EECS/CSE 70A Network Analysis I

Homework #6

Solution Key

Problem 1 (20pts)

Part (a): $u = (A + jB)(C + jD) = AC + jAD + jBC - BD$

$$\mathrm{Re}\{u\} = AC - BD$$
$$\mathrm{Im}\{u\} = AD + BC$$

$$u = x + jy =$$
$$= (AC - BD) + j(AD + BC)$$

$$r = |u| = \sqrt{u(u^*)} = \sqrt{(\mathrm{Re}\{u\})^2 + (\mathrm{Im}\{u\})^2}$$
$$= \sqrt{(AC - BD)^2 + (AD + BC)^2}$$
$$\phi = \tan^{-1}\frac{\mathrm{Im}\{u\}}{\mathrm{Re}\{u\}} = \tan^{-1}\frac{AD + BC}{AC - BD}$$
$$u = \sqrt{(AC - BD)^2 + (AD + BC)^2}\, e^{j\tan^{-1}\frac{AD+BC}{AC-BD}}$$

$$\mathrm{Re}\{ue^{j\omega t}\} = \sqrt{(AC - BD)^2 + (AD + BC)^2}\cos\left[\omega t + \tan^{-1}\frac{AD + BC}{AC - BD}\right]$$

Problem 1 cont'd

Part (a) 2nd way:

$$A+jB=\sqrt{A^2+B^2}\,e^{j\tan^{-1}\frac{B}{A}},\quad C+jD=\sqrt{C^2+D^2}\,e^{j\tan^{-1}\frac{D}{C}}$$

$$u=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\,e^{j\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)}$$

$$\operatorname{Re}\{u\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\cos\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)$$

$$\operatorname{Im}\{u\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\sin\left(\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right)$$

$$r=|u|=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}$$

$$\phi=\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}$$

$$\operatorname{Re}\left\{ue^{j\omega t}\right\}=\sqrt{\left(A^2+B^2\right)\left(C^2+D^2\right)}\cos\left[\omega t+\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}\right]$$

Problem 1 cont'd

Part (b):

$$u=\frac{A+jB}{C+jD}=\frac{(A+jB)(C+jD)^*}{(C+jD)(C+jD)^*}=\frac{(A+jB)(C-jD)}{C^2+D^2}=\frac{AC-jAD+jBC+BD}{C^2+D^2}$$

$$\operatorname{Re}\{u\}=\frac{AC+BD}{C^2+D^2}$$

$$\operatorname{Im}\{u\}=\frac{-AD+BC}{C^2+D^2}$$

$$u=x+jy=$$

$$=\left(\frac{AC+BD}{C^2+D^2}\right)+j\left(\frac{-AD+BC}{C^2+D^2}\right)$$

$$r=|u|=\frac{\sqrt{(AC+BD)^2+(-AD+BC)^2}}{C^2+D^2}$$

$$\phi=\tan^{-1}\frac{\operatorname{Im}\{u\}}{\operatorname{Re}\{u\}}=\tan^{-1}\frac{-AD+BC}{AC+BD}$$

$$u=\frac{\sqrt{(AC+BD)^2+(-AD+BC)^2}}{C^2+D^2}e^{j\tan^{-1}\frac{-AD+BC}{AC+BD}}$$

$$\operatorname{Re}\{ue^{j\omega t}\}=\frac{\sqrt{(AC+BD)^2+(-AD+BC)^2}}{C^2+D^2}\cos\left[\omega t+\tan^{-1}\frac{-AD+BC}{AC+BD}\right]$$

Problem 1 cont'd

Part (b) 2nd way:

$$A + jB = \sqrt{A^2 + B^2}\, e^{j\tan^{-1}\frac{B}{A}}, \quad C + jD = \sqrt{C^2 + D^2}\, e^{j\tan^{-1}\frac{D}{C}}$$

$$u = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}\, e^{j\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)}$$

$$\mathrm{Re}\{u\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \cos\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)$$

$$\mathrm{Im}\{u\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \sin\left(\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right)$$

$$r = |u| = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}$$

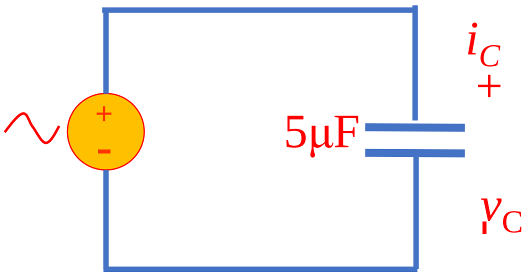
$$\phi = \tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}$$

$$\mathrm{Re}\left\{u e^{j\omega t}\right\} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \cos\left[\omega t + \tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}\right]$$

Problem 2 (20pts.)

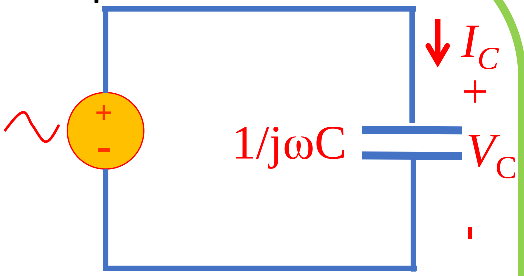
Part (a): Find the current $i_C(t)$ at the frequency 80Hz.

$$v(t) = 4\cos\left(\omega t - \frac{\pi}{3}\right)$$



Convert to phasor domain

$$V = 4e^{-j\frac{\pi}{3}}$$



$$V_C = V$$

$$\omega = 2\pi f = 2\pi 80 \text{ rad/s} = 160\pi \text{ rad/s}$$

$$V_C = I_C\left(\frac{1}{j\omega C}\right)$$

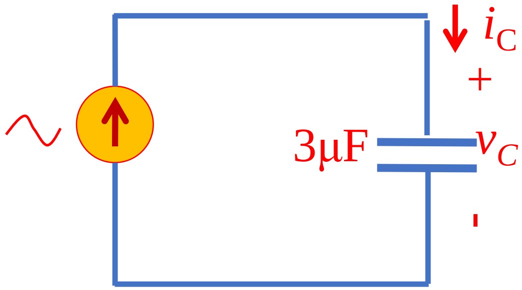
$$v(t) = 4\cos\left(\omega t - \frac{\pi}{3}\right) \Leftrightarrow V = 4e^{-j\frac{\pi}{3}}$$

$$I_C = Vj\omega C = 4e^{-j\frac{\pi}{3}}(j8\pi\times10^{-4}) = 32\pi\times10^{-4}e^{j\left(-\frac{\pi}{3}+\frac{\pi}{2}\right)} = 32\pi\times10^{-4}e^{j\frac{\pi}{6}}\text{A}$$

$$i_C(t) = \text{Re}\left\{I_C e^{j\omega t}\right\} = \text{Re}\left\{32\pi\times10^{-4}e^{j\frac{\pi}{6}}e^{j160\pi t}\right\} = 32\pi\times10^{-4}\cos\left(160\pi t + \frac{\pi}{6}\right)\text{A} = 3.2\pi\cos(160\pi t + 30^\circ)\text{ mA}$$

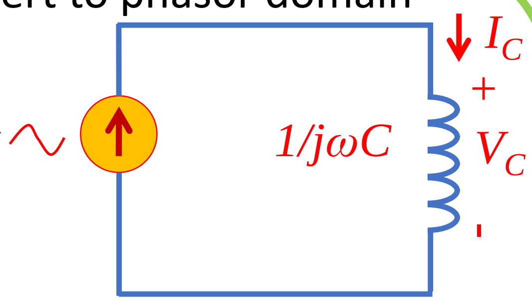
Part (b): Find the voltage $v_C(t)$ at the frequency 30Hz.

$$i(t) = 7\cos\left(\omega t + \frac{\pi}{8}\right)$$



Convert to phasor domain

$$I = 7e^{j\frac{\pi}{8}}$$



$$i(t) = 7\cos\left(\omega t + \frac{\pi}{8}\right) \Leftrightarrow I = 7e^{j\frac{\pi}{8}}$$

