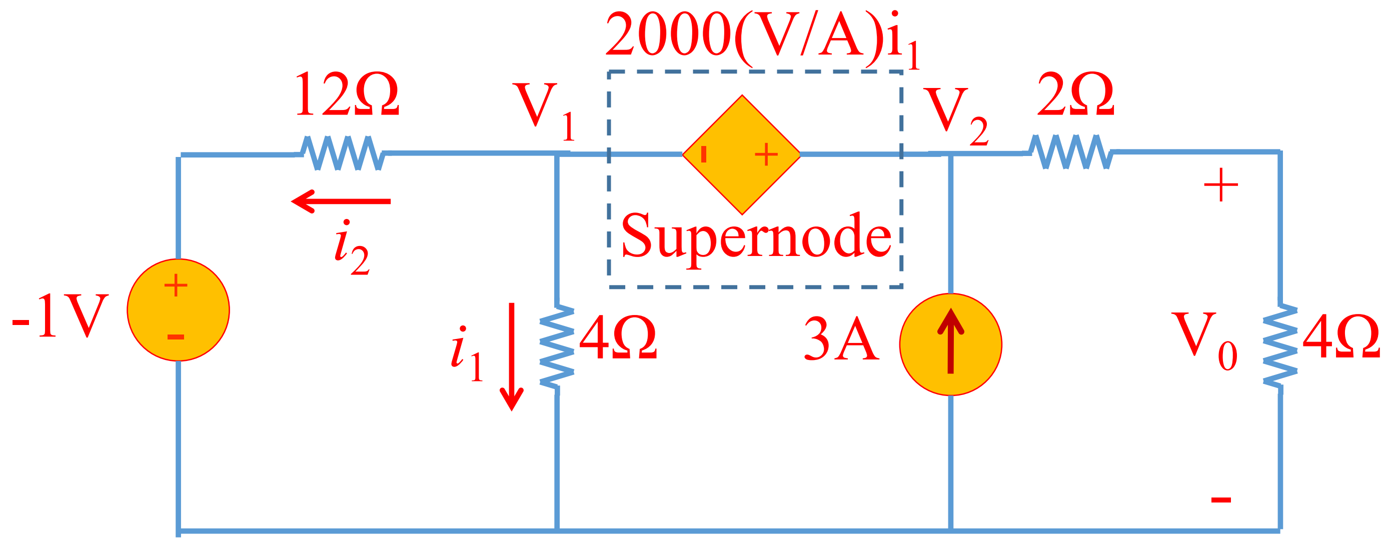
KCL @ node 1: $54\text{ A} + i_2 = i_1 \rightarrow 54 + (V_2 - V_1)/12\ \Omega = V_1/12\ \Omega$

KCL @ node 2: $5i_1 + i_2 + i_3 = 0 \rightarrow 5V_1/12\ \Omega + (V_2 - V_1)/12\ \Omega + V_2/12\ \Omega = 0$

→ Solving the system of two equations and two unknowns:

→ $V_1 = 162\text{V}$ and $V_2 = -324\text{V} \rightarrow i_3 = -27\text{A}$

Problem 3: Use nodal analysis and find V_0. (10pts.)



Solution:

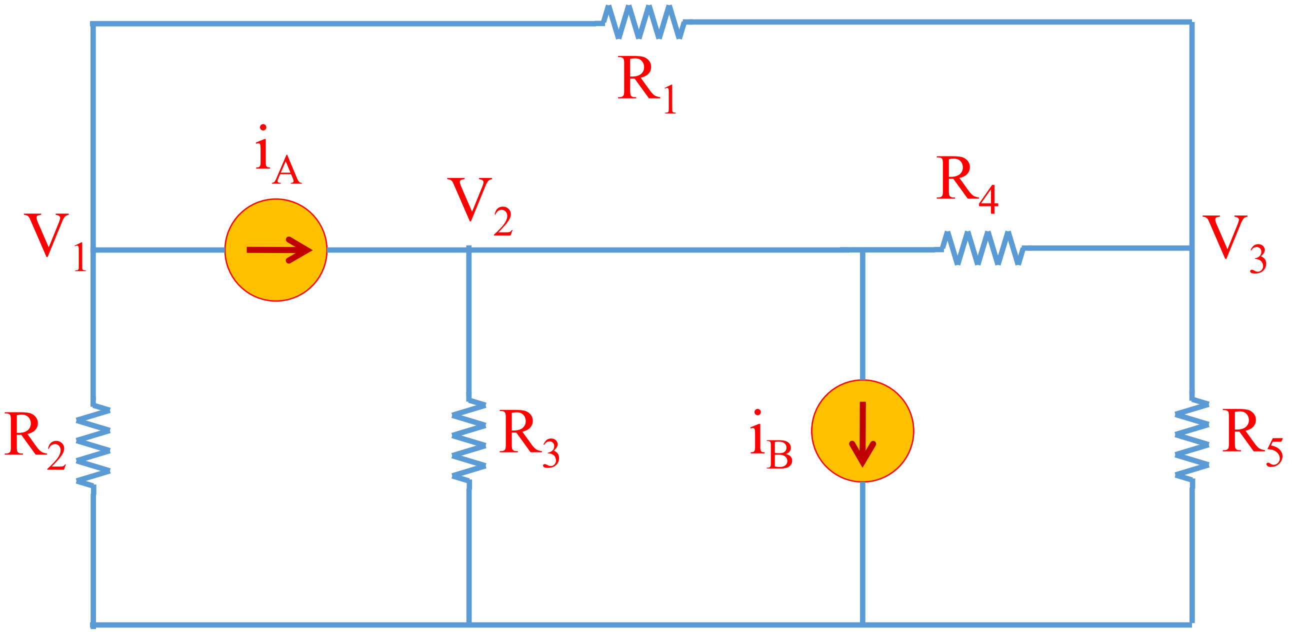
$i_1 = V_1/4\ \Omega$, $i_2 = (V_1 + 1)/12\ \Omega$, $V_2 = V_1 + 2000i_1 = V_1 + 500V_1 = 501V_1$

