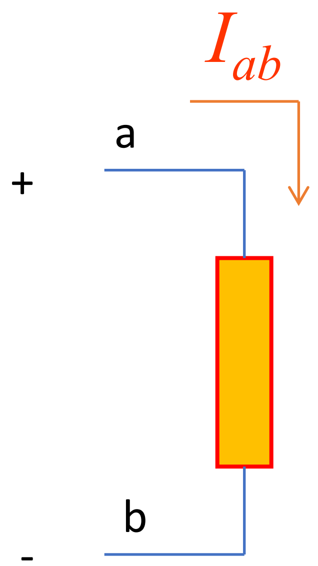
Sign convention

$P = I_{ab} \times V_{ab}$



V_{ab} positive => $V_a > V_b$

I_{ab} positive => current flows from a to b

V_{ab} negative => $V_a < V_b$

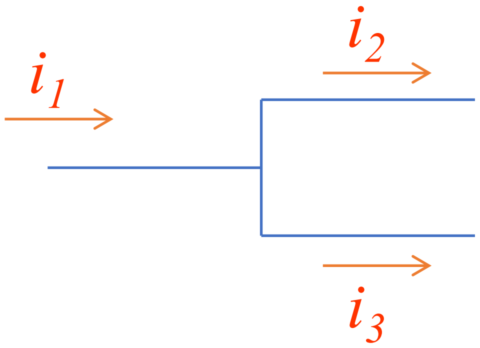
I_{ab} negative => current flows from b to a

Define convention first, then solve problem.

P > 0 means power flowing into element (e.g. resistor)

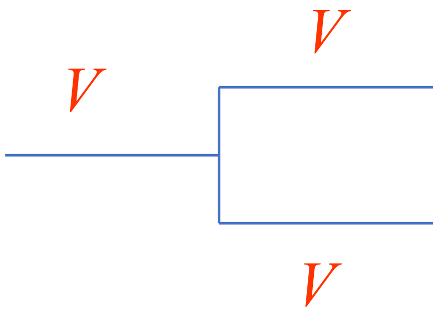
P < 0 means power flowing out of element (e.g. battery)

Topology



$$i_1 = i_2 + i_3$$

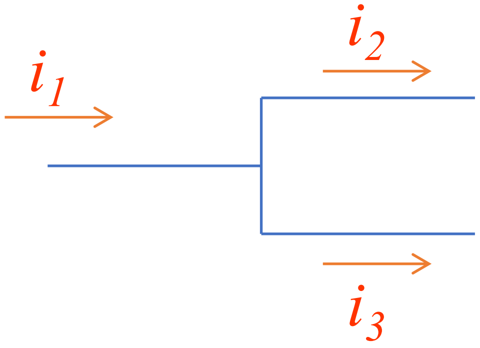
Like water in a river…



Voltage same everywhere….
Concept of a node

Kirchoff's current law

You have already seen:



$$i_1 = i_2 + i_3$$

Like water in a river…

More generally:

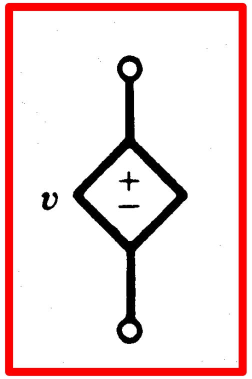
Sum of currents *entering* node = sum of currents *leaving* node.

Stated as Kirchoff's current law (KCL):

$$\sum_{n=1}^{N} i_n = 0$$

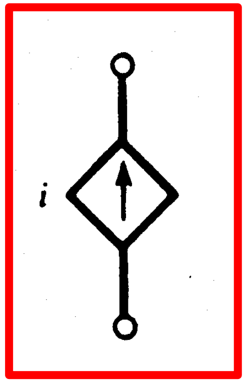
Current *entering* a node: i_n positive
Current *leaving* a node: i_n negative