$$I_C = I$$

$$\omega = 2\pi f = 2\pi 30 \text{ rad/s} = 60\pi \text{ rad/s}$$

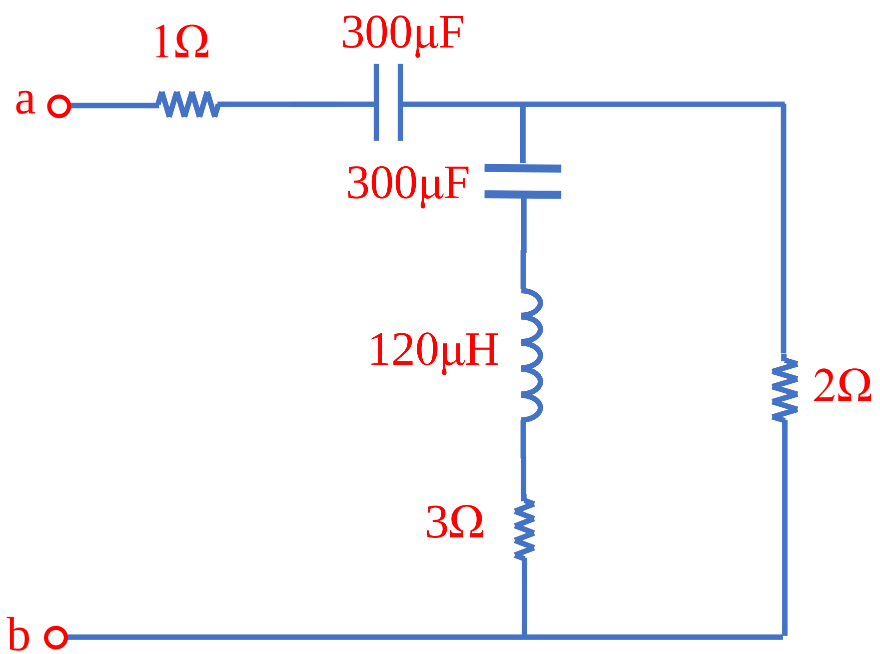
$$V_C = \frac{I_C}{j\omega C}$$

$$= 7e^{j\frac{\pi}{8}}\left(\frac{1}{j180\pi \times 10^{-6}}\right) = 12.38\times 10^3 e^{j\left(\frac{\pi}{8}-\frac{\pi}{2}\right)} = 12.38e^{-j\frac{3\pi}{8}} \text{ kV}$$

$$v_C(t) = \text{Re}\left\{V_C e^{j\omega t}\right\} = \text{Re}\left\{12.38\times 10^4 e^{-j\frac{3\pi}{8}} e^{j60\pi t}\right\} = 12.38\times 10^4 \cos\left(60\pi t - \frac{3\pi}{8}\right) \text{V}$$

Problem 3 (30pts.)

Part (a): Find the impedance seen from terminals a-b as a function of the angular frequency ω.



$$Z_{eq} = 1 + \frac{1}{j\omega C} + [2 \,||\, (3 + j\omega L + \frac{1}{j\omega C})]$$

$$= 1 + \frac{1}{j\omega C} + \frac{2 - 2\omega^2 LC + 6j\omega C}{1 - \omega^2 LC + 5j\omega C}$$

$$= 1 + \frac{1}{j\omega 3\times10^{-4}} + \frac{2 - \omega^2 7.2\times10^{-8} + j\omega 18\times10^{-4}}{1 - \omega^2 3.6\times10^{-8} + j\omega 15\times10^{-4}}$$

Part (b): Evaluate the impedance at 750Hz

$$\omega = 2\pi 750 \text{ rad/s} = 1500\pi \text{ rad/s} \rightarrow \text{Z}_{eq} = 2.20 - 0.73j \;\; \Omega$$

Part (c): Evaluate the impedance at 3kHz

$$\omega = 2\pi 3000 \text{ rad/s} = 6000\pi \text{ rad/s} \rightarrow \text{Z}_{eq} = 2.31 + 0.10j \;\; \Omega$$

Problem 4: (20 pts)

Find the relationship of the phase shift between i(t) and v(t) in terms of the impedance Z for an element.

Suppose for an element we have:

$$v(t) = A_v \cos(\omega t + \varphi_v) \xrightarrow{Phasor} V = A_v e^{j\varphi_v}$$

$$i(t) = A_i \cos(\omega t + \varphi_i) \xrightarrow{Phasor} I = A_i e^{j\varphi_i}$$

So for the impedance Z we have:

$$Z = \frac{V}{I} = \frac{A_v e^{j\varphi_v}}{A_i e^{j\varphi_i}} = \frac{A_v}{A_i} e^{j(\varphi_v - \varphi_i)} \Rightarrow |Z| = \frac{A_v}{A_i}, \angle Z = \varphi_z = \varphi_v - \varphi_i$$

For example for common passive elements:

$$R \qquad \varphi_Z = 0 \rightarrow \varphi_v = \varphi_i$$

$$L \qquad \varphi_Z = \frac{\pi}{2} \rightarrow \varphi_v = \varphi_i + \frac{\pi}{2}$$

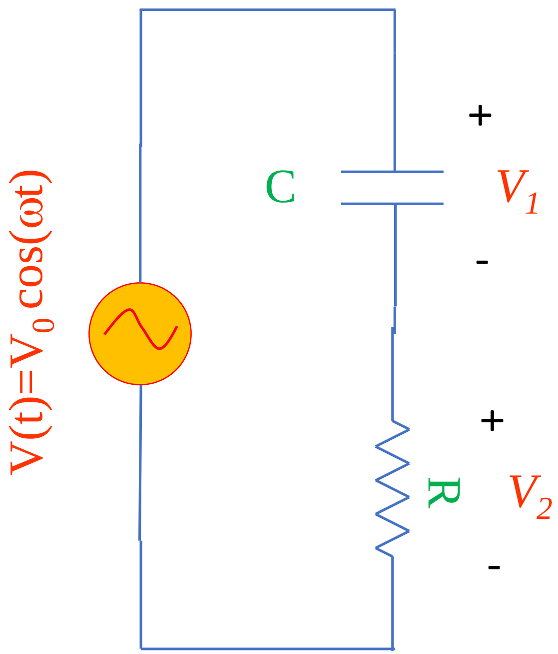
$$C \qquad \varphi_Z = -\frac{\pi}{2} \rightarrow \varphi_v = \varphi_i - \frac{\pi}{2}$$

Problem 5: (40 pts)

For the circuit shown below, by writing all the steps, prove that

Needs correction

$$V_1(t) = \frac{V_0}{\sqrt{1+(\omega\tau)^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{1}{\omega\tau}\right)\right) \qquad ;\tau = RC$$



$$v(t) = V_0\cos(\omega t) \xrightarrow{Phasor} V = V_0$$

$$I = \frac{V_0}{R+\frac{1}{j\omega C}} = \frac{j\omega C V_0}{1+j\omega CR}$$

$$V_1 = \frac{1}{j\omega C} I = \frac{V_0}{1+j\omega CR} \Rightarrow |V_1| = \frac{V_0}{\sqrt{1+(\omega CR)^2}}, \angle V_1 = -\tan^{-1}(\omega CR)$$

$$v_1(t) = \mathrm{Re}\{V_1 e^{j\omega t}\} = \mathrm{Re}\{\frac{V_0}{\sqrt{1+(\omega CR)^2}} e^{-j\tan^{-1}(\omega CR)} e^{j\omega t}\}$$

$$= \frac{V_0}{\sqrt{1+(\omega CR)^2}}\cos(\omega t - \tan^{-1}(\omega CR))$$

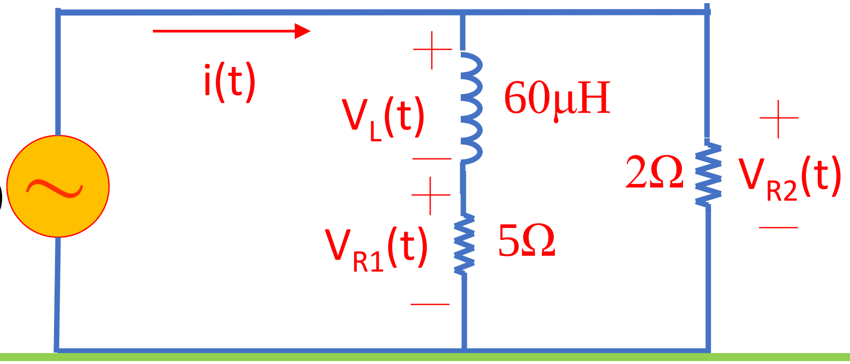
$$\tau = RC \Rightarrow v_1(t) = \frac{V_0}{\sqrt{1+(\omega\tau)^2}}\cos(\omega t - \tan^{-1}(\omega\tau))$$

correction

Problem 6: (20 pts)

For the circuit shown below, find i(t), $v_L(t)$, $v_{R1}(t)$ and $v_{R2}(t)$

$$v_S(t) = 12\cos(10 \times t + 30^\circ)$$



$$v_s(t) = 12\cos(10t + 30^\circ) \xrightarrow{Phasor} V_s = 12e^{j\frac{\pi}{6}}, \omega = 10\, rad/s$$

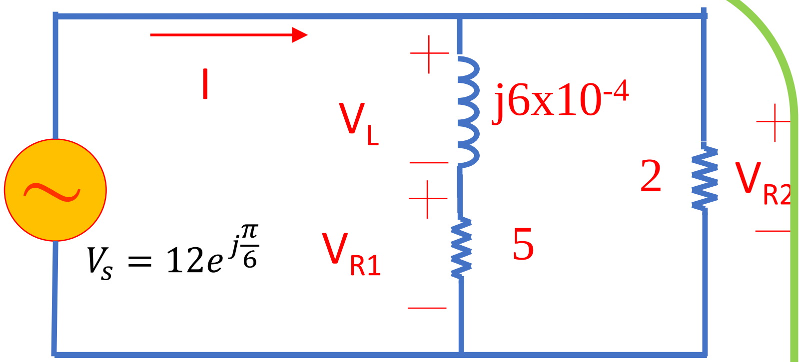
$$Z_{eq} = 2\Omega \,||\, (5\Omega + j6 \times 10^{-4}\Omega) = \frac{10 + j1.2 \times 10^{-3}}{7 + j6 \times 10^{-4}} = 1.42e^{j3.42\times10^{-5}}\Omega$$

$$I = \frac{V_s}{Z_{eq}} = \frac{12e^{j\frac{\pi}{6}}}{1.42e^{j3.42\times10^{-5}}} = 8.4e^{j0.52}\text{ A} \Rightarrow i(t) = 8.4\cos(10t + 30^\circ)\text{ A}$$

$$V_{R_2} = V_s = 12e^{j\frac{\pi}{6}}\text{ V} \Rightarrow v_{R_2}(t) = 12\cos(10t + 30^\circ)\text{ V}$$

$$V_L = V_s \frac{j6 \times 10^{-4}}{5 + j6 \times 10^{-4}} = 0.0014e^{j2.094}\text{ V} \Rightarrow v_L(t) = 0.0014\cos(10t + 120^\circ)\text{ V}$$

$$V_{R_1} = V_s \frac{5}{5 + j6 \times 10^{-4}} = 12e^{j0.523}\text{ V} \Rightarrow v_{R_1}(t) = 12\cos(10t + 30^\circ)\text{ V}$$



