EECS/CSE 70A Network Analysis I

Homework #5

Solution Key

Problem 1: (Phasor) (40pts)

a) Given v(t) = 5cos(ωt-$\pi/3$). Find the phasor **V** that represents v(t). Express **V** as *x+jy* and as $re^{j\varphi}$

Phasor:

$$\mathbf{V} = x + jy = re^{j\varphi} \quad ; x = r\cos\varphi, \quad y = r\sin\varphi$$

$$r = 5,\ \varphi = -\pi/3 \Rightarrow \mathbf{V} = 5e^{-j\pi/3}$$

$$\left.\begin{array}{l} x = 5\cos(-\pi/3) = 2.5 \\ y = 5\sin(-\pi/3) = -4.33 \end{array}\right\} \Rightarrow \mathbf{V} = 2.5 - j4.33$$

b) Given i(t) = 10sin(3t+ $\pi/4$). Find the phasor **I** that represents i(t). Express **I** as *x+jy* and as $re^{j\varphi}$

Phasor:

$$i(t) = 10\sin(3t + \frac{\pi}{4}) = 10\cos(3t + \frac{\pi}{4} - \frac{\pi}{2}) = 10\cos(3t - \frac{\pi}{4})$$

$$\mathbf{I} = x + jy = re^{j\varphi} \quad ; x = r\cos\varphi, \quad y = r\sin\varphi$$

$$r = 10,\ \varphi = -\pi/4 \Rightarrow \mathbf{I} = 10e^{-j\pi/4}$$

$$\left.\begin{array}{l} x = 10\cos(-\pi/4) = 7.07 \\ y = 10\sin(-\pi/4) = -7.07 \end{array}\right\} \Rightarrow \mathbf{I} = 7.07 - j7.07$$

c) Convert the phasor **V** = 3+7j to time domain expression v(t).

$$\left.\begin{aligned} |\mathbf{V}| = |3 + j7| = \sqrt{9+49} = 7.6158 \\ \angle\mathbf{V} = \arctan(\frac{7}{3}) = 1.16 rad \end{aligned}\right\} \Rightarrow \mathbf{V} = 7.6158 e^{j1.16}$$

$$v(t) = \mathrm{Re}\{\mathbf{V} e^{j\omega t}\} = 7.6158 \cos(\omega t + 1.16)$$

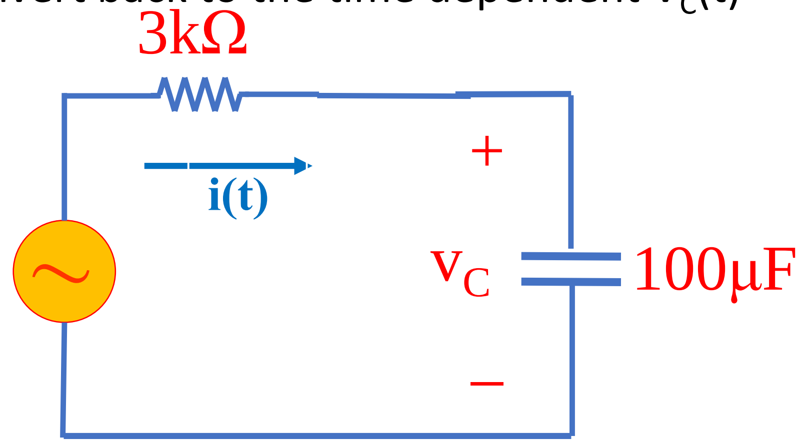
d) Convert the phasor **I** = 16-9j to time domain expression i(t).

$$\left.\begin{aligned} |\mathbf{I}| = |16 - j9| = \sqrt{256+81} = 18.35 \\ \angle\mathbf{I} = \arctan(\frac{-9}{16}) = -0.51 rad \end{aligned}\right\} \Rightarrow \mathbf{I} = 18.35 e^{-j0.51}$$

$$i(t) = \mathrm{Re}\{\mathbf{I} e^{j\omega t}\} = 18.35 \cos(\omega t - 0.51)$$

Problem 2: Find $V_C(t)$. Hint: convert the voltage source into a phasor, then find the voltage phasor for the capacitor, then convert back to the time dependent $V_C(t)$ (30pts)

$$v_S(t) = 12\cos(10 \times t + 30^\circ)$$



Convert to the Phasor domain:

$$v_S(t) = 12\cos(10 \times t + 30^\circ) \Rightarrow \mathbf{V}_s = 12e^{j\frac{\pi}{6}} \qquad \omega = 10(rad/s)$$