Dependent sources



VCVS: Voltage controlled voltage source

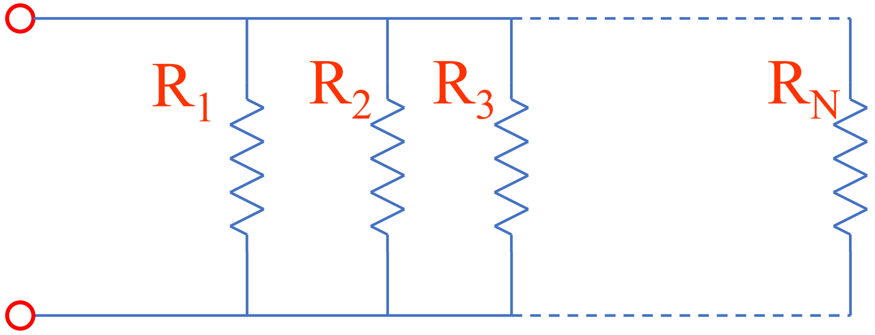
CCVS: Current controlled voltage source



VCCS: Voltage controlled current source

CCCS: Current controlled current source

Generalize: N resistors in parallel



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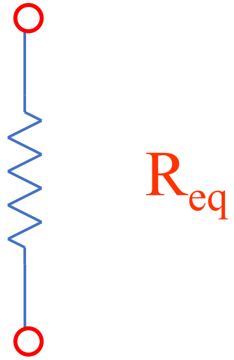
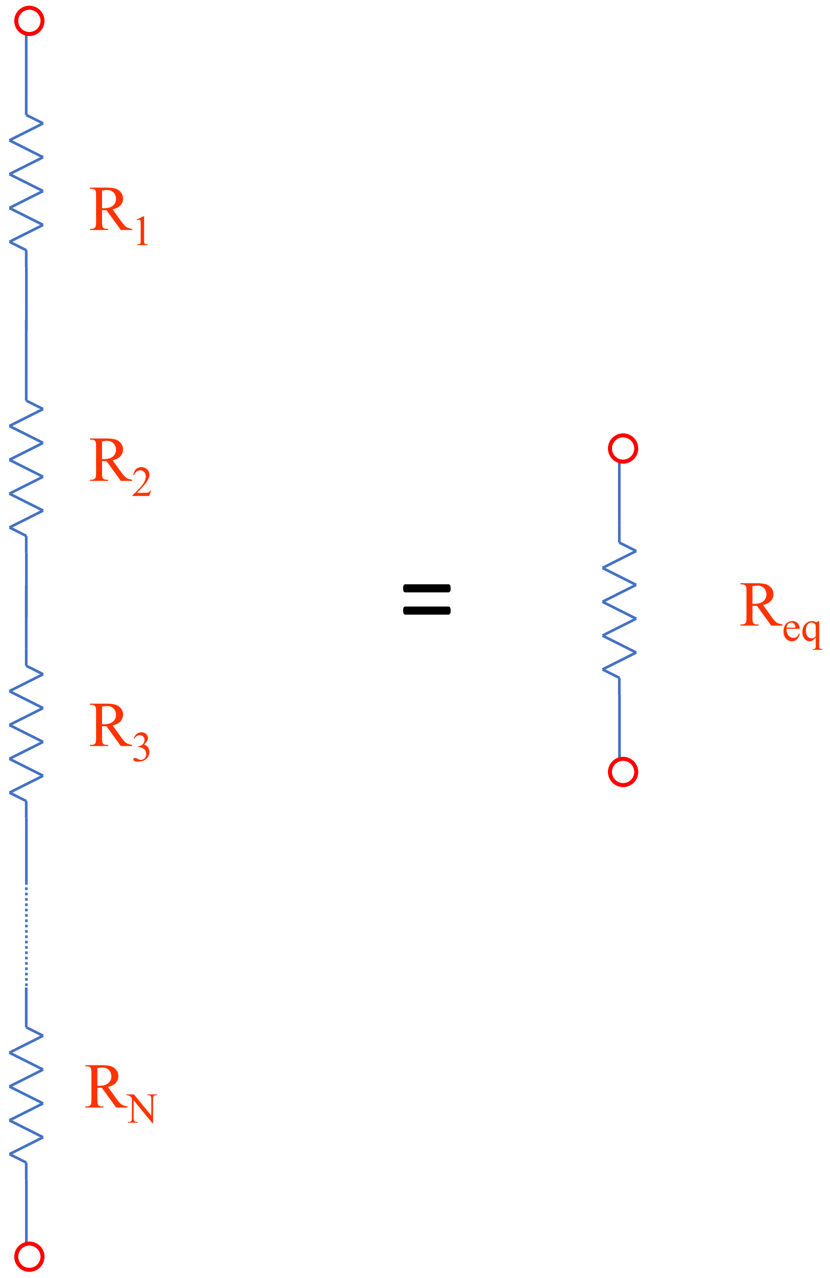


$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$R_1 \parallel R_2$ is notation for "R_1 in parallel with R_2"

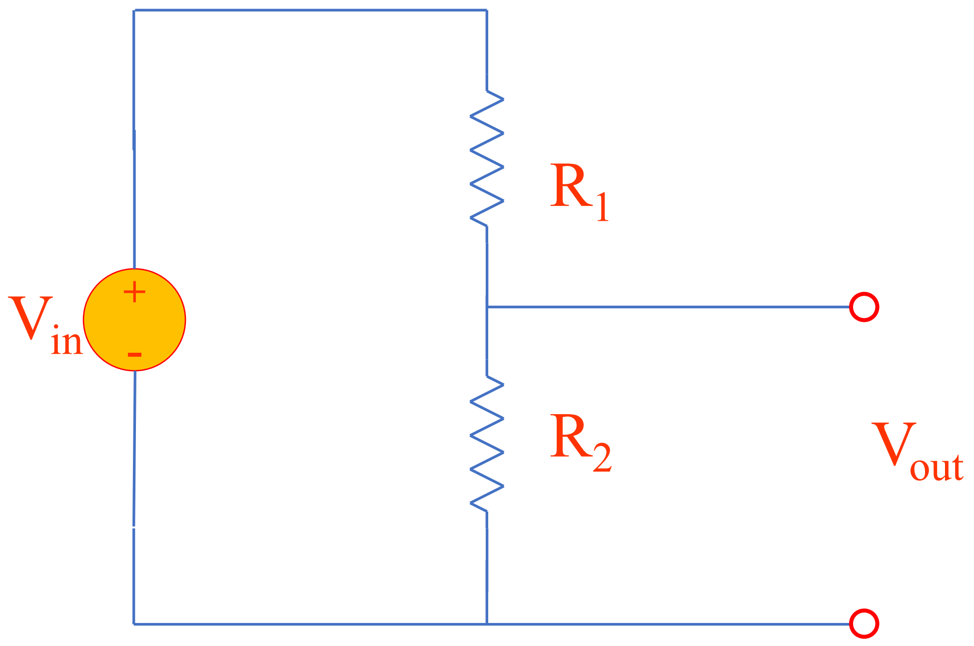
Generalize: N resistors in series



$$R_{eq} = \sum_{i=1}^{N} R_i$$

Voltage divider

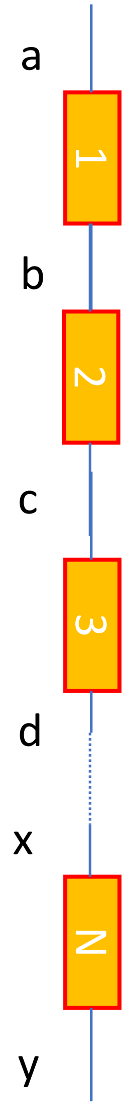
Derivation:



Why important?
Concept of source/load. (Thevenin…)