Convert to phasor domain

Problem 7: (20 pts)

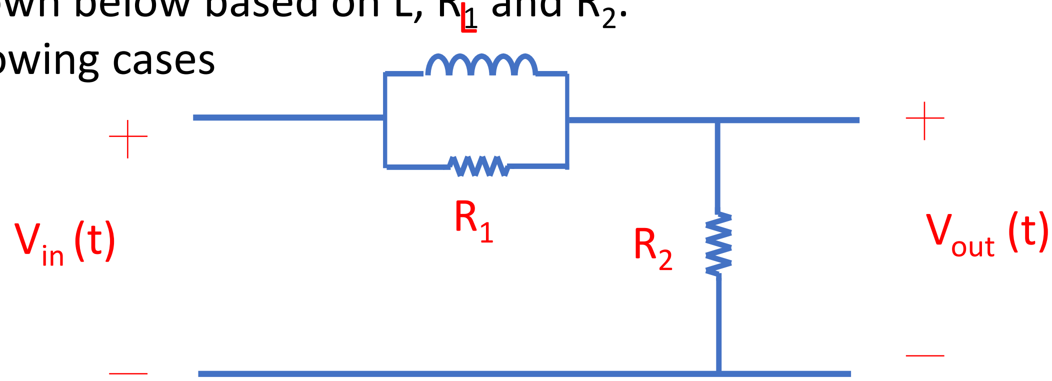
a) Determine the type of the filter shown below based on L, R_1 and R_2.

b) Plot V_{out} (t) versus V_{in}(t) for the following cases

i) $\omega \to 0$

ii) $\omega = 1/\tau$

iii) $\omega \to \infty$



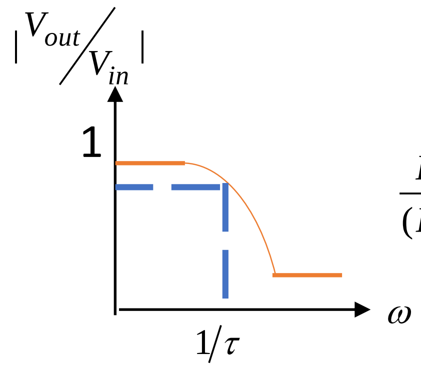
Based on voltage division, in phasor domain:

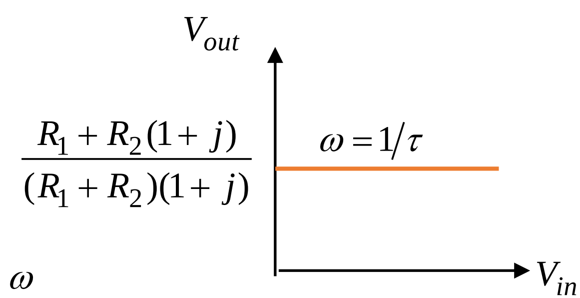
$$V_{out} = V_{in}\frac{R_2}{R_2 + (R_1 \parallel j\omega L)} = V_{in}\frac{R_1R_2 + j\omega LR_2}{R_1R_2 + j\omega L(R_2 + R_1)}$$

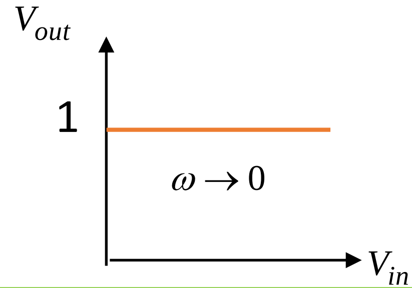
The filter is low pass.

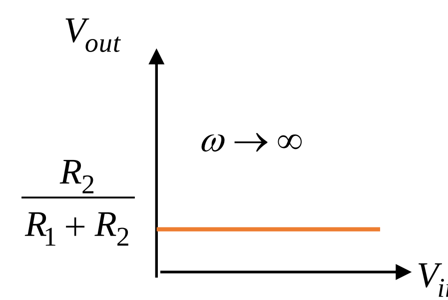
$$\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 \parallel R_2} = \frac{L(R_1 + R_2)}{R_1R_2}\ s$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1R_2 + j\omega LR_2}{R_1R_2 + j\omega L(R_2 + R_1)} \Rightarrow \begin{cases} \lim_{\omega\to 0}\left(\frac{V_{out}}{V_{in}}\right) = 1 \\ \lim_{\omega\to 1/\tau}\left(\frac{V_{out}}{V_{in}}\right) = \frac{R_1 + R_2(1+j)}{(R_1 + R_2)(1+j)} \\ \lim_{\omega\to\infty}\left(\frac{V_{out}}{V_{in}}\right) = \frac{R_2}{R_1 + R_2} \end{cases}$$





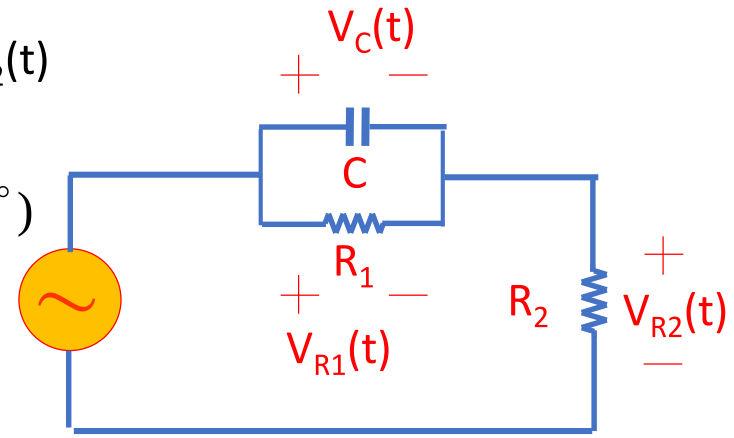




Problem 8: (20 pts)

For the circuit shown below, find $i_C(t)$, $v_C(t)$, $v_{R1}(t)$ and $v_{R2}(t)$

$$v_S(t) = 12\cos(20 \times t + 45^\circ)$$



$$v_s(t) = 12\cos(20t + 45^\circ) \xrightarrow{Phasor} V_s = 12e^{j\frac{\pi}{4}}, \omega = 20\,rad/s$$

Based on voltage devision:

$$V_{R_2} = V_s \frac{R_2}{R_2 + (R_1 \| \frac{1}{j20 \times C})} = 12e^{j\frac{\pi}{4}} \frac{R_2 + j20 \times CR_1R_2}{R_1 + R_2 + j20 \times CR_1R_2}$$

$$|V_{R_2}| = 12 \times \frac{\sqrt{R_2^2 + (20 \times CR_1R_2)^2}}{\sqrt{(R_1 + R_2)^2 + (20 \times CR_1R_2)^2}}$$

$$\angle V_{R_2} = \frac{\pi}{4} + \tan^{-1}(20 \times R_1C) - \tan^{-1}(\frac{20 \times R_1R_2C}{R_1 + R_2})$$

$$\longrightarrow v_{R_2}(t) = \mathrm{Re}\{V_{R_2} e^{j20t}\} = |V_{R_2}| \cos(20t + \angle V_{R_2})$$

Problem 8 cont'd

$$V_C = V_{R_1} = V_s \frac{R_1 \| \frac{1}{j20\times C}}{R_2 + (R_1 \| \frac{1}{j20\times C})} = 12e^{j\frac{\pi}{4}} \frac{R_1}{R_1 + R_2 + j20\times CR_1R_2}$$

$$|V_C| = |V_{R_1}| = 12 \times \frac{R_1}{\sqrt{(R_1 + R_2)^2 + (20\times CR_1R_2)^2}}$$

$$\angle V_C = \angle V_{R_1} = \frac{\pi}{4} - \tan^{-1}(\frac{20\times R_1R_2C}{R_1 + R_2})$$

$$v_{R_1}(t) = \mathrm{Re}\{V_{R_1} e^{j20t}\} = |V_{R_1}| \cos(20t + \angle V_{R1})\ \mathrm{V}$$

$$v_C(t) = v_{R_1}(t)$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = -20C|V_{R_1}|\sin(20t + \angle V_{R1}) = 20C|V_{R_1}|\cos(20t + \angle V_{R1} + \frac{\pi}{2})\ \mathrm{A}$$