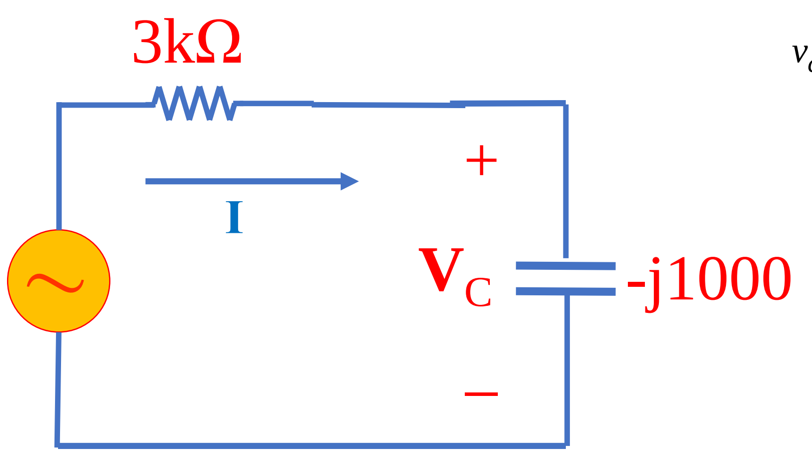
$$Z_c = \frac{1}{j\omega c} = \frac{1}{j10 \times 100 \times 10^{-6}} = -j1000\Omega$$

$$Z_{eq} = 3000 - j1000 \quad (\Omega)$$

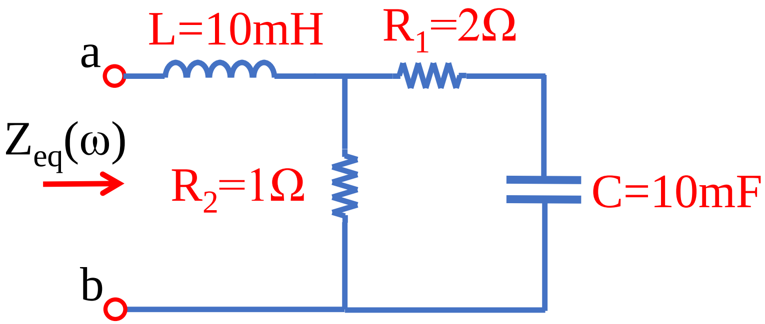
$$\mathbf{I} = \frac{\mathbf{V}_s}{Z_{eq}} \rightarrow \mathbf{V}_c = \mathbf{I} \times Z_c = \frac{Z_c}{Z_{eq}}\mathbf{V}_s = \frac{-j1000}{3000 - j1000} 12e^{-j\frac{\pi}{6}} = 3.7949e^{-j0.7254} \ V$$

$$v_c(t) = \mathrm{Re}\{\mathbf{V}_c e^{j\omega t}\} = \mathrm{Re}\{3.7949e^{-j0.7254}e^{j10t}\}$$
$$= 3.7949\cos(10t - 0.7254)$$

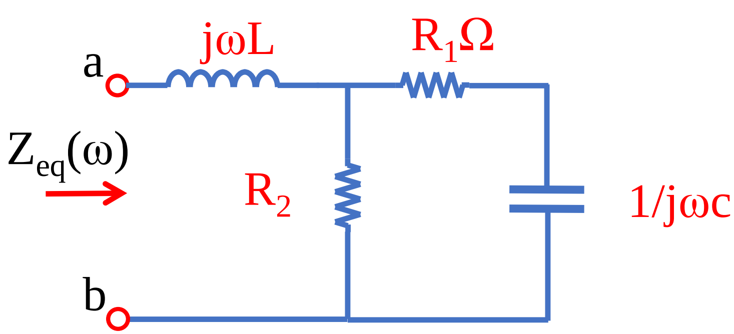
$$\mathbf{V}_s = 12e^{j\frac{\pi}{6}}$$



Problem 3: $Z_{eq}(\omega)$ is the equivalent impedance between terminals a-b. (30pts)
Find the parametric expression for $Z_{eq}(\omega)$ as a function of the angular frequency ω and circuit elements (R_1, R_2, C and L). You do not need to simplify the expression.



Phasor Domain:



$$Z_{eq} = j\omega L + [R_2 \,||\, (R_1 + \frac{1}{j\omega C})]$$

$$= j\omega L + R_2 \,||\, \frac{1 + j\omega C R_1}{j\omega C}$$

$$= j\omega L + \frac{R_2 + j\omega C R_1 R_2}{1 + j\omega C(R_1 + R_2)}$$