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

Generalize loop to N-elements:



$$V_{ay} = V_{ab} + V_{bc} + V_{cd} + \ldots + V_{xy}$$

V_{ab} = “voltage drop” across element # 1

V_{bc} = “voltage drop” across element # 2

V_{cd} = “voltage drop” across element # 3

V_{xy} = “voltage drop” across element # N

Kirchoff's voltage law

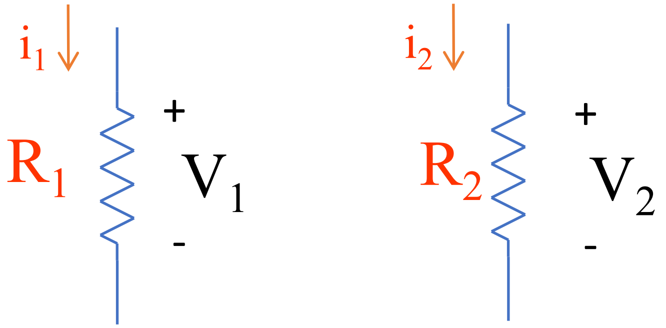
$$\sum_{n=1}^{N} v_n = 0$$

around *any* closed loop.

voltage *drops*

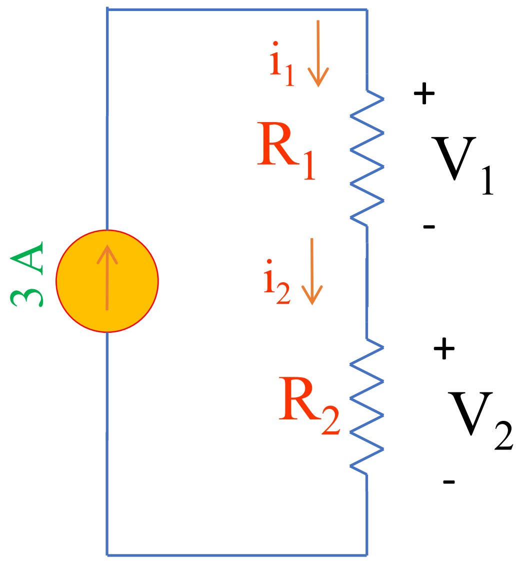
Nodal vs. mesh analysis

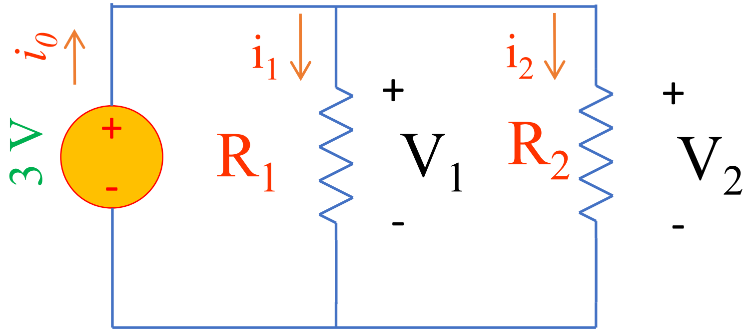
Nodal Analysis → write KCL for every node. If a resistor connects to nodes, express the currents in terms of voltage across the element. Your goal is to find all the node voltages first. If a voltage source connects two nodes, circuit can become easier to solve (super node). From there, you can solve for whatever quantity you maybe interested in.



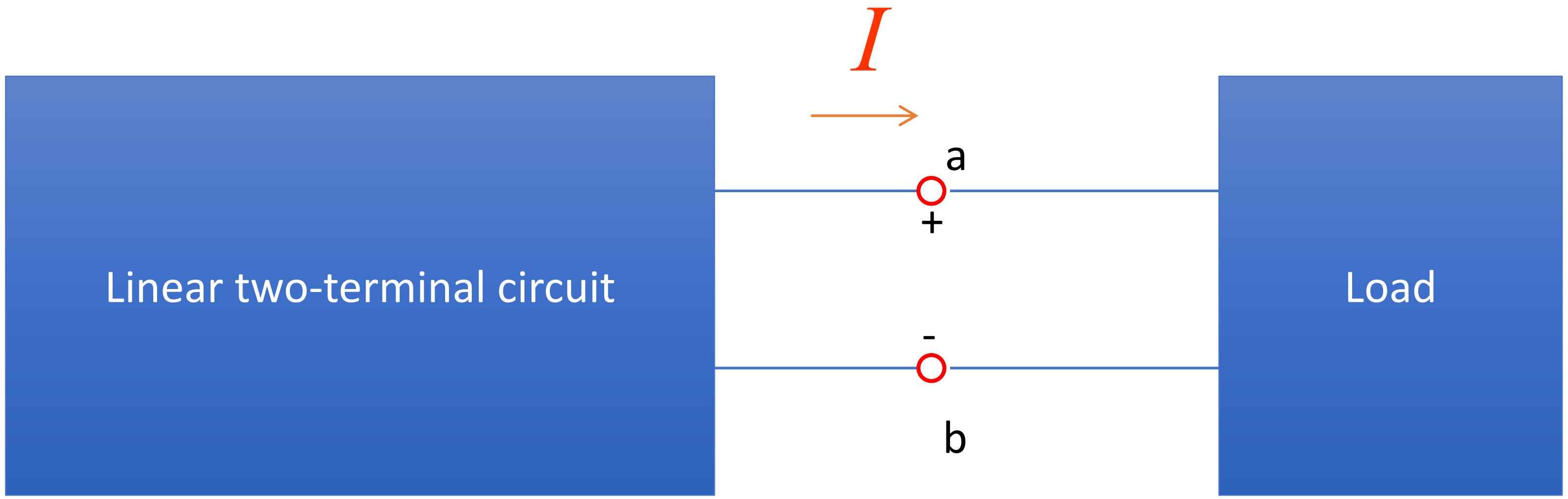
Mesh Analysis: Assign current to all the loops. Then write KVL across each loop. Your goal is to find the loop currents. From there, you can solve for whatever quantity you maybe interested in.