Problem 9: (20 pts)

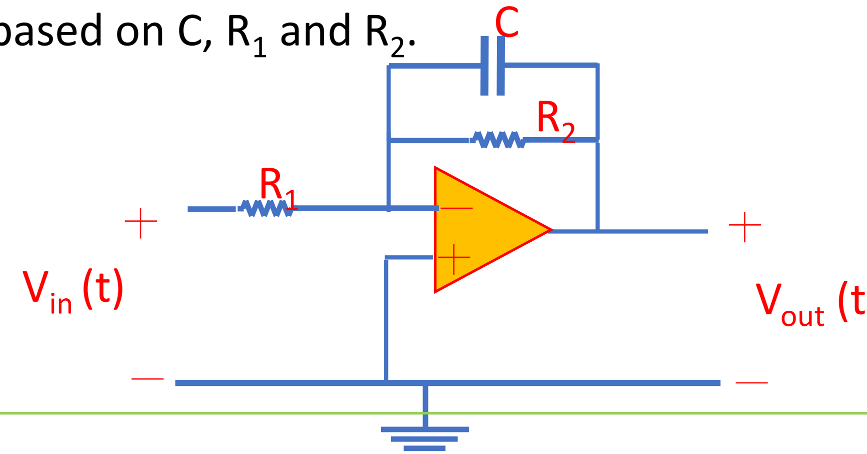
a) Determine the type of the filter shown below based on C, R_1 and R_2.

b) Plot V_{out} (t) versus V_{in}(t) for the following cases

i) $\omega \to 0$

ii) $\omega = 1/\tau$

iii) $\omega \to \infty$



We assume the op-amp is ideal, so

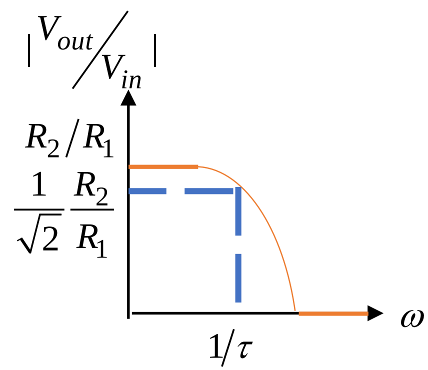
$$v^-(t) = v^+(t) = 0$$

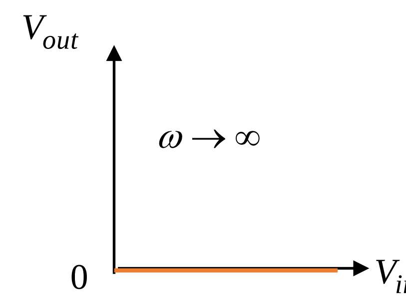
By solving in phasor domain and writing KCL at negative input node of op-amp:

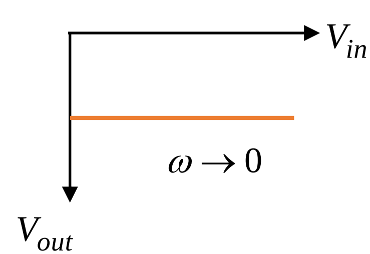
$$\frac{V^- - V_{in}}{R_1} + \frac{V^- - V_{out}}{R2 \| \frac{1}{j\omega C}} = 0 \to \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1(1 + j\omega C R_2)}$$

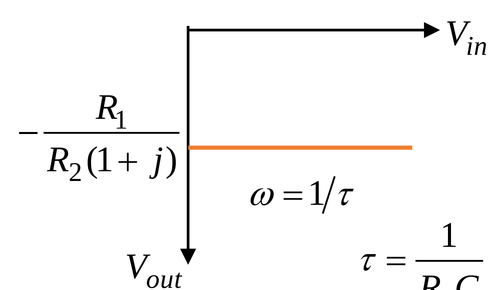
$$\begin{cases} \lim_{\omega \to 0} \left(\frac{V_{out}}{V_{in}}\right) = -\frac{R_2}{R_1} \\ \lim_{\omega \to 1/\tau} \left(\frac{V_{out}}{V_{in}}\right) = -\frac{R_2}{R_1}\left(\frac{1}{1+j}\right) \\ \lim_{\omega \to \infty} \left(\frac{V_{out}}{V_{in}}\right) = 0 \end{cases}$$

The filter is low pass









$\tau = \frac{1}{R_2 C}$

Problem 10: (90 pts)

For each of the circuits shown below

i) Find the transfer function H(ω), |H(ω)| and ∠ H(ω)

ii) Plot |H(ω)| for linear-linear and log-log scales.

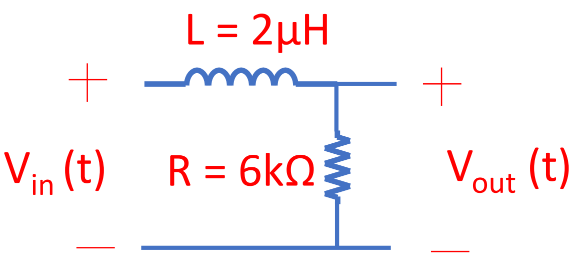
iii) Plot ∠ H(ω) for linear-log scales.

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L} = \frac{6\times10^{3}}{6\times10^{3} + j\omega 2\times10^{-6}}$$

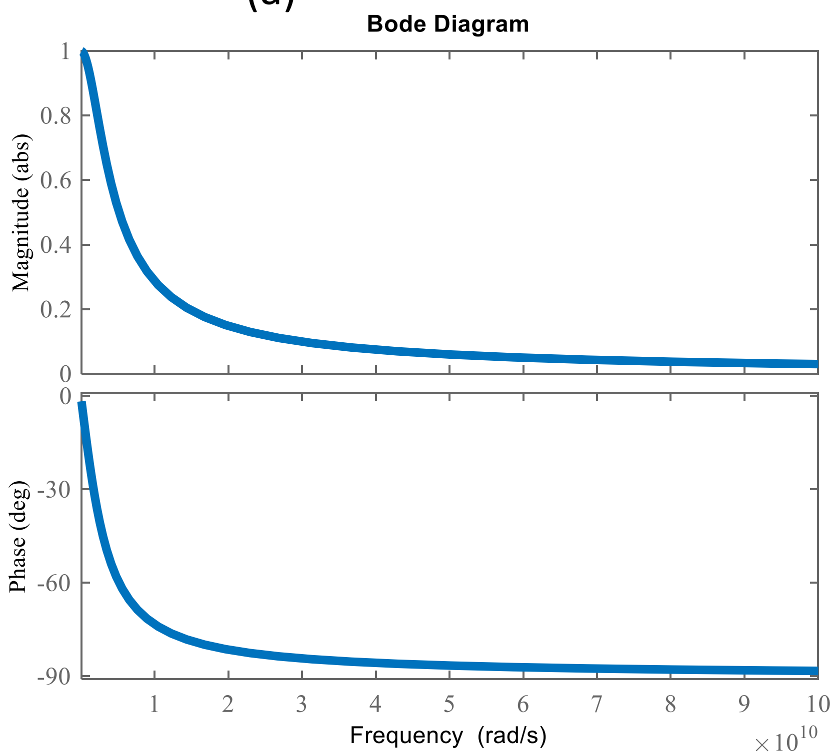
$$|H(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

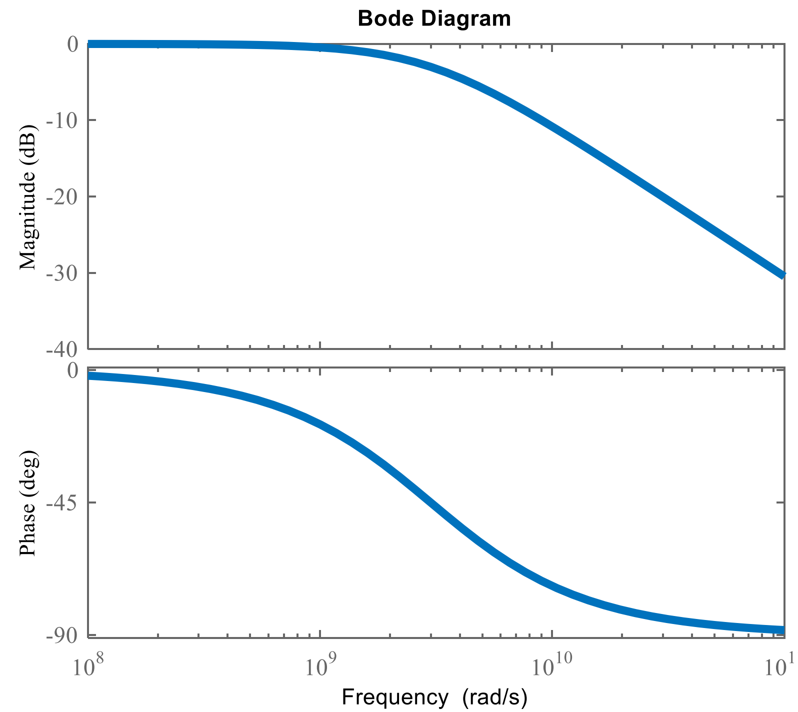
$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\omega_0 = \frac{1}{\tau} = 3\times10^{9} (rad/s)$$



(a)





Problem 10: (90 pts)

For each of the circuits shown below

i) Find the transfer function H(ω), |H(ω)| and ∠ H(ω)

ii) Plot |H(ω)| for linear-linear and log-log scales.

