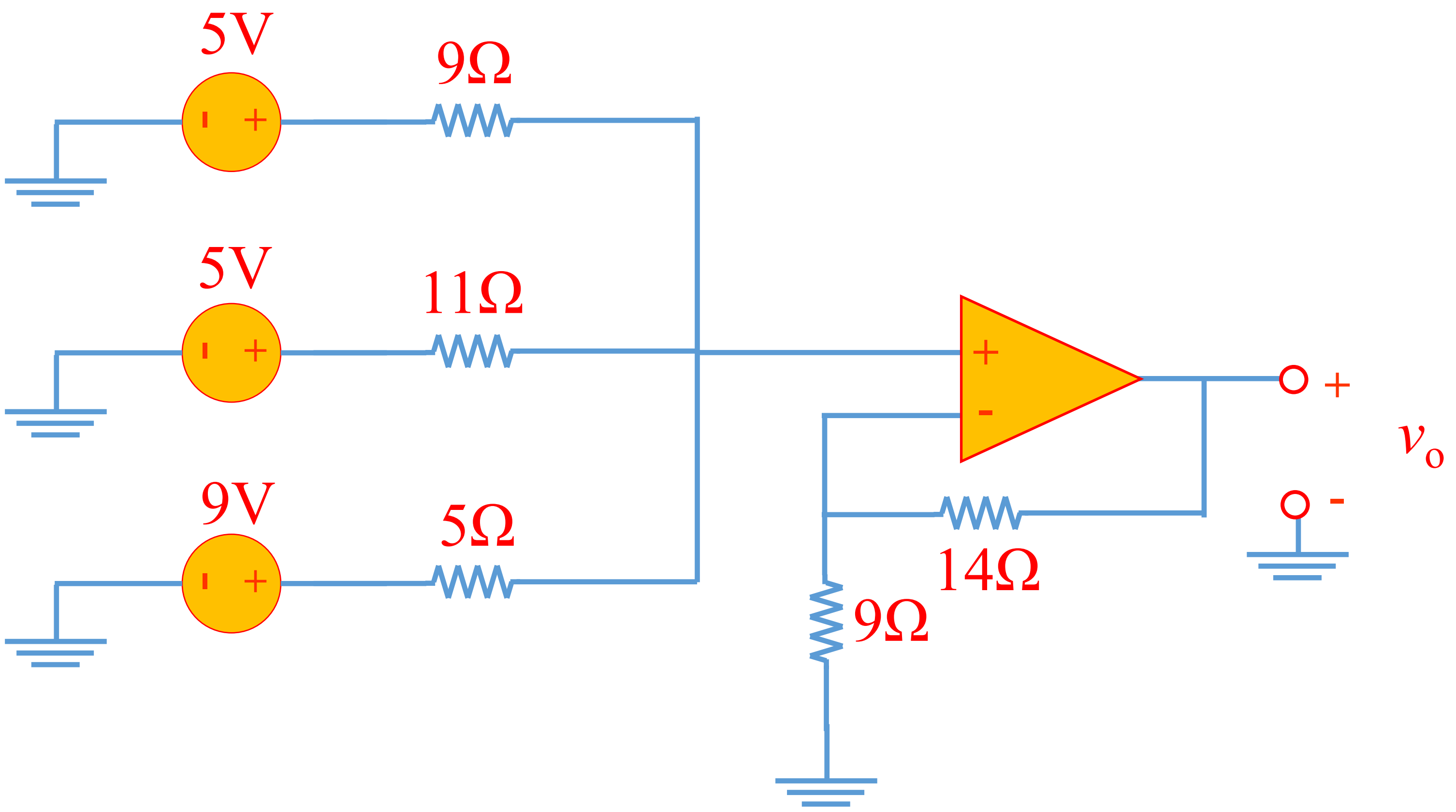
EECS/CSE 70A Network Analysis I

Homework #4

Solution Key

Problem 1: (Ideal Opamp) Find the output voltage v_o (20pts.)