KCL @ supernode: $i_2 + i_1 = 3A - V_2/6 \rightarrow V_1/4 + (V_1 + 1)/12\ \Omega = 3A - V_2/6 = 3A - 501V_1/6$

$\rightarrow V_2 = 17.43V$

$V_0 = 4V_2/(4+2) = 11.62V$

Problem 4: Write all nodal voltage equations and put them in the matrix form. (You do not need to solve.) (10pts.)



Solution. Write KCL for all three nodes of the circuit:

$(V_1-V_3)/R_1 + V_1/R_2 + i_A = 0$, $V_2/R_3 + i_B + (V_2-V_3)/R_4 = 0$,

$(V_3-V_1)/R_1 + (V_3-V_2)/R_4 + V_3/R_5 = 0$

Rearranging the equations in the matrix form:

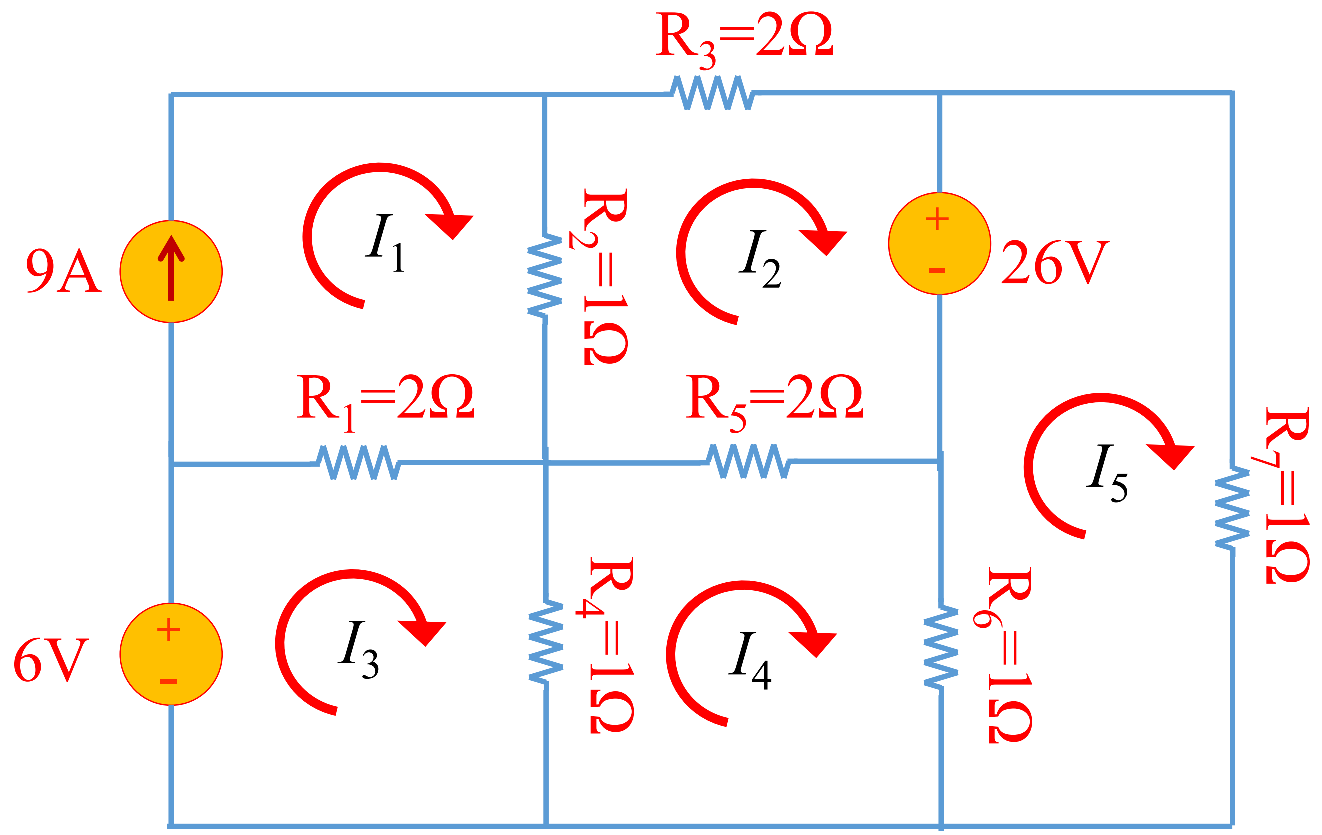
$V_1(1/R_1+1/R_2) - V_2\times 0 - V_3/R_1 = -i_A$

$-V_1\times 0 + V_2(1/R_3+1/R_4) - V_3/R_4 = i_A - i_B$

$-V_1(1/R_1) - V_2(1/R_4) + V_3(1/R_1+1/R_4+1/R_5) = 0$

$$\rightarrow \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & 0 & -\frac{1}{R_1} \\ 0 & \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R4} \\ -\frac{1}{R_1} & -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -i_A \\ i_A - i_B \\ 0 \end{bmatrix}$$

Problem 5: Write all mesh current equations and put them in the matrix form. (You do not need to solve.) (10pts.)