Remember voltage is a relative quantity.

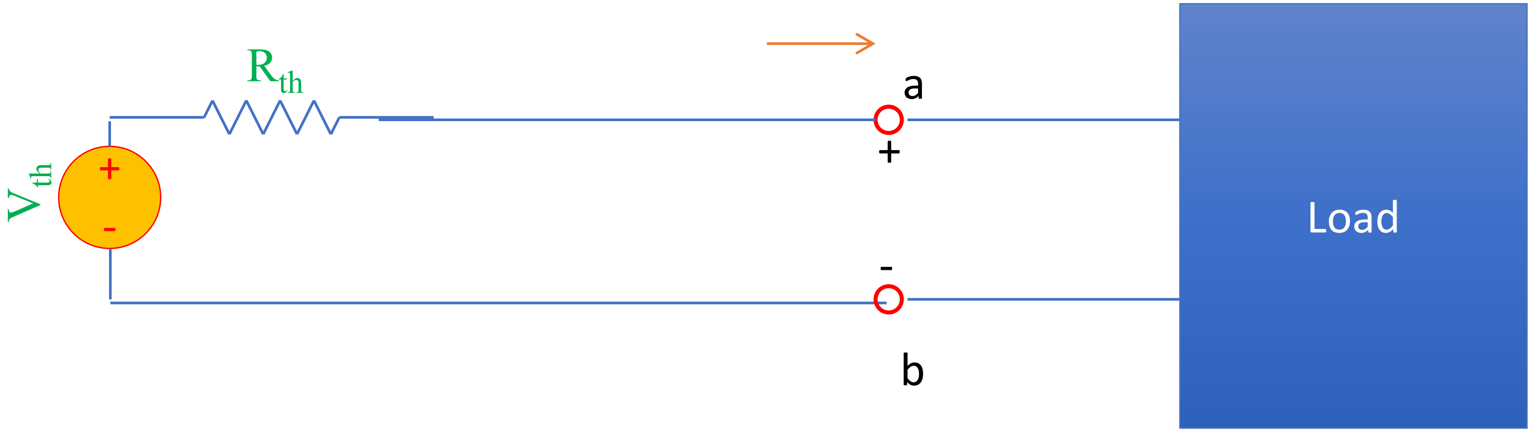




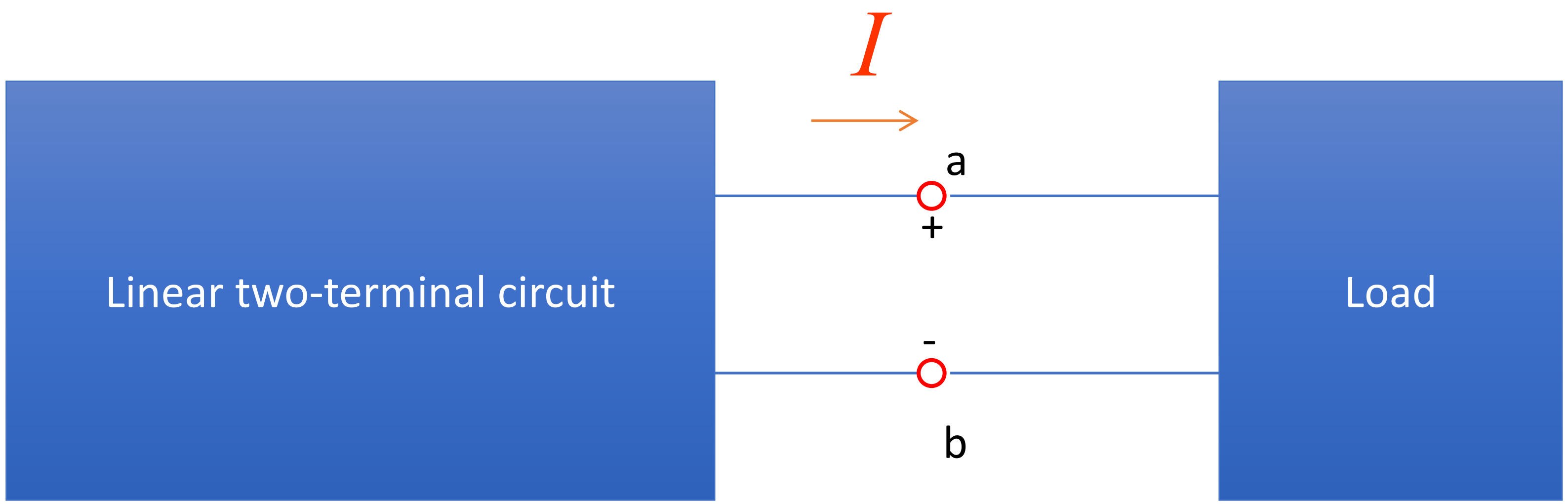
Thevenin's Theorem



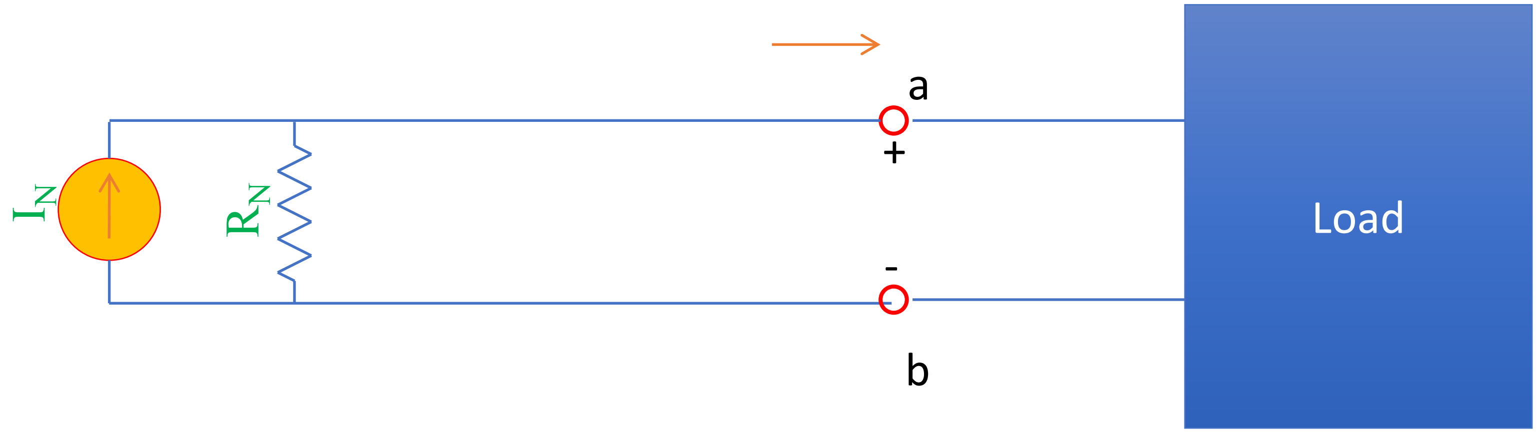
Equivalent to:



Norton's Theorem

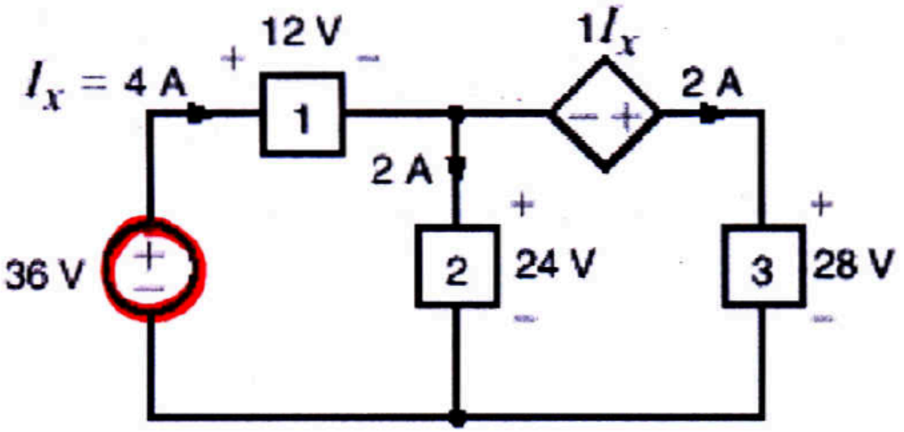


Equivalent to:



Examples

Compute the elements power (sinked or sourced?)



$P_{36V} = -36(4) = -144\,W$

$P_{36V} = 144\,W$ supplied