Solution:

Ideal opamp → A = ∞, opamp input resistance R_{in} = ∞ and opamp output resistance R_o = 0.

KCL at V_+ : $\frac{5-V_+}{9} + \frac{5-V_+}{11} + \frac{9-V_+}{5} = 0$ →

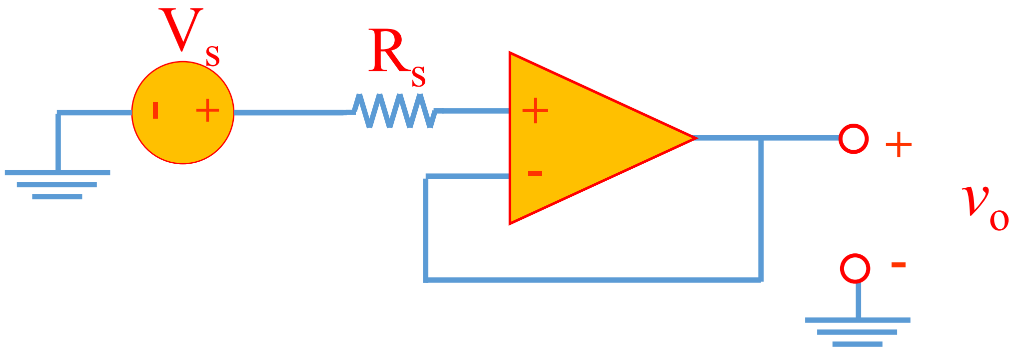
$V_+ = 7V$

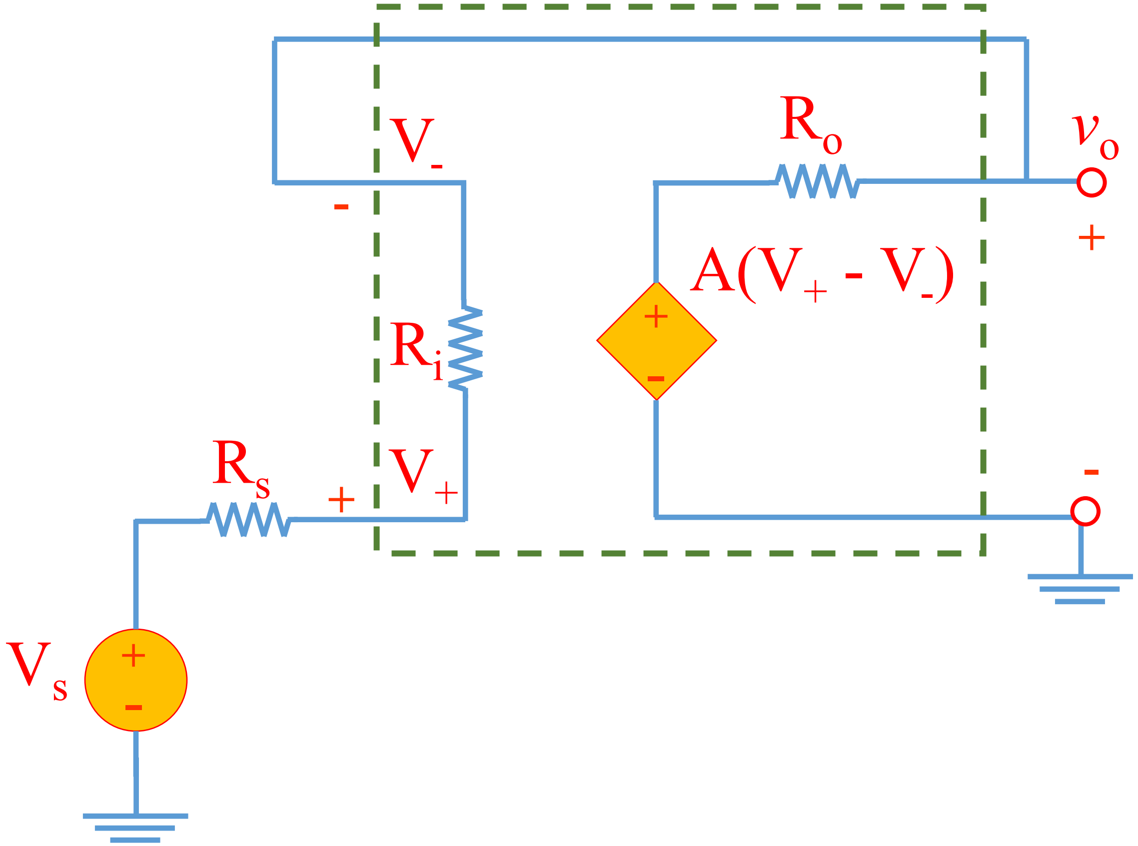
A = ∞ → $V_+ = V_- = 7V$ (Virtual ground at the input)