Solution. Write KVL for loops I_2 through I_5:

Loop I_2: $(I_2 - I_1)R_2 + I_2R_3 + 26 + (I_2 - I_4)R_5 = 0$

Loop I_3: $-6 + (I_3 - I_1)R_1 + (I_3 - I_4)R_4 = 0$

Loop I_4: $(I_4 - I_3)R_4 + (I_4 - I_2)R_5 + (I_4 - I_5)R_6 = 0$

Loop I_5: $-26 + I_5R_7 + (I_5 - I_4)R_6 = 0$

Rearranging the equations in the matrix form:

Loop I_2 : $-I_1 + 5I_2 - 2I_4 = -26$

Loop I_2 : $-2I_1 + 3I_3 - I_4 = 6$

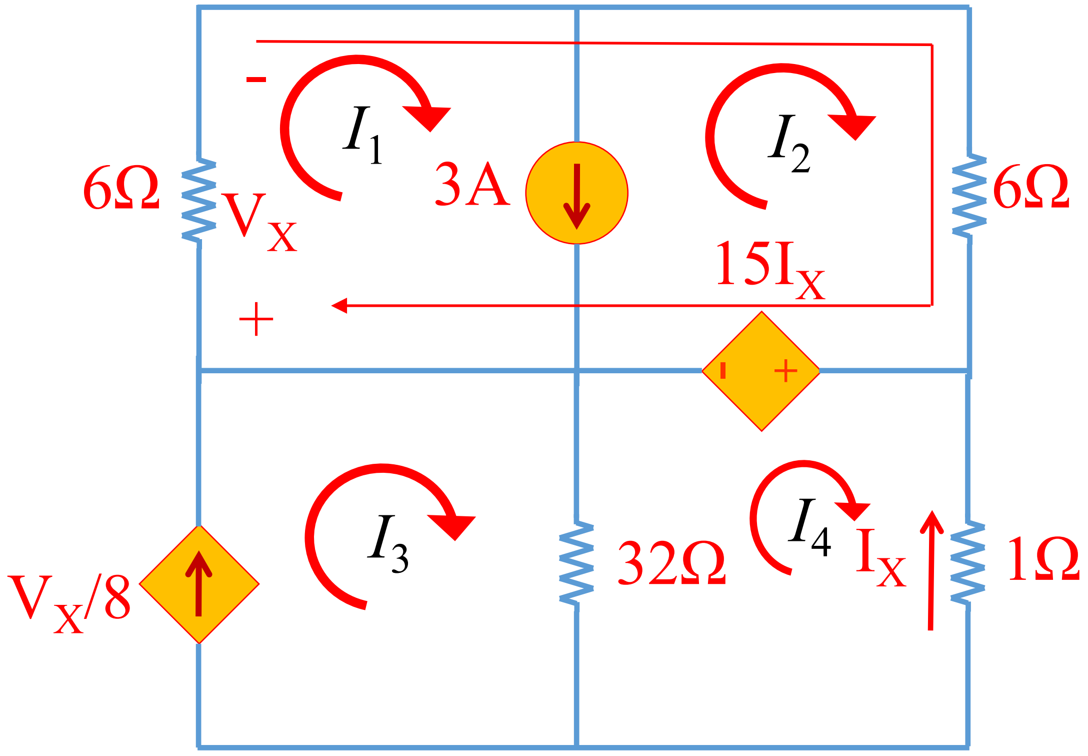
Loop I_2 : $-2I_2 - I_3 + 4I_4 - I_5 = 0$

Loop I_2 : $-I_4 + 2I_5 = 26$

So in the matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 5 & 0 & -2 & 0 \\ -2 & 0 & 3 & -1 & 0 \\ 0 & -2 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 9 \\ -26 \\ 6 \\ 0 \\ 26 \end{bmatrix}$$

Problem 6: Use mesh analysis to find the power delivered/consumed by the CCVS (10pts.)



Solution: Note we have a current source in the loop 1, we have a "supermesh" in this circuit, reducing the number of the equations.

$I_1 - I_2 = 3A$, $V_X = 6I_1$, $I_X = -I_4$, $I_3 = V_X/8 = 3I_1/4$

Write KVL for the loops:

Loop I_4: $-32(I_4 - I_3) + 15I_X - I_4 = 0$

→ $-48I_4 + 32I_3 = 0$ and since $I_3 = 3I_1/4$ →

→ $-48I_4 + 24I_1 = 0$ or $I_1 = 2I_4$ (*)