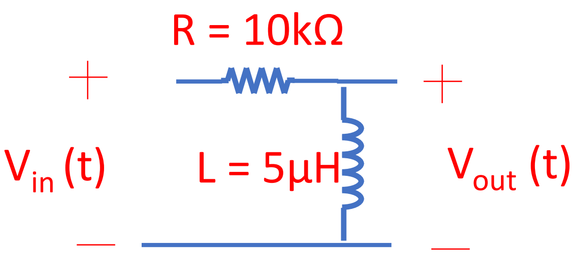
iii) Plot ∠ H(ω) for linear-log scales.

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega 5\times10^{-6}}{10\times10^{3} + j\omega 5\times10^{-6}}$$

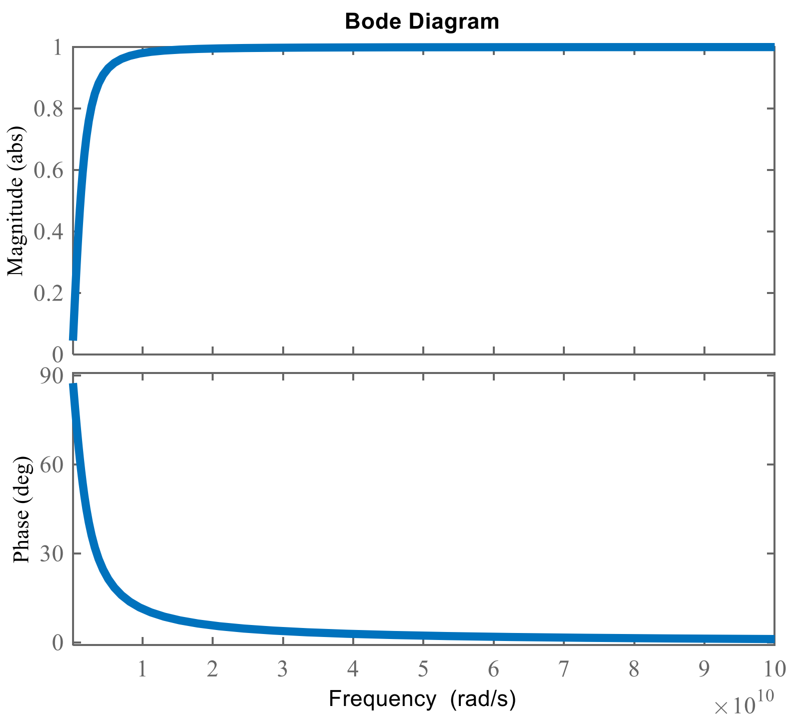
$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

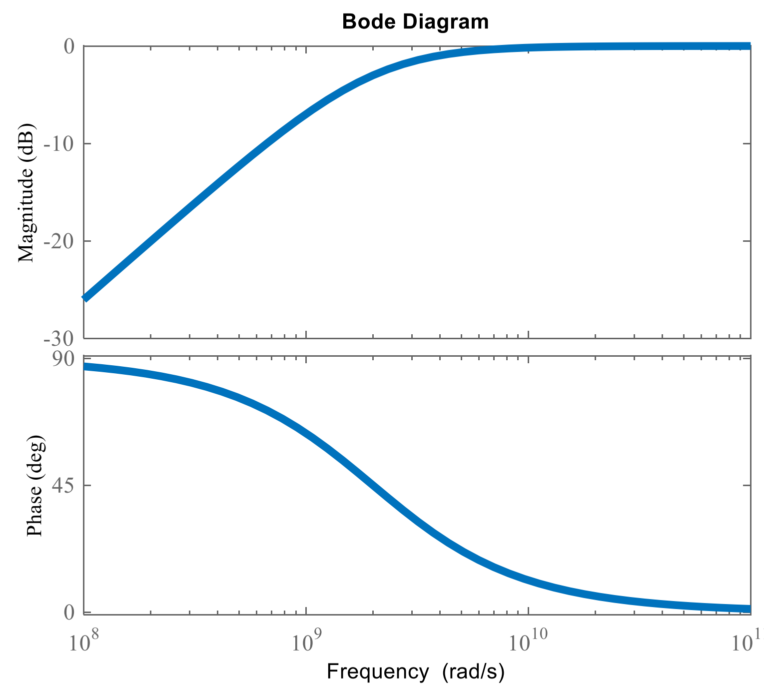
$$\angle H(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\omega_0 = \frac{1}{\tau} = 2\times10^{9} (rad/s)$$



(b)





Problem 10: (90 pts)

For each of the circuits shown below

i) Find the transfer function H(ω), |H(ω)| and ∠ H(ω)

ii) Plot |H(ω)| for linear-linear and log-log scales.

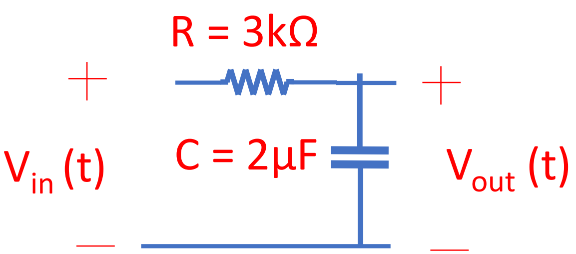
iii) Plot ∠ H(ω) for linear-log scales.

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega 6 \times 10^{-3}}$$

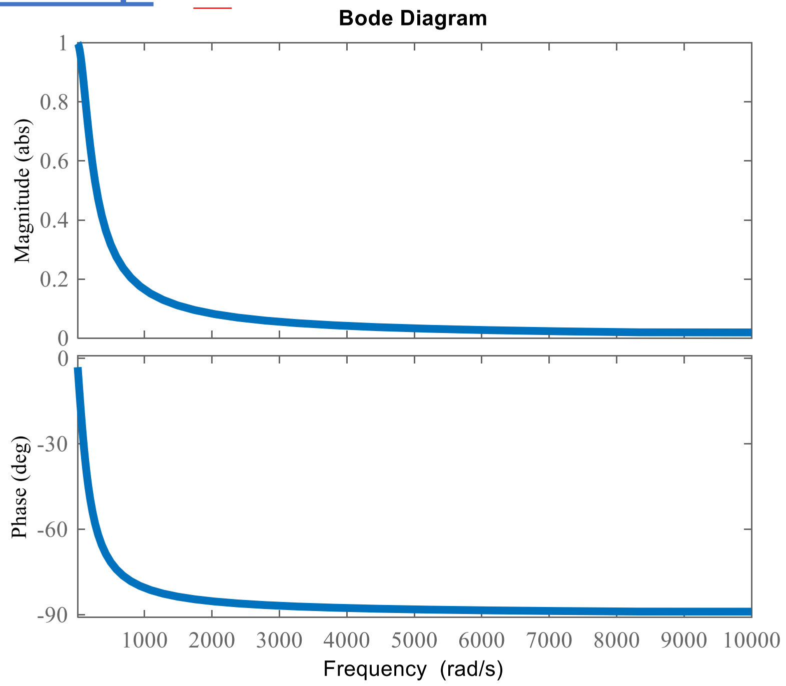
$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

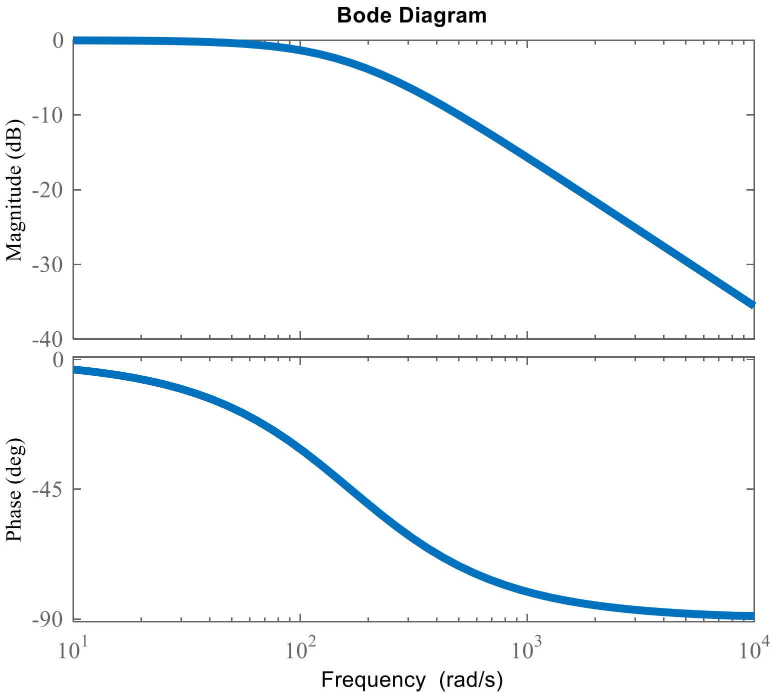
$$\angle H(\omega) = -\tan^{-1}(\omega RC)$$

$$\omega_0 = \frac{1}{\tau} = \frac{1}{6} \times 10^3 (rad/s)$$



(c)





Problem 11: (30 pts)

For the circuit shown below

i) Find the transfer function H(ω), |H(ω)| and ∠ H(ω)

ii) Plot |H(ω)| for linear-linear and log-log scales.

iii) Plot ∠ H(ω) for linear-log scales.