KCL at V_-: $\frac{V_-}{9} = \frac{v_O - V_-}{14}$ → v_o = 17.9V

Note that since R_{in} is infinity, no current is entering either of the opamp inputs.

Problem 2: Find the equivalent Thevenin output resistance if the opamp is modeled as the circuit in the green box in terms of R_i, R_o and A (10pts.)





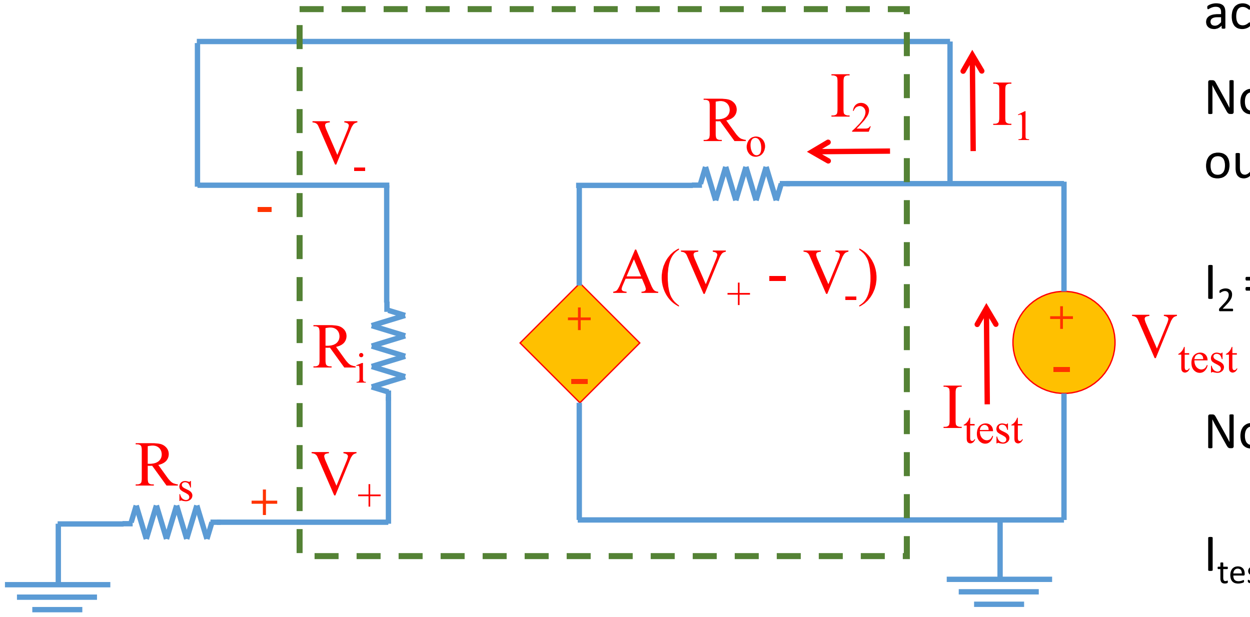
Solution:

First we nullify the independent sources. Second we apply a test voltage source at the output and determine how much current is drawn from it. The ratio of the test voltage and the drawn current is the Thevenin resistance.