KVL @ supermesh: $-V_X - 6I_2 - 15I_X = 0$ →

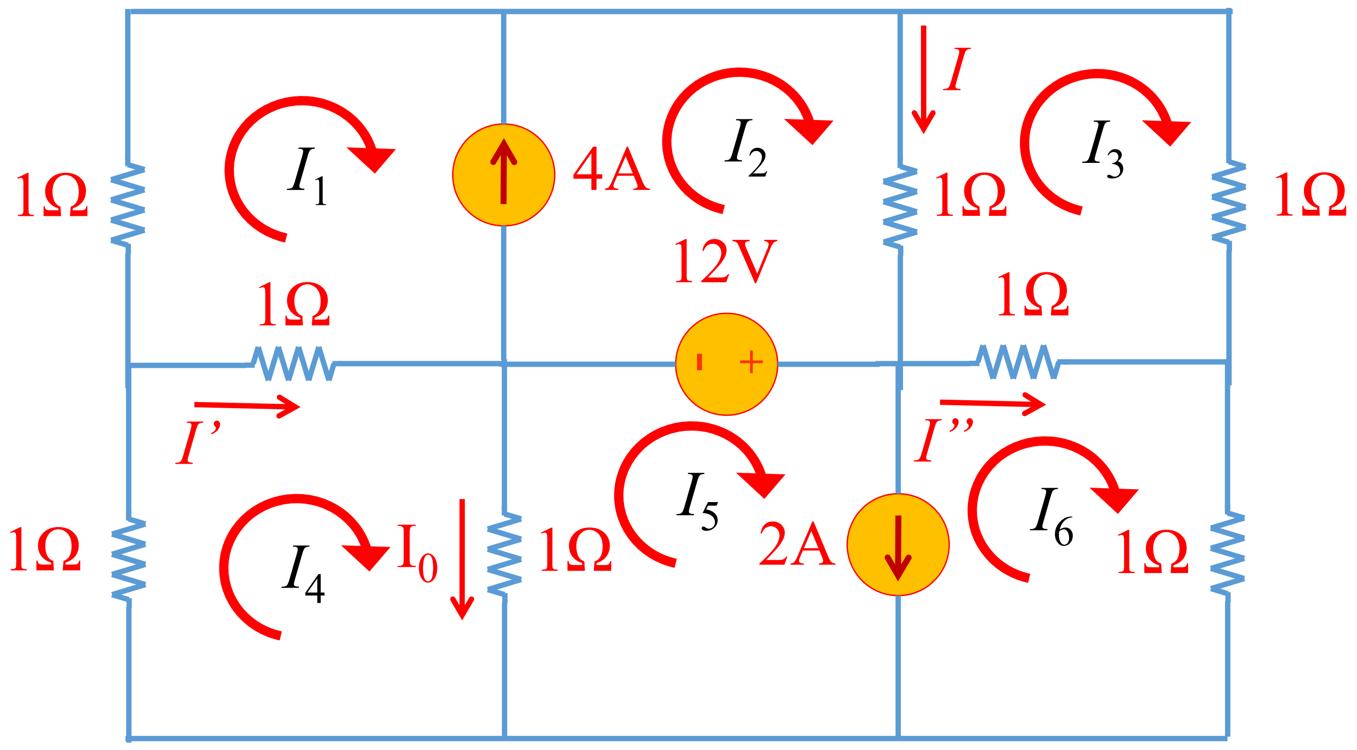
$-6I_1 - 6I_2 + 15I_4 = 0$ and since $I_1 - I_2 = 3A$ →

$-12I_1 + 15I_4 = 18A$ (**)

→ From (*) and (**): $I_1 = 4A$, $I_4 = 2A$, $I_2 = 1A$

→ The current passing through CCVS (with a voltage of $15I_X$=-30V)from its positive polarity to its negative polarity is $I_2 - I_4 = -1A$ so it is consuming +30W

Problem 7: Use mesh analysis to find I_0 (10pts.)



Solution in the next page

KCL: $I_2 = I + I_3 \rightarrow I = I_2 - I_3$

KCL: $I_3 + I'' = I_6 \rightarrow I'' = I_6 - I_3$

KCL: $I_4 = I_1 + I' \rightarrow I' = I_4 - I_1$

KCL: $I_o + I_5 = I_4 \rightarrow I_o = I_4 - I_5$

KVL: $12 = 1 \times I'' + 1 \times I_6 - 1 \times I_o$ → replacing for I_o and I'' from KCL's above → $\boxed{-I_3 - I_4 + I_5 + 2I_6 = 12}$

KVL: $1 \times I' + 1 \times I_o + 1 \times I_4 = \phi \rightarrow 1 \times (I_4 - I_1) + 1 \times (I_4 - I_5) + 1 \times I_4 = \phi$

$\rightarrow \boxed{-I_1 + 3I_4 - I_5 = \phi}$

KCL: $I_6 - I_5 = -2A$, $I_2 - I_1 = 4A$

KVL: $1 \times I_1 + 1 \times I_3 - 1 \times I'' + 12 + 1 \times (-I') = \phi$

$\rightarrow 1 \times I_1 + 1 \times I_3 - 1 \times (I_6 - I_3) - 1 \times (I_4 - I_1) = \phi$

$\rightarrow \boxed{2I_1 + 2I_3 - I_4 - I_6 = -12}$

KVL: $1 \times I_3 + 1 \times (-I'') + 1 \times (-I) = \phi$

$\rightarrow I_3 - (I_6 - I_3) - (I_2 - I_3) = \phi \rightarrow \boxed{-I_2 + 3I_3 - I_6 = \phi}$

Arrange in matrix form and solve:

$0I_1 + 0I_2 - I_4 + I_5 + 2I_6 = 12$

$-I_1 + 0I_2 + 0I_3 + 3I_4 - I_5 + 0I_6 = \phi$

$0I_1 + 0I_2 + 0I_3 + 0I_4 - I_5 + I_6 = -2$

$I_1 - I_2 + 0I_3 + 0I_4 + 0I_5 + 0I_6 = -4$

$2I_1 + 0I_2 + 2I_3 - I_4 + 0I_5 - 1I_6 = -12$

$0I_1 - I_2 + 3I_3 + 0I_4 + 0I_5 - I_6 = \phi$

→

$I_1 = -4.93\ A$

$I_2 = -0.933\ A$

$I_3 = 0.933\ A$

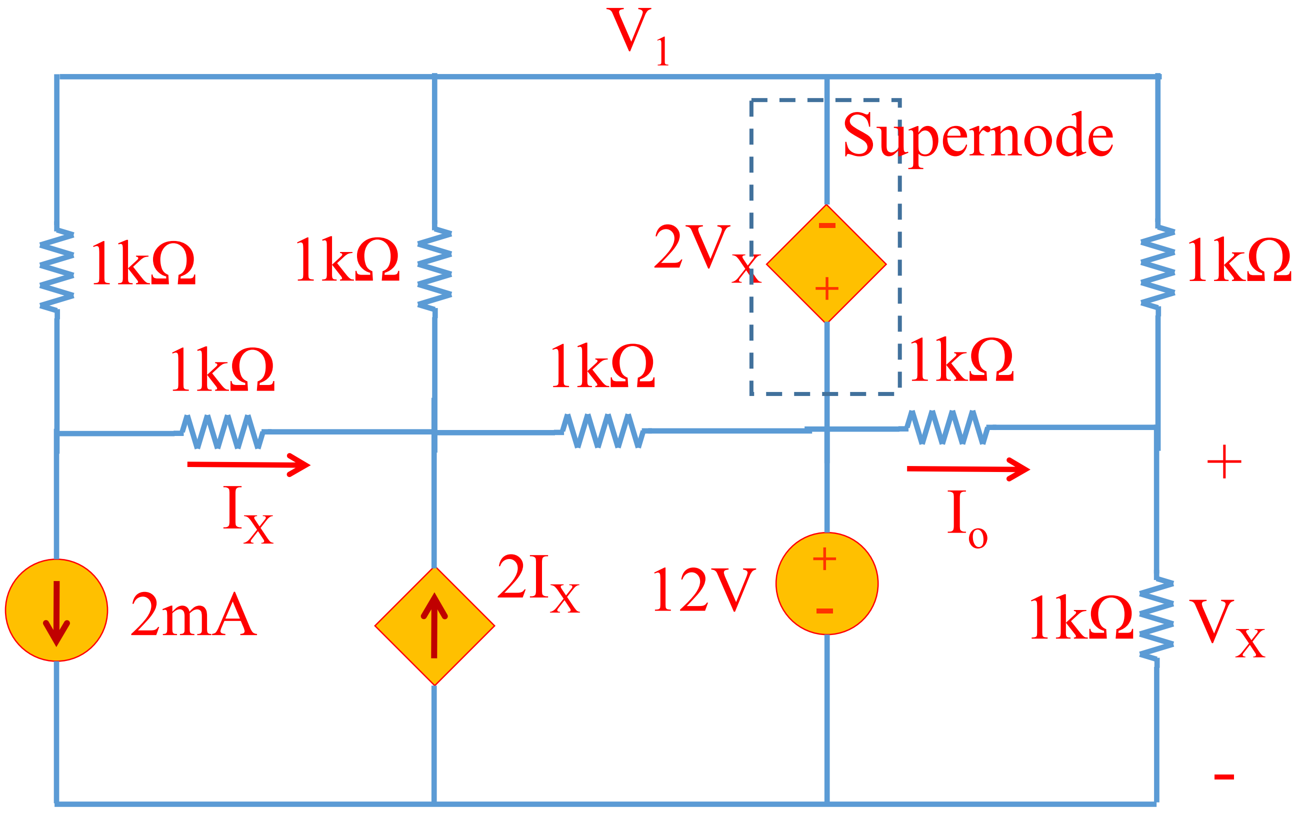
$I_4 = 0.267\ A$

$I_5 = 5.73\ A$

$I_6 = 3.73\ A$

$I_o = I_4 - I_5 = -5.46A$

Problem 8: Use both nodal and mesh analyses to find I_o (10pts.)



KCL @ V_x: $\frac{V_1 - V_x}{1k\Omega} + I_o = \frac{V_x}{1k\Omega}$

$I_o = \frac{12V - V_x}{1k\Omega} \rightarrow \frac{V_1}{1k\Omega} = \frac{3V_x - 12V}{1k\Omega}$