$P_1 = 12(4) = 48\,W$ absorbed

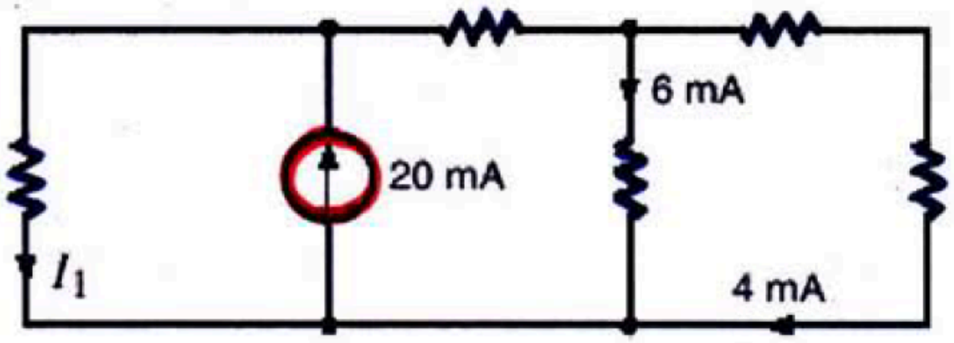
$P_2 = 24(2) = 48\,W$ absorbed

$P_{1I_x} = (-I_x)(2) = -4(2) = -8\,W$

$P_{1I_x} = 8\,W$ supplied

$P_3 = 28(2) = 56\,W$ absorbed

Find I_1



KCL at mode B: $I_2 = 6m + 4m$

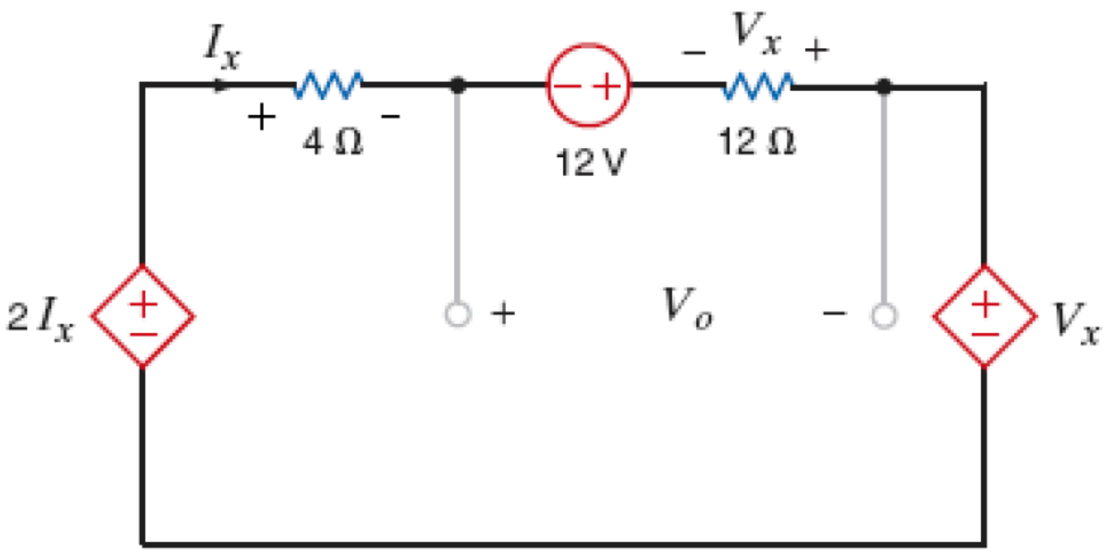
$I_2 = 10m\,A$

KCL at mode A: $I_1 + I_2 = 20m$

$I_1 = 20m - 10m$

$I_1 = 10m\,A$

Find Vo



KVL:

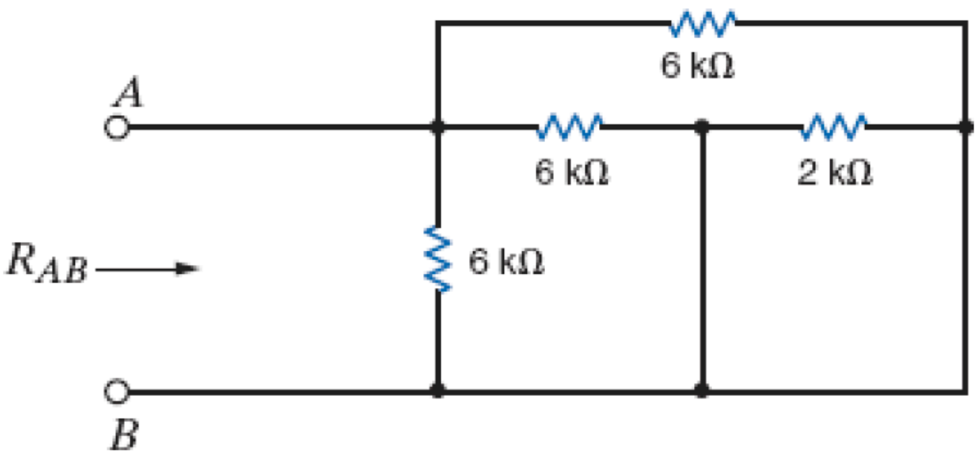
$$V_o + 12 + V_x = 0$$
$$V_o = -V_x - 12$$
$$V_x = -12 I_x$$

KVL around outer loop:

$$2I_x + 12 + V_x = 4I_x + V_x$$
$$2I_x + 12 + 12I_x = 4I_x + 12I_x$$
$$2I_x = 12$$
$$I_x = 6A$$

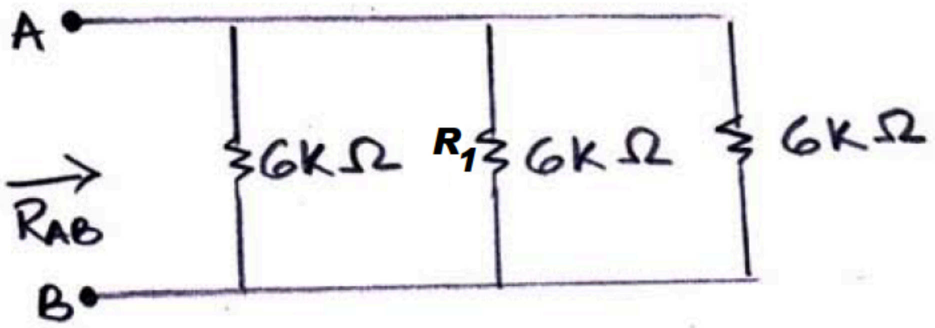
$$V_x = -12(6) = -72\text{ V}$$
$$V_o = -(-72) - 12$$
$$V_o = 60\text{ V}$$

Find R_{AB}



$$R_1 = (2K \parallel 0) + 6K$$

$$R_1 = 6K\,\Omega$$

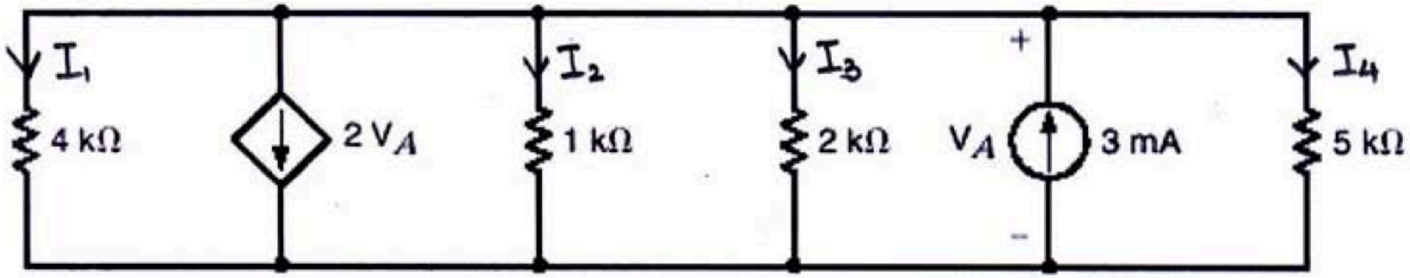


$$R_{AB} = (6K \parallel 6K) \parallel 6K$$

$$R_{AB} = 3K \parallel 6K$$

$$R_{AB} = 2K\,\Omega$$

Find the power absorbed/dissipated by the VCCS



KCL: $3m = I_1 + 2V_A + I_2 + I_3 + I_4$

$I_1 = \frac{V_A}{4K}$, $I_2 = \frac{V_A}{1K}$, $I_3 = \frac{V_A}{2K}$, and $I_4 = \frac{V_A}{5K}$