Problem 2: Find the equivalent Thevenin output resistance if the opamp is modeled as the circuit in the green box in terms of R_i, R_o and A (10pts.)

Solution Continued:

The equivalent circuit is shown below:



The current passing R_i is $I_1 = V_{test}/(R_i + R_s)$. Therefore, voltage across the R_i is: $V_+ - V_- = \frac{-R_i}{R_i+R_s} V_{test}$

Now we can calculate I_2 (the current flowing into the opamp output):

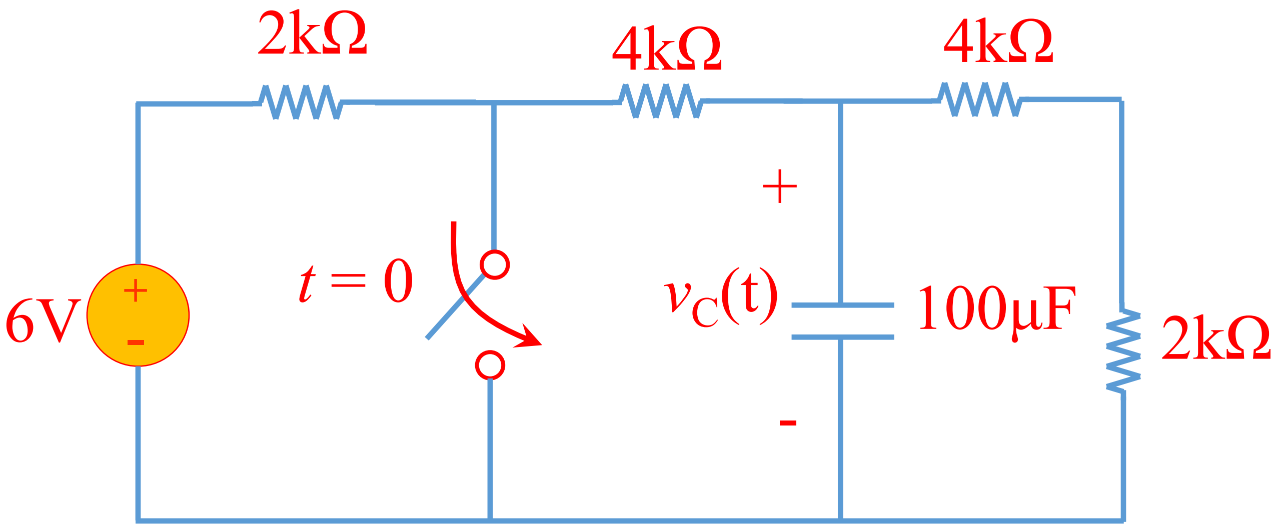
$$I_2 = [V_{test} - A(V_+ - V_-)]/R_o = \frac{\left(V_{test} - \frac{-AR_i}{R_i+R_s} V_{test}\right)}{R_o} = \frac{V_{test}\left(1+\frac{AR_i}{R_i+R_s}\right)}{R_o}$$

Now from KCL we have:

$$I_{test} = I_1 + I_2 = V_{test}\left[\frac{\left(1+\frac{AR_i}{R_i+R_s}\right)}{R_o} + \frac{1}{R_i+R_s}\right]$$