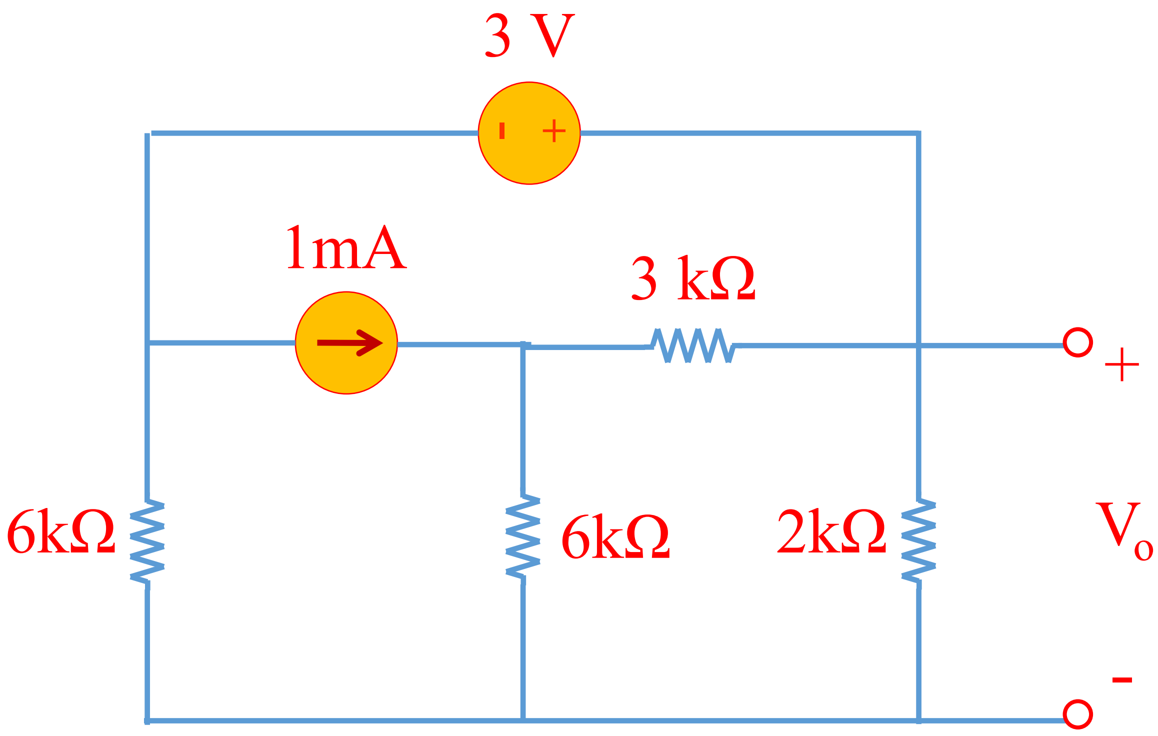
$\rightarrow \boxed{V_1 = 3V_x - 12V}$ (*)

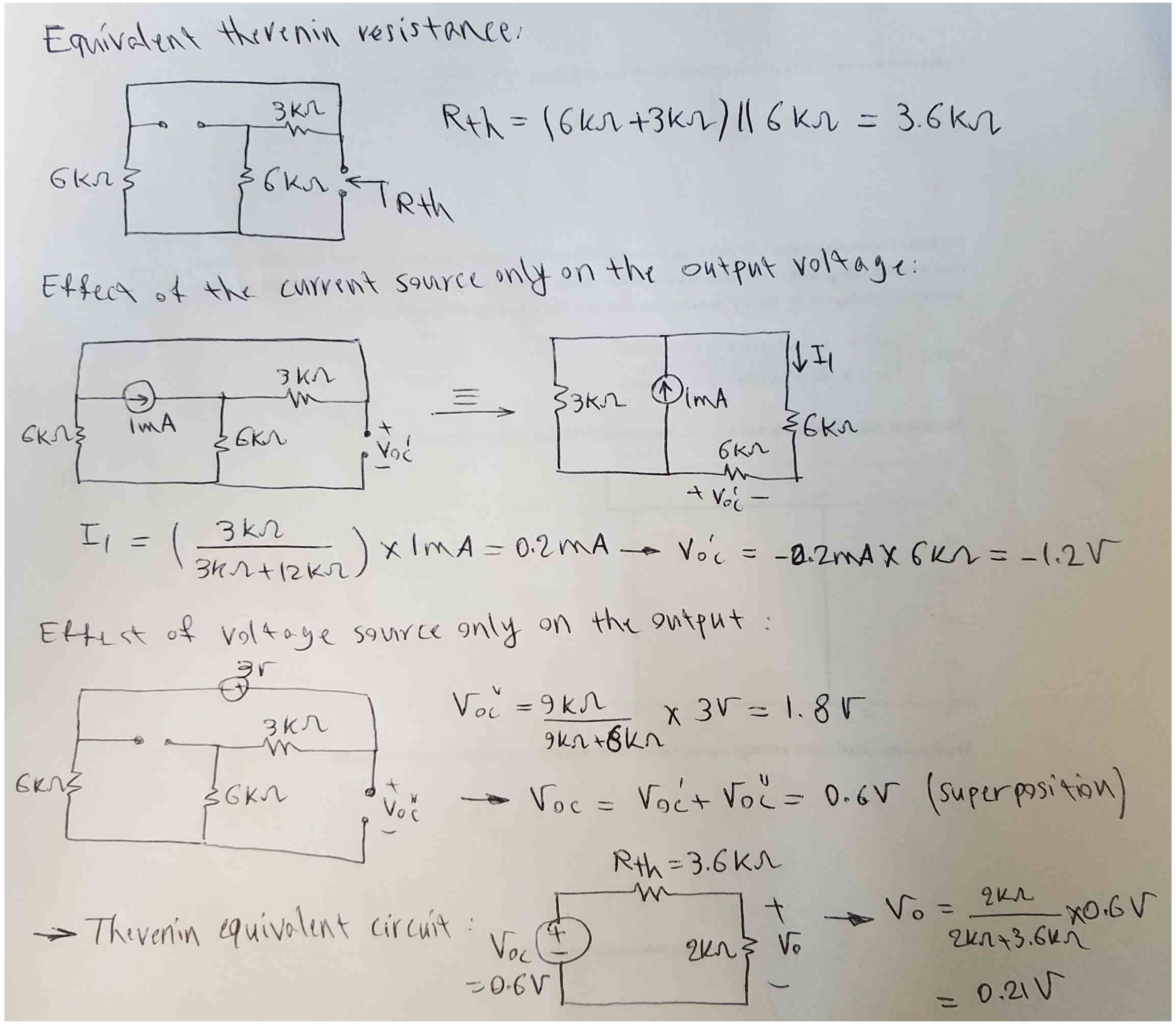
KVL @ supernode: $\boxed{V_1 + 2V_x = 12}$ (**)