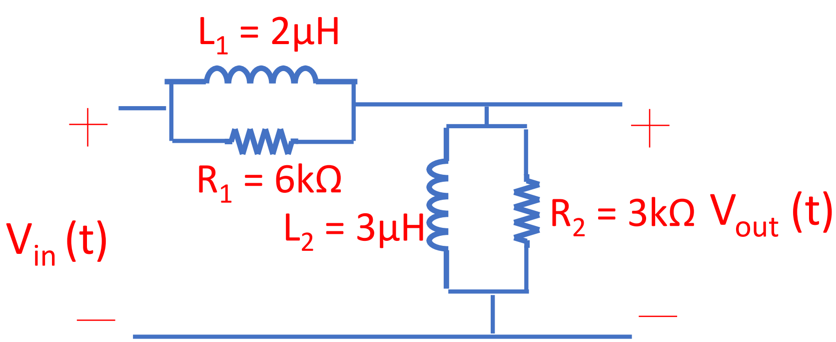
$$Z_1 = R_1 \parallel j\omega L_1 = \frac{j\omega R_1 L_1}{R_1 + j\omega L_1}$$

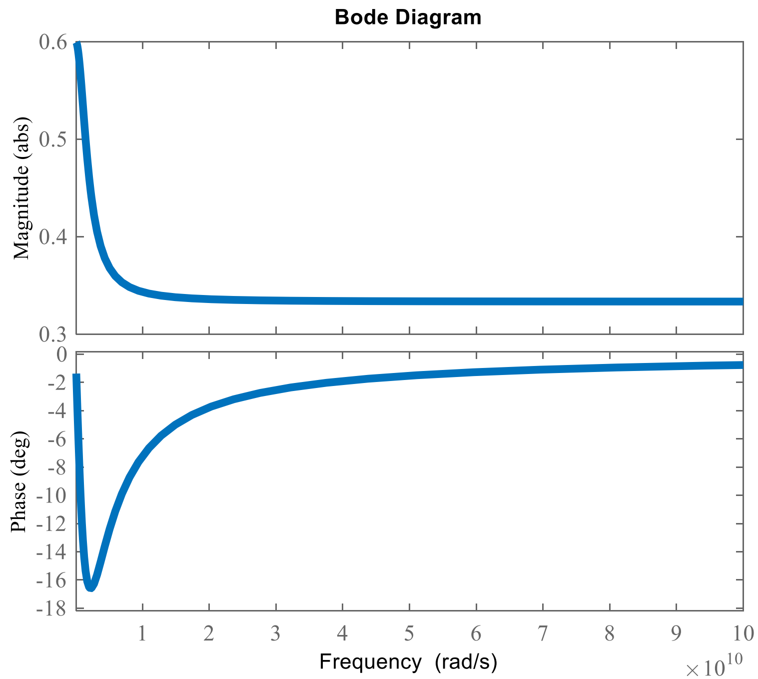
$$Z_2 = R_2 \parallel j\omega L_2 = \frac{j\omega R_2 L_2}{R_2 + j\omega L_2}$$

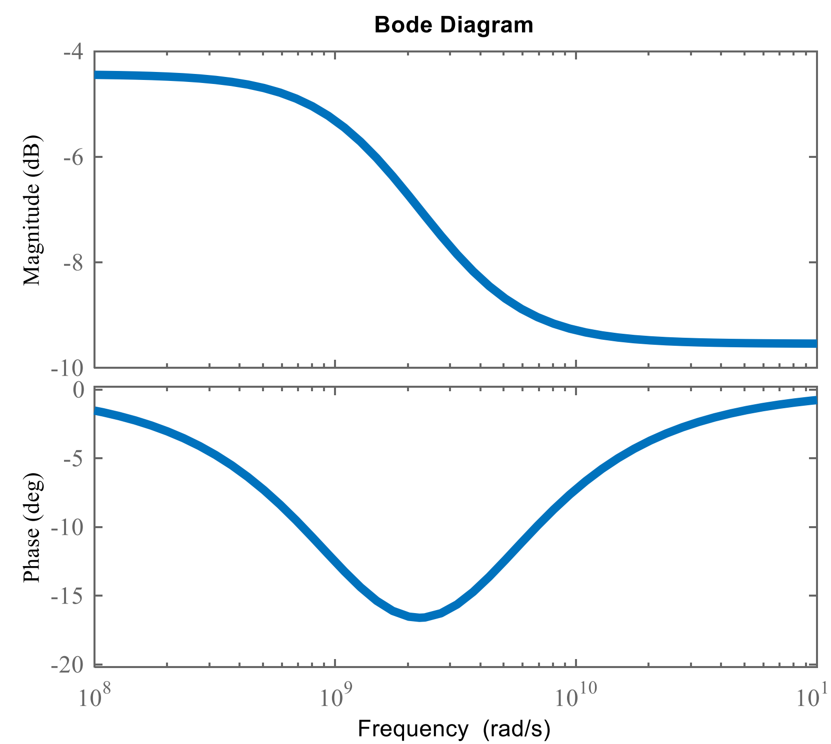
$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{-\omega^2 L_1 L_2 R_2 + j\omega R_1 R_2 L_2}{-\omega^2 L_1 L_2 (R_2 + R_1) + j\omega R_1 R_2 (L_2 + L_1)}$$

$$|H(\omega)| = \frac{\sqrt{(\omega^2 L_1 L_2 R_2)^2 + (\omega R_1 R_2 L_2)^2}}{\sqrt{(\omega^2 L_1 L_2 (R_2 + R_1))^2 + (\omega R_1 R_2 (L_2 + L_1))^2}}$$

$$\angle H(\omega) = \tan^{-1}\left(-\frac{R_1}{\omega L_1}\right) - \tan^{-1}\left(\frac{\omega R_1 R_2 (L_2 + L_1)}{\omega^2 L_1 L_2 (R_2 + R_1)}\right)$$







Problem 12: (30 pts)

For the circuit shown below

i) Find the transfer function H(ω), |H(ω)| and ∠ H(ω)

ii) Plot |H(ω)| for linear-linear and log-log scales.

iii) Plot ∠ H(ω) for linear-log scales.

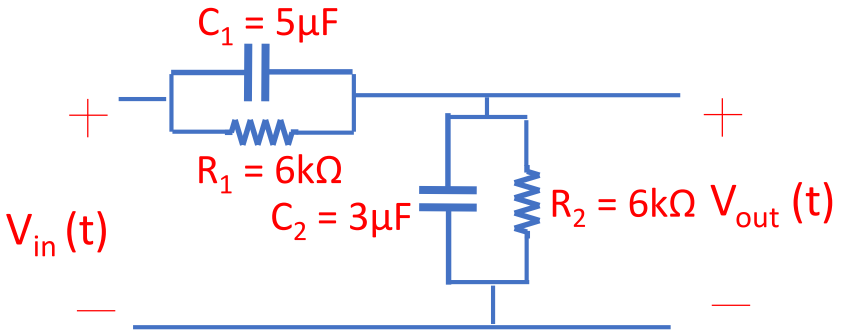
$$Z_1 = R_1 \| \frac{1}{j\omega C_1} = \frac{1}{1 + j\omega R_1 C_1}$$

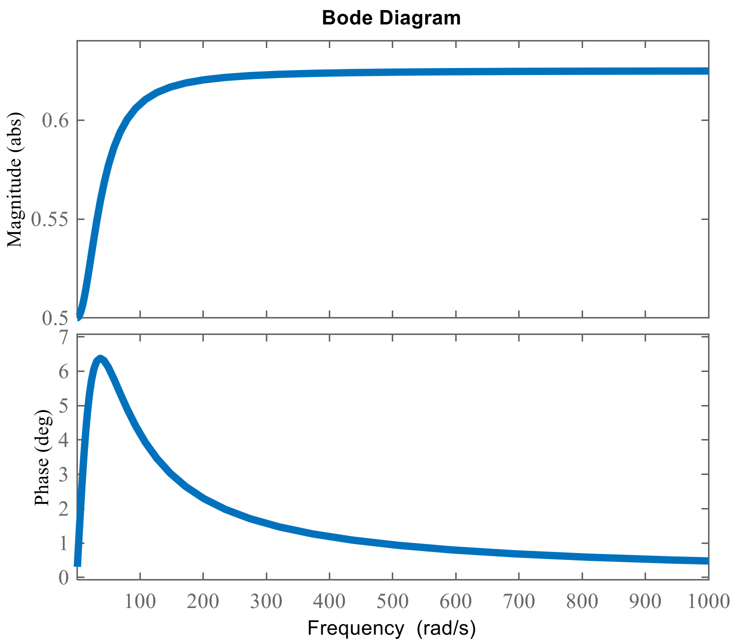
$$Z_2 = R_2 \| \frac{1}{j\omega C_2} = \frac{1}{1 + j\omega R_2 C_2}$$