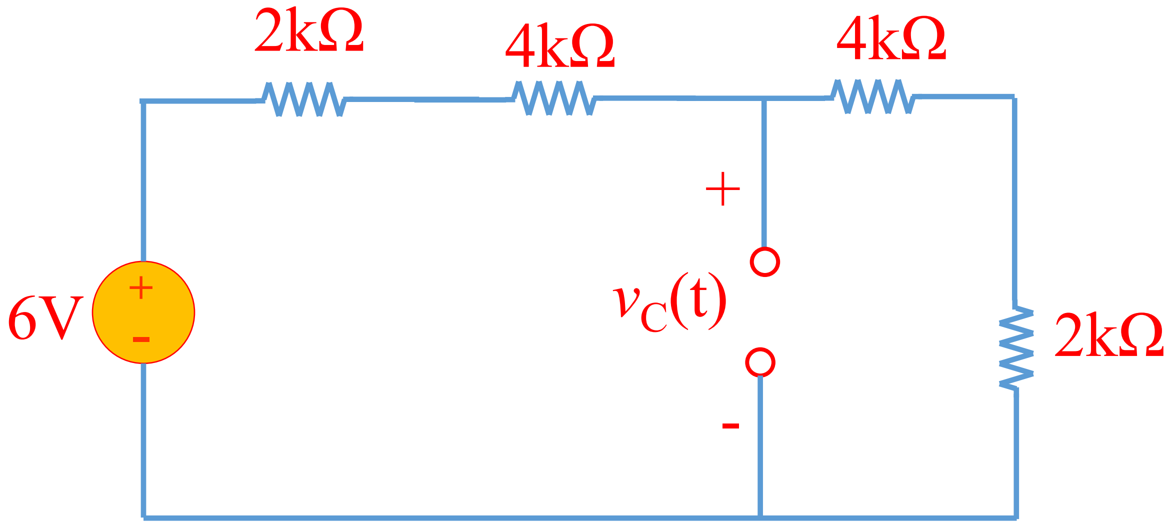
$$\rightarrow R_{th} = \frac{V_{test}}{I_{test}} = 1/\left[\frac{\left(1+\frac{AR_i}{R_i+R_s}\right)}{R_o} + \frac{1}{R_i+R_s}\right]$$

Problem 2: (RC circuit) Find the expression of $v_C(t)$ for $t > 0$. What is the circuit time constant after switch is closed? Plot the $v_C(t)$ for $-\infty<t<\infty$ (35pts.)



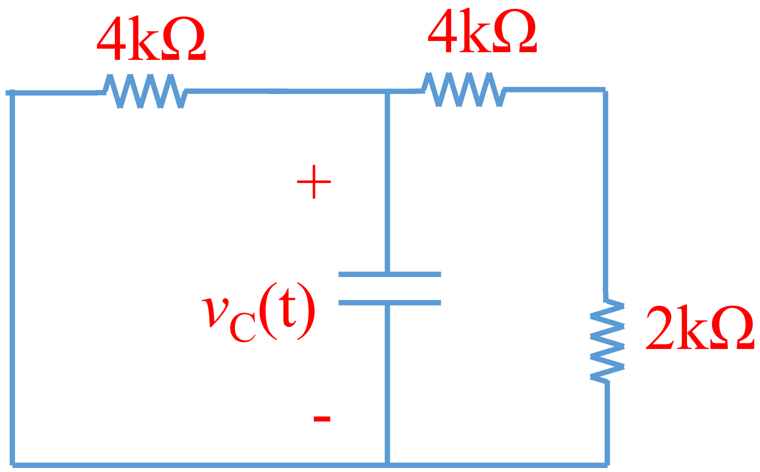
Solution:

At $t < 0$, switch is open and the capacitor is open circuit. The voltage across the capacitor at $t < 0$ can be derived using the equivalent circuit below:

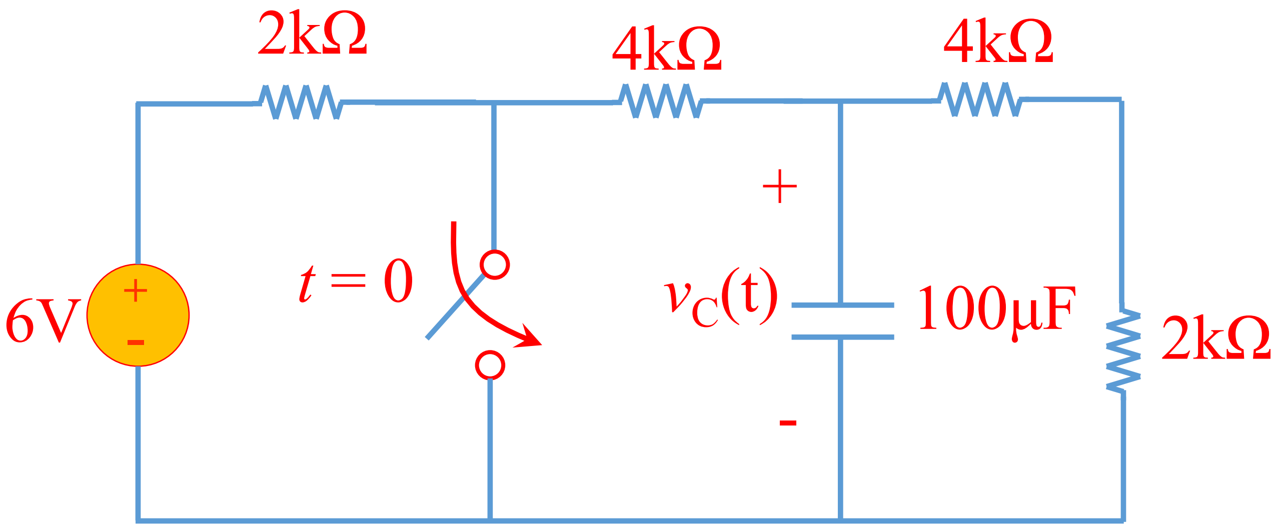


→ $v_C(t < 0) = 3V$

for $t > 0$, switch is closed and the capacitor is getting discharged. The equivalent circuit after switch is closed:



Problem 2: (RC circuit) Find the expression of $v_C(t)$ for t > 0. What is the circuit time constant after switch is closed? Plot the $v_C(t)$ for $-\infty < t < \infty$ (35pts.)



Solution Continued:

Circuit time constant T is $R_{eq}C$, where R_{eq} is the total resistance seen by the capacitor:

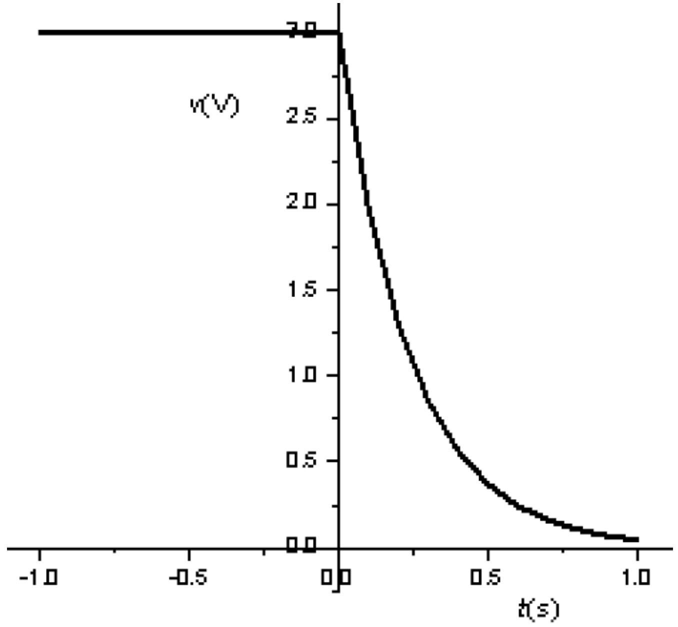
R_{eq} = 4 kΩ || (4 kΩ + 2 kΩ) = 2.4 kΩ

→ T = 0.24

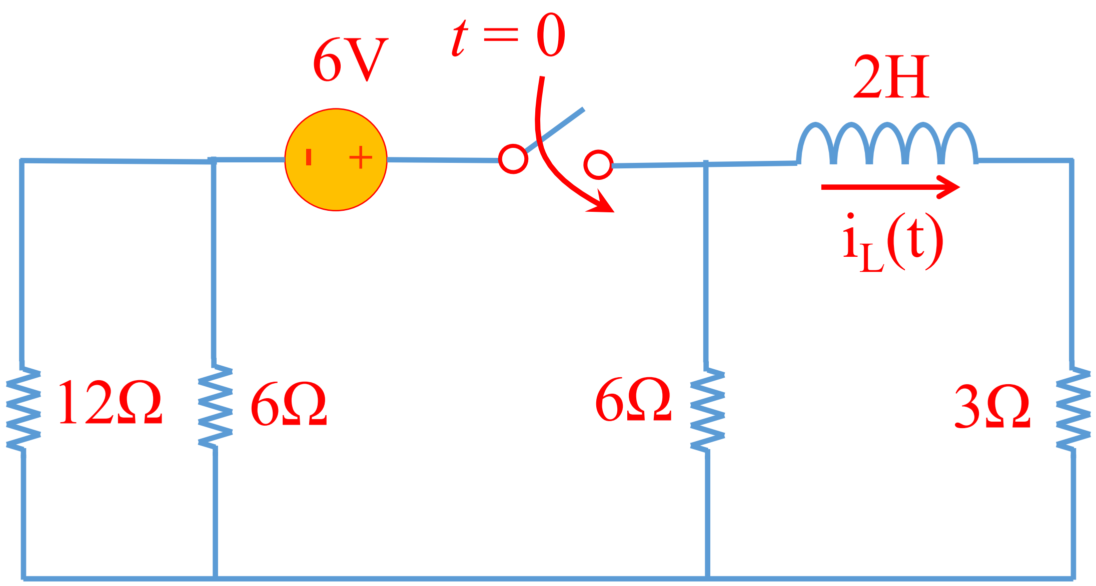
The current through capacitor vanishes, and the voltage decays towards zero at t = ∞ → $v_C(t=\infty) = 0$

$$v_C(t) = v_C(t=\infty) + [v_C(t=0) - v_C(t=\infty)]\, e^{\frac{-t}{T}}$$

$$v_C(t) = 3e^{\frac{-t}{0.24}}$$



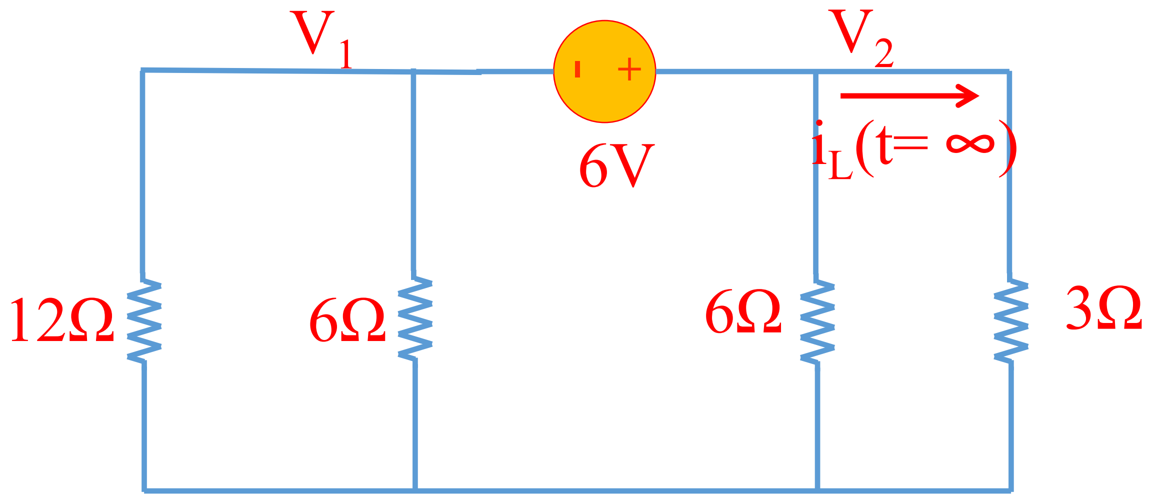
Problem 3: (RL circuit) Find the expression of $i_L(t)$ for $t > 0$. What is the circuit time constant after switch is closed? Plot the $i_L(t)$ for $-\infty<t<\infty$ (35pts.)