→ from the two equations above:

$V_x = \frac{24}{5} V = 4.8V \rightarrow I_o = \frac{12 - 4.8V}{1k\Omega} = 7.2mA$

Problem 9: Find V_o using Thévenin theorem. (10pts.)





Equivalent thevenin resistance:

$R_{th} = (6k\Omega + 3k\Omega) \| 6k\Omega = 3.6k\Omega$

Effect of the current source only on the output voltage:

$I_1 = \left(\frac{3k\Omega}{3k\Omega + 12k\Omega}\right) \times 1mA = 0.2mA \rightarrow V'_{oc} = -0.2mA \times 6k\Omega = -1.2V$

Effect of voltage source only on the output:

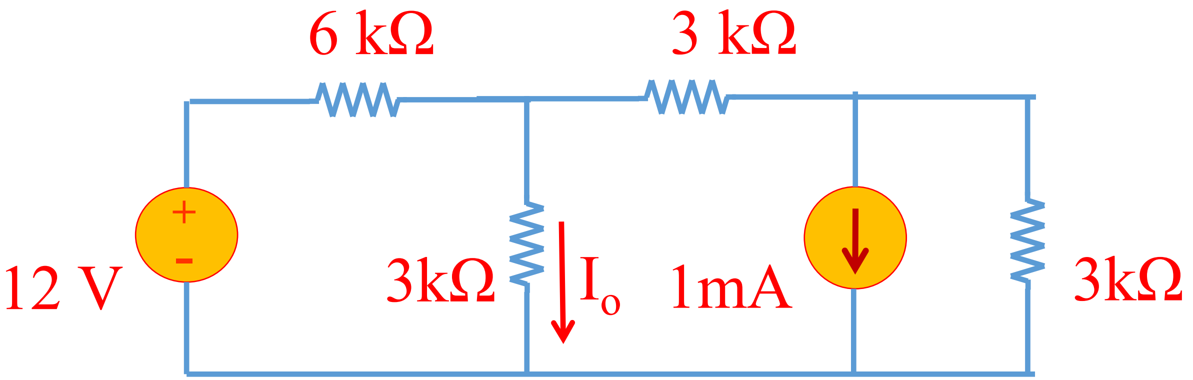
$V''_{oc} = \frac{9k\Omega}{9k\Omega + 6k\Omega} \times 3V = 1.8V$

$\rightarrow V_{oc} = V'_{oc} + V''_{oc} = 0.6V$ (superposition)

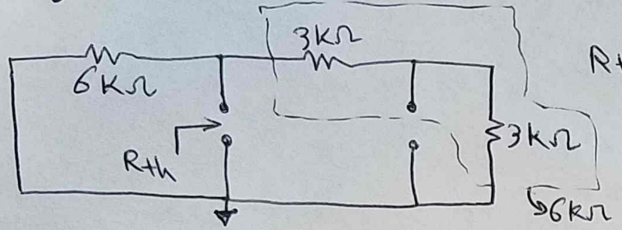
→ Thevenin equivalent circuit: $V_{oc} = 0.6V$, $R_{th} = 3.6k\Omega$, $2k\Omega$, V_o

$\rightarrow V_o = \frac{2k\Omega}{2k\Omega + 3.6k\Omega} \times 0.6V = 0.21V$

Problem 10: Find I_o using Norton theorem. (10pts.)

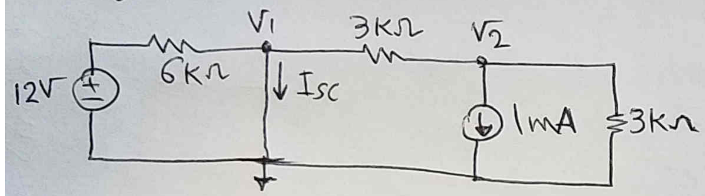


Solving for equivalent Norton/Thevenin resistance:



$R_{th} = 6k\Omega \,||\, 6k\Omega = 3k\Omega$

Finding the short circuit current (I_{sc}):



$V_1 = \phi$

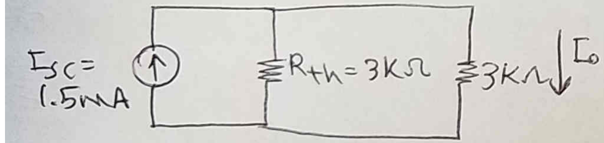
KCL @ V_1: $\dfrac{12 - V_1}{6k\Omega} = \dfrac{V_1 - V_2}{3k\Omega} + I_{sc}$