$3m = \frac{V_A}{4K} + 2V_A + \frac{V_A}{1K} + \frac{V_A}{2K} + \frac{V_A}{5K}$

$60 = 5V_A + 40KV_A + 20V_A + 10V_A + 4V_A$

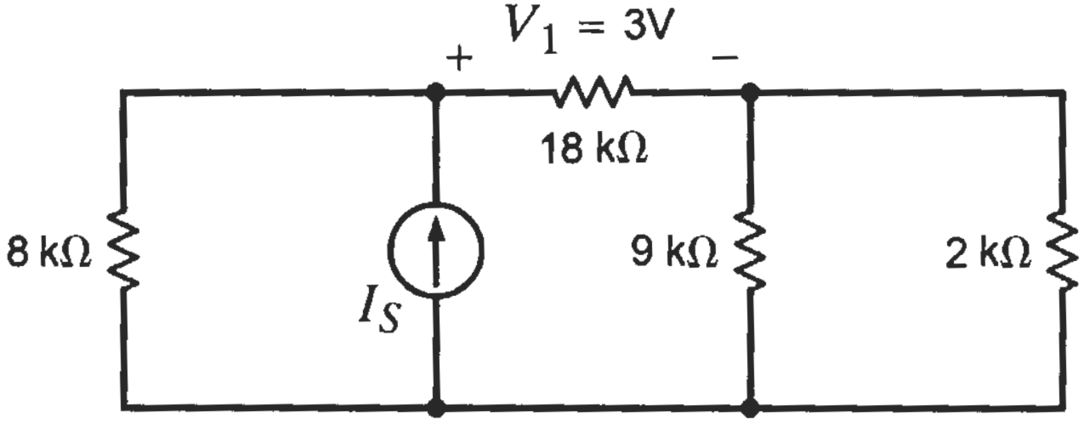
$V_A = 1.5mV$

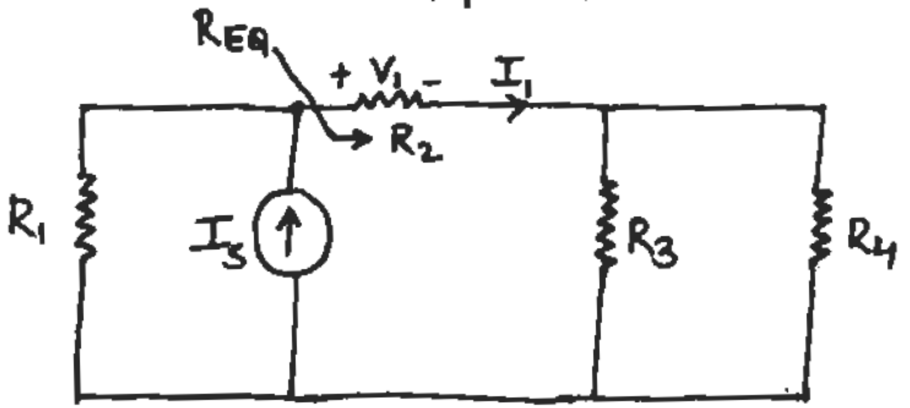
$P_{2VA} = V_A I = V_A(2V_A)$

$P_{2VA} = 1.5m(2)(1.5m)$

$P_{2VA} = 4.5\mu W$

Find I_s





$R_1 = 8\,k\Omega,\ R_2 = 18\,k\Omega,\ R_3 = 9\,k\Omega,\ R_4 = 2\,k\Omega$

$V_1 = 3V$

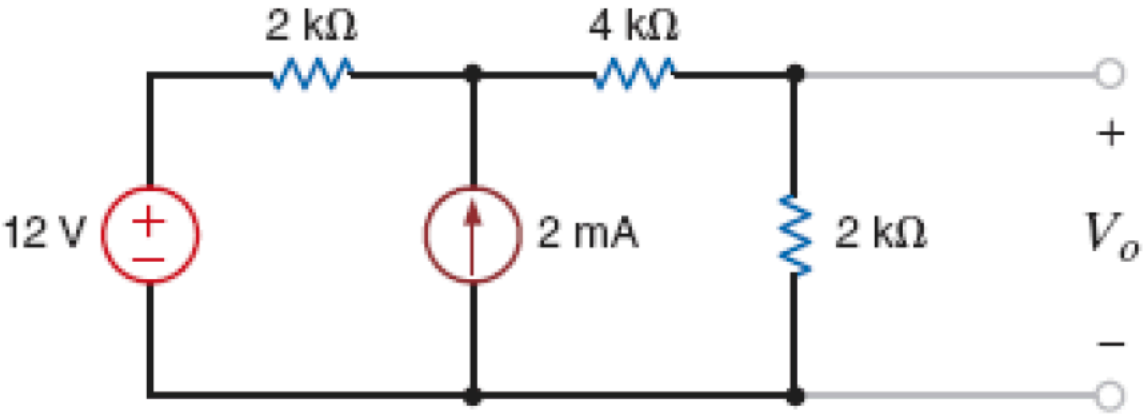
$R_{EQ} = R_2 + (R_3 \| R_4) = 19.636 = 19.64\,k$

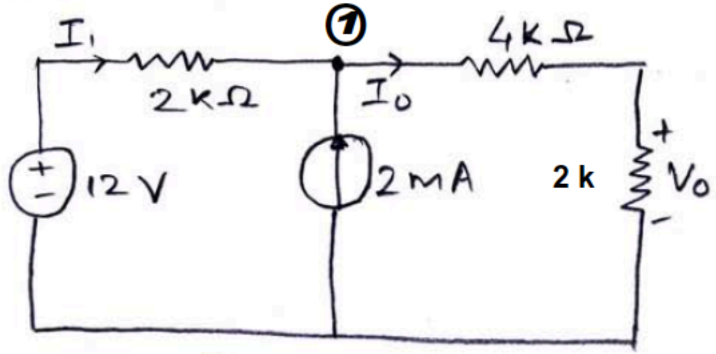
$$I_1 = \frac{V_1}{R_2} = \frac{3}{18} = \frac{1}{6}\,mA$$