Solution:

At $t < 0$, switch is open and the inductor has is discharged so $i_L(t < 0) = 0$.

After switch closes and at $t = \infty$, the inductor will become short circuit and the equivalent circuit below results:

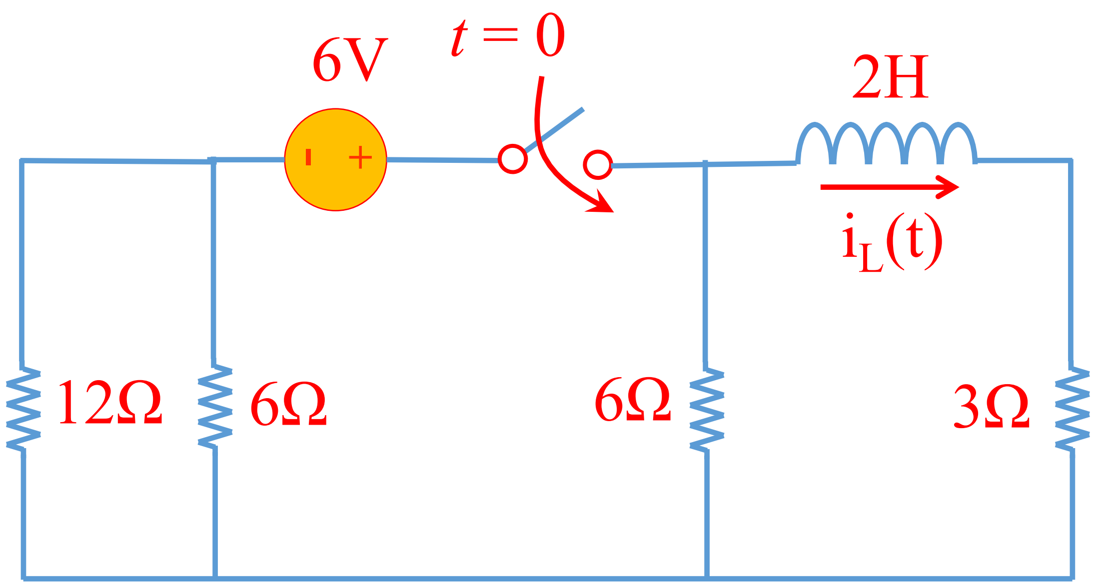


Using nodal analysis we can find $i_L(t = \infty)$:

$V_1(1/6 + 1/12) = -V_2(1/6 + 1/3)$, $V_2 = V_1 + 6$

$\rightarrow V_1 = 4V$, $V_2 = 2V \rightarrow i_L(t = \infty) = V_2/3 = 2/3A$

Problem 3: (RL circuit) Find the expression of $i_L(t)$ for $t > 0$. What is the circuit time constant after switch is closed? Plot the $i_L(t)$ for $-\infty<t<\infty$ (35pts.)



Solution Continued:

The circuit time constant is $\tau = L/R_{eq}$ where R_{eq} is the total resistance as seen from the inductor:

$R_{eq} = 12\Omega \,||\, 6\Omega \,||\, 6\Omega + 3\Omega = 5.4\Omega$

→ $\tau = 0.37s$

$$i_L(t) = i_L(t=\infty) + [i_L(t=0) - i_L(t=\infty)]\, e^{\frac{-t}{\tau}}$$

$$i_L(t) = 2/3 + (-2/3)e^{\frac{-t}{0.37}}$$