KCL @ V_2: $\dfrac{V_2 - V_1}{3k\Omega} + 1mA + \dfrac{V_2}{3k\Omega} = \phi$

$V_2 = -1.5V$

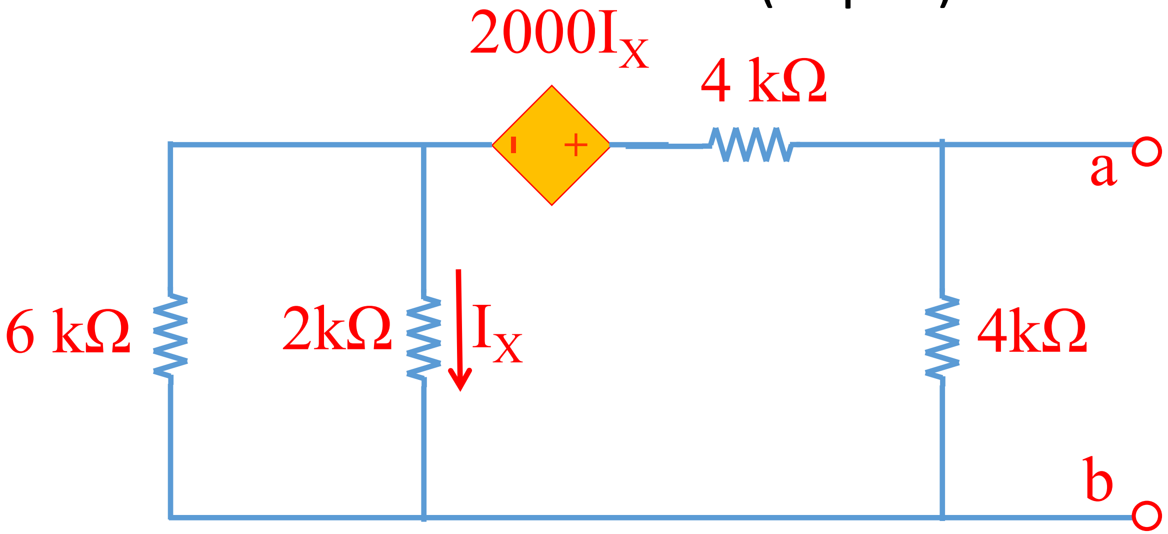
$I_{sc} = 1.5mA$

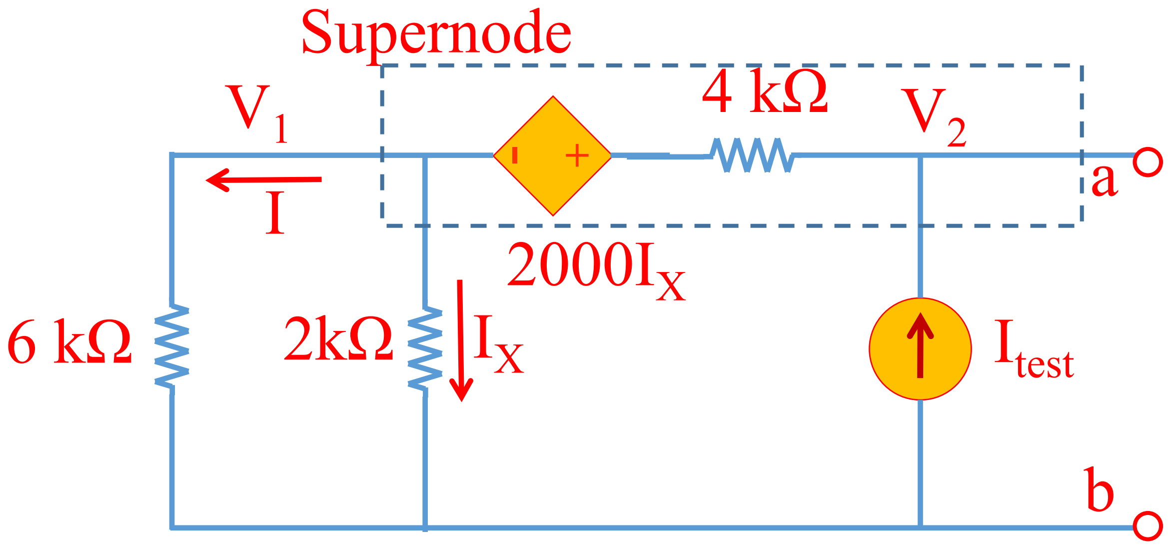
→ Equivalent Norton Circuit:



$I_o = \dfrac{3k\Omega}{3k\Omega + 3k\Omega} \times I_{sc} = 0.75\ mA$

Problem 11: Obtain the Thévenin and Norton equivalent network representations as seen from the terminals a-b. (10pts.)





Solution: Note we do not have any independent source, so $V_{th} = 0$. We only need to find the Thevenin resistor seen from terminal a-b. To do so, we apply a test current at the port and find the voltage appearing across it (V_2). The ratio of this voltage (V_2) and the applied current results in resistor R. We have a resistor parallel to the port (4kΩ). Finally R_{th} will be R||4kΩ.

KCL at supernode: $V_1/6k\Omega + V_1/2k\Omega = I_{test}$

$\rightarrow 2V_1/3k\Omega = I_{test} \rightarrow V_1 = 3k \times I_{test}/2$

$V_2 - V_1 = 2000I_X + 4k\Omega \times I_{test} = 2000(V_1/2k\Omega) + 8V_1/3 \rightarrow V_2 = 14V_1/3 = 7k \times I_{test} \rightarrow V_2/I_{test} = R = 7k\Omega$

$\rightarrow R_{th} = 4k\Omega \,||\, 7k\Omega = 2.54k\Omega$