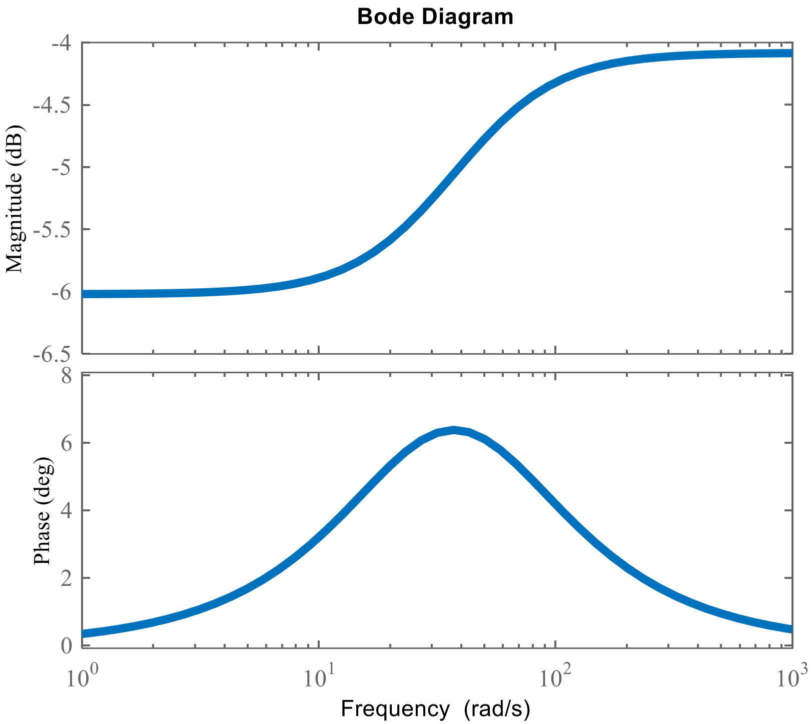
$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{1 + j\omega R_1 C_1}{2 + j\omega(R_1 C_1 + R_2 C_2)}$$

$$|H(\omega)| = \frac{\sqrt{1 + (\omega R_1 C_1)^2}}{\sqrt{4 + \omega^2 (R_2 C_2 + R_1 C_1)^2}}$$

$$\angle H(\omega) = \tan^{-1}(\omega R_1 C_1) - \tan^{-1}\left(\frac{\omega(R_1 C_1 + R_2 C_2)}{2}\right)$$

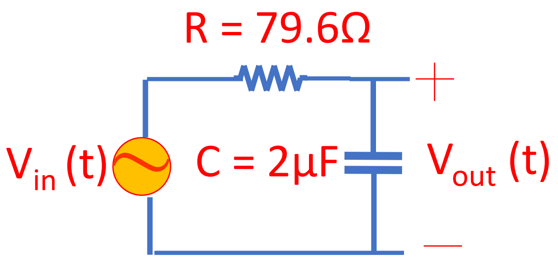






Problem 13: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\mathrm{V}}\cos(2\pi ft + \varphi)$ where φ is the phase and for $f = 1, 10, 100, 1\mathrm{k}, 10\mathrm{k}, 100\,\mathrm{kHz}$ if the input voltage is $V_{in}(t) = 1^{\mathrm{mV}}\sin(2\pi ft)$ for each of the following circuits.



$$v_{in}(t) = 1\times10^{-3}\sin(2\pi ft) = 1\times10^{-3}\cos(2\pi ft - \frac{\pi}{2}) \xrightarrow{phasor} V_{in} = 0.001e^{-j\frac{\pi}{2}}$$

$$V_{out} = V_{in}\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_{in}\frac{1}{1 + j\omega RC} = 0.001e^{-j\frac{\pi}{2}}\frac{1}{1 + j\omega\times159.2\times10^{-6}}\,\mathrm{V}$$

(a)

$$f = 1\,\mathrm{Hz} \rightarrow \omega = 2\pi f = 2\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j10^{-3}} = 10^{-3}e^{-j1.57} \Rightarrow v_{out}(t) = 10^{-3}\cos(2\pi t - 90.05^\circ)\,\mathrm{V}$$

$$f = 10\,\mathrm{Hz} \rightarrow \omega = 2\pi f = 20\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j10^{-2}} = 10^{-3}e^{-j1.58} \Rightarrow v_{out}(t) = 10^{-3}\cos(20\pi t - 90.57^\circ)\,\mathrm{V}$$

$$f = 100\,\mathrm{Hz} \rightarrow \omega = 2\pi f = 200\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j10^{-1}} = 0.995\times10^{-3}e^{-j1.67} \Rightarrow v_{out}(t) = 0.995\times10^{-3}\cos(200\pi t - 95.71^\circ)\,\mathrm{V}$$

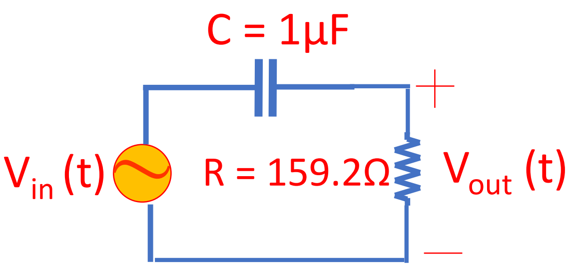
$$f = 1000\,\mathrm{Hz} \rightarrow \omega = 2\pi f = 2000\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j} = 0.707\times10^{-3}e^{-j2.35} \Rightarrow v_{out}(t) = 0.7\times10^{-3}\cos(2000\pi t - 135^\circ)\,\mathrm{V}$$

$$f = 10\,\mathrm{k\,Hz} \rightarrow \omega = 2\pi f = 20000\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j10^{1}} = 0.0995\times10^{-3}e^{-j3.04} \Rightarrow v_{out}(t) = 0.0995\times10^{-3}\cos(20000\pi t - 174.2^\circ)\,\mathrm{V}$$

$$f = 100000\,\mathrm{Hz} \rightarrow \omega = 2\pi f = 200000\pi(rad/s) \Rightarrow V_{out} = \frac{-j10^{-3}}{1 + j10^{2}} = 0.01\times10^{-3}e^{-3.13} \Rightarrow v_{out}(t) = 0.01\times10^{-3}\cos(200000\pi t - 179.9^\circ)\,\mathrm{V}$$

Problem 13: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\mathrm{V}}\cos(2\pi ft + \varphi)$ where φ is the phase and for $f = 1, 10, 100, 1\mathrm{k}, 10\mathrm{k}, 100\,\mathrm{kHz}$ if the input voltage is $V_{in}(t) = 1^{\mathrm{mV}}\sin(2\pi ft)$ for each of the following circuits.



$$v_{in}(t) = 1\times10^{-3}\sin(2\pi ft) = 1\times10^{-3}\cos(2\pi ft - \frac{\pi}{2}) \xrightarrow{phasor} V_{in} = 0.001e^{-j\frac{\pi}{2}}$$

$$V_{out} = V_{in}\frac{R}{R+\frac{1}{j\omega C}} = V_{in}\frac{j\omega RC}{1+j\omega RC} = 0.001e^{-j\frac{\pi}{2}}\frac{j\omega\times159.2\times10^{-6}}{1+j\omega\times159.2\times10^{-6}}\mathrm{V}$$

(b)

$$f = 1\,\mathrm{Hz} \to \omega = 2\pi f = 2\pi(rad/s) \Rightarrow V_{out} = 10^{-5}e^{-j0.001} \Rightarrow v_{out}(t) = 10^{-5}\cos(2\pi t - 0.057^\circ)\,\mathrm{V}$$

$$f = 10\,\mathrm{Hz} \to \omega = 2\pi f = 20\pi(rad/s) \Rightarrow V_{out} = 10^{-4}e^{-j0.1} \Rightarrow v_{out}(t) = 10^{-4}\cos(20\pi t - 0.057^\circ)\,\mathrm{V}$$

$$f = 100\,\mathrm{Hz} \to \omega = 2\pi f = 200\pi(rad/s) \Rightarrow V_{out} = 0.09\times10^{-3}e^{-j0.09} \Rightarrow v_{out}(t) = 0.09\cos(200\pi t - 5.71^\circ)\,\mathrm{m\,V}$$

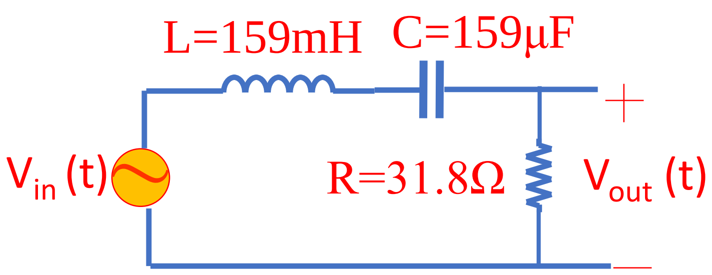
$$f = 1000\,\mathrm{Hz} \to \omega = 2\pi f = 2000\pi(rad/s) \Rightarrow V_{out} = 0.707\times10^{-3}e^{-j0.78} \Rightarrow v_{out}(t) = 0.7\cos(2000\pi t - 45^\circ)\,\mathrm{m\,V}$$

$$f = 10\,\mathrm{k\,Hz} \to \omega = 2\pi f = 20000\pi(rad/s) \Rightarrow V_{out} = 0.995\times10^{-3}e^{-j1.47} \Rightarrow v_{out}(t) = 0.995\cos(20000\pi t - 84^\circ)\,\mathrm{m\,V}$$

$$f = 100000\,\mathrm{Hz} \to \omega = 2\pi f = 200000\pi(rad/s) \Rightarrow V_{out} = 0.001e^{-1.56} \Rightarrow v_{out}(t) = 0.001\cos(200000\pi t - 89^\circ)\,\mathrm{V}$$