$$I_1 = I_s \left[\frac{R_1}{R_1 + R_{EQ}}\right]$$

$$\boxed{I_s = 0.576\ mA}$$

Find Vo by nodal analysis





KCL at ①: $I_1 + 2m = I_o$

$$\frac{12 - V_1}{2K} + 2m = \frac{V_1}{4K + 2K}$$

$$36 - 3V_1 + 12 = V_1$$

$$4V_1 = 48$$

$$V_1 = 12 \text{ V}$$

$$I_o = \frac{V_1}{4K+2K}$$

$$= \frac{12}{6K}$$

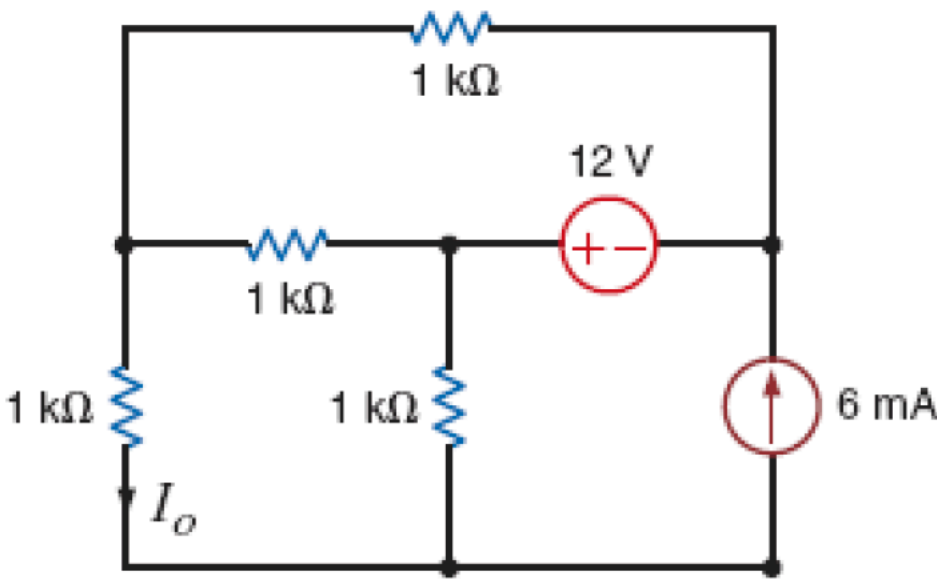
$$I_o = 2 \text{ mA}$$

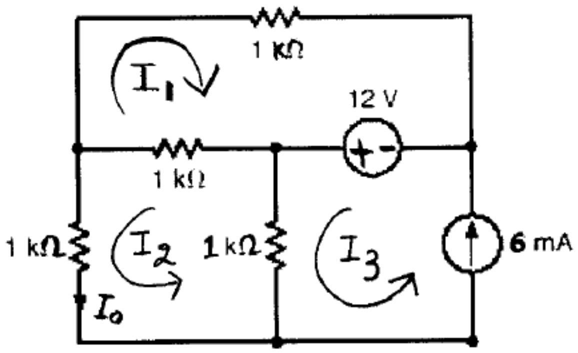
$$V_o = I_o (2K)$$

$$= 2m (2K)$$

$$V_o = 4 \text{ V}$$

Find I_o by mesh analysis





Let I_1, I_2 and I_3 be the loop currents (in mA).

KVL in the first loop:

$$12 = 1(I_1 + I_2) + 1 \times I_1$$

$$\Rightarrow 12 = 2I_1 + I_2 \quad \text{—— (1)}$$

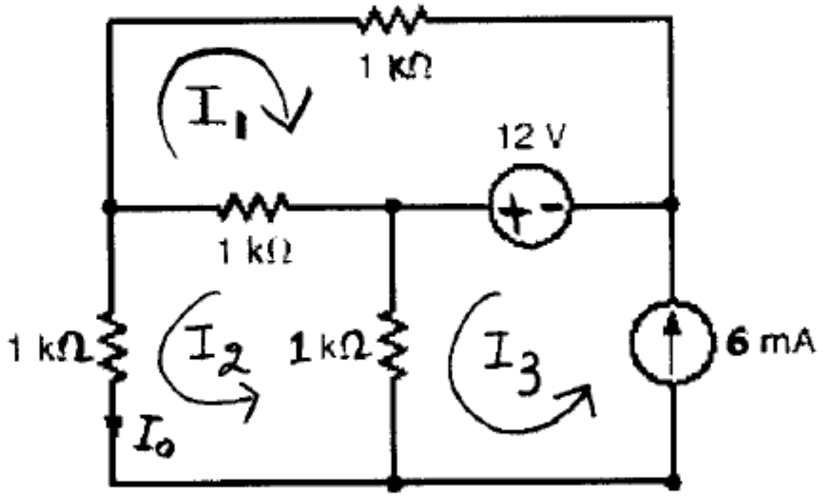
From the third loop:

$$I_3 = 6 \text{ mA} \quad \text{— (2)}$$

KVL in the second loop:

$$1(I_1 + I_2) + 1(I_2) + 1(I_2 - I_3) = 0$$

Solution Cont.



$$\Rightarrow I_1 + 3I_2 - I_3 = 0 \quad \text{— (3)}$$

From equation (2) & (3), we get :

$$I_1 + 3I_2 = I_3 = 6 \quad \text{— (4)}$$

From equation (1) & (4) :

$$12 = 2I_1 + I_2 \quad \text{— (1)}$$

$$6 = I_1 + 3I_2$$

$$\Rightarrow 12 = 2I_1 + 6I_2 \quad \text{— (5)}$$

$$5I_2 = 0 \quad \{eq.(5) - eq.(1)\}$$

$$\text{or} \quad \boxed{I_2 = 0\,mA}$$

Find The Thevenin and Norton equivalent seen from the port