Problem 13: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\mathrm{V}}\cos(2\pi ft + \varphi)$ where φ is the phase and for $f = 1, 10, 100, 1\,\mathrm{k}, 10\,\mathrm{k}, 100\,\mathrm{kHz}$ if the input voltage is $V_{in}(t) = 1^{\mathrm{mV}}\sin(2\pi ft)$ for each of the following circuits.



$$v_{in}(t) = 1\times10^{-3}\sin(2\pi ft) = 1\times10^{-3}\cos(2\pi ft - \frac{\pi}{2}) \xrightarrow{phasor} V_{in} = 0.001e^{-j\frac{\pi}{2}}$$

$$V_{out} = V_{in}\frac{R}{R + \frac{1}{j\omega C} + j\omega L} = V_{in}\frac{j\omega RC}{1 - \omega^2 LC + j\omega RC} = \frac{\omega\times5.05\times10^{-6}}{1 - \omega^2\times2.5\times10^{-5} + j\omega\times5.05\times10^{-3}}\,\mathrm{V}$$

(c)

$$f = 1\,\mathrm{Hz} \to \omega = 2\pi f = 2\pi(rad/s) \Rightarrow V_{out} = 0.031\times10^{-3}e^{-j0.031} \Rightarrow v_{out}(t) = 0.031\times10^{-3}\cos(2\pi t - 1.81^{\circ})\,\mathrm{V}$$

$$f = 10\,\mathrm{Hz} \to \omega = 2\pi f = 20\pi(rad/s) \Rightarrow V_{out} = 0.33\times10^{-3}e^{-j0.33} \Rightarrow v_{out}(t) = 0.33\times10^{-3}\cos(20\pi t - 19.39^{\circ})\,\mathrm{V}$$

$$f = 100\,\mathrm{Hz} \to \omega = 2\pi f = 200\pi(rad/s) \Rightarrow V_{out} = 0.33\times10^{-3}e^{-j2.79} \Rightarrow v_{out}(t) = 0.33\cos(200\pi t - 160.31^{\circ})\,\mathrm{m\,V}$$

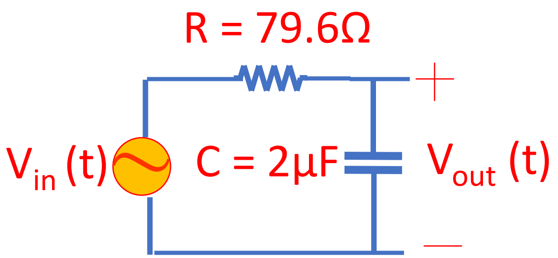
$$f = 1000\,\mathrm{Hz} \to \omega = 2\pi f = 2000\pi(rad/s) \Rightarrow V_{out} = 0.032\times10^{-3}e^{-j3.1} \Rightarrow v_{out}(t) = 0.032\cos(2000\pi t - 178.15^{\circ})\,\mathrm{m\,V}$$

$$f = 10\,\mathrm{k\,Hz} \to \omega = 2\pi f = 20000\pi(rad/s) \Rightarrow V_{out} = 0.0032\times10^{-3}e^{-j3.13} \Rightarrow v_{out}(t) = 0.0032\cos(20000\pi t - 179.81^{\circ})\,\mathrm{m\,V}$$

$$f = 100000\,\mathrm{Hz} \to \omega = 2\pi f = 200000\pi(rad/s) \Rightarrow V_{out} = 0.003\times10^{-3}e^{-3.14} \Rightarrow v_{out}(t) = 0.003\times10^{-3}\cos(200000\pi t - 179.98^{\circ})\,\mathrm{V}$$

Problem 14: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\mathrm{V}} \cos(2\pi ft + \varphi)$ where φ is the phase, if the input voltage is $V_{in}(t) = \sum_i 1^{\mathrm{mV}} \sin(2\pi f_i t);\ f_i = 1, 10, 100, 1\mathrm{k}, 10\,\mathrm{k}, 100\,\mathrm{kHz}$ for each of the following circuits.



Since the circuit is linear, the output is the superposition of the input signals. So the output consists of multiple frequencies which can be calculated as the summation of all the output signals found in problem 13(a).

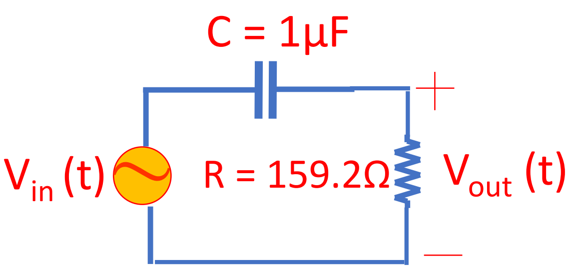
(a)

$$v_{out}(t) = \sum_i v_{out_i}(t)$$

$$v_{out}(t) = 10^{-3}\cos(2\pi t - 90.05^\circ) + 10^{-3}\cos(20\pi t - 90.57^\circ) + 0.995\times10^{-3}\cos(200\pi t - 95.71^\circ)$$
$$+ 0.7\times10^{-3}\cos(2000\pi t - 135^\circ) + 0.0995\times10^{-3}\cos(20000\pi t - 174.2^\circ) + 0.01\times10^{-3}\cos(200000\pi t - 179.9^\circ) \quad \mathrm{V}$$

Problem 14: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\mathrm{V}}\cos(2\pi ft + \varphi)$ where φ is the phase, if the input voltage is $V_{in}(t) = \sum_i 1^{\mathrm{mV}} \sin(2\pi f_i t);\ f_i = 1, 10, 100, 1\mathrm{k}, 10\mathrm{k}, 100\,\mathrm{kHz}$ for each of the following circuits.



Since the circuit is linear, the output is the superposition of the input signals. So the output consists of multiple frequencies which can be calculated as the summation of all the output signals found in problem 13(b).

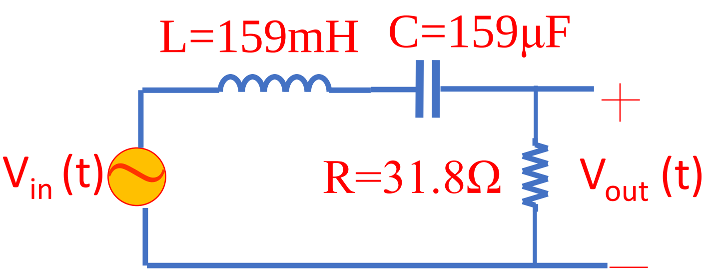
(b)

$$v_{out}(t) = \sum_i v_{out_i}(t)$$

$$v_{out}(t) = 10^{-5}\cos(2\pi t - 0.057^{\circ}) + 10^{-4}\cos(20\pi t - 0.057^{\circ}) + 0.09\times 10^{-3}\cos(200\pi t - 5.71^{\circ}) + 0.7\times 10^{-3}\cos(2000\pi t - 45^{\circ}) + 0.995\times 10^{-3}\cos(20000\pi t - 84^{\circ}) + 0.001\cos(200000\pi t - 89^{\circ}) \quad \mathrm{V}$$

Problem 14: (60 pts)

Find the output voltage as $V_{out}(t) = A^{\text{V}}\cos(2\pi ft + \varphi)$ where φ is the phase, if the input voltage is $V_{in}(t) = \sum_i 1^{\text{mV}} \sin(2\pi f_i t);\ f_i = 1, 10, 100, 1\text{k}, 10\text{k}, 100\,\text{kHz}$ for each of the following circuits.



Since the circuit is linear, the output is the superposition of the input signals. So the output consists of multiple frequencies which can be calculated as the summation of all the output signals found in problem 13(c).

(c)

$$v_{out}(t) = \sum_i v_{out_i}(t)$$

$$v_{out}(t) = 0.031\times10^{-3}\cos(2\pi t - 1.81^\circ) + 0.33\times10^{-3}\cos(20\pi t - 19.39^\circ) + 0.33\times10^{-3}\cos(200\pi t - 160.31^\circ) +$$
$$0.032\times10^{-3}\cos(2000\pi t - 178.15^\circ) + 0.0032\times10^{-3}\cos(20000\pi t - 179.81^\circ) + 0.003\times10^{-3}\cos(200000\pi t - 179.98^\text{o})\quad \text{V}$$

Problem 15: (20 pts)

Draw the Bode plot (magnitude only) for the following transfer function.

$$H(\omega) = \frac{1}{(1+j\omega\tau)(1+j\omega\tau)}$$

Extra credit: Try to design a circuit having such a transfer function. (40